MATHEMATICS

Class-X

Topic-2

POLYNOMIALS

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CLASSROOM

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POLYNOMIALS

(A) INTRODUCTION TO POLYNOMIALS

An algebraic expression f(x) of the form **$f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$**, where **$a_0$, a_1, a_2,............,a_n** are real numbers and all the index of **'x'** are non-negative integers is called a **polynomial** in x.

(a) Identification of Polynomial

For this, we have following examples :

(i) $\sqrt{3}\,x^2 + x – 5$ is a polynomial in variable x as all the exponents of x are non negative integers.

(ii) $\sqrt{3}\,x^2 + \sqrt{x} – 5x$ is not a polynomial as the exponent of second term ($\sqrt{x} = x^{1/2}$) is not a non–negative integer.

(iii) $5x^3 + 2x^2 + 3x – \frac{5}{x} + 6$ is not a polynomial as the exponent of fourth term $\left[-\frac{5}{x} = -5x^{-1}\right]$ is not non–negative integer.

(b) Degree of Polynomial

Highest index of **x** in algebraic expression is called the **degree of the polynomial**, here a_0, a_1x, a_2x^2,............., a_nx^n, are called the terms of the polynomial and a_0, a_1, a_2,.........., a_n are called various coefficients of the polynomial f(x).
For example:

(i) $p(x) = 3x^4 – 5x^2 + 2$ is a polynomial of degree 4

(ii) $q(x) = 5x^4 + 2x^5 – 6x^6 – 5$ is a polynomial of degree 6

(iii) $f(x) = 2x^3 + 7x – 5$ is a polynomial of degree 3.

(c) Classification of Polynomial

Generally, we divide the polynomials in the following categories.

(i) Based on degrees :
There are four types of polynomials based on degrees. These are listed below :

(I) Zero degree polynomial : Any non-zero number (constant) is regarded as a polynomial of degree zero or **zero degree polynomial**. i.e. f(x) = a, where a0 is a zero degree polynomial, since we can write f(x) = a as $f(x) = ax^o$.

(II) Linear Polynomial : A polynomial of degree one is called a **linear polynomial**. The general form of linear polynomial is **ax + b**, where **a** and **b** are any real constant and **$a \neq 0$**.

(III) Quadratic Polynomial : A polynomial of degree two is called a **quadratic polynomial**. The general form of a quadratic polynomial is **$ax^2 + bx + c$**, where **$a \neq 0$**.

(IV) Cubic Polynomial : A polynomial of degree three is called a **cubic polynomial**. The general form of a cubic polynomial is **$ax^3 + bx^2 + cx + d$**, where **$a \neq 0$**.

(V) Biquadratic (or quartic) Polynomial : A polynomial of degree four is called a **biquadratic (quartic) polynomial**. The general form of a biquadratic polynomial is **$ax^4 + bx^3 + cx^2 + dx + e$**, where **$a \neq 0$**.

(ii) Based on number of terms :

There are three types of polynomials based on number of terms. These are as follows :

(I) Monomial : A polynomial is said to be a **monomial** if it has only one term.
e.g. x, $9x^2$, $5x^3$ all are monomials.

(II) Binomial : A polynomial is said to be a **binomial** if it contains two terms.

e.g. $2x^2 + 3x$, $\frac{1}{2}x + 5x^3$, $-8x^3 + 3$, all are binomials.

(III) Trinomials : A polynomial is said to be a **trinomial** if it contains three terms.

e.g. $3x^3 - 8x + \frac{5}{2}$, $5 - 7x + 8x^9$, $x^{10} + 8x^4 - 3x^2$ are all trinomials.

NOTE :

(i) A polynomial having four or more than four terms does not have particular name. These are simply called **polynomials**.
(ii) $f(x) = 0$ is called as zero polynomial and its degree is not defined.

(d) Important Formulae

(i) $(x + a)^2 = x^2 + 2ax + a^2$
(ii) $(x - a)^2 = x^2 - 2ax + a^2$
(iii) $x^2 - a^2 = (x + a)(x - a)$
(iv) $x^3 + a^3 = (x + a)(x^2 - ax + a^2) = (x + a)^3 - 3xa(x + a)$
(v) $x^3 - a^3 = (x - a)(x^2 + ax + a^2) = (x - a)^3 + 3xa(x - a)$
(vi) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
(vii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
(viii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
(ix) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
Special Case : If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.

(e) Value of a Polynomial

The value of a polynomial **f(x)** at **$x = \alpha$** is obtained by substituting **$x = \alpha$** in the given polynomial and is denoted by **$f(\alpha)$**.
For example : If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then its value at $x = 1$ is
$f(1) = 2(1)^3 - 13(1)^2 + 17(1) + 12 = 2 - 13 + 17 + 12 = 18$.

(f) Zeros of a Polynomial

A real number **'a'** is a zero of a polynomial **f(x),** if **f(a) = 0**.

Solved Examples

Example 1.
Show that $x = 2$ is a zero of $2x^3 + x^2 - 7x - 6$

Solution.
$p(x) = 2x^3 + x^2 - 7x - 6$.
Then, $p(2) = 2(2)^3 + (2)^2 - 7(2) - 6 = 16 + 4 - 14 - 6 = 0$
Hence $x = 2$ is a zero of $p(x)$.

Example 2.

If $x = \frac{4}{3}$ is a zero of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Solution.
$f(x) = 6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$$

$\Rightarrow \quad 6\left(\frac{64}{27}\right) - 11\left(\frac{16}{9}\right) + \frac{4k}{3} - 20 = 0$

$\Rightarrow \quad 128 - 176 + 12k - 180 = 0$

$\Rightarrow \quad 12k + 128 - 356 = 0$

$\Rightarrow \quad 12k = 228 \Rightarrow \quad k = 19.$

Example 3.

If x = 2 & x = 0 are zeros of the polynomials $f(x) = 2x^3 - 5x^2 + ax + b$, then find the values of a and b.

Solution. $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow \quad 16 - 20 + 2a + b = 0 \Rightarrow 2a + b = 4$ ......(i)

$\Rightarrow \quad f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\Rightarrow \quad b = 0$

$\Rightarrow \quad 2a = 4 \Rightarrow a = 2, b = 0.$

Example 4.

If $4x + 5y = 10$ and $xy = 12$, then evaluate $64x^3 + 125y^3$.

Solution.

$4x + 5y = 10$

$x y = 12$

$64x^3 + 125y^3 = ?$

$64x^3 + 125y^3 = (4x)^3 + (5y)^3$

$= (4x + 5y)^3 - 3 \times 4x \times 5y\,(4x + 5y)$

$= (10)^3 - 60 \times 12 \times 10$

$= 1000 - 7200$

$= -6200$ **Ans.**

Example 5.

If $x - y = 1$, then find the value of $x^3 - y^3 - 3xy$

Solution.

$\because \quad x^3 - y^3 - 3xy$

$= (x - y)(x^2 + xy + y^2) - 3xy$

$= 1(x^2 + xy + y^2) - 3xy \qquad [\because x - y = 1]$

$= x^2 - 2xy + y^2$

$= (x - y)^2 = 1$

Check Your Level

1. Is x = – 4 a zero of the polynomial $2x^2 + 5x - 12$.

2. If (x + 5) is a zero of $x^3 + 2x^2 - 14x + K + 2$, then find K.

3. Find the value of $125p^3 - 8q^3$ if $5p - 2q = 1$ and $pq = 2$.

4. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then prove that a = b = c.

5. If $x + y = -4$, then find the value of $x^3 + y^3 - 12xy + 64$

Answers

1. Yes **2.** 3 **3.** 61 **5.** 0

(B) IMPORTANT THEOREM RELATED TO POLYNOMIALS

(a) Remainder Theorem

Let **p(x)** be any polynomial of degree greater than or equal to one and **'a'** be any real number. If **p(x)** is divided by **(x – a),** then the remainder is equal to **p(a)**.

(b) Factor Theorem

Let **p(x)** be a polynomial of degree greater than or equal to 1 and **'a'** be a real number such that **p(a) = 0**, then **(x – a)** is a factor of **p(x)**. Conversely, if **(x – a)** is a factor of **p(x)**, then **p(a) = 0**.

(c) Division Algorithm for Polynomial

If p(x) and g(x) are any two polynomials with $g(x) \neq 0$, then we can find polynomials r(x) and q(x) such that

$p(x) = g(x) \times q(x) + r(x)$

i.e. Dividend = (Divisor x Quotient) + Remainder

where r(x)=0 or degree of r(x) < degree of g(x).

