SOLVED EXAMPLES

Ex.1 Which of the following is a function?

(A) {(2,1), (2,2), (2,3), (2,4)}

(B) {(1,4), (2,5), (1,6) , (3,9)}

(C) {(1,2), (3,3), (2,3), (1,4)}

(D) { (1,2), (2,2), (3,2), (4,2)}

Sol. We know that for a relation to be function every element of first set should be associated with one and only one element of second set but elements of first set can have same f-image in second set which is given in (D).

Ans.[D]

Ex.2 If $f(x) = \frac{x}{x-1} = \frac{1}{y}$, then f (y) equals

(A) x (B) x – 1

(C) x + 1 (D) 1 – x

Sol. $f(y) = \frac{y}{y-1} = \frac{(x-1)/x}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x} = 1 - x.$

Ans.[D]

Ex.3 The domain of $f(x) = \frac{1}{x^3 - x}$ is -

(A) R – {–1,0,1} (B) R

(C) R – {0,1} (D) None of these

Sol. Domain = $\{x;\ x \in R;\ x^3 - x \neq 0\}$

$= R - \{-1, 0, 1\}$ **Ans.[A]**

Ex.4 The range of $f(x) = \cos \frac{\pi[x]}{2}$ is -

(A) {0,1} (B) {–1,1}

(C) {–1,0,1} (D) [–1,1]

Sol. [x] is an integer, cos (–x) = cos x and

$\cos\left(\frac{\pi}{2}\right) = 0$, $\cos 2\left(\frac{\pi}{2}\right) = -1$.

$\cos 0\left(\frac{\pi}{2}\right) = 1$, $\cos 3\left(\frac{\pi}{2}\right) = 0$,.....

Hence range = {–1,0,1} **Ans.[C]**

Ex.5 If $f : R^+ \to R^+$, $f(x) = x^2 + 2$ and $g : R^+ \to R^+$, $g(x) = \sqrt{x+1}$ then (f + g) (x) equals -

(A) $\sqrt{x^2+3}$ (B) x + 3

(C) $\sqrt{x^2+2} + (x+1)$ (D) $x^2 + 2 + \sqrt{(x+1)}$

Sol. (f + g) (x) = f(x) + g(x)

$= x^2 + 2 + \sqrt{x+1}$ **Ans. [D]**

Ex.6 Function $f(x) = x^{-2} + x^{-3}$ is -

(A) a rational function

(B) an irrational function

(C) an inverse function

(D) None of these

Sol. $f(x) = \frac{1}{x^2} + \frac{1}{x^3} = \frac{x+1}{x^3}$

= ratio of two polynomials

∴ f(x) is a rational function. **Ans.[A]**

Ex.7 The period of | sin 2x | is-

(A) π/4 (B) π/2 (C) π (D) 2π

Sol. Here $|\sin 2x| = \sqrt{\sin^2 2x}$

$= \sqrt{\frac{(1-\cos 4x)}{2}}$

Period of cos 4 x is π/2

Period of |sin 2x | will be π/2. **Ans.[B]**

Ex.8 If $f(x) = \frac{x-3}{x+1}$, then f [f {f (x)}] equals -

(A) x (B) 1/x (C) –x (D) –1/x

Sol. Here $f\{f(x)\} = f\left(\frac{x-3}{x+1}\right) = \frac{\left(\frac{x-3}{x+1}\right)-3}{\left(\frac{x-3}{x+1}\right)+1} = \frac{x+3}{1-x}$

$\therefore f[f\{f(x)\}] = \frac{\frac{x+3}{1-x}-3}{\frac{x+3}{1-x}+1} = \frac{4x}{4} = x$ **Ans. [A]**

Ex.9 If f(x) = 2|x – 2| – 3|x – 3|, then the value of f(x) when 2 < x < 3 is -

(A) 5 – x (B) x – 5

(C) $5x - 13$ (D) None of these

Sol. $2 < x < 3 \Rightarrow |x - 2| = x - 2$

$|x - 3| = 3 - x$

$f(x) = 2(x - 2) - 3(3 - x) = 5x - 13$.**Ans. [C]**

Ex.10 Which of the following functions defined from R to R are one-one -

(A) $f(x) = |x|$ (B) $f(x) = \cos x$

(C) $f(x) = e^x$ (D) $f(x) = x^2$

Sol. $x_1 \neq x_2 \Rightarrow e^{x_1} \neq e^{x_2}$

$\Rightarrow f(x_1) \neq f(x_2)$

$\therefore f(x) = e^x$ is one-one. **Ans. [C]**

Ex.11 The function $f : R \to R$, $f(x) = x^2$ is -

(A) one-one but not onto

(B) onto but not one-one

(C) one-one onto

(D) None of these

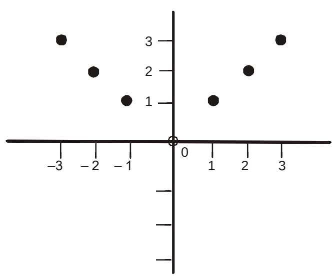
Sol. $\because$ $4 \neq -4$, but $f(4) = f(-4) = 16$

