MATHEMATICS

Class-X

Topic-3

LINEAR EQUATION IN TWO VARIABLES

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LINEAR EQUATION IN TWO VARIABLES

(A) INTRODUCTION TO LINEAR EQUATION IN TWO VARIABLE AND METHODS OF SOLVING

An equation of the form Ax + By + C = 0 is called a linear equation.
Where A is called coefficient of x, B is called coefficient of y and C is the constant term (free from x & y)
A, B, C, $\in$ R [$\in \rightarrow$ belongs to, R $\rightarrow$ Real No.] But A and B can not be simultaneously zero.

If $A \neq 0$, B = 0 equation will be of the form Ax + C = 0. [Line || to Y-axis]

If A = 0, $B \neq 0$, equation will be of the form By + C = 0. [Line || to X-axis]

If $A \neq 0$, $B \neq 0$, C = 0 equation will be of the form A x + By = 0. [Line passing through origin]

If $A \neq 0$, $B \neq 0$, $C \neq 0$ equation will be of the form A x + By + C = 0.

It is called a linear equation in two variable because the two unknowns (x & y) occurs only in the first power, and the product of two unknown quantities does not occur.

Since it involves two variables therefore a single equation will have infinite set of solution i.e. indeterminate solution. So we require a pair of equation i.e. simultaneous equations.

Standard form of linear equation :

$a_1x + b_1y + c_1 = 0$ ....(i)

$a_2x + b_2y + c_2 = 0$ ....(ii)

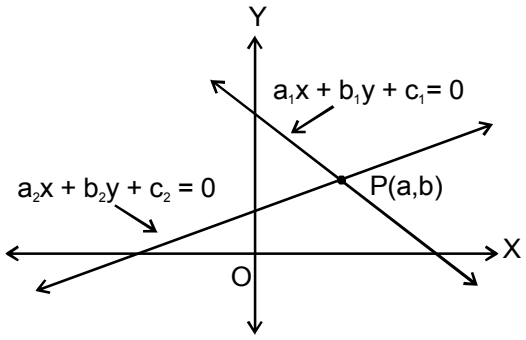
For solving such equations we have four methods.

(a) Graphical method
(b) Elimination by substitution
(c) Elimination by equating the coefficients
(d) Elimination by cross multiplication.

(a) Graphical solution of linear equations in two variables

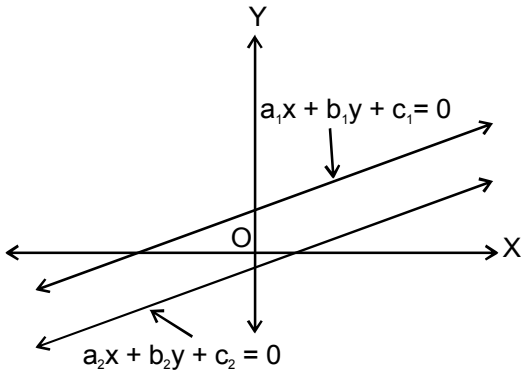
Let equations of two lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.
Find atleast two solutions for each of the two equations by assuming value of one variable and then calculating the other variable.
Plot these points of both equations in the same co - ordinate axes to get two straight line, one for each equation.
While plotting the graph , the following three cases arises:

Case I : The two lines intersect at a point P



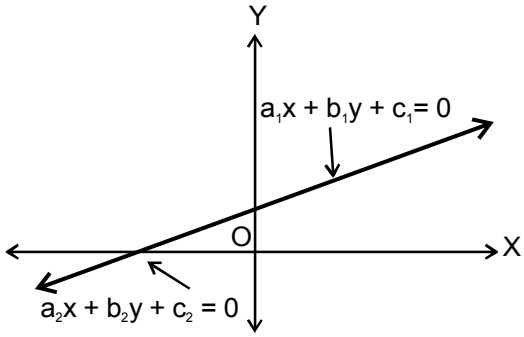
Then the two equations have unique solution given by x=a and y=b. The equations are said to be consistent.

Case II : The two lines are parallel to each other.



Then the two equations have no solutions and are said to be inconsistent.

Case III : The two lines are coincident



Then the two equations have infinitely many solutions and are said to be consistent.

(b) Elimination By Substitution

The procedure is as follows.

Step 1: From first equation find x in terms of y.

Step 2: Substitute the value of x obtained as above in the second equation

Step 3: Second equation reduces to an equation in single variable y. Solve for y.

Step 4: Using this value of y in any of the given equations, find x.

(c) Elimination by Equating the Coefficients

In this method, the coefficients of y in both the equations were same and hence we could eliminate y by subtraction. If the coefficients are different, then we multiply one or both equations by a suitable number and make the coefficients equal. Furthermore, we may eliminate either x or y. If we eliminate x, then we get y first.