(i) If r(x) = 0, g(x) is a factor of p(x)

(ii) If deg(p(x)) > deg(g(x)),

then deg(q(x)) = deg(p(x)) –deg(g(x))

(iii) If deg(p(x)) = deg(g(x)),

then deg(q(x)) = 0 and deg(r(x)) < deg(g(x))

Solved Examples

Example. 6

Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.

Sol. $1 - 2x = 0 \quad \Rightarrow \quad 2x = 1 \quad \Rightarrow \quad x = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{8} - \frac{3}{2} + 1 - 4 = \frac{1 - 12 + 8 - 32}{8} = -\frac{35}{8}.$$

Example. 7

Show that x + 1 and 2x – 3 are factors of $2x^3 - 9x^2 + x + 12$.

Sol. To prove that (x + 1) and (2x – 3) are factors of $p(x) = 2x^3 - 9x^2 + x + 12$ it is sufficient to show that $p(-1)$ and $p\left(\frac{3}{2}\right)$ both are equal to zero.

$p(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = -2 - 9 - 1 + 12 = -12 + 12 = 0$

$$\text{And } p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + 12$$

$$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{27 - 81 + 6 + 48}{4} = \frac{-81 + 81}{4} = 0.$$

Example. 8

Find α and β if x + 1 and x + 2 are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.

Sol. x + 1 and x + 2 are the factor of p(x).

Then, p(–1) = 0 & p(–2) = 0

Therefore, $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow \quad -1 + 3 + 2\alpha + \beta = 0$

$\Rightarrow \quad \beta = -2\alpha - 2$ ....(i)

$\Rightarrow \quad p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow \quad -8 + 12 + 4\alpha + \beta = 0$

$\Rightarrow \quad \beta = -4\alpha - 4$ ....(ii)

From equation (1) and (2)

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow \quad 2\alpha = -2$

$\Rightarrow \quad \alpha = -1$

Put $\alpha = -1$ in equation (1)

$\Rightarrow \quad \beta = -2(-1) - 2 = 2 - 2 = 0.$

Hence $\alpha = -1, \beta = 0.$

Example. 9

What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$.

Sol: Let $p(x) = 3x^3 + x^2 - 22x + 9$ and $q(x) = 3x^2 + 7x - 6$

We know if p(x) is divided by q(x) which is quadratic polynomial then the remainder be r(x) and degree of r(x) is less than q(x) or Divisor.

By long division method

Let we added ax + b (linear polynomial) in p(x), so that p(x) + ax + b is exactly divisible by $3x^2 + 7x - 6$.

Hence, $p(x) + ax + b = s(x) = 3x^3 + x^2 - 22x + 9 + ax + b \quad = 3x^3 + x^2 - x(22 - a) + (9 + b).$

$$\begin{array}{r|l} & x-2 \\ \hline 3x^2+7x-6 & 3x^3+x^2-x(22-a)+9+b \\ & \underset{-}{}3x^3 \underset{-}{+} 7x^2 \underset{+}{-} 6x \\ \hline & -6x^2+6x-(22-a)x+9+b \\ & \text{or} \\ & -6x^2+x(-16+a)+9+b \\ & \underset{+}{-}6x^2 \underset{+}{-} 14x \underset{-}{+} 12 \\ \hline & x(-2+a)+(b-3)=0 \end{array}$$

Hence, $x(a - 2) + b - 3 = 0.x + 0$

$\Rightarrow \quad a - 2 = 0$ & $b - 3 = 0$

$\Rightarrow \quad a = 2$ and $b = 3$

Hence if in p(x) we added $2x + 3$ then it is exactly divisible by $3x^2 + 7x - 6$.

Illustration. 10

What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$.

Sol. Let ax + b be subtracted from $p(x) = x^3 - 6x^2 - 15x + 80$ so that it is exactly divisible by $x^2 + x - 12$.

$s(x) = x^3 - 6x^2 - 15x + 80 - (ax + b)$

$= x^3 - 6x^2 - (15 + a)x + (80 - b)$

Dividend = Divisor × quotient + remainder

But remainder will be zero.

Dividend = Divisor × quotient

$\Rightarrow \quad s(x) = (x^2 + x - 12) \times \text{quotient}$

$\Rightarrow \quad s(x) = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$$\begin{array}{r|l} & x-7 \\ \hline x^2+x-12 & x^3-6x^2-x(15+a)+80-b \\ & \underline{_{-}x^3 \ {}_{-}+x^2 \ {}_{+}^{-}12x} \\ & -7x^2+12x-(15+a)x+80-b \\ & \text{or} \\ & -7x^2+x(-3-a)+80-b \\ & \underline{{}_{+}^{-}7x^2 {}_{+}^{-}7x \quad {}_{-}^{+}84} \\ & x(4-a)+(-4-b)=0 \end{array}$$

Hence, $x(4-a)+(-4-b) = 0.x + 0$

$\Rightarrow \quad 4-a = 0 \ \& \ (-4-b) = 0 \quad \Rightarrow \quad a = 4$ and $b = -4$

Hence, if in p(x) we subtract $4x - 4$ then it is exactly divisible by $x^2 + x - 12$.

Example. 11

Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$ by $g(x) = 2x^2 - x + 1$.

Sol.

$$\begin{array}{r|l} & 5x^2+11x-28 \\ \hline 2x^2-x+1 & 10x^4+17x^3-62x^2+30x-3 \\ & \underline{\underset{-}{10x^4}\ \underset{+}{-5x^3}\ \underset{-}{+5x^2}} \\ & 22x^3-67x^2+30x-3 \\ & \underline{\underset{-}{22x^3}\ \underset{+}{-11x^2}\ \underset{-}{+11x}} \\ & -56x^2+19x-3 \\ & \underline{\underset{+}{-56x^2}\ \underset{-}{+28x}\ \underset{+}{-28}} \\ & -9x+25 \end{array}$$

So, quotient $q(x) = 5x^2 + 11x - 28$ and remainder $r(x) = -9x + 25$.

Now, dividend = Quotient × Divisor + Remainder

$= (5x^2 + 11x - 28)(2x^2 - x + 1) + (-9x + 25)$

$= 10x^4 - 5x^3 + 5x^2 + 22x^3 - 11x^2 + 11x - 56x^2 + 28x - 28 - 9x + 25$

$= 10x^4 + 17x^3 - 62x^2 + 30x - 3$

Hence, the division algorithm is verified.

Example. 12

If 1 and – 2 are zeros of $x^4 - 4x^3 - x^2 + 16x - 12$ find the other zeros.

Solution.

The quadratic polynomial for which 1 and – 2 are zeros is $(x - 1)(x + 2) = x^2 + x - 2$ dividing the given polynomial by $x^2 + x - 2$

	$x^2 - 5x + 6$
$x^2 + x - 2$	$x^4 - 4x^3 - x^2 + 16x - 12$
	$x^4 + x^3 - 2x^2$
	$-5x^3 + x^2 + 16x$
	$-5x^3 - 5x^2 + 10x$
	$+6x^2 + 6x - 12$
	$6x^2 + 6x - 12$
	0

To find the zeros of $x^2 - 5x + 6$

$x^2 - 3x - 2x + 6$

$x(x - 3) - 2(x - 3)$

$(x - 3)(x - 2)$

$\therefore$ The other two zeros are 2 and 3.

Example. 13

Find all the zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if two of its zeros are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

Sol. Since, $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeros of f(x).

Therefore, $\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right) = \left(x^2-\frac{3}{2}\right) = \frac{2x^2-3}{2}$ or $2x^2 - 3$ is a factor of f(x).

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\
 & \underline{\underset{-}{2x^4} \quad \underset{+}{\mp} 3x^2} \\
 & -2x^3 - 4x^2 + 3x + 6 \\
 & \underline{\underset{+}{\mp} 2x^3 \quad \underset{-}{+} 3x} \\
 & -4x^2 + 6 \\
 & \underline{\underset{+}{-} 4x^2 \underset{-}{+} 6} \\
 & 0
\end{array}
$$

$\therefore$ $2x^4 - 2x^3 - 7x^2 + 3x + 6 = (2x^2 - 3)(x^2 - x - 2)$

$= (2x^2 - 3)(x - 2)(x + 1)$

$= 2\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)(x - 2)(x + 1)$

So, the zeros are $-\sqrt{\frac{3}{2}}$, $\sqrt{\frac{3}{2}}$, 2, -1.