$\therefore$ f is many one function.

Again $f(R) = R^+ \cup \{0\}$ R, therefore f is into.

Ans. [D]

Ex.12 If $f : I_0 \to N$, $f(x) = |x|$, then f is -



(A) one-one (B) onto

(C) one-one onto (D) none of these

Sol. Observing the graph of this function, we find that every line parallel to x-axis meets its graph at more than one point so it is not one-one.

Now range of f = N = Co-domain, so it is onto.

Ans. [B]

Ex.13 If $f : R - \{3\}$ $R - \{1\}$, $f(x) = \frac{x-2}{x-3}$ then function f(x) is -

(A) Only one-one (B) one-one into

(C) Many one onto (D) one-one onto

Sol. $\because$ $f(x) = \frac{x-2}{x-3}$

$\therefore$ $f'(x) = \frac{(x-3).1 - (x-2).1}{(x-3)^2} = \frac{-1}{(x-3)^2}$

$\therefore$ $f'(x) < 0 \ \forall \ x \in R - \{3\}$

$\therefore$ f (x) is monotonocally decreasing function

$\Rightarrow$ f is one-one function.

onto/ into : Let $y \in R - \{1\}$ (co-domain)

Then one element $x \in R - \{3\}$ is domain is such that

$f(x) = y \Rightarrow \frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$

$\Rightarrow x = \left(\frac{3y-2}{y-1}\right) = x \in R - \{3\}$

$\therefore$ the pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\}$.

$\Rightarrow$ f is onto. **Ans. [D]**

Ex.14 Function $f : N \to N$, $f(x) = 2x + 3$ is -

(A) one-one onto (B) one-one into

(C) many one onto (D) many one into

Sol. f is one-one because for any $x_1, x_2 \in N$

$x_1 \neq x_2 \Rightarrow 2x_1 + 3 \neq 2x_2 + 3 \Rightarrow f(x_1) \neq f(x_2)$

Further $f^{-1}(x) = \frac{x-3}{2} \notin N$ (domain) when $x = 1, 2, 3$ etc.

$\therefore$ f is into which shows that f is one- one into.

<u>Alter</u>

$f(x) = 2x + 3$

$f'(x) = 2 > 0 \ \forall \ x \in N$

$\therefore$ f(x) is increasing function

$\therefore$ f(x) is one-one function

& $\because$ $x = 1, 2, 3, ...$

$\therefore$ min value of f(x) is $2.1 + 3 = 5$

$\therefore$ $f(x) \neq \{1, 2, 3, 4\}$

$\therefore$ Co Domain $\neq$ Range

$\therefore$ f(x) is into function **Ans. [B]**

Ex.15 Function $f : R \to R$, $f(x) = x^3 - x$ is -

(A) one-one onto (B) one-one into

(C) many-one onto (D) many-one into

Sol. Since $-1 \neq 1$, but $f(-1) = f(1)$, therefore f is many-one.

Also let, $f(x) = x^3 - x = \alpha \Rightarrow x^3 - x - \alpha = 0$. This is a cubic equation in x which has at least one real root because complex roots always occur in pairs. Therefore each element of co-domain R has pre-image in R. Thus function f in onto .

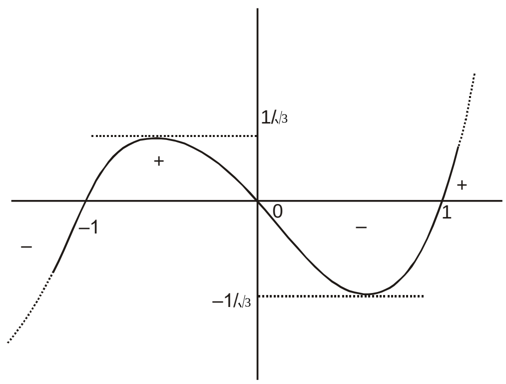
$\therefore$ function f is many-one onto.

