MATHEMATICS

Class-X

Topic-1

REAL NUMBERS

INDEX

CH-01
REAL NUMBERS

(A) INTRODUCTION AND EUCLID'S DIVISION LEMMA

(a) Classification of numbers

In earlier classes you have studied different types of numbers such as natural numbers, whole numbers, integers, rational and irrational numbers. All these together are called Real numbers. In this chapter, we shall study some properties of numbers, especially valid for integers

(i) Real numbers: Numbers which can represent actual physical quantities in a meaningful way are known as real numbers. These can be represented on the number line. Number line is geometrical straight line with arbitrarily defined zero (origin).Real numbers includes all rational and irrational numbers.

(ii) Prime numbers : All natural numbers that have one and itself only as their factors are called prime numbers i.e. prime numbers are exactly divisible by 1 and themselves. e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23,...etc.

(iii) Composite numbers : All natural numbers having more than two distinct factors.
Note that 1 is neither prime nor composite number.

(iv) Co-prime Numbers : If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers. e.g. 4, 9 are co-prime as H.C.F. of (4, 9) = 1.
Note : Any two consecutive numbers will always be co-prime. Any two prime numbers are always coprime.

(b) Divisibility

A non-zero integer 'b' is said to divide an integer 'a' if there exists an integer 'q' such that a = bq. The integer 'a' is called dividend, integer 'b' is known as the divisor and integer 'q' is known as the quotient.

For example : 5 divides 35 because there is an integer 7 such that 35 = 5 × 7.
If a non-zero integer 'b' divides an integer a, then it is written as b | a and read as " b divides a", $b \nmid a$ is written to indicate that a is not divisible by b.

(c) Lemma

Lemma is a proven statement used to prove another statement or theorem.

(d) Statement of Euclid's Division Lemma

Let 'a' and 'b' be any two positive integers. Then, there exists unique integers 'q' and 'r' such that a = b q + r, where $0 \le r < b$. If b|a, then r = 0.
This can easily be remembered as follows

$$b\overline{\smash{)}a}^{\,q} \quad \underline{}\; r$$

This can be restated as follows : Dividend = Divisor × Quotient + Remainder.

Note : In Division Lemma, q or r may be 0 but r is always less than b.

Solved Examples

Example.1
Prove that any positive odd integer can be written in the form 4q + 1 or 4q + 3 where q is an integer.

Solution :
Let a be any positive odd integer. Taking 4 as a divisor, we can write a as, a = 4q + r, where $0 \le r < 4$ (division Lemma). Now let us put r = 0, 1, 2, 3. Then,

$a = 4q$ … (i)
$a = 4q + 1$ … (ii)
$a = 4q + 2$ … (iii)
$a = 4q + 3$ ... (iv)
(i) and (iii) give only even numbers.
Since a is odd, it must be of the form (ii) or (iv) namely $4q + 1$ or $4q + 3$.

Example.2
Prove that the square of any positive integer of the form $5q + 1$ is of the same form.
Solution.
Let x be any positive integer of the form $5q + 1$.
When $x = 5q + 1$
$x^2 = 25q^2 + 10q + 1$
$x^2 = 5q(5q + 2) + 1$
Let $m = q(5q + 2)$.
$x^2 = 5m + 1$.
Hence, x^2 is of the same form i.e. $5m + 1$.

Example.3
Show that one and only one out of **n**, **n + 2** or **n + 4** is divisible by **3**, where **n** is any positive integer.
Solution.
Let n is any positive integer of form $3q + r$ where $0 \le r < 3$
Case I When $r = 0$
$n = 3q$, which is divisible by 3.
$n + 2 = 3q + 2$
$\Rightarrow$ $n + 2$ leaves remainder 2, when divided by 3
$\Rightarrow$ $n + 2$ is not divisible by 3
$n + 4 \Rightarrow 3q + 4 = 3(q + 1) + 1$
$\Rightarrow$ $n + 4$ is not divisible by 3
Thus, n is divisible by 3 but $n + 2$ and $n + 4$ is not divisible by 3.
Case II When $r = 1$
$n = 3q + 1$
$n + 2 = 3q + 3$
and $n + 4 = 3q + 5$
Thus $n + 2$ is divisible by 3 but n and $n + 4$ are not divisible by 3.
Case III When $r = 2$
$n = 3q + 2$
$n + 2 = 3q + 4$
and $n + 4 = 3q + 6$
Thus $n + 4$ is divisible by 3 but n and $n + 2$ is not divisible by 3.

Example.4
Use Euclid's Division Lemma to show that the cube of any positive integer is of the form 9 m, 9 m + 1 or 9 m + 8, for some integer m.
Solution.
Let x be any positive integer. Then, it is of the form 3q or, $3q + 1$ or, $3q + 2$.
Case -I When $x = 3q$
$\Rightarrow$ $x^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 3q^3$.
Case-II when $x = 3q + 1$
$\Rightarrow$ $x^3 = (3q + 1)^3$
$\Rightarrow$ $x^3 = 27q^3 + 27q^2 + 9q + 1$
$\Rightarrow$ $x^3 = 9q(3q^2 + 3q + 1) + 1$
$\Rightarrow$ $x^3 = 9m + 1$, where $m = q(3q^2 + 3q + 1)$.
Case-III when $x = 3q + 2$
$\Rightarrow$ $x^3 = (3q + 2)^3$
$\Rightarrow$ $x^3 = 27q^3 + 54q^2 + 36q + 8$
$\Rightarrow$ $x^3 = 9q(3q^2 + 6q + 4) + 8$
$\Rightarrow$ $x^3 = 9m + 8$, where $m = q(3q^2 + 6q + 4)$
Hence, x^3 is either of the form 9 m or 9 m + 1 or 9 m + 8.