(d) Elimination by Cross Multiplication

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ $\quad \left[\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$

$b_1 \quad c_1 \quad a_1 \quad b_1$

$b_2 \quad c_2 \quad a_2 \quad b_2$

[Write the coefficient in this manner]

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1} \Rightarrow \frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$\Rightarrow \quad x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

Also $\quad \dfrac{y}{a_2c_1 - a_1c_2} = \dfrac{1}{a_1b_2 - a_2b_1}$

$\therefore \quad y = \dfrac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Solved Examples

Example 1.

Solve the following system of linear equations graphically : $x - y = 1$ and $2x + y = 8$. Shade the area bounded by these two lines and y-axis. Also, determine this area.

Sol. **(i)** $x - y = 1 \quad ; \quad x = y + 1$

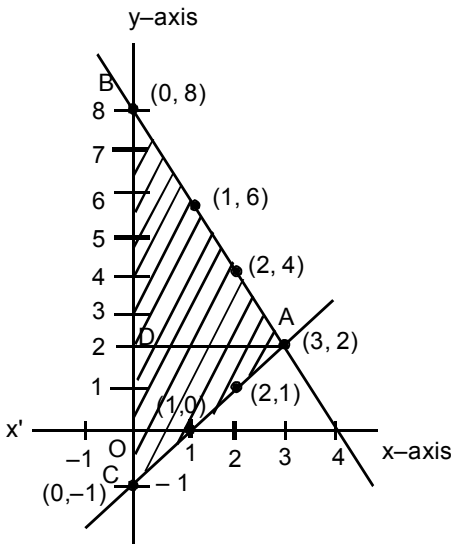
x	0	1	2
y	– 1	0	1

(ii) $2x + y = 8 \quad ; \quad y = 8 - 2x$

x	0	1	2
y	8	6	4

Solution is $x = 3$ and $y = 2$

Area of $\Delta ABC = \dfrac{1}{2} \times BC \times AD = \dfrac{1}{2} \times 9 \times 3 = 13.5$ Sq. unit.



Example 2.

Show graphically that the system of equations $2x + 4y = 10$ and $3x + 6y = 12$ has no solution.

Sol. Graph of $2x + 4y = 10$

We have,

$2x + 4y = 10 \qquad 4y = 10 - 2x \Rightarrow y = \dfrac{5 - x}{2}$

When $x = 1$, we have : $y = \dfrac{5 - 1}{2} = 2$

When $x = 3$, we have $y = \dfrac{5 - 3}{2} = 1$

Thus, we have the following table

x	1	3
y	2	1

Plot the points A (1,2) and B (3,1) on a graph paper. Join A and B and extend it on both sides to obtain the graph of 2x + 4y = 10 as shown in figure.

Graph of 3x + 6y = 12

We have,

$3x + 6y = 12 \Rightarrow 6y = 12 - 3x \Rightarrow y = \frac{4 - x}{2}$

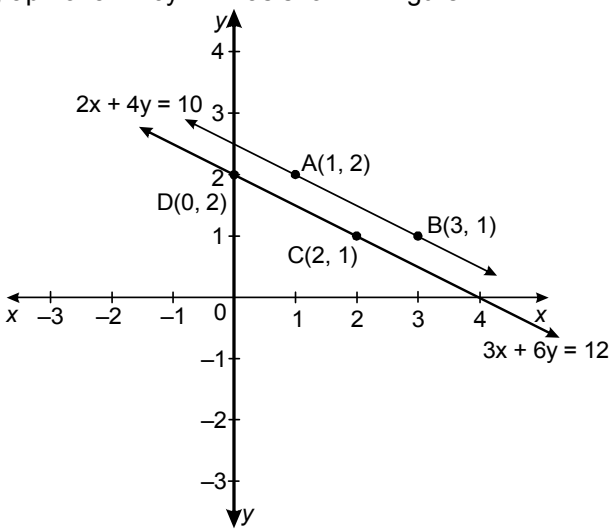
When x = 2, we have : $y = \frac{4-2}{2} = 1$

When x = 0, we have : $y = \frac{4-0}{2} = 2$

Thus, we have the following table

x	2	0
y	1	2

Plot the point C (2,1) and D (0,2) on the same graph paper. Join C and D and extend it on both sides to obtain the graph of 3x + 6y = 12 as shown in figure.



We find the lines represented by given equations are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

Example 3.

Solve using substitution method : x + 4y = 14 & 7x – 3y = 5.

Sol. x + 4y = 14 .....(i)

7x – 3y = 5 .....(ii)

From equation (i), x = 14 – 4y .....(iii)

Substitute the value of x in equation (ii)

$\Rightarrow$ 7 (14 – 4y) – 3y = 5 $\Rightarrow$ 98 – 28y – 3y = 5 $\Rightarrow$ 98 – 31y = 5

$\Rightarrow$ 93 = 31y $\Rightarrow$ $y = \frac{93}{31} \Rightarrow y = 3.$

Now substitute value of y in equation (ii)

$\Rightarrow$ 7x – 3 (3) = 5 $\Rightarrow$ 7x – 3 (3) = 5

$\Rightarrow$ 7x = 14 $\Rightarrow$ $x = \frac{14}{7} = 2$

So, solution is x = 2 and y = 3.