Alter

$f(x) = x^3 - x$

$= x(x-1)(x+1)$

graph of f(x) is



from graph function is many one- onto function **Ans. [C]**

Ex.16 If $f : R \to R$, $f(x) = 2x - 1$ and $g : R \to R$, $g(x) = x^2 + 2$, then (gof) (x) equals-

(A) $2x^2 - 1$ (B) $(2x-1)^2$

(C) $2x^2 + 3$ (D) $4x^2 - 4x + 3$

Sol. Here $(gof)(x) = g[f(x)] = g(2x-1)$

$= (2x-1)^2 + 2 = 4x^2 - 4x + 3$. **Ans. [D]**

Ex.17 If $f : R \to R$, $f(x) = 4x^3 + 3$, then $f^{-1}(x)$ equals-

(A) $\left(\frac{x-3}{4}\right)^{1/3}$ (B) $\left(\frac{x^{1/3}-3}{4}\right)$

(C) $\frac{1}{4}(x-3)^{1/3}$ (D) None of these

Sol. Since f is a bijection, therefore f^{-1} exists. Now if f-image of x is y, then $f^{-1}: R \to R$ defined as follows :

$f^{-1}(y) = x \Rightarrow f(x) = y$

But $f(x) = 4x^3 + 3 \Rightarrow y = 4x^3 + 3 \Rightarrow x = \left(\frac{y-3}{4}\right)^{1/3}$

Therefore $f^{-1}(y) = \left(\frac{y-3}{4}\right)^{1/3}$

$\Rightarrow f^{-1}(x) = \left(\frac{x-3}{4}\right)^{1/3}$ **Ans. [A]**

Ex.18 $f(x) = \sqrt{|x-1|}$ and g (x) = sin x then (fog) (x) equals -

(A) $\sin\left\{\sqrt{|x-1|}\right\}$

(B) $|\sin x/2 - \cos x/2|$

(C) $|\sin x - \cos x|$

(D) None of these

Sol. $(fog)(x) = f[g(x)] = f[\sin x]$

$= \sqrt{|\sin x - 1|}$

$= \sqrt{|1 - \sin x|}$

$= \sqrt{|\sin^2 x/2 + \cos^2 x/2 - 2\sin x/2 \cos x/2|}$

$= \sqrt{|(\sin x/2 - \cos x/2)^2|}$

$= |\sin x/2 - \cos x/2|$ **Ans.[B]**

Ex.19 If $f : R \to R$, $f(x) = 2x + 1$ and $g : R \to R$, $g(x) = x^3$, then $(gof)^{-1}(27)$ equals -

(A) –1 (B) 0 (C) 1 (D) 2

Sol. Here $f(x) = 2x + 1$ $f^{-1}(x) = \frac{x-1}{2}$

and $g(x) = x^3 \Rightarrow g^{-1}(x) = x^{1/3}$

$\because (gof)^{-1}(27) = (f^{-1}og^{-1})(27)$

$= f^{-1}[g^{-1}(27)] = f^{-1}[(27)^{1/3}]$

$= f^{-1}(3) = \frac{3-1}{2} = 1$ **Ans.[C]**

Ex.20 The domain of function $f(x) = \sqrt{2^x - 3^x}$ is -

(A) $(-\infty, 0]$ (B) R

(C) $[0, \infty)$ (D) No value of x

Sol. Domain $= \{x ; 2^x - 3^x \ge 0\} = \{x ; (2/3)^x \ge 1\}$

$= x \in (-\infty, 0]$ **Ans.[A]**

Ex.21 The domain of the function

$f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is -

(A) $[-2, 2] - (-1, 1)$ (B) $[-1, 2] - \{0\}$

(C) $[1, 2]$ (D) $[-2, 2] - \{0\}$

Sol. We know that the domain of $\sin^{-1}x$ is [–1,1]. So for f(x) to be meaningful , we must have

$$-1 \le \log_2 \frac{x^2}{2} \le 1$$

$\Rightarrow 2^{-1} \le x^2/2 \le 2 \quad x \ne 0$

$\Rightarrow 1 \le x^2 \le 4, x \ne 0$

$\Rightarrow x \in [-2, -1] \cup [1,2]$

$\Rightarrow x \in [-2, 2] - (-1, 1)$ **Ans.[A]**

Ex.22 The range of function $f(x) = \frac{x^2}{1+x^2}$ is -

(A) $R - \{1\}$ (B) $R^+ \cup \{0\}$

(C) [0, 1] (D) None of these

Sol. Range is containing those real numbers y for which f(x) = y where x is real number.