Check Your Level

1. The polynomial $5x^2 + 7x + 3$ is divided by $x - 2$. Find the remainder by using remainder theorem.

2. Examine whether $(a - 3)$ is a factor of $a^3 - a^2 - 5a - 3$. If so, what are the other two factors?

3. If $(x + 2)$ and $(x - 2)$ are two factors of $x^4 + 8x^3 + 3x^2 - 32x - 28$, then the remaining factors are
(A) $(x + 1), (x + 7)$ (B) $(x - 1), (x + 7)$ (C) $(x + 1), (x - 7)$ (D) $(x - 1), (x - 7)$

4. If $(x - 2)$ is a factor of $(x - 1)^5 - (2x + 3k)^2$, then the value of k is
(A) 1 (B) -1 (C) 2 (D) -2

5. If $-1, -2$ are the two zeros of $2x^4 + 9x^3 + 14x^2 + 9x + 2$ then the other zeros are
(A) $+\frac{1}{2}, +1$ (B) $-\frac{1}{2}, -1$ (C) $2, -1$ (D) $-2, +1$

Answers

1. 37 **2.** (a + 1) and (a + 1) **3.** (A) **4.** (B) **5.** (B)

(C) RELATIONSHIP BETWEEN ZEROS AND COEFFICIENTS OF A POLYNOMIAL

(a) Relationship between Zeros and coefficient of a quadratic polynomial:

Let α and β be the zeros of a quadratic polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$. By factor theorem $(x - \alpha)$ and $(x - \beta)$ are the factors of f(x).

$\therefore$ $f(x) = k(x - \alpha)(x - \beta)$ are the factors of f(x)

$\Rightarrow$ $ax^2 + bx + c = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

$\Rightarrow$ $ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + k\alpha\beta$

Comparing the coefficients of x^2, x and constant terms on both sides, we get

$a = k$, $b = -k(\alpha + \beta)$ and $c = k\alpha\beta$

$\Rightarrow$ $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ $\Rightarrow$ $\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$ Hence,

Sum of the zeros $= -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$.

REMARKS :

If α and β are the zeros of a quadratic polynomial f(x). Then, the polynomial f(x) is given by

$f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

or, $f(x) = k\{x^2 - (\text{Sum of the zeros})\,x + \text{Product of the zeros}\}$

(b) Relationship between Zeros and coefficient of a cubic polynomial

Let α, β, γ be the zeros of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. Then, by factor theorem, $x - \alpha$, $x - \beta$ and $x - \gamma$ are factors of f(x) . Also, f(x) being a cubic polynomial, cannot have more than three linear factors.

$\therefore$ $f(x) = k(x - \alpha)(x - \beta)(x - \gamma)$

$\Rightarrow$ $ax^3 + bx^2 + cx + d = k(x - \alpha)(x - \beta)(x - \gamma)$

$\Rightarrow$ $ax^3 + bx^2 + cx + d = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$

$\Rightarrow$ $ax^3 + bx^2 + cx + d = kx^3 - k(\alpha + \beta + \gamma)x^2 + k(\alpha\beta + \beta\gamma + \gamma\alpha)x - k\alpha\beta\gamma$

Comparing the coefficients of x^3, x^2, x and constant terms on both sides, we get

$a = k$, $b = -k(\alpha + \beta + \gamma)$, $c = k(\alpha\beta + \beta\gamma + \gamma\alpha)$ and $d = -k(\alpha\beta\gamma)$

$\Rightarrow$ $\alpha + \beta + \gamma = -\frac{b}{a}$

$\Rightarrow$ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

And, $\alpha\beta\gamma = -\frac{d}{a}$

$\Rightarrow$ Sum of the zeros $= -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow$ Sum of the products of the zeros taken two at a time $= \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\Rightarrow$ Product of the zeros $= -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$.

❖ **REMARKS :**

Cubic polynomial having α, β and γ as its zeros is given by

$f(x) = k (x - \alpha) (x - \beta) (x - \gamma)$

$f(x) = k\{x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha) x - \alpha\beta\gamma\}$, where k is any non-zero real number.

Solved Examples

Example. 14

Find the zeros of the quadratic polynomial $f(x) = x^2 - 2x - 8$ and verify the relationship between the zeros and their coefficients.

Solution.

$f(x) = x^2 - 2x - 8$

$\Rightarrow$ $f(x) = x^2 - 4x + 2x - 8$

$\Rightarrow$ $f(x) = [x(x - 4) + 2(x - 4)]$

$\Rightarrow$ $f(x) = (x - 4)(x + 2)$

Zeros of f(x) are given by f(x) = 0

$\Rightarrow$ $x^2 - 2x - 8 = 0$

$\Rightarrow$ $(x - 4)(x + 2) = 0$

$\Rightarrow$ $x = 4$ or $x = -2$

So, $\alpha = 4$ and $\beta = -2$

$\therefore$ sum of zeros $= \alpha + \beta = 4 - 2 = 2$

Also, sum of zeros $= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \dfrac{-(-2)}{1} = 2$

So, sum of zeros $= \alpha + \beta = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Now, product of zeros $= \alpha\beta = (4)(-2) = -8$

Also, product of zeros $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-8}{1} = -8$

$\therefore$ Product of zeros $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \alpha\beta.$

Example. 15

Find a quadratic polynomial whose zeros are $5 + \sqrt{2}$ and $5 - \sqrt{2}$.

Solution.

Given, $\alpha = 5 + \sqrt{2}$, $\beta = 5 - \sqrt{2}$

$\therefore$ $f(x) = k\{x^2 - x(\alpha + \beta) + \alpha\beta\}$

Here, $\alpha + \beta = 5 + \sqrt{2} + 5 - \sqrt{2} = 10$

and $\alpha\beta = (5 + \sqrt{2})(5 - \sqrt{2}) = 25 - 2 = 23$

$\therefore$ $f(x) = k\{x^2 - 10x + 23\}$, where, k is any non-zero real number.

Example. 16

Sum and product of zeros of quadratic polynomial are 5 and 17 respectively. Find the polynomial.

Solution.

Given : Sum of zeros = 5 and product of zeros = 17

So, quadratic polynomial is given by

$\Rightarrow$ $f(x) = k\{x^2 - x$ (sum of zeros) + product of zeros$\}$

$\Rightarrow$ $f(x) = k\{x^2 - 5x + 17\}$, where, k is any non-zero real number.

Example. 17

Form the quadratic polynomial whose zeros are squares of the zeros of the polynomial $x^2 - 2x - 15$

Solution.

$x^2 - 2x - 15 = x^2 - 5x + 3x - 15 = x(x - 5) + 3(x - 5) = (x - 5)(x + 3)$

$\therefore$ zeroes are 5 and - 3

Squares of zeros are 5^2 and $(-3)^2$ i.e. 25 and 9

$\therefore$ The quadratic polynomial whose zeros are 25 and 9 is

x^2 – (sum of the zeros)x + product of the zeros $= x^2 - (25 + 9)x + 25 \times 9 = x^2 - 34x + 225$

Example. 18

Verify that $\frac{1}{2}$, 1, – 2 are zeros of cubic polynomial $2x^3 + x^2 - 5x + 2$. Also verify the relationship between, the zeros and their coefficients.

Solution.

$f(x) = 2x^3 + x^2 - 5x + 2$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = 0$

$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0,$

$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = -16 + 4 + 10 + 2 = 0.$

Let $\alpha = \frac{1}{2}$, $\beta = 1$, and $\gamma = -2$

Now, Sum of zeros $= \alpha + \beta + \gamma = \frac{1}{2} + 1 - 2 = -\frac{1}{2}$

Also, sum of zeros $= -\frac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = -\frac{1}{2}$

So, sum of zeros $= \alpha + \beta + \gamma = -\frac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$

Sum of product of zeros taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha$

$= \frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2} = -\frac{5}{2}$

Also, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{-5}{2}$

So, sum of product of zeros taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

Now, Product of zeros $= \alpha\beta\gamma = \left(\frac{1}{2}\right)(1)(-2) = -1$

Also, product of zeros $= -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = \frac{-2}{2} = -1$

$\therefore$ Product of zeros $= \alpha\beta\gamma = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$.