Check Your Level

1. Let 'a' and 'b' be any two positive integers. Then, there exists unique integers 'q' and 'r' such that a = bq + r. If b = 5, then find the possible values of r.

2. Check whether the number 21q + 18 is of the form 7q + 4, for some integer q.

3. If n = 3q + 2, then check whether n + 7 is divisible by 3, for some integer q.

4. If n = 5q + 4, then check whether $n^2 - 1$ is divisible by 5, for some integer q.

5. Show that cube of the number of the form 4q + 3 is of the form 4q + 3, for some integer q.

6. Out of the numbers n, n + 1 and n + 2, show that only one number is divisible by 3.

Answers

1. 0, 1, 2, 3, 4 **2.** Yes **3.** Yes **4.** Yes

(B) EUCLID'S DIVISION ALGORITHM

(a) Algorithm

You may be familiar with computer program which is a sequence of steps to do a given task, the order of steps being very important.

(b) Euclid's Division Algorithm

If **'a'** and **'b'** are positive integers such that **a = bq + r,** then every common divisor of **'a'** and **'b'** is a common divisor of **'b'** and **'r'**, and vice-versa. The HCF of positive integers a and b where a > b is obtained as follows.

Step 1: Apply Euclid's division Lemma to a and b. That is, find whole numbers q and r such that $a = bq + r, 0 \leq r < b$

Step 2: If r = 0, then b is the HCF of a and b. If $r \neq 0$, apply division Lemma to b and r.

Step 3: Continue the process till r is 0. The divisor at this stage is the HCF of a and b.

This procedure has to work because the HCF of a and b is same as HCF of b and r.

Solved Examples

Use Euclid's division algorithm to find the H.C.F. of 196 and 38318.

Solution.

Applying Euclid's division lemma to 196 and 38318.

38318 = 195 × 196 + 98

196 = 98 × 2 + 0

The remainder at the second stage is zero. So, the H.C.F. of 38318 and 196 is 98.

Example.6

Use Euclid's division algorithm to find the HCF of **(i)** 56 and 814 **(ii)** 6265 and 76254

Solution :

(i) HCF of 56 and 814

814 = 56 × 14 + 30

56 = 30 × 1 + 26

30 = 26 × 1 + 4

26 = 4 × 6 + 2

4 = 2 × 2 + 0

Hence, the HCF of 56 and 814 = 2.

(ii) HCF of 6265 and 76254
76254 = 6265 × 12 + 1074
6265 = 1074 × 5 + 895
1074 = 895 × 1 + 179
895 = 179 × 5 + 0
Hence, the HCF of 6265 and 76254 is 179.

Example.7
If the H.C.F of 657 and 963 is expressible in the form 657 x + 963 × (– 15), find x.

Solution.
Applying Euclid's division lemma on 657 and 963.
963 = 657 × 1 + 306
657 = 306 × 2 + 45
306 = 45 × 6 + 36
45 = 36 × 1 + 9
36 = 9 × 4 + 0
So, the H.C.F of 657 and 963 is 9.
Given : 657 x + 963 ×(– 15) = H.C.F of 657 and 963.
657 x + 963 × (– 15) = 9 ⇒ 657 x = 9 + 963 × 15 ⇒ 657 x = 14454

$x = \frac{14454}{657} = 22.$

Example 8.
If d is the HCF of 468 and 222, find x, y satisfying d = 468x + 222y. Also, show that x and y are not unique.

Sol. Applying Euclid's division lemma to 468 and 222, we get
468 = 222 × 2 + 24 ... (i)
Since the remainder 24 ≠ 0. So, we consider the divisor 222 and the remainder 24 and apply division lemma to get
222 = 24 × 9 + 6 ... (ii)
We consider the divisor 24 and the ramainder 6 and apply division algorithm to get
24 = 6 × 4 + 0 ... (iii)
We observe that the remainder at this stage is zero. Therefore, last divisor 6 (or the remainder at the earlier state) is the HCF of 468 and 222.
6 = 222 – 24 × 9
⇒ 6 = 222 – (468-222×2)×9 [∵ 24 = 468 - 222 × 2 (from (i))]
⇒ 6 = 222 – 468×9 + 222×18 ⇒ 6 = 19×222 – 468×9
∴ x = – 9 and y = 19.
Now, 6 = 19×222 – 468×9 + 222×468–222×468
6 = 19×222 + 222×468 – 468×9 –222×468
⇒ 8 = (19+468)×222 –(9+222)×468 ⇒ 8 = 487 ×222 – 231 × 468
∴ x = – 231 and y = 487. Hence, x and y are not unique.

Example.9
144 cartons of coke cans and 90 cartons of pepsi cans are to be stacked in a canteen. If each stack is of same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have ?

Solution.
In order to arrange the cartons of the same drink in the same stack, we have to find the greatest number that divides 144 and 90 exactly. Using Euclid's division algorithm, to find the H.C.F. of 144 and 90.
144 = 90 × 1 + 54
90 = 54 × 1 + 36
54 = 36 × 1 + 18
36 = 18 × 2 + 0
So, the H.C.F. of 144 and 90 is 18.
Number of cartons in each stack = 18.

Check Your Level

1. Find the HCF of 1650 and 847.

2. Find the HCF of 2781 and 1242.

3. Find HCF of 13281 and 15844.

4. Find HCF of 97 and 101.

5. If the H.C.F. of 408 and 1032 is expressible in the form 1032 m – 408 × 5, find m.

6. 105 goats, 140 sheeps have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip.

Answers

1. 11 **2.** 27 **3.** 233 **4.** 1 **5.** m = 2.

6. 35

(C) FUNDAMENTAL THEOREM OF ARITHMETIC

(a) Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique, except for the order in which the prime factors occurs.