Example 4.

Solve using equating the coefficients: $9x - 4y = 8$ & $13x + 7y = 101$.

Sol. $9x - 4y = 8$ .....(i)

$13x + 7y = 101$ .....(ii)

Multiply equation (i) by 7 and equation (ii) by 4, we get

$$\begin{array}{rlll} & 63x & -28y & = 56 \\ \text{Add} & 52x & +28y & = 404 \\ \hline & 115x & & = 460 \end{array} \quad \Rightarrow \quad x = \frac{460}{115} \Rightarrow x = 4.$$

Substitute $x = 4$ in equation (i)

$9(4) - 4y = 8$

$$\Rightarrow \quad 36 - 8 = 4y \quad \Rightarrow \quad 28 = 4y \quad \Rightarrow \quad y = \frac{28}{4} = 7$$

So, solution is $x = 4$ and $y = 7$.

Example 5.

Solve using equating the coefficients: $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0$.

Sol. $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$

$b^2x - a^2y + a^2b + ab^2 = 0$ ....(i)

$bx - ay + 2ab = 0$ ....(ii)

Multiply equation (ii) by b

$$\begin{array}{rlll} & b^2x & -a^2y & +a^2b + ab^2 = 0 \\ - & b^2x & -aby & +2ab^2 = 0 \\ \hline & -a^2y & +aby & +a^2b - ab^2 = 0 \end{array}$$

$ay(-a + b) = -a^2b + ab^2$

$ay(-a + b) = ab(b - a)$

$ay = ab$

$y = b$

Substitute $y = b$ in equation (i)

$b^2x - a^2b + a^2b + ab^2 = 0$

$b^2x = -ab^2$

$x = -a$

So, $x = -a$ and $y = b$.

Example 6.

Solve using equating the coefficients:

$\frac{42}{3x+4y} + \frac{52}{4y-3x} = 5$ and $\frac{7}{3x+4y} + \frac{13}{4y-3x} = 1$ where $3x + 4 \neq 0$, $4y - 3x \neq 0$

Sol. Let $\frac{1}{3x+4y} = a$ and $\frac{1}{4y-3x} = b$

Then, $42a + 52b = 5$ ....(i)

$7a + 13b = 1$ ....(ii)

Multiplying (ii) by 6 we get

$42a + 78b = 6$ ....(iii)

(iii) – (i) we get

$26b = 1 \Rightarrow b = \frac{1}{26}$

By putting $b = \frac{1}{26}$ in (ii) we get

$a = \frac{1}{14}$

$a = \frac{1}{14} = \frac{1}{3x+4y} \Rightarrow 3x + 4y = 14$ ...(iv)

$b = \frac{1}{26} = \frac{1}{4y-3x} \Rightarrow 4y-3x = 26$ ...(v)

Adding (iv) & (v) we get

$8y = 40$

$y = 5$

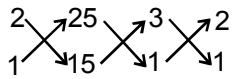
Similarly $x = -2$.

Example 7.

Solve using cross multiplication method : $3x + 2y + 25 = 0$ & $x + y + 15 = 0$.

Sol. Here, $a_1 = 3, b_1 = 2, c_1 = 25$

$a_2 = 1, b_2 = 1, c_2 = 15$

$\therefore$ 

$$\frac{x}{2\times15-25\times1} = \frac{y}{25\times1-15\times3} = \frac{1}{3\times1-2\times1}$$

$$\frac{x}{30-25} = \frac{y}{25-45} = \frac{1}{3-2}$$

$$\frac{x}{5} = \frac{y}{-20} = \frac{1}{1}$$

$\frac{x}{5} = 1, \quad \frac{y}{-20} = \frac{1}{1}$

$x = 5, y = -20$

So, solution is $x = 5$ and $y = -20$.

Check Your Level

1. Solve graphically: $\begin{cases} 3x + 2y = 12 \\ 5x - 3y = 1 \end{cases}$

2. Ram is walking along line $y = 3x$ and Shyam is walking along line $y = 3x + 5$. Represent the situation graphically. Will they meet?

3. Solve using substitution :

(i) $3x + 2y = 13;\ 5x - 3y = 9$

(ii) Solve $2x + 3y = 22, 3x - 2y = 7$

4. Solve using elimination by equating the coefficients :

(i) $11x - 6y = 28, 2x + 3y = 1$

(ii) $\frac{6}{x} - \frac{5}{y} = 22, \frac{5}{x} + \frac{4}{y} = 2$

5. Solve using cross multiplication method :

(i) $4x + 5y + 9 = 0$

$3x + 4y + 8 = 0$

(ii) $8x - 7y = 19$

$10x - 9y = 23$

Answers

1. x = 2, y = 3. **2.** No **3.** **(i)** x = 3 and y = 2 **(ii)** x = 5 and y = 4

4. **(i)** x = 2 , y = –1 **(ii)** $x = \frac{1}{2}, y = -\frac{1}{2}$

5. **(i)** x = 4 and y = –5 **(ii)** x = 5, y = 3

(B) CONDITION FOR SOLVABILITY

(a) Unique Solution

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if the denominator $a_1b_2 - a_2b_1 \neq 0$ then the given system of equations have **unique solution** (i.e. only one solution) and solutions are said to be **consistent**.