Example. 19

Find a polynomial with the sum, sum of the product of its zeros taken two at a time, and product of its zeros as 3, –1 and – 3 respectively.

Solution.

Given, $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -1$ and $\alpha\beta\gamma = -3$

So, polynomial $f(x) = k\{x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma\}$

$f(x) = k\{x^3 - 3x^2 - x + 3\}$, where k is any non-zero real number.

Check Your Level

1. Find the cubic polynomial whose zeros are given below
(a) 1, 2 and 3 **(b)** 2, – 3 and 4

2. Find the polynomial whose zeros are squares of the zeros of polynomial $x^2 + x - 50$.

3. Find the polynomial whose zeros are double the zeros of the polynomial $x^2 - x - 42$.

4. Find the polynomial whose zeros are reciprocal of the zeros of polynomial $2x^2 + 13x + 15$.

5. Find the polynomial whose zeros are one more than the zeros of the polynomial $6x^2 + 17x + 12$..

Answers

1. **(a)** $x^3 - 6x^2 + 11x – 6$ **(b)** $x^3 - 3x^2 - 10x + 24$ **2.** $x^2 - 61x + 900$

3. $x^2 - 2x – 168$ **4.** $15x^2 + 13x + 2$ **5.** $6x^2 + 5x + 1$

(D) GEOMETRICAL MEANING OF ZEROES OF A POLYNOMIAL

(a) Graph of a Linear Polynomial

Consider a linear polynomial **f(x) = ax + b**, $a \neq 0$. Graph of **y = ax + b** is a straight line. That is why f(x) = ax + b is called a **linear polynomial**. Since two points determine a straight line, so only two points need to be plotted to draw the line **y = ax + b**. The line represented by **y = ax + b** crosses the x-axis at exactly one point, namely $\left(-\frac{b}{a},\ 0\right)$.

(b) Graph of a Quadratic Polynomial

Let a, b, c be real numbers and Then $f(x) = ax^2 + bx + c$, $a \neq 0$ is known as a **quadratic polynomial** in x. Graph of the quadratic polynomial i.e. the curve whose equation is $y = ax^2 + bx + c$, is always a parabola.

The graph of $y = ax^2 + bx + c$ is of the two shapes either opening upwards like  or opening downwards like  depending on whether $a > 0$ or $a < 0$.

(c) Number of zeroes

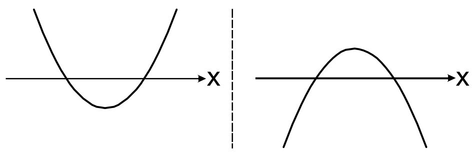
Let α be a real number such that $a\alpha^2 + b\alpha + c = 0$. Thus, point $(\alpha, 0)$ lies on $y = ax^2 + bx + c$. Thus, every real value of x satisfying $ax^2 + bx + c = 0$ represents a point of intersection of the parabola with the X-axis.

Conversely, if the parabola $y = ax^2 + bx + c$ intersects the X-axis at a point $(\alpha, 0)$, then $(\alpha, 0)$ satisfies the equation $y = ax^2 + bx + c$

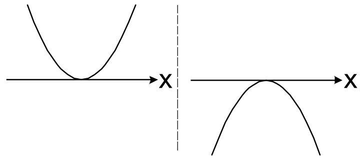
$$a\alpha^2 + b\alpha + c = 0$$

Thus, the intersection of the parabola $y = ax^2 + bx + c$ with X-axis gives all the real zeroes of $ax^2 + bx + c$. Following conclusions may be drawn :

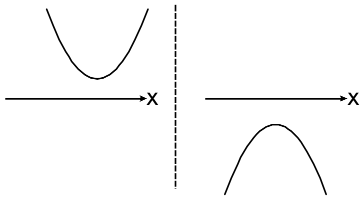
(i) If the parabola intersects the x-axis in two distinct points, then the x-coordinates of these points are the two zeroes of the polynomial $ax^2 + bx + c$



(ii) If the parabola just touches the x-axis at one point ,i.e, at two coincident points, then the x-coordinates of the point is the only zeroes of the polynomial $ax^2 + bx + c$



(iii) If the parabola does not intersect x-axis at all, then the polynomial $ax^2 + bx + c$ has no zeroes in this case.



NOTE : As we can see that a quadratic polynomial has either two zeroes, one zero or no zero. Hence, we can conclude that a polynomial of degree n has atmost two zeroes.

Solved Examples

Example. 20

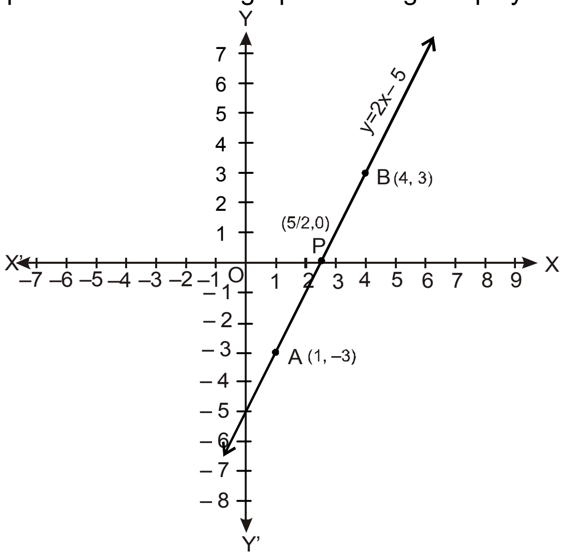
Draw the graph of the polynomial $f(x) = 2x - 5$. Also, find the coordinates of the point where it crosses X-axis.

Sol. Let $y = 2x - 5$.

The following table list the values of **y** corresponding to different values of **x**.

x	1	4
y	–3	3

The points A (1, – 3) and B (4, 3) are plotted on the graph paper on a suitable scale. A line is drawn passing through these points to obtain the graphs of the given polynomial.



Example. 21

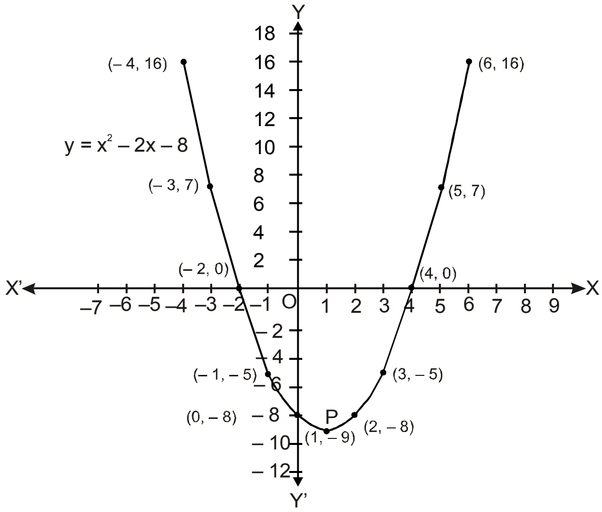
Draw the graph of the polynomial $f(x) = x^2 - 2x - 8$.

Sol. Let $y = x^2 - 2x - 8$.

The following table gives the values of y or f(x) for various values of x.

x	– 4	– 3	– 2	– 1	0	1	2	3	4	5	6
$y = x^2 - 2x - 8$	16	7	0	– 5	– 8	– 9	– 8	– 5	0	7	16

Let us plot the points (–4,16), (–3, 7), (–2, 0), (–1, –5), (0, –8), (1, –9), (2, –8), (3, –5), (4, 0), (5, 7) and (6, 16) on a graphs paper and draw a smooth free hand curve passing through these points. The curve thus obtained represents the graphs of the polynomial f(x) = x^2 – 2x – 8. This is called a parabola.



Example 22.

Draw the graphs of the quadratic polynomial f(x) = 3 – 2x – x^2.

Solution.

Let y = f(x) or, y = 3 – 2x – x^2.