Now $f(x) = y = \Rightarrow \frac{x^2}{1+x^2} = y$

$\Rightarrow x = \sqrt{\frac{y}{(1-y)}}$ ....(1)

by (1) clearly $y \ne 1$, and for x to be real

$\frac{y}{1-y} \ge 0 \Rightarrow$ $y \ge 0$ and $y < 1$.

($\because$ If y = 2 then $\frac{y}{1-y} = \frac{2}{1-2} = (-2)$ and

$\sqrt{\frac{y}{(1-y)}} = \sqrt{-2} \notin R$)

$\therefore \; 0 \le y < 1$

$\therefore$ Range of function = $(0 \le y < 1) = [0,1)$

Ans.[D]

Ex.23 If f(x) = cos (log x), then

f(x) f(y) – 1/2 [f (x/y) + f(xy)] is equal to

(A) –1 (B) 1/2

(C) –2 (D) 0

Sol. cos (log x) cos (log y)

$$-\frac{1}{2}[\cos(\log x/y) + \cos(\log xy)]$$

$= \frac{1}{2}$ [cos (log x + log y) + cos (log x – log y)]

$-\frac{1}{2}$ [cos (log x – log y) + cos (log x + log y)]

= 0 **Ans.[D]**

Ex.24 If $f(x) = \frac{2^x + 2^{-x}}{2}$, then f (x + y) . f (x – y) is equal to -

(A) $\frac{1}{2}$ [f (x+ y) + f(x – y)]

(B) $\frac{1}{2}$ [f (2x) + f (2y)]

(C) $\frac{1}{2}$ [f(x+ y). f(x – y)]

(D) None of these

Sol. $f(x + y). f(x - y) = \frac{2^{x+y} + 2^{-x-y}}{2} . \frac{2^{x+y} + 2^{-x-y}}{2}$

$$= \frac{2^{2x} + 2^{2y} + 2^{-2x} + 2^{-2y}}{4}$$

$$= \frac{1}{2}\left[\frac{2^{2x} + 2^{-2x}}{2} . \frac{2^{2y} + 2^{-2y}}{2}\right]$$

$= \frac{1}{2}$ [f (2x) + f(2y)] **Ans.[B]**

Ex.25 If $f : R \to R$. f (x) = 2x + |x| , then f (3x) – f (–x) – 4x equals -

(A) f(x) (B) – f(x)

(C) f (–x) (D) 2f(x)

Sol. f (3x) – f (–x) – 4x

= 6x + |3x| – {–2x + | –x|} – 4x

= 6x + 3 |x| + 2x – |x| – 4x

= 4x + 2 |x| = 2 f(x). **Ans.[D]**

Ex.26 If $g(x) = x^2 + x - 2$ and $\frac{1}{2}$ (gof) (x) = $2x^2 - 5x + 2$, then f(x) is equal to -

(A) 2x – 3 (B) 2x + 3

(C) $2x^2 + 3x + 1$ (D) $2x^2 - 3x - 1$

Sol. $g(x) = x^2 + x - 2$

$\Rightarrow$ (gof) (x) = g[f(x)] = $[f(x)]^2 + f(x) - 2$

Given, $\frac{1}{2}$ (gof) (x) = $2x^2 - 5x + 2$

$\therefore \frac{1}{2}[f(x)]^2 + \frac{1}{2} f(x) - 1 = 2x^2 - 5x + 2$

$\Rightarrow [f(x)]^2 + f(x) = 4x^2 - 10x + 6$

$\Rightarrow$ f(x) [f(x) + 1] = (2x – 3) [(2x – 3) + 1]

$\Rightarrow f(x) = 2x - 3$ **Ans.[A]**

Required value = 1 + 0 = 1. **Ans.[B]**

Ex.27 If f (x) = |x| and g(x) = [x], then value of

$fog\left(-\frac{1}{4}\right) + gof\left(-\frac{1}{4}\right)$ is -

(A) 0 (B) 1

(C) –1 (D) 1/4

Sol. $fog = f\left[g\left(-\frac{1}{4}\right)\right] f(-1) = 1$

and $gof\left(-\frac{1}{4}\right) = g\left[f\left(-\frac{1}{4}\right)\right] = g\left(\frac{1}{4}\right) = [1/4] = 0$