(b) HCF and LCM

HCF and LCM of numbers can be determined by prime factorization. This is nothing but an application of the fundamental theorem of arithmetic.

HCF = Product of the smallest power of each common factor.

LCM = Product of the biggest power of each prime factor

Let a and b be natural numbers. Then their

HCF × LCM = a × b

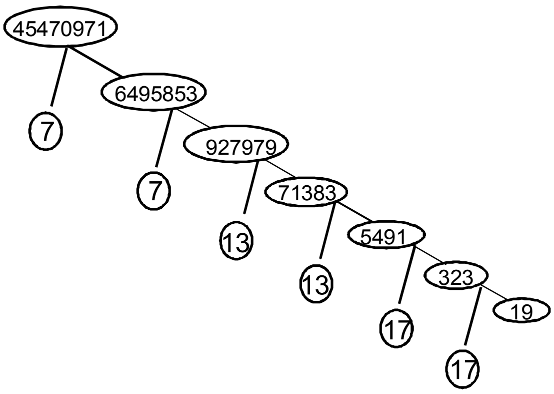
Note : LCM is always divisible by HCF.

Solved Examples

Example.10

Determine the prime factors of 45470971.

Solution.



∴ $45470971 = 7^2 \times 13^2 \times 17^2 \times 19$.

Example.11

Check whether 6^n can end with the digit 0 for any natural number.

Solution.

Any positive integer ending with the digit zero is divisible by 5 and so its prime factorisations must contain the prime 5.

$6^n = (2 \times 3)^n = 2^n \times 3^n$

$\Rightarrow$ The prime in the factorisation of 6^n is 2 and 3.

$\Rightarrow$ 5 does not occur in the prime factorisation of 6^n for any n.

$\Rightarrow$ 6^n does not end with the digit zero for any natural number n.

Example.12

Find the LCM and HCF of 84, 90 and 120 by applying the prime factorisation method.

Sol. $84 = 2^2 \times 3 \times 7$, $90 = 2 \times 3^2 \times 5$ and $120 = 2^3 \times 3 \times 5$.

Prime factors	Least exponent
2	1
3	1
5	0
7	0

$\therefore$ HCF $= 2^1 \times 3^1 = 6$.

Common prime factors	Greatest exponent
2	3
3	2
5	1
7	1

$\therefore$ LCM $= 2^3 \times 3^2 \times 5^1 \times 7^1 = 8 \times 9 \times 5 \times 7 = 2520$.

Example.13

In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that they can cover the distance in complete steps ?

Solution.

Required minimum distance each should walk so, that they can cover the distance in complete step is the L.C.M. of 80 cm, 85 cm and 90 cm

$80 = 2^4 \times 5$

$85 = 5 \times 17$

$90 = 2 \times 3^2 \times 5$

LCM $= 2^4 \times 3^2 \times 5^1 \times 17^1$

LCM $= 16 \times 9 \times 5 \times 17$

LCM = 12240 cm = 122 m 40 cm.

Check Your Level

1. Express the following numbers as a product of prime factors:

(a) 1771 **(b)** 8232 **(c)** 10584

2. Find the HCF and LCM of the following pairs of numbers by prime factorization and verify that LCM $\times$ HCF = product of the numbers.

(a) 1080 and 252 **(b)** 252 and 294

3. If the LCM of two numbers is 252, HCF is 2 and one of the numbers is 28 find the other number.

4. Check whether 4^n can end with the digit 0 for any natural number.

5. Leena has music class on alternate days, dance class once in 3 days and yoga once in 5 days. On the 1st of January she had all the three classes. When will she have all the 3 classes again?

6. There is an oval shaped park with a pathway running round it.
Babu and Raju start jogging at P at the same time and jog in the same direction. If Babu can complete a full round in 16 minutes while Raju can do it in 20 minutes, after how many minutes will they meet at P again?

7. A rope of length 140 cm has to be cut into 2 pieces in the ratio 3 : 4. What is the maximum length of the measuring stick which should be used to measure both the lengths ?

Answers

1. **(a)** $7 \times 11 \times 23$ **(b)** $2^3 \times 3 \times 7^3$ **(c)** $7^2 \times 3^3 \times 2^3$

2. **(a)** 36 and 7560 **(b)** 42 and 1764

3. 18 **5.** 31st January **6.** 80 min **7.** 20 cm

(D) PROOF OF IRRATIONALITY AND DECIMAL REPRESENTATION

(a) Some important results

(i) Let **'p'** be a prime number and '**a**' be a positive integer. If **'p'** divides $\mathbf{a^2}$, then **'p'** divides **'a'**.
(ii) Let **x** be a rational number whose decimal expansion terminates. Then, **x** can be expressed in the form $\frac{p}{q}$, where **p** and q are co-primes, and prime factorisations of **q** is of the form $\mathbf{2^m \times 5^n}$, where **m**, **n** are non-negative integers.

(iii) Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of **q** is not of the form $2^m \times 5^n$, where **m, n** are non-negative integers. Then, **x** has a decimal expansion which is non -terminating repeating.

In earlier classes you have studied different types of numbers such as natural numbers, whole numbers, integers, rational and irrational numbers. All these together are called Real numbers. In this chapter, we shall study some properties of numbers, especially valid for integers

Solved Examples

Example.14

Prove that $\sqrt{2}$ is an irrational number.

Solution.

Let us assume on the contrary that $\sqrt{2}$ is a rational number.

Then, there exists positive integer a and b such that $\sqrt{2} = \frac{a}{b}$ where, a and b are coprimes i.e. their HCF is1.

$\Rightarrow \quad (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2 \quad \Rightarrow \quad 2 = \frac{a^2}{b^2}$

$\Rightarrow \quad a^2 = 2b^2 \quad \Rightarrow \quad a^2$ is a multiple of 2

$\Rightarrow$ a is a multiple of 2 .....(i)

a = 2c for some integer c.