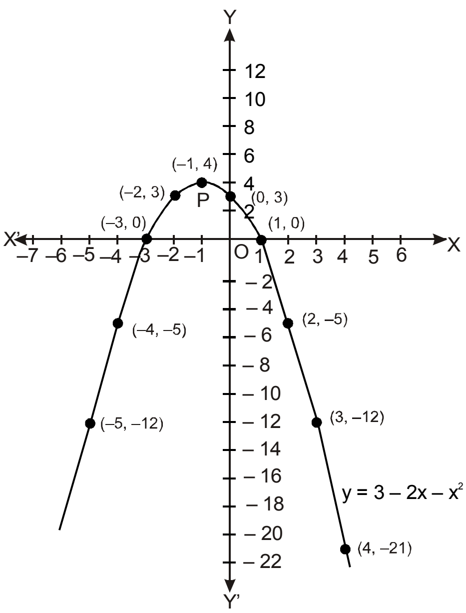
Let us list a few values of y = 3 – 2x – x^2 corresponding to a few values of x as follows :

x	– 5	– 4	– 3	– 2	– 1	0	1	2	3	4
y = 3 – 2x – x^2	– 12	– 5	0	3	4	3	0	– 5	– 12	– 21

Thus, the following points lie on the graph of the polynomial y = 3 – 2x – x^2 :

(– 5, – 12), (– 4, – 5), (– 3, 0), (– 2, 3), (– 1, 4), (0, 3), (1, 0), (2, – 5), (3, – 12) and (4, – 21).

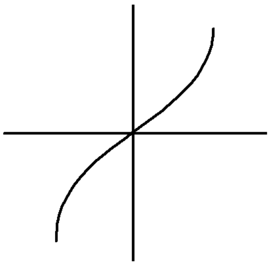
Let plot these points on a graph paper and draw a smooth free hand curve passing through these points to obtain the graphs of y = 3 – 2x – x^2. The curve thus obtained represents a parabola, as shown in figure. The highest point P (– 1, 4), is called a maximum points, is the vertex of the parabola. Vertical line through P is the axis of the parabola. Clearly, parabola is symmetric about the axis.



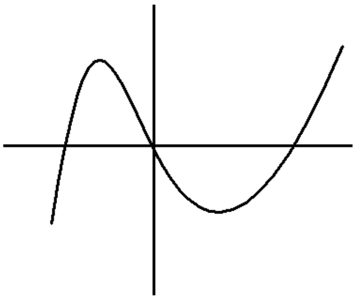
Check Your Level

1. Check whether the graph of $3(x - 2)(2 - x) = x^2 + 2x(1 - x)$ open upwards or downwards.

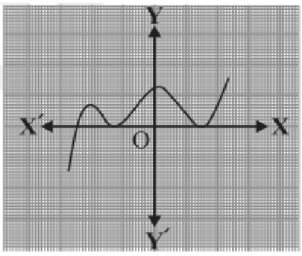
2. Find the number of zeroes



3. Find the number of zeroes



4. Find the number of zeroes



Answers

2. one **3.** Three **4.** Three

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]

1. Find a quadratic polynomial whose zeroes are –3 and 4.

2. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then find the product of the other two zeroes.

3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then find a and b.

4. Find the number of polynomials having zeroes as –2 and 5.

5. If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, find the product of the other two zeroes.

6. Show that zeroes of the quadratic polynomial $x^2 + 99x + 127$ are both negative.

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

Directions Q 7 to 12 : Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials:

7. $4x^2 - 3x - 1$

8. $3x^2 + 4x - 4$

9. $2x^2 + \frac{7}{2}x + \frac{3}{4}$

10. $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

11. $v^2 + 4\sqrt{3}v - 15$

12. $t^3 - 2t^2 - 15t$

13. Find a quadratic polynomial whose sum and product of zeroes are as given.

(i) $\frac{-8}{3}, \frac{4}{3}$ **(ii)** $\frac{21}{8}, \frac{5}{16}$ **(iii)** $-2\sqrt{3}, -9$ **(iv)** $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

TYPE (III) : LONG ANSWER TYPE QUESTIONS: [04 MARK EACH]

14. If $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, then find all the zeroes of the polynomial.

15. If the remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

16. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[05 MARK EACH]**

17. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, a + b, a + 2b for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

18. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials

Previous Year Problems

1. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3, then the value of k is **[2 marks / CBSE 10TH BOARD: 2013]**

(A) $-\frac{4}{3}$ (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

2. The value of p for which the polynomial $x^3 + 4x^2 - px + 8$ is exactly divisible by $(x - 2)$ is **[2 marks / CBSE 10TH BOARD: 2013]**

(A) 1 (B) 0 (C) 15 (D) 16

3. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - ax + b$, then find the value of $\alpha^2 + \beta^2$ **[2 marks / CBSE 10TH BOARD: 2013]**

4. Divide $30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $(3x^2 + 2x - 4)$ and verify the result by division algorithm. **[4 marks / CBSE 10TH BOARD: 2013]**

5. If the product of zeroes of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a'. **[4 marks / CBSE 10TH BOARD: 2014]**

6. If α and β the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate

(i) $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$ **(ii)** $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right]$

[3 marks / CBSE 10TH BOARD: 2014]

7. Find the zeroes of the quadratic polynomial $8x^2 - 21 - 22x$ and verify the relationship between the zeroes and the coefficients of the polynomial. **[2 marks / CBSE 10TH BOARD: 2015]**

8. If α and β are the zeroes of the quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$, find a quadratic polynomial having α and β as its zeros **[3 marks / CBSE 10TH BOARD: 2015]**

9. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$, then find the value of k, If $\alpha + \beta = \frac{1}{2}\alpha\beta$. **[2 marks / CBSE 10TH BOARD: 2016]**

10. Find a quadratic polynomial with zeroes $3 + \sqrt{2}$ and $3 - \sqrt{2}$. **[2 marks / CBSE 10TH BOARD: 2016]**

11. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$. **[3 marks / CBSE 10TH BOARD: 2016]**

12. Find the zeroes of the polynomial $f(x) = x^3 - 5x^2 - 16x + 80$, If its two zeroes are equal in magnitude but opposite in sign. **[4 marks / CBSE 10TH BOARD: 2016]**

13. If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, then evaluate $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

[2 marks / CBSE10TH BOARD:2017]

14. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, prove that

$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$ **[3 marks / CBSE 10TH BOARD: 2017]**

15. Find all zeroes of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if it's two zeroes are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

[4 marks / CBSE 10TH BOARD: 2017]

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction to polynomials

A–1. Give one example of polynomial which is cubic as well as binomial.

A–2. Find the maximum and minimum number of terms in the polynomial in single variable of degree 100.

A–3. If $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$.

A–4. Find the value of $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$.

A–5. Prove that $a^2 + b^2 + c^2 - ab - bc - ca$ is always non – negative for all values of a, b & c.

A–6. Prove that : $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

A–7. If $f(x) = x^3 - x^2 + x + 1$ then value of $\frac{f(1)+f(-1)}{2}$

Section (B) : Important theorem related to polynomials

B–1. Apply the division algorithm to find the quotient and remainder on dividing $p(x) = x^4 - 3x^2 + 4x + 5$ by $g(x) = x^2 + 1 - x$.

B–2. Obtain all the zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

B–3. What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisible by $x^2 + x - 6$?

B–4. What must be subtracted from $x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the result is exactly divisible by $x^2 - 4x + 3$?

B–5. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$.

Section (C) : Relationship between zeroes and coefficients of a polynomial

C–1. Find a quadratic polynomial whose zeros are 5 and – 5.

C–2. Sum and product of zeros of a quadratic polynomial are 2 and $\sqrt{5}$ respectively. Find the quadratic polynomial.

C–3. If the product of zeros of the polynomial $ax^2 - 3x - 3$ is 12, find the value of 'a'.

C–4. Find the quadratic polynomial sum of whose zeros is 8 and their product is 12. Hence find the zeros of the polynomial.

C–5. Find a quadratic polynomial whose zeros are $3+\sqrt{5}$ and $3-\sqrt{5}$.

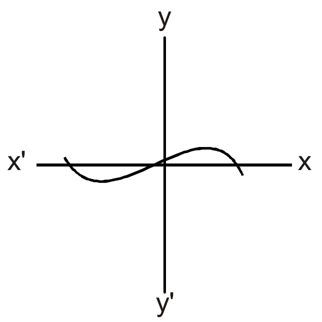
C–6. If α, β are zeros of $x^2 + 5x + 5$, find the value of $\alpha^{-1} + \beta^{-1}$.

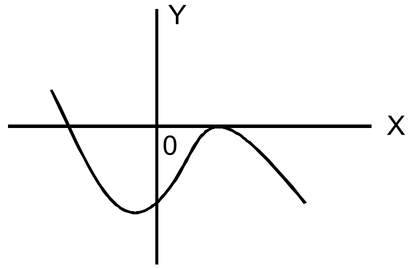
C–7. Find the zeros of quadratic polynomial $p(x) = 4x^2 + 24x + 36$ and verify the relationship between the zeros and their coefficients.