$\therefore \quad a_1b_2 - a_2b_1 \neq 0 \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(b) No Solution

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if the denominator $a_1b_2 - a_2b_1 = 0$ then the given system of equations have **no solution** and solutions are said to be **inconsistent**.

i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(c) Many Solution (Infinite Solutions)

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then system of equations has **many solution** and solutions are said to be **consistent**.

Solved Examples

Example 8.

Find the value of 'P' for which the given system of equation has only one solution (i.e. unique solution).

Px – y = 2 & 6x – 2y = 3.

Sol. $a_1 = P, \ b_1 = -1, \ c_1 = -2$

$a_2 = 6, \ b_2 = -2, \ c_2 = -3$

Condition for unique solution is $\neq$

$\Rightarrow \quad \frac{P}{6} \neq \frac{-1}{-2} \quad \Rightarrow \quad P \neq \frac{6}{2} \quad \Rightarrow \quad P \neq 3$

$\therefore$ P can have all real values except 3.

Example 9.

Find the value of k for which the system of linear equation kx + 4y = k – 4 & 16x + ky = k has infinite solution.

Sol. $a_1 = k, \ b_1 = 4, \ c_1 = -(k-4)$

$a_2 = 16, \ b_2 = k, \ c_2 = -k$

Here condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{k}{16} = \frac{4}{k} = \frac{(k-4)}{(k)}.$

$\Rightarrow \quad \frac{k}{16} = \frac{4}{k}$ and $\frac{4}{k} = \frac{k-4}{k}$

$\Rightarrow \quad k^2 = 64 \quad \Rightarrow \quad 4k = k^2 - 4k$

$\Rightarrow \quad k = \pm 8 \quad \Rightarrow \quad k^2 - 8k = 0$

$\Rightarrow \quad k(k - 8) = 0$

$\Rightarrow \quad k = 0$ or $k = 8$.

But $k = 0$ is not possible other wise equation will be one variable.

$\therefore$ $k = 8$ is correct value for infinite solution.

Example 10.

Determine the value of k so that the following linear equations has no solution

$(3k + 1)x + 3y - 2 = 0$ & $(k^2 + 1)x + (k - 2)y - 5 = 0$.

Sol. Here $a_1 = 3k + 1$, $b_1 = 3$ and $c_1 = -2$

$a_2 = k^2 + 1$, $b_2 = k - 2$ and $c_2 = -5$

For no solution, condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3k+1}{k^2+1} = \frac{3}{k-2}$

$\Rightarrow \quad \frac{3k+1}{k^2+1} = \frac{3}{k-2}$ and $\frac{3}{k-2} \neq \frac{2}{5}$

Now, $\frac{3k+1}{k^2+1} = \frac{3}{k-2}$

$\Rightarrow \quad (3k + 1)(k - 2) = 3(k^2 + 1) \quad \Rightarrow \quad 3k^2 - 5k - 2 = 3k^2 + 3$

$\Rightarrow \quad -5k - 2 = 3 \quad \Rightarrow \quad -5k = 5$

$\Rightarrow \quad k = -1$

Clearly, $\frac{3}{k-2}$ for $k = -1$. $\frac{3}{k-2} \neq \frac{5}{2}$ for $k = -1$

Hence, the given system of equations will have no solution for $k = -1$.

Check Your Level

1. Which of the following pairs of equations have exactly unique solution, no solution and infinitely many solutions?

(a) $5x - y + 6 = 0$, $4x + 7y + 1 = 0$ **(b)** $9x + 7y = 3$, $7x + 9y = 3$

(c) $2x - 5y = 6$, $4x - 10y = 3$ **(d)** $x + 4y - 4 = 0$, $3x + 12y = 12$

(e) $x + y = 10$, $x - y = 10$

2. Find the value of p if the equations $6x + py = 5$, $3x + 4y = 2$ has

(i) unique solution **(ii)** no solution.

3. Find value of k if $4x + 2y = k$ and $2x + 3y = 6$ have unique solution.

4. Find value of k if $kx - y = 5$ and $x + 3y = 8$ have infinitely many solutions.

5. Find value of k if $3x + ky = 2$ and $5x + y = 7$ have no solution.

Answers

1. **(a)** Unique **(b)** Unique **(c)** No solution **(d)** Infinite solution **(e)** Unique

2. **(i)** $p \neq 8$ **(ii)** $p = 8$ **3.** Any real value

4. No real value **5.** $k = 3/5$

(C) WORD PROBLEMS

For solving daily – life problems with the help of simultaneous linear equation in two variables or equations reducible to them proceed as :-

(i) Represent the unknown quantities by same variable x and y, which are to be determined.