$\Rightarrow \quad a^2 = 4c^2 \quad \Rightarrow \quad 2b^2 = 4c^2$

$\Rightarrow \quad b^2 = 2c^2 \quad \Rightarrow \quad b^2$ is a multiple of 2

$\Rightarrow$ b is a multiple of 2 ....(ii)

From (i) and (ii), a and b have at least 2 as a common factor. But this contradicts the fact that a and b are co-prime. This means that $\sqrt{2}$ is an irrational number.

Example. 15

Prove that $5\sqrt{3}$ is not rational

Solution.

If possible let $5\sqrt{3}$ be rational

Let $5\sqrt{3} = \frac{a}{b}$ where a and b are coprime integers $\sqrt{3} = \frac{a}{5b}$

This means that $\sqrt{3}$ which is irrational is equal to a rational number $\frac{a}{5b}$.

$\therefore$ $5\sqrt{3}$ cannot be rational.

Example.16

Prove that $3 - \sqrt{5}$ is an irrational number.

Solution.

Let us assume that on the contrary that $3 - \sqrt{5}$ is rational.

Then, there exist co-prime positive integers a and b such that,

$3 - \sqrt{5} = \frac{a}{b}$ $\Rightarrow$ $3 - \frac{a}{b} = \sqrt{5}$

$\Rightarrow$ $\frac{3b - a}{b} = \sqrt{5}$ $\Rightarrow$ $\sqrt{5}$ is rational

[a, b are integer $\therefore$ $\frac{3b-a}{b}$ is a rational number]

This contradicts the fact that $\sqrt{5}$ is irrational.

Hence, $3 - \sqrt{5}$ is an irrational number.

Example 17.

Prove that $\sqrt{2} + \sqrt{5}$ is irrational.

Sol. Let us assume on the contrary that $\sqrt{2} + \sqrt{5}$ is a rational number, Then, there exist co-prime positive integers a and b such that $\sqrt{2} + \sqrt{5} = \frac{a}{b}$

$\Rightarrow$ $\sqrt{5} = \frac{a}{b} - \sqrt{2}$

Squaring both sides

$\Rightarrow$ $\left(\frac{a}{b} - \sqrt{2}\right)^2 = \left(\sqrt{5}\right)^2$

$\Rightarrow$ $\frac{a^2}{b^2} - \frac{2a}{b}\sqrt{2} + 2 = 5$

$\Rightarrow$ $\frac{a^2}{b^2} - 3 = \frac{2a}{b}\sqrt{2}$

$\Rightarrow$ $\frac{a^2 - 3b^2}{2ab} = \sqrt{2}$

$\Rightarrow$ $\sqrt{2}$ is a rational number. [$\because$ $\frac{a^2 - 3b^2}{2ab}$ is a rational number]

$\Rightarrow$ $\sqrt{2}$ is an irrational number.

So, our assumption is wrong. Hence, $\sqrt{2} + \sqrt{5}$ is irrational.

Example.18

Without actually performing the long division, state whether $\frac{13}{3125}$ has terminating decimal expansion or not.

Sol. $\frac{13}{3125} = \frac{13}{2^0 \times 5^5}$

This, shows that the prime factorisation of the denominator is of the form $2^m \times 5^n$.

Hence, it has terminating decimal expansion.

Example.19

What can you say about the prime factorisations of the denominators of the following rationals :

(i) 43.123456789 **(ii)** $43.\overline{123456789}$

Sol.

(i) Since, 43 .123456789 has terminating decimal, so prime factorisations of the denominator is of the form $2^m \times 5^n$, where m, n are non - negative integers.

(ii) Since, $43.\overline{123456789}$ has non - terminating repeating decimal expansion. So, its denominator has factors other than 2 or 5.

Check Your Level

1. Show that $\sqrt{11}$ is an irrational number.

2. Show that $4 - \sqrt{3}$ is an irrational number.

3. Show that $2\sqrt{5}$ is an irrational number.

4. Show that $\sqrt{8}$ is an irrational number.

5. Show that $\sqrt{5} - \sqrt{3}$ is an irrational number.

6. Without actually performing the long division, state whether $\frac{343}{875}$ has terminating decimal expansion or not.

Answers

6. Terminating decimal expansion.

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : **[01 MARK EACH]**

1. Find the largest number which divides 70 and 125, leaving remainders 5 and 8, respectively.

2. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x,y are prime numbers, then find the HCF (a, b)

3. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then find the LCM (p, q)

4. Find the least number that is divisible by all the numbers from 1 to 10 (both inclusive).

5. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after how many decimal places ?

6. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : **[02 MARKS EACH]**

7. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

8. Prove that $\sqrt{3}+\sqrt{7}$ is irrational.

9. Explain why 3 × 5 × 7 + 7 is a composite number.

10. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

11. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form ?

TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[03 MARK EACH]**

12. Show that cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3, for some integer m.

13. Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.

14. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.

15. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.

16. Prove that $\sqrt{7}$ is irrational.

17. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

18. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[04 MARK EACH]**

19. Show that the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

20. Prove that one of any three consecutive positive integers must be divisible by 3.

21. Show that one and only one out of n, n + 4, n + 8, n + 12 and n + 16 is divisible by 5, where n is any positive integer.
[Hint: Any positive integer can be written in the form 5q, 5q + 1, 5q + 2, 5q + 3, 5q + 4].