C–8. Find a cubic polynomial whose zeros are 3,5 and –2.

Section (D) : Geometrical meaning of zeroes of a polynomial

D–1. Write the number of zeros of the polynomial y = f(x) whose graph is given figures

(i) 

(ii) 

D–2. Draw the graph of following polynomials f(x) = – 3

D–3. Draw the graph of following polynomials f(x) = x – 4

D–4. Draw the graph of following polynomials $f(x) = x^2 - 9$

D–5. Check whether the graph of $4(x - 3)(1 + x) = x^2 + 2x(1 + x)$ open upwards or downwards.

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Introduction to polynomials

A–1 If – 4 is a zero of the polynomial $x^2 – x – (2 + 2k)$, then the value of k is :
(A) 3 (B) 9 (C) 6 (D) – 9

A–2. It 2 and 3 are the zeros of the polynomial $3x^2 – 2kx + 2m$, find the values of **k** and **m**.
(A) $k = -\frac{15}{2}$, m = 9 (B) $k = \frac{15}{2}$, m = 9 (C) $k = \frac{15}{2}$, m = – 9 (D) $k = -\frac{15}{2}$, m = –9

A–3. If $p(x) = 2 + \frac{x}{2} + x^2 - \frac{x^3}{3}$ then p (–1) is :
(A) $\frac{15}{6}$ (B) $\frac{17}{6}$ (C) $\frac{1}{6}$ (D) $\frac{13}{6}$

A–4. $\sqrt{2}$ is a polynomial of degree :
(A) 2 (B) 0 (C) 1 (D) $\frac{1}{2}$

A–5. Which of the following is a quadratic polynomial in one variable ?
(A) $\sqrt{2x^3} + 5$ (B) $2x^2 + 2x^{-2}$ (C) x^2 (D) $2x^2 + y^2$

Section (B) : Important theorem related to polynomials

B–1. If $4x^4 – 3x^3 – 3x^2 + x – 7$ is divided by 1 – 2x then remainder will be :
(A) $\frac{57}{8}$ (B) $-\frac{59}{8}$ (C) $\frac{55}{8}$ (D) $-\frac{55}{8}$

B–2. The polynomials $ax^3 + 3x^2 – 3$ and $2x^3 – 5x + a$ when divided by (x – 4) leaves remainders R_1 & R_2 respectively then value of **'a'** if $2R_1 – R_2 = 0$.
(A) $-\frac{18}{127}$ (B) $\frac{18}{127}$ (C) $\frac{17}{127}$ (D) $-\frac{17}{127}$

B–3. A quadratic polynomial is exactly divisible by (x + 1) & (x + 2) and leaves the remainder 4 after division by (x + 3) then that polynomial is :
(A) $x^2 + 6x + 4$ (B) $2x^2 + 6x + 4$ (C) $2x^2 + 6x – 4$ (D) $x^2 + 6x – 4$

B–4. The values of a & b so that the polynomial $x^3 – ax^2 – 13x + b$ is divisible by (x – 1) & (x + 3) are :
(A) a = 15, b = 3 (B) a = 3, b = 15 (C) a = –3, b = 15 (D) a = 3, b = –15

B–5. The value of p for which the polynomial $px^3 + 4x^2 – px + 8$ is exactly divisible by (x + 2) is :
(A) 0 (B) 3 (C) 5 (D) 4

Section (C) : Relationship between zeroes and coefficients of a polynomial

C–1. If one zero of $2x^2 – 3x + k$ is reciprocal to the other, then the value of k is :
(A) 2 (B) $-\frac{2}{3}$ (C) $-\frac{3}{2}$ (D) – 3

C–2. The quadratic polynomial whose sum of zeroes is 3 and product of zeroes is 2 is :
(A) $x^2 + 3x - 2$ (B) $x^2 - 2x + 3$ (C) $x^2 - 3x + 2$ (D) $x^2 - 3x - 2$

C–3. If α and β are the zeroes of $x^2 - 4x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ is :
(A) 3 (B) 5 (C) – 5 (D) – 3

C–4. If α and β are the zeroes of the polynomial $5x^2 - 7x + 2$, then sum of their reciprocals is :
(A) $\frac{7}{2}$ (B) $\frac{7}{5}$ (C) $\frac{2}{5}$ (D) $\frac{14}{25}$

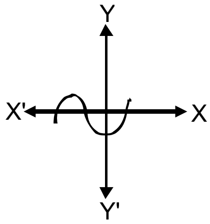
Section (D) : Geometrical meaning of zeroes of a polynomial

D–1. Graph of a quadratic equation is always a :
(A) straight line (B) circle (C) parabola (D) Hyperbola

D–2. The graph of polynomial $y = x^3 - x^2 + x$ is always passing through the point :
(A) (0, 0) (B) (3, 2) (C) (1, – 2) (D) all of these

D–3. How many time, graph of the polynomial $f(x) = x^3 - 1$ will intersect X–axis :
(A) 0 (B) 1 (C) 2 (D) 4

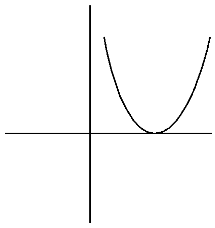
D–4. In figure, the graph of a polynomial p(x) is shown. The number of zeroes of p(x) is :



(A) 3 (B) 2 (C) 4 (D) 0

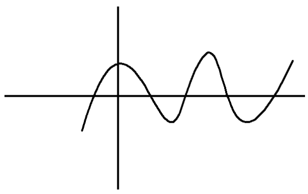
D–5. If the sign of 'a' is positive in a quadratic equation then its graph should be :
(A) parabola open upwards (B) parabola open downwards
(C) parabola open leftwards (D) can't be determined

D–6. Which one of the following is true for the given graph?



(A) Zeros are identical and positive (B) Zeros are zero
(C) No zeros (D) Zeros are distinct and positive

D–7. The number of zeros for the curve



(A) 5 (B) 4 (C) 1 (D) 6

Exercise-2

OBJECTIVE QUESTIONS

1. F(x) is a polynomial in x. When F(x) is divided by (x – 2), the remainder obtained is 3, when the same polynomial is divided by (x – 3), the remainder obtained is 2. What is the remainder when F(x) is divided by (x – 3) (x – 2)

(A) $-x + 5$ (B) $-\frac{5}{3}x + 7$ (C) 0 (D) 5

2. Determine the value of **a** for which the polynomial $2x^4 - ax^3 + 4x^2 + 2x + 1$ is divisible by $1 - 2x$.

(A) 25 (B) 26 (C) 28 (D) 30

3. If α, β are zeros of quadratic polynomial $kx^2 + 4x + 4$, find the value of k such that $(\alpha+\beta)^2 - 2\alpha\beta = 24$.

(A) – 1 (B) $\frac{2}{3}$ (C) both (A) and (B) (D) None of these

4. The equation $x^2 + Bx + C = 0$ has 5 as the sum of its roots, and 15 as the sum of the square of its roots. The value of 'C' is :

(A) 5 (B) 7.5 (C) 10 (D) 12.5

5. If the sum of the zeros of the quadratic polynomial $f(t) = kt^2 + 2t + 3k$ is equal to their product, then the value of k is :

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{3}$ (D) none of these

6. If α, β are the zeroes of $x^2 - 6x + k = 0$. What is the value of k if $3\alpha + 2\beta = 20$.

(A) –16 (B) 8 (C) – 2 (D) – 8

7. Minimum value for the polynomial $4x^2 - 6x + 1$ is :

(A) $-\frac{3}{4}$ (B) $-\frac{5}{4}$ (C) $-\frac{5}{16}$ (D) $-\infty$

8. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(A) 10 (B) – 10 (C) 5 (D) – 5

9. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then the value of a is

(A) 0 (B) 1 (C) 2 (D) 3

10. Find the other zero of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

(A) – 3 (B) 3 (C) 2 (D) None of these

11. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

(A) $g(x) = x^2 + x + 1$ (B) $g(x) = -x^2 - x + 1$ (C) $g(x) = x^2 - x + 1$ (D) $g(x) = x^2 - x - 1$

12. If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$.

(A) – 1 (B) 1 (C) 0 (D) None of these

13. If the zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $(a - b)$, a, $(a + b)$, find the sum of all values of b