(ii) Find the conditions given in the problem and translate the verbal conditions into a pair of simultaneous linear equation.

(iii) Solve these equations & obtain the required quantities with appropriate units.

Type of Problems :

(i) Determining two numbers when the relation between them is given.

(ii) Problems regarding fractions, digits of a number, ages of persons.

(iii) Problems regarding current of a river, regarding time & distance.

(iv) Problems regarding mensuration and geometry.

(v) Problems regarding time & work.

(vi) Problems regarding mixtures, cost of articles, profit & loss, discount etc.

Solved Examples

Example 11.

Find two numbers such that the sum of twice the first and thrice the second is 89 and four times the first exceeds five times the second by 13.

Sol. Let the two numbers be x and y.
Then, equation formed are

$2x + 3y = 89$ ....(i)

$4x - 5y = 13$ ....(ii)

On solving equation (i) & (ii) we get

$x = 22$ and $y = 15$

Hence, required numbers are 22 & 15.

Example 12.

The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator is eight times the numerator, find the fraction.

Sol. Let the numerator and denominator of a fraction be x and y
Then, equation formed are

$y - x = 4$ ....(i)

$y + 1 = 8(x - 2)$ ....(ii)

On solving equation (i) & (ii) we get

$x = 3$ and $y = 7$

Hence, fraction is $\frac{3}{7}$.

Example 13.

A number consists of two digits, the sum of the digits being 12. If 18 is substracted from the number, the digits are reversed. Find the number.

Sol. Let the two digits number be $10y + x$.
Then, equations formed are $10y + x - 18 = 10x + y$

$\Rightarrow$ $y - x = 2$ ...(i)
and $x + y = 12$ ...(ii)
On solving equation (i) & (ii) we get
$x = 5$ and $y = 7$
Hence number is 75.

Example 14.
The sum of a two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number

Sol. Let unit digit be x ad ten's digit be y. Then number = $10y + x$.
According to problem $(10y + x) + (10x + y) = 165$
$\Rightarrow$ $x + y = 15$ ... (i)
and $x - y = 3$ ... (ii)
or $y - x = 3$ ... (iii)
On solving equation (i) and (ii) $x = 9$ and $y = 6$
$\therefore$ The number will be 69.
On solving equation (i) and (iii) $x = 6$ and $y = 9$
$\therefore$ The number will be 96.

Example 15.
Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son . Find their present ages.

Sol. Let man's present age be 'x' yrs & son's present age be 'y' yrs.
According to problem
$x + 6 = 3(y + 6)$
$x - 3y = 12$ ... (i)
and $x - 3 = 9(y - 3)$
$x - 9y = -24$ ... (ii)
On solving equation (i) & (ii) $x = 30$ and $y = 6$.
So, the present age of man = 30 years and present age of son = 6 years.

Example 16.
A boat goes 12 km upstream and 40 km downstream in 8 hrs. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

Sol. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.
Speed of boat in downstream = (x + y) km/hr
and speed of boat in upstream = (x – y) km/hr.

Time taken to cover 12 km upstream = $\frac{12}{x-y}$ hrs.

Time taken to cover 40 km downstream = $\frac{40}{x+y}$ hrs.

But, total time taken = 8 hr.

$\therefore$ $\frac{12}{x-y} + \frac{40}{x+y} = 8$ ..... (i)

Time taken to cover 16 km upstream = $\frac{16}{x-y}$ hrs.

Time taken to cover 32 km downstream = $\frac{32}{x+y}$ hrs.

Total time taken = 8 hrs

$\therefore$ $\frac{16}{x-y} + \frac{32}{x+y} = 8$ ..... (ii)

Solving equation (i) & (ii) $x = 6$ and $y = 2$.
Hence, speed of boat in still water = 6 km/hr and speed of stream = 2 km/hr.

Example 17.

Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hr, if he travels 160 km by train and the rest by car. He takes 12 minutes more, if he travels 240 km by train and the rest by car. Find the speed of train and the car.

Sol. Let the speed of train be x km/hr & car be y km/hr respectively.

According to problem

$\frac{160}{x} + \frac{600}{y} = 8$ ..... (i) $\frac{240}{x} + \frac{520}{y} = \frac{41}{5}$ .....(ii)

Solving equation (i) & (ii) x = 80 and y = 100.

Hence, speed of train = 80 km/hr and speed of car = 100 km/hr.

Example 18.

Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hrs and if they go in opposite directions, they meet in $\frac{9}{7}$ hrs . Find their speeds.

Sol. Let the speeds of the cars starting from A and B be x km/hr and y km/hr respectively.