Previous Years Problems

1. Which of the following numbers has terminating decimal expansion ?
[1 MARK / CBSE 10TH BOARD2013]

(A) $\frac{37}{45}$ (B) $\frac{21}{2^3 5^6}$ (C) $\frac{17}{49}$ (D) $\frac{89}{2^2 3^2}$

2. If a rational number x is expressed as $x = \frac{p}{q}$, where p,q are integer , $q \neq 0$ and p,q have no common factor (except 1), then the decimal expansion of x is terminating if and only if q has a prime factorization of the form: **[1 MARKS /CBSE 10TH BOARD: 2013]**
(A) $2^m . 5^n$ (B) $2^m . 3^n$ (C) $2^m . 7^n$ (D) $5^m . 3^n$
Where m and n are non-negative integers.

3. Which of the following numbers has non-terminating repeating decimal expansion ?
[1 mark CBSE 10TH BOARD: 2013]

(A) $\frac{7}{80}$ (B) $\frac{17}{320}$ (C) $\frac{20}{100}$ (D) $\frac{93}{420}$

4. Use Euclid's division algorithm to find HCF of 870 and 225. **[2 marks CBSE 10TH BOARD: 2013]**

5. Explain $5 \times 4 \times 3 \times 2 \times 1 + 3$ is a composite number. **[2 marks CBSE 10TH BOARD: 2013]**

6. Prove that $3 + \sqrt{2}$ is an irrational number
OR
Prove that $5\sqrt{2}$ is irrational number. **[3 marks CBSE 10TH BOARD: 2013]**

7. Show that 5^n can't end with the digit 2 for any natural number n.
[3 marks CBSE 10TH BOARD: 2013]

8. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, then the other number is **[1 marks CBSE 10TH BOARD: 2014]**
(A) 415 (B) 425 (C) 435 (D) 445

9. If HCF (96, 404) =4, then LCM (96, 404) is **[1 marks CBSE 10TH BOARD: 2014]**
(A) 9626 (B) 9696 (C) 9656 (D) 9676

10. Check whether 6^n can end with the digit 0 for any natural number n.
[2 marks CBSE 10TH BOARD: 2014]

11. Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number
OR
Prove that $5 + \sqrt{2}$ is an irrational number **[3 marks CBSE 10TH BOARD: 2014]**

12. Use Euclid's division algorithm to find the HCF of 10224 and 9648.
[3 marks CBSE 10TH BOARD: 2015]

13. Which of the following is not a rational number ? **[1 marks CBSE 10TH BOARD: 2015]**
(A) $\sqrt{3}$ (B) $\sqrt{9}$ (C) $\sqrt{16}$ (D) $\sqrt{25}$

14. Prove that $\sqrt{7}$ is an irrational number
OR
Prove that $3 - \sqrt{5}$ is an irrational number **[3 marks CBSE 10TH BOARD: 2015]**

15. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m. **[3 marks CBSE 10TH BOARD: 2015]**

16. The [HCF × LCM] for the number 50 and 20 is **[1 marks CBSE 10TH BOARD: 2016]**
(A) 10 (B) 1000 (C) 100 (D) 110

17. If the HCF of 210 and 55 is expressible in the form 210 × 5 + 55 × p, then the value of p is
[1 marks CBSE 10TH BOARD: 2016]
(A) – 17 (B) – 18 (C) – 20 (D) – 19

18. Prove that $\sqrt{5}$ is an irrational number
OR
Prove that $5 + 3\sqrt{2}$ is an irrational number **[3 marks CBSE 10TH BOARD: 2016]**

19. Prove that for any positive integer n, $n^3 - n$ is divisible by 6.**[3 marks CBSE 10TH BOARD: 2016]**

20. Find the LCM and HCF of 510 and 92 and verify that LCM × HCF = product of the two numbers
[1 marks CBSE 10TH BOARD: 2017]

21. The largest number that will divide 398, 436 and 542 leaving remainder 7,11 and 15 respectively is
[1 marks CBSE 10TH BOARD: 2017]
(A) 11 (B) 17 (C) 34 (D) 51

22. Prove that $\frac{7}{3}\sqrt{5}$ is irrational number.
OR
Prove that $5 - 2\sqrt{3}$ is an irrational number **[3 marks CBSE 10TH BOARD: 2017]**

23. Prove that $n^2 - n$ is divisible by 2 for any positive integer n.**[3 marks CBSE 10TH BOARD: 2017]**

Exercise-1

SUBJECTIVE QUESTIONS

SUBJECTIVE EASY, ONLY LEARNING VALUE PROBLEMS

Section (A) : Introduction and Euclid's Division Lemma

A-1 Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

A-2 Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational number.

A-3 Use Euclid's Division Lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

A-4. Show that of the numbers n , n+2 and n+4 , only one of them is divisible by 3.

A-5. Let n = 640640640643, without actually computing n^2 prove that n^2 leave remainder 1 when divided by 8.

A-6. There is remainder of 3 when a number is divided by 6. what will be the remainder if the square of the same number is divided by 6?

Section (B) : Euclid's Division Algorithm

B-1. Using Euclid's Division algorithm, find the HCF of 210 and 55.

B-2. Using Euclid's Division algorithm, find the HCF of 101 and 1277.

B-3. If d is the HCF of 56 and 72, find x, y satisfying d = 56x + 72y. Also, show that x and y are not unique.

B-4. Using Euclid's Division algorithm, find the greatest number that divides 445, 572 and 699 leaving remainder 4, 5 and 6 respectively.

B-5 Find the largest number that divides 245 and 1029 leaving a remainder of 5 in each case.

Section (C) : Fundamental Theorem of Arithmetic

C-1 Can we have any natural number n, where 7^n ends with the digit zero.

C-2 Find the [HCF × LCM] for the numbers 105 and 120.

C-3. Find the HCF and LCM of following using Fundamental Theorem of Arithmetic method.
(i) 426 and 576 **(ii)** 625, 1125 and 2125

C-4. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

C-5 An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march ?

C-6 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?