(A) 1 (B) 0 (C) $\sqrt{2}$ (D) None of these

14. If α, β, γ are the zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.
(A) 0 (B) 1 (C) 5 (D) – 5

15. If $(x + 1)$ is a factor of $x^2 - 3ax + 3a - 7$, then the value of a is :
(A) 1 (B) – 1 (C) 0 (D) – 2

16. If α,β,γ are the zeros of $x^3 - 5x^2 + 6x - 1$ then value of $\alpha^3 + \beta^3 + \gamma^3$
(A) 38 (B) –38 (C) 19 (D) –19

17. If one of the zeros of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeros is
(A) $a - b - 1$ (B) $b - a - 1$ (C) $1 - a + b$ (D) $1 + a - b$

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. One of the factors of the expression $x^4 + 8x$ is: **[Raj. NTSE Stage–1 2006]**
(A) $x^2 + 2$ (B) $x^2 + 8$ (C) $x + 2$ (D) $x - 2$

2. If $x + y + z = 1$, $x^2 + y^2 + z^2 = 2$ and $x^3 + y^3 + z^3 = 3$ then the value of xyz is ___________. **[Orissa NTSE Stage – 1 2012]**
(A) 1/5 (B) 1/6 (C) 1/7 (D) 1/8

3. If $a + b = 6$ and $ab = 8$, then $a^3 + b^3$ = ... **[Gujarat NTSE Stage – 1 2013]**
(A) 18 (B) 36 (C) 54 (D) 72

4. If polynomial $P(x) = 3x^3 - x^2 - ax - 45$ has one zero of 3, then a = ... **[Gujarat NTSE Stage – 1 2013]**
(A) 3 (B) 6 (C) 9 (D) 12

5. If one factor of $27x^3 + 64y^3$ is $(3x + 4y)$ what is the second factor ? **[Gujarat NTSE Stage – 1 2013]**
(A) $(3x^2 - 4y)$ (B) $(3x^2 + 12xy + 4y^2)$ (C) $(9x^2 + 12xy - 16y^2)$ (D) $(9x^2 - 12xy + 16y^2)$

6. If the zero of the polynomial $f(x) = k^2x^2 - 17x + k + 2 (k > 0)$ are reciprocal of each other, then the value of k is : **[Delhi NTSE Stage–1 2013]**
(A) 2 (B) – 1 (C) – 2 (D) 1

7. If $x + \frac{1}{x} = 3$, then the value of $x^6 + \frac{1}{x^6}$ is : **[Delhi NTSE Stage – 1 2013]**
(A) 927 (B) 114 (C) 364 (D) 322

8. If x + 3, divides $x^3 + 5x^2 + kx$, then k is equal to : **[Orissa NTSE Stage – 1 2013]**
(A) 2 (B) 4 (C) 6 (D) 8

9. Which one of the following is a factor of the expression $(a + b)^3 - (a - b)^3$? **[MP NTSE Stage – 1 2013]**
(A) a (B) $3a^2 - b$ (C) 2b (D) $(a + b)(a - b)$

10. If α, β are the zeros of polynomial $f(x) = x^2 - p(x + 1) - c$, then $(\alpha + 1)(\beta + 1) =$ **[Raj. NTSE Stage–1 2014]**
(A) $c - 1$ (B) $1 - c$ (C) c (D) $1 + c$

11. If $x + \frac{1}{x} = 5$, then $x^3 - 5x^2 + x + \frac{1}{x^3} - \frac{5}{x^2} + \frac{1}{x} =$ .... **[Bihar NTSE Stage–1 2014]**

(A) –5 (B) 0 (C) 5 (D) 10

12. If a + b + c = 0 and $a^2 + b^2 + c^2 = k(a^2 - bc)$ then k = ... **[Bihar NTSE Stage–1 2014]**

(A) 0 (B) 1 (C) 2 (D) 3

13. If (x – 2) is a factor of polynomial $x^3 + 2x^2 - kx + 10$. Then the value of k will be :

[Chattisgarh NTSE Stage–1 2014]

(A) 10 (B) 13 (C) 16 (D) 9

14. If $\frac{x+a}{b+c} + \frac{x+b}{c+a} + \frac{x+c}{a+b} + 3 = 0$, a > 0, b > 0, c > 0, then the value of x is :

[Delhi NTSE Stage – 1 2014]

(A) $-(a^2 + b^2 + c^2)$ (B) (a + b + c) (C) – (a + b + c) (D) $\sqrt{a+b+c}$

15. If $x = \frac{1}{1+\sqrt{2}}$, then value of $x^2 + 2x + 3$ is : **[Delhi NTSE Stage – 1 2014]**

(A) 3 (B) 0 (C) 4 (D) 1

16. If x + y = 1 then $x^3 + y^3 + 3xy =$ ..... **[Jharkhand NTSE Stage – 1 2014]**

(A) 0 (B) 1 (C) 2 (D) None of these

17. If α and β are the zeroes of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, the value of K is :

[Delhi NTSE Stage – 1 2014]

(A) 8 (B) 6 (C) $\frac{13}{2}$ (D) 4

18. If p–q = – 8 and p.q. = – 12 then the value of $p^3 - q^3$ is : **[MP NTSE Stage – 1 2014]**

(A) 224 (B) – 224 (C) 242 (D) – 242

19. If $2 \pm \sqrt{3}$ are zeros of $x^4 - 6x^3 - 26x^2 + 138x - 35$ then the other zeros are

[MP NTSE Stage – 1 2014]

(A) – 5, –7 (B) 5, – 7 (C) – 5, 7 (D) 5, 7

20. (a + b + c) (ab + bc + ca) – abc is equal to the **[MP NTSE Stage – 1 2014]**

(A) (a + b) (c + b) (c + a) (B) (a – b) (b + c) (c + a)

(C) (a + b) (b – c) (c + a) (D) (a + b) (b + c) (c – a)

21. If $x + \frac{1}{x} = 2$ then $\sqrt{x} + \frac{1}{\sqrt{x}}$ will be – **[UP NTSE Stage – 1 2014]**

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{2} + 1$ (D) 1

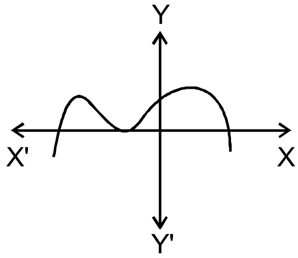
22. If x + y = 8, xy = 15, then $x^2 + y^2$ will be **[UP NTSE Stage – 1 2014]**

(A) 32 (B) 34 (C) 36 (D) 38

23. If x – y = 5, xy = 24 then the value of $x^2 + y^2$ will be **[UP NTSE Stage – 1 2015]**

(A) 23 (B) 73 (C) 65 (D) 74

24. The graph of y = p(x) is given below. The number of zeroes of polynomial p(x), is **[Raj. NTSE Stage–1 2015]**



(A) 3 (B) 2 (C) 1 (D) 0

25. If $\frac{p}{q} + \frac{q}{p} = 2$, what is the value of $\left(\frac{p}{q}\right)^{23} + \left(\frac{q}{p}\right)^{7}$ **[Delhi NTSE Stage – 1 2015]**

(A) 0 (B) 2 (C) –2 (D) none of these

26. If $x^{47}+1$ is divided by $x^2 - 1$,the remainder will be **[Delhi NTSE Stage – 1 2015]**

(A) x –1 (B) x +1 (C) x (D) –x

27. Value of $x\left[\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)-1\right]$ is **[Delhi NTSE Stage – 1 2015]**

(A) 3 (B) 2x (C) 5x (D) 1

28. Simplify the value of $\frac{3.75\times3.75+1.25\times1.25-2\times3.75\times1.25}{3.75\times3.75-1.25\times1.25}$ **[Delhi NTSE Stage – 1 2015]**

(A) 5.0 (B) 0.5 (C) 2.5 (D) 1.5

29. If $p(x)= 2x^3 - 3x^2 + 5x - 4$ is divided by (x – 2), what is remainder? **[Gujarat NTSE Stage – 1 2015]**

(A) 12 (B) 8 (C) 10 (D) –10

30. What is the co–efficient of xy in the expansion of $(x + y)^2$? **[Gujarat NTSE Stage – 1 2015]**

(A) 3 (B) 4 (C) 5 (D) 2

31. Zeroes of which quadratic polynomial are 4 and 3. **[Gujarat NTSE Stage – 1 2015]**

(A) $x^2 + 7x + 12$ (B) $x^2 - 7x + 12$ (C) $x^2 + 7x - 12$ (D) $x^2 - 7x - 12$

32. If α, β be the zeros of the polynomial $2x^2 + 5x + k$ such that $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, than K = ? **[Jharkhand NTSE Stage – 1 2015]**