According to problem

$9x - 90 = 9y$ .....(i)

and $\frac{9}{7}x + \frac{9}{7}y = 90$ .....(ii)

Solving equation (i) & (ii) x = 40 & y = 30.

Hence, speed of car starting from point A = 40 km/hr & speed of car starting from point B = 30 km/hr.

Example 19.

In a cyclic quadrilateral ABCD, $\angle A = (2x + 11)^0$, $\angle B = (y + 12)^0$, $\angle C = (3y + 6)^0$ and $\angle D = (5x - 25)^0$, find the angles of the quadrilateral.

Sol. According to problem

$(2x + 11)^0 + (3y + 6)^0 = 180^0$ .....(i)

and $(y + 12)^0 + (5x - 25)^0 = 180^0$ .....(ii)

Solving equation (i) and (ii) x = 32° and y = 33°

$\therefore$ $\angle A = 75^0$, $\angle B = 45^0$, $\angle C = 105^0$, $\angle D = 135^0$.

Example 20.

A vessel contain's mixture of 24 ℓ milk and 6 ℓ water and a second vessel contains a mixture of 15 ℓ milk & 10 ℓ water. How much mixture of milk and water should be taken from the first and the second vessel separately and kept in a third vessel so that the third vessel may contain a mixture of 25 ℓ milk and 10 ℓ water ?

Sol. Let x ℓ of mixture be taken from 1st vessel & y ℓ of the mixture be taken from 2nd vessel and kept in 3rd vessel so that (x + y) ℓ of the mixture in third vessel may contain 25 ℓ of milk & 10 ℓ of water.

A mixture of x ℓ form 1st vessel contains = $\frac{4}{5}x$ litre of milk & $\frac{x}{5}$ litre of water and a mixture of y ℓ from 2nd vessel contains $\frac{3y}{5}$ ℓ of milk & $\frac{2y}{5}$ ℓ of water.

$\therefore$ $\frac{4}{5}x + \frac{3}{5}y = 25$ ....(i)

$\frac{x}{5} + \frac{2}{5}y = 10$ ....(ii)

Solving equation (i) & (ii) x = 20 litres and y = 15 litres.

Example 21.

A lady has 25 p and 50 p coins in her purse. If in all she has 40 coins totalling Rs. 12.50, find the number of coins of each type she has.

Sol. Let the lady has x coins of 25 p and y coins of 50 p.
Then, according to problem

$x + y = 40$ .....(i)

and $25x + 50y = 1250$ .....(ii)

Solving equation (i) and (ii)
x = 30 (25 p coins) & y = 10 (50 p coins).

Example 22.

Students of a class are made to stand in rows. If one student is extra in a row , there would be 2 rows less. If one student is less in row, there would be 3 rows more. Find the total number of students in the class.

Sol. Let x be the original number of rows & y be the original number of students in each row.

$\therefore$ Total number of students = xy.

According to problem

$(y + 1)(x - 2) = xy$ .....(i)

and $(y - 1)(x + 3) = xy$ .....(ii)

Solving (i) & (ii) to get

$x = 12$ & $y = 5$

$\therefore$ Total no. of students = 60.

Example 23.

A man started his job with a certain monthly salary and earned a fixed increment every year. If his salary was Rs.4500 after 5 years. of service and Rs. 5550 after 12 years of service, what was his starting salary and what was his annual increment.

Sol. Let his initial monthly salary be Rs x and annual increment be Rs y.
Then, Acc. to problem

$x + 5y = 4500$ ..... (i)

$x + 12y = 5550$ .....(ii)

Solving these two equations, we get x = Rs 3750, y = Rs 150.
So, starting salary = Rs. 3750 and annual increment = Rs. 150.

Example 24.

A dealer sold a VCR and a TV for Rs. 38560 making a profit of 12% on VCR and 15% on TV. By Selling them for Rs. 38620, he would have realised a profit of 15% on VCR and 12% on TV. Find the cost price of each.

Sol. Let C.P. of VCR be Rs x & C.P. of T.V. be Rs y.
According to problem

$$\frac{112}{100}x + \frac{115}{100}y = 38560 \quad \text{..... (i)}$$

and $$\frac{115}{100}x + \frac{112}{100}y = 38620 \quad \text{..... (ii)}$$

Solving equation (i) and (ii) x = Rs. 18000 & y = Rs. 16000.
So, C.P. of VCR = Rs. 18000 and C.P. of T.V. = Rs. 16000.

Example 25.

$2x + 3y = 10$. How many integral values of x and y are possible ?

(A) 1 (B) 3 (C) 4 (D) 2

Sol. $2x + 3y = 10$

x	5	2	-1
y	0	2	4

only two values can satisfy.

Check Your Level

1. The cost of 5 pencils and 4 pens together is Rs.35 while the cost of 2 pencils and 3 pens together is Rs.21. What is the price of a pencil and a pen?