C-7 Aakash, Kushal and Harish go for a morning walk. They step off together and their steps measure 40cm, 42cm and 45cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

Section (D) : Proof of Irrationality and Decimal Representation

D-1 Prove that $\sqrt{3}$ is an irrational number.

D-2. Prove that $5 - 2\sqrt{3}$ is an irrational number.

D-3. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

D-4. Without actually performing the long division, state whether the following rational number will have a terminating decimal expansion or non - terminating decimal expansion :

(i) $\frac{77}{210}$ **(ii)** $\frac{15}{1600}$

D-5 What can you say about the prime factorisations of the denominators of the following rationals :

(i) 25.234567 **(ii)** $25.\overline{345678}$

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Introduction and Euclid's Division Lemma

A-1. Which one of the following is true ?
(A) π is a rational number.
(B) All rational numbers are irrational numbers.
(C) All real numbers can be represented on a number line.
(D) $\frac{\sqrt{7}}{8}$ is a rational number.

A-2 Which is not an irrational number ?
(A) $5-\sqrt{3}$ (B) $\sqrt{2}+\sqrt{5}$ (C) $4+\sqrt{2}$ (D) $6+\sqrt{9}$

A-3. If least prime factor of a is 3 and the least prime factor of b is 7, the least prime factor of (a + b) is :
(A) 2 (B) 3 (C) 5 (D) 11

A-4. Euclid's division lemma state that for any positive integers a and b, there exist unique integers q and r such that a = bq + r, where r must satisfy
(A) $1 < r < b$ (B) $0 < r \;\; b$ (C) $0 \le r < b$ (D) $0 < r < b$

A-5. $(n^2 - 1)$ is divisible by 8, if n is
(A) any natural number (B) any integer
(C) any odd positive integer (D) any even positive integer

A-6.. A positive integer n when divided by 9, gives 7 as remainder. What will be the remainder when (3n – 1) is divided by 9 ?
(A) 1 (B) 2 (C) 3 (D) 4

Section (B) : Euclid's Division Algorithm

B-1. If the HCF of 85 and 153 is expressible in the form 85n – 153, then value of n is :
(A) 3 (B) 2 (C) 4 (D) 1

B-2. If the HCF of 79 and 97 is expressible in the form 97n – 79m, then value of m – n is :
(A) 3 (B) 2 (C) 5 (D) 1

B-3. If the HCF of 144 and 90 is expressible in the form 144x + 90y, then value of x – y = :
(A) 3 (B) 2 (C) 5 (D) 1

B-4. If the HCF of 420 and 130 is expressible in the form 420p + 130q, then value of p + q is :
(A) 17 (B) 6 (C) 5 (D) 9

B-5. For positive integers a and b, if a = bq + r, then
(A) every common divisor of 'a' and 'q' is a common divisor of 'b' and 'r', and vice-versa.
(B) every common divisor of 'a' and 'b' is a common divisor of 'q' and 'r', and vice-versa.
(C) every common divisor of 'a' and 'b' is a common divisor of 'b' and 'r', and vice-versa.
(D) None of these

Section (C) : Fundamental Theorem of Arithmetic

C-1 The [HCF × LCM] for the numbers 125 and 80 is :
(A) 100 (B) 1000 (C) 10000 (D) 500

C-2. If $x = 2^3 \times 3 \times 5^2$, $y = 2^2 \times 3^3$, then HCF (x, y) is :
(A) 12 (B) 108 (C) 6 (D) 36

C-3. Given that HCF (253,440) = 11 and LCM (253, 440) = 253 × R. The value of R is :
(A) 400 (B) 40 (C) 440 (D) 253

C-4. If least prime factor of a is 5 and the least prime factor of b is 11, the least prime factor of (a + b) is :
(A) 2 (B) 3 (C) 5 (D) 11

C-5. How many prime factors are there in prime factorization of 5005.
(A) 2 (B) 4 (C) 6 (D) 7

C-6. The product of the HCF and LCM of the smallest prime number and the smallest composite number is :
(A) 2 (B) 4 (C) 6 (D) 8

Section (D) : Proof of Irrationality and Decimal Representation

D-1 The decimal expansion of the rational number $\frac{31}{2^2 \times 5}$ will terminate after :
(A) one decimal place (B) two decimal places
(C) three decimal places (D) more than three decimal places

D-2. Which of the following is a non-terminating repeating decimal ?
(A) $\frac{35}{14}$ (B) $\frac{14}{35}$ (C) $\frac{1}{7}$ (D) $\frac{7}{8}$

D-3. The decimal representation of $\frac{27}{400}$ is :
(A) Terminating (B) Non terminating recurring
(C) Non terminating non recurring (D) None of these

D-4. How many rational numbers exist between any two distinct rational numbers?
(A) 2 (B) 3 (C) 11 (D) Infinite

D-5. 3.24636363..... is
(A) an integer (B) A rational number (C) an irrational number (D) None of these

D-6. A rational number can be expressed as terminating decimal if the denominator has factor
(A) 2,3 or 5 (B) 3 or 5 (C) 2 or 3 (D) 2 or 5

Exercise-2

OBJECTIVE QUESTIONS

1. The positive integers A, B, A – B and A + B are all prime numbers. The sum of these four primes is
(A) even (B) divisible by 3 (C) divisible by 5 (D) prime

2. V is product of first 41 natural numbers. A = V + 1. The number of primes among A + 1, A + 2, A + 3, A + 4 .... A + 39, A + 40 is :
(A) 1 (B) 2 (C) 3 (D) 0