(A) 3 (B) –3 (C) – 2 (D) 2

33. If $x^2 - 3x + 1 = 0$, then the value of $x^5 + \frac{1}{x^5}$ **[Jharkhand NTSE Stage – 1 2015]**

(A) 87 (B) 123 (C) 135 (D) 201

34. If $\frac{xy}{x+y} = a$, $\frac{xz}{x+z} = b$ and $\frac{yz}{y+z} = c$, where a, b, c are non–zero numbers, then the value of x ? **[Jharkhand NTSE Stage – 1 2015]**

(A) $\frac{2abc}{ab+ac-bc}$ (B) $\frac{2abc}{ac+bc-ab}$ (C) $\frac{abc}{ab+bc+ac}$ (D) $\frac{2abc}{ab+bc-ac}$

35. (ab + bc + ca) can be expressed as .... **[MP NTSE Stage – 1 2015]**

(A) abc (a+b+c) (B) ab (a+c) (C) $abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ (D) $c\left(\frac{1}{a}+\frac{1}{b}\right)$

36. If pqr = 1, then the value of $\left(\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}\right)$ **[Orissa NTSE Stage – 1 2015]**

(A) 0 (B) pq (C) 1 (D) pq

37. If (x+ 2), is a factor of $2x^3 - 5x + K$, then the value of k is **[Raj. NTSE Stage–1 2016]**

(A) 6 (B) – 6 (C) 26 (D) – 26

38. If a + b + c = 0, then the value of $\frac{(a+b)^2}{ab}+\frac{(b+c)^2}{bc}+\frac{(c+a)^2}{ca}$ is **[Raj. NTSE Stage–1 2016]**

(A) 1 (B) 2 (C) 3 (D) –3

39. The value of $\frac{(0.03)^2-(0.01)^2}{0.03-0.01}$ is **[Bihar NTSE Stage–1 2016]**

(A) 0.02 (B) 0.004 (C) 0.4 (D) 0.04

40. The simplified form of the expression given below is **[Delhi NTSE Stage – 1 2016]**

$$\frac{\frac{y^4-x^4}{x(x+y)}-\frac{y^3}{x}}{y^2-xy+x^2}$$

(A) 1 (B) 0 (C) –1 (D) 2

41. If a = $\frac{4xy}{x+y}$, the value of $\frac{a+2x}{a-2x}$ + $\frac{a+2y}{a-2y}$ in most simplified form is **[Delhi NTSE Stage – 1 2016]**

(A) 0 (B) 1 (C) –1 (D) 2

42. If x, y, z are real numbers such that $\sqrt{x-1}+\sqrt{y-2}+\sqrt{z-3}$ = 0 then the values of x, y, z are respectively **[Delhi NTSE Stage – 1 2016]**

(A) 1, 2, 3 (B) 0, 0, 0 (C) 2, 3, 1 (D) 2, 4, 1

43. If x – 2 is a factor of $3x^4 - 2x^3 + 7x^2 - 21x + k$ then the value of K is **[Gujarat NTSE Stage – 1 2016]**

(A) 2 (B) 9 (C) 18 (D) –18

44. If 2x +3y + z = 0 then $8x^3 + 27y^3 + z^3 - 18xyz$ is equal to **[UP NTSE Stage – 1 2017]**

(A) 0 (B) 6 (C) 18 (D) 9

45. If $p = x+\frac{1}{x}$ then the value of $p-\frac{1}{p}$ will be– **[UP NTSE Stage – 1 2017]**

(A) 3x (B) $\frac{3}{x}$ (C) $\frac{x^4+x^2+1}{x^3+x}$ (D) $\frac{x^4+3x^2+1}{x^3+x}$

46. Factors of $\frac{1}{3}c^2 - 2c - 9$ are– **[UP NTSE Stage – 1 2017]**

(A) $\left(\frac{1}{3}c+3\right)(c+3)$ (B) $\left(\frac{1}{3}c-3\right)(c-3)$ (C) $\left(\frac{1}{3}c-3\right)(c+3)$ (D) $\left(c-\frac{1}{3}\right)(3c+1)$

Answer Key

BOARD LEVEL EXERCISE

TYPE (I)

1. $x^2 - x - 12$ **2.** $b - a + 1$ **3.** $a = 0, b = -6$ **4.** Infinite

5. $\frac{c}{a}$

TYPE (II)

7. $x = 1, -\frac{1}{4}$ **8.** $x = -2, \frac{2}{3}$ **9.** $x = -\frac{3}{2}, -\frac{1}{4}$

10. $s = \frac{1}{2}, \sqrt{2}$ **11.** $v = 5\sqrt{3}, \sqrt{3}$ **12.** $t = 0, 5, -3$

13. **(i)** $3x^2 + 8x + 4$ **(ii)** $16x^2 - 42x + 5$ **(iii)** $x^2 + 2\sqrt{3}\,x - 9$ **(iv)** $2\sqrt{5}\,x^2 + 3x - \sqrt{5}$

TYPE (III)

14. $\sqrt{5}, (\sqrt{5} + \sqrt{2}), (\sqrt{5} - \sqrt{2})$ **15.** $x^2 + 5x + 6$, $k = -9$, zeroes are $3, -2, -3$ **16.** $\frac{-1}{\sqrt{2}}, \frac{-4}{3\sqrt{2}}$

TYPE (IV)

17. $a = -1, b = 3$ or $a = 5, b = -3$ and zeroes are $-1, 2, 5$ **18.** $k = -3$, zeroes are $1, -3, 2, \frac{-1}{2}$

PREVIOUS YEAR PROBLEMS

1. (B) **2.** (D) **3.** $a^2 - 2b$

4. quotient = $10x^2 - 3x - 12$, remainder = 0 **5.** $-3/2$

6. **(i)** b/ac **(ii)** b **7.** $7/2$ & $-3/4$ **8.** $x^2 - 24x + 128$

9. $k = 7$ **10.** $x^2 - 6x + 7$ **11.** $x^2 + 4x + 4$ **12.** $4, -4$ & 5

13. $(b^2 - 2ac) / ac$ **15.** $2, -1, -\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A–1. $x^3 + 7$ **A–2.** Max. 101 terms, Min. 1 terms **A–3.** 370.

A–4. 0 **A–7.** 0

Section (B)

B–1. quotient $q(x) = x^2 + x - 3$ and remainder $r(x) = 8$

B–2. $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.

B–3. $2x + 5$ **B–4.** $2x - 3$ **B–5.** $-\sqrt{3}, \sqrt{3}$ and $\frac{-1}{2}$.

Section (C)

C–1. $k\{x^2 - 25\}$ **C–2.** $k\{x^2 - 2x + \sqrt{5}\}$. **C–3.** $\frac{-1}{4}$

C–4. $x^2 - 8x + 12 = 0$, 6 and 2. **C–5.** $k\{x^2 - 6x + 4\}$ **C–6.** -1

C–7. -3 and -3. **C–8.** $k(x^3 - 6x^2 - x + 30)$

Section (D)

D–1. **(i)** 3 **(ii)** 2 **D–5.** Open upwards

OBJECTIVE QUESTIONS

Section (A)

A–1 (B) **A–2.** (B) **A–3.** (B) **A–4.** (B) **A–5.** (C)

Section (B)

B–1. (B) **B–2.** (B) **B–3.** (B) **B–4.** (B) **B–5.** (D)

Section (C)

C–1. (A) **C–2.** (C) **C–3.** (A) **C–4.** (A)

Section (D)

D–1. (C) **D–2.** (A) **D–3.** (B) **D–4.** (A) **D–5.** (A)

D–6. (A) **D–7.** (A)

Exercise-2

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Ans.	A	A	C	A	C	A	B	B	B	A	C	A	B	C	A	A	C

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	D	C	D	A	D	C	C	B	B	C	B	C	C	B	B	B	C	A
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	B	B	B	A	B	B	A	B	C	D	B	D	B	B	C	C	A	C	D	C
Ques.	41	42	43	44	45	46														
Ans.	D	A	D	A	C	C														