2. In a farm there are some cows and hens. If the total number of heads is 35 and the total number of legs is 98, how many cows and hens are there?

3. Find the fraction which becomes $\frac{2}{3}$ when numerator and denominator are increased by 1 and $\frac{1}{2}$ when numerator and denominator are diminished by 1.

4. A certain number of two digits is four times the sum of its digits. If 9 is added to the number the digits in the number are reversed. Find the number.

5. Six years ago a man was three times as old as his son. In six years, he will be twice as old as his son. Find their present ages.

6. If the length of a rectangle is increased by 8 metres and the breadth by 3 metres its area will be increased by 200 square metres. Its length is increased by 3 metres and breadth increased by 8 metres its area will be increased by 255 square metres. Find the length and breadth of the rectangle.

7. In a pen (a small enclosure) there are rabbits and pheasants (game birds). They have between them 35 heads and 98 feet. How many rabbits are there?

Answers

1. Rs.3 per pencil , Rs.5 per pen **2.** 14 cow and 21 hens **3.** $\frac{3}{5}$

4. 12 **5.** 42 years and 18 years

6. Length of rectangle = 24 m, Breadth of rectangle = 13 m

7. 14 rabbits and 21 pheasants

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : **[01 MARK EACH]**

1. Do the following pair of linear equations have no solution ? Justify your answer.

(i) $2x + 4y = 3$ $\quad 12y + 6x = 6$

(ii) $x = 2y$ $\quad y = 2x$

(iii) $3x + y - 3 = 0$ $\quad 2x + \frac{2}{3}y = 2$

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$

(ii) $-2x - 3y = 1$ and $6y + 4x = -2$

(iii) $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \quad = 0$

3. Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$ and $4y + 3x = 12$

(ii) $\frac{3}{5}x - y = \frac{1}{6}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$

(iii) $2ax + by = a$ and $4ax + 2by - 2a = 0$; $a, b \neq 0$

(iv) $x + 3y = 11$ and $2(2x + 6y) = 22$

4. For all real values of c, the pair of equations
$x - 2y = 8$
$5x - 10y = c$
have a unique solution. Justify whether it is true or false.

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : **[02 MARKS EACH]**

5. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ?

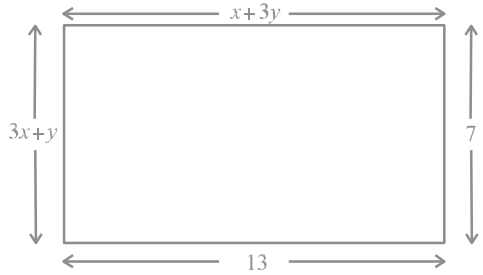
6. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find the value of k.

7. Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions.

8. For which value(s) of k will the pair of equations $kx + 3y = k - 3$ $12x + ky = k$ have no solution ?

9. If $2x + y = 23$ and $4x - y = 19$, find the values of $5y - 2x$ and $\frac{y}{x} - 2$.

10. Find the values of x and y in the following rectangle. **(see Figure).**



TYPE (III) : LONG ANSWER TYPE QUESTIONS: [03 MARK EACH]

11. In a competitive examination, one mark is awarded for each correct answer while $\frac{1}{2}$ mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions she answer correctly?

12. If x + 1 is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that 2a – 3b = 4

13. Graphically, solve the following pair of equations :
2x + y = 6
2x – y + 2 = 0
Find the ratio of the areas of the two triangles formed by the lines representing these equations with x-axis and the lines with the y-axis.

14. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

15. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

16. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs Rs 2530. Also, one reserved first class ticket from A to B costs Rs 3810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS [04 MARK EACH]

17. Draw the graphs of the equations x = 3, x = 5 and 2x – y – 4 = 0. Also find the area of the quadrilateral formed by the lines and the x–axis.

18. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.

19. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received Rs 20 more as annual interest. How much money did she invest in each scheme ?

20. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of Rs 2 for 3 bananas and the second lot at the rate of Re 1 per banana, and got a total of Rs 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of Rs 4 for 5 bananas, his total collection would have been Rs 460. Find the total number of bananas he had.

Previous Year Problems

1. Solve 37x + 43y = 123, 43x + 37y = 117
OR
Solve $x + \frac{6}{y} = 6$, $3x - \frac{8}{y} = 5$. **[2 MARKS/CBSE 10TH BOARD: 2013, 2015]**

2. A and B are friends and their ages differ by a year. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B.
OR
Solve the following pair of equations :

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} + \frac{3}{y-2} = 1$$

[3 MARKS/CBSE 10TH BOARD: 2013]

3. Form the pair of linear equations in the following problem, and find their solutions graphically. 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

[4 MARKS/CBSE 10TH BOARD: 2013]

4. If the pair of linear equations $2x + 3y = 7$ and $2\alpha x + (\alpha + \beta) y = 28$ has infinitely many solutions, then the values of α and β are : **[1 MARK/CBSE 10TH BOARD: 2014]**