3. If $a^2 - b^2 = 13$ where a and b are natural numbers, then value of a is :
(A) 6 (B) 7 (C) 8 (D) 9

4. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M. is $2^4 \times 3^5 \times 5^2 \times 7^2$. Then the third number is
(A) $2^2 \times 3^5 \times 7^2$ (B) $2^2 \times 5^3 \times 7^2$ (C) $2^5 \times 5^2 \times 7^2$ (D) $2^3 \times 3^5 \times 7^2$

5. The number of ordered pairs (a, b) of positive integers such that a + b = 90 and their greatest common divisor is 6 equals.
(A) 5 (B) 4 (C) 8 (D) 10

6. If HCF (p, q) = 12 and p × q = 1800×n, where n belongs to natural number then LCM (p, q) is :
(A) 3600 (B) 900 (C) 150 (D) 90

7. The value of the digit d for which the number d456d is divisible by 18, is :
(A) 3 (B) 4 (C) 6 (D) 9

8. Which of the following number is divisible by 99 ?
(A) 3572404 (B) 135792 (C) 913464 (D) 114345

9. There is an N digit number (N > 1). If the sum of digits is subtracted from the number then the resulting number will be divisible by :
(A) 7 (B) 2 (C) 11 (D) 9

10. If x is a positive integer such that 2x + 12 is perfectly divisible by 'x', then the number of possible values of 'x' is :
(A) 2 (B) 5 (C) 6 (D) 12

11. The least number which on division by 35 leaves a remainder 25 and on division by 45 leaves the remainder 35 and on division by 55 leaves the remainder 45 is :
(A) 2515 (B) 3455 (C) 2875 (D) 2785

12. A number divided by 14 gives a remainder 8. What is the remainder, if this number is divided by 7 ?
(A) 1 (B) 2 (C) 3 (D) 4

13. The sum of the digits of two digit number is 11, if the digits are reversed the number decreases by 45. The number is :
(A) 38 (B) 65 (C) 74 (D) 83

14. One hundred monkeys have 100 apples to divide. Each adult gets three apples while three children share one. Number of adult monkeys are :
(A) 20 (B) 25 (C) 30 (D) 33

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. If $2^x = 4^y = 8^z$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$, then the value of z is - **(Rajasthan NTSE Stage-1 2005)**
(A) $\frac{7}{16}$ (B) $\frac{7}{32}$ (C) $\frac{7}{48}$ (D) $\frac{7}{64}$

2. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$ then the value of x is - **(Rajasthan NTSE Stage-1 2005)**
(A) 1 (B) 2 (C) 3 (D) 4

3. If $a^x = b$, $b^y = c$ and $c^z = a$, then value of xyz is **(Rajasthan NTSE Stage-1 2007)**
(A) 1 (B) 0 (C) –1 (D) a + b + c.

4. Rationalising the denominator of $\frac{5}{\sqrt{3}-\sqrt{5}}$ is: **(Rajasthan NTSE Stage-1 2013)**
(A) $\left(\frac{5}{2}\right)(\sqrt{3}+\sqrt{5})$ (B) $\left(-\frac{5}{2}\right)(\sqrt{3}+\sqrt{5})$ (C) $\left(\frac{5}{2}\right)(\sqrt{3}-\sqrt{5})$ (D) $\left(-\frac{5}{2}\right)(\sqrt{3}-\sqrt{5})$

5. Value of $\frac{2^{100}}{2}$ is : **(Rajasthan NTSE Stage-1 2013)**
(A) 1 (B) 50^{100} (C) 2^{50} (D) 2^{99}

6. Number of zero's in the product of 5 × 10 × 25 × 40 × 50 × 55 × 65 × 125 × 80, is **(M.P. NTSE Stage-1 2013)**

(A) 8 (B) 9 (C) 12 (D) 13

7. A farmer divides his herd of x cows among his 4 son's such that first son gets one-half of the herd, the second son gets one fourth, the third son gets one-fifth and the fourth son gets 7 cows, then the value of x is : **(M.P. NTSE Stage-1 2013)**

(A) 100 (B) 140 (C) 160 (D) 180

8. H.C.F. (28, 35, 91) = ... **[Gujarat NTSE Stage-1 2013]**

(A) 1 (B) 5 (C) 7 (D) 14

9. Which real number lies between 2 and 2.5 **(Chandigarh NTSE Stage-1 2014)**

(A) $\sqrt{11}$ (B) $\sqrt{8}$ (C) $\sqrt[3]{7}$ (D) $\sqrt[3]{9}$

10. Which of the following time expressions is right for the fraction $\frac{1}{4}$? **[Gujarat NTSE Stage-1 2014]**

(A) 15 minute (B) 30 minute (C) 45 minute (D) 10 minute

11. The HCF of any two prime numbers a and b, is **(Rajasthan NTSE Stage-1 2015)**

(A) a (B) ab (C) b (D) 1

12. Which number is the inverse of the opposite of $-\frac{5}{8}$? **[Gujarat NTSE Stage-1 2015]**

(A) $\frac{5}{8}8$ (B) $1\frac{3}{5}$ (C) $2\frac{2}{5}$ (D) $-\frac{8}{5}$

13. If $x = \sqrt[4]{16} + \sqrt[4]{625}$ than what is x = ? **[Gujarat NTSE Stage-1 2015]**

(A) 7 (B) 29 (C) 12 (D) 5

14. Find HCF of $\frac{6}{5}, \frac{4}{15}, \frac{2}{5}$ **[Delhi NTSE Stage-1 2015]**

(A) $\frac{6}{15}$ (B) $\frac{2}{15}$ (C) $\frac{2}{5}$ (D) $\frac{4}{15}$

15. The simplified value of $\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$ is **[Delhi NTSE Stage-1 2015]**

(A) 1 (B) 0 (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

16. Raj wanted to type the first 200 natural numbers, how many times does he have to press the keys **[Delhi NTSE Stage-1 2015]**