(A) 3 & 5 (B) 4 & 8 (C) 4 & 7 (D) 4 & 5

5. The lines representing the linear equations $2x - y = 3$ and $4x - y = 5$ **[1 MARK/CBSE 10TH BOARD: 2014]**

(A) Intersect at a point (B) Are parallel
(C) Are coincident (D) Intersect at exactly two points

6. If the pair of linear equations $10x + 5y - (k - 5) = 0$ and $20x + 10y - k = 0$ have infinitely many solutions, then the value of k is **[1 MARK/CBSE 10TH BOARD: 2014]**

(A) 2 (B) 11 (C) 10 (D) 8

7. 2 tables and 3 chairs together cost Rs. 3500 whereas 3 tables and 2 chairs together cost Rs. 4000. Find the cost of a table and a chair. **[2 MARKS/CBSE 10TH BOARD: 2014]**

8. Solve for x and y

$$4x + \frac{y}{3} = \frac{8}{3}$$

$$\frac{x}{2} + \frac{3y}{4} = -\frac{5}{2}$$

OR

The sum of the numerator and the denominator of a fraction is 8. If 3 is added to both the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction.

[3 MARKS/CBSE 10TH BOARD: 2014]

9. Solve the following system of equations graphically and find the vertices of the triangle formed by these lines and the x-axis.

$4x - 3y + 4 = 0$, $4x + 3y - 20 = 0$ **[4 MARKS/CBSE 10TH BOARD: 2014]**

10. If $x = a$, $y = b$ is the solution of the equations $x + y = 50$ and $4x + 5y = 225$, then the values of a and b are respectively . **[1 MARK/CBSE 10TH BOARD: 2015]**

(A) 10 & 40 (B) 25 & 25 (C) 23 & 27 (D) 20 & 30

11. For what value of 'k' will the following pair of linear equations have infinitely many solution.

$kx + 3y = k - 3$

$12x + ky = k$ **[2 MARKS/CBSE 10TH BOARD: 2015]**

12. Solve the following pair of equations :

$$\frac{5}{x+y} - \frac{2}{x-y} = -1$$

$$\frac{15}{x+y} - \frac{7}{x-y} = 10$$

OR

Ram travels 760 km to his home, partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by the car. He takes 12 minutes more if he travels 240 km by train and the rest by car. Find the speed of the train and the car separately.

[3 MARKS/CBSE 10TH BOARD: 2015]

13. The value of k for which the pair of linear equations $4x + 6y - 1 = 0$ and $2x + ky - 7 = 0$, represents parallel lines is **[1 MARK/CBSE 10TH BOARD: 2016]**

(A) 3 (B) – 3 (C) 2 (D) – 2

14. If the pair of linear equations

$3x + 2y = 1$

$(2k + 1)x + (k + 2)y = k - 1$

Has infinitely many solution, then the value of k is **[1 MARK/CBSE 10TH BOARD: 2016]**

(A) 2 (B) 4 (C) 3 (D) 5

15. Solve for x and y $\left.\begin{array}{l}\frac{5}{x}+\frac{1}{y}=2\\ \frac{6}{x}-\frac{3}{y}=1\end{array}\right\} x \neq 0, y \neq 0$ **[2 MARKS/CBSE 10TH BOARD: 2016]**

16. A and B each have certain number of oranges. A says to B, "If you give me 10 of your oranges, I will have twice the number of oranges left with you". B replies, "If you give me 10 of your oranges, I will have the same number of oranges as left with you". Find the number of oranges with A and B separately. **[2 MARKS/CBSE 10TH BOARD: 2016]**

17. A two-digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the number.

[3 MARKS/CBSE 10TH BOARD: 2016]

18. Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils. **[2 MARKS/CBSE 10TH BOARD: 2017]**

19. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars ?

OR

Solve the following pair of equations :

$$\frac{10}{x+y}+\frac{2}{x-y}=4$$

$$\frac{15}{x+y}+\frac{5}{x-y}=-2$$

[4 MARKS/CBSE 10TH BOARD: 2017]

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction to Linear equation in two variables and method of solving

A-1. If (2, – 3) is one of the solution of the equation $5x-ay=2$ then find the value of a.

A-2. Solve the linear equation by substitution method :

(i) $x + 2y = -1$ and $2x + 3y = 12$ **(ii)** $\frac{x}{a} + \frac{y}{b} = 2$ and $ax - by = a^2 - b^2$

A-3. Solve the linear equation by elimination method :

(i) $0.2x + 0.3y - 0.11 = 0$ and $0.7x - 0.5y + 0.08 = 0$.

(ii) $\frac{x}{3} + \frac{y}{12} = \frac{7}{2}$ and $\frac{x}{6} - \frac{y}{8} = \frac{6}{8}$.

A-4. Solve the linear equation by cross multiplication method.