(A) 489 (B) 492 (C) 400 (D) 365

17. Which is the greatest among $\sqrt[6]{100}$, $\sqrt[3]{12}$ and $\sqrt{3}$ **[Delhi NTSE Stage-1 2015]**

(A) $\sqrt{3}$ (B) $\sqrt[6]{100}$ (C) $\sqrt[3]{12}$ (D) cannot be determined

18. The traffic lights at three different signals change after 48 seconds, 72 seconds and 108. If they change at 7 a.m. simultaneously. How many times they will change between 7 a.m. to 7 : 30 a.m. simultaneously ? **(Haryana NTSE Stage-1 2015)**

(A) 3 (B) 4 (C) 5 (D) 2

19. If $x = 2 + \sqrt{3}$ and xy = 1 then $\frac{x}{\sqrt{2}+\sqrt{x}} + \frac{y}{\sqrt{2}-\sqrt{y}}$ = ... **[Bihar NTSE Stage-1 2015]**

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 1 (D) None of these

20. Raj wanted to type the first 200 natural numbers, how many times does he have to press the keys **(Delhi NTSE Stage-1 2016)**

(A) 489 (B) 492 (C) 400 (D) 365

21. If a number m is divided by 5 leaves a remainder 2, while another number n is divided by 5 leaves a remainder 4, then the remainder, when (m + n) is divided by 5 is : **(Haryana NTSE Stage-1 2016)**

(A) 1 (B) 2 (C) 3 (D) 4

22. What is the square root of $9 + 2\sqrt{14}$? **[Bihar NTSE Stage-1 2016]**

(A) $1+2\sqrt{2}$ (B) $\sqrt{3} + \sqrt{6}$ (C) $\sqrt{2} + \sqrt{7}$ (D) $\sqrt{2} + \sqrt{5}$

23. $\sqrt[3]{1-\frac{127}{343}}$ is equal to **[Bihar NTSE Stage-1 2016]**

(A) $\frac{5}{9}$ (B) $1-\frac{1}{7}$ (C) $\frac{4}{7}$ (D) $1-\frac{2}{7}$

24. What is the value of $2.\overline{6} - 1.\overline{9}$? **[Bihar NTSE Stage-1 2016]**

(A) $0.\overline{6}$ (B) $0.\overline{9}$ (C) $0.\overline{7}$ (D) 0.7

25. An equivalent expression of $\frac{5}{7+4\sqrt{5}}$ after rationalizing the denominator is **[Gujarat NTSE Stage-1 2016]**

(A) $\frac{20\sqrt{5}-35}{31}$ (B) $\frac{20\sqrt{5}-35}{129}$ (C) $\frac{35-20\sqrt{5}}{31}$ (D) $\frac{35-20\sqrt{5}}{121}$

26. Four positive integers sum to 125. If the first of these numbers is increased by 4, the second is decreased by 4. the third is multiplied by 4 and the fourth is divided by 4 we find four equal numbers then four original integers are **[Delhi NTSE Stage-1 2016]**

(A) 16, 24, 5, 80 (B) 8, 22, 38, 57 (C) 7, 19, 46, 53 (D) 12, 28, 40, 45

27. If $a = \sqrt{6}+\sqrt{5};$, $b = \sqrt{6}-\sqrt{5}$ the find the value of **[Maharashtra NTSE Stage-1 2016]**

(A) 36 (B) 37 (C) 39 (D) 41

28. $\sqrt{m^4n^4} \times \sqrt[6]{m^2n^2} \times \sqrt[3]{m^2n^2} = (mn)^k$, then find the value of k. **[Maharashtra NTSE Stage-1 2017]**

(A) 6 (B) 3 (C) 2 (D) 1

Answer Key

BOARD LEVEL EXERCISE

TYPE (I)

1. 13 **2.** xy^2 **3.** a^3b^2 **4.** 2520

5. four **6.** No

TYPE (II)

7. True. **10.** it is terminating decimal.

TYPE (III)

15. 625 **18.** 2520 cm

PREVIOUS YEAR PROBLEMS

1. (B)

2. (A) **3.** (D) **4.** 15 **8.** (C)

9. (B) **12.** 144 **13.** (A) **16.** (B)

17. (D) **20.** HCF = 2, LCM = 23460 **21.** (B)

EXERCISE - 1

SUBJECTIVE QUESTIONS

Section (A)

A-1 $\frac{3}{2}$ **A-2** a rational number. **A-6.** remainder = 3

Section (B)

B-1. 5 **B-2.** 1 **B-3** x = 76 and y = (59)

B-4. 63 **B-5** 16.

Section (C)

C-2 12600 **C-3.** **(i)** HCF = 6, LCM = 40896 **(ii)** HCF = 125, LCM = 95625

C-5 8 **C-6** 36 minutes

C-7 2520 cm

Section (D)

D-4. **(i)** non-terminating **(ii)** terminating

OBJECTIVE QUESTIONS

Section (A)

A-1. (C) **A-2** (D) **A-3.** (A) **A-4.** (C)

A-5. (C) **A-6..** (B)

Section (B)

B-1. (B) **B-2.** (C) **B-3.** (C) **B-4.** (D)

B-5. (C)

Section (C)

C-1 (C) **C-2.** (A) **C-3.** (B) **C-4.** (A)

C-5. (B) **C-6.** (D)

Section (D)

D-1 (B) **D-2.** (C) **D-3.** (A) **D-4.** (D)

D-5. (B) **D-6.** (D)

EXERCISE - 2

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	D	D	B	A	C	B	C	D	D	C	B	A	D	B

EXERCISE - 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	A	B	D	B	B	C	D	A	D	B	A	B	D	B	C	B	A	B
Ques.	21	22	23	24	25	26	27	28												
Ans.	A	C	B	A	A	A	C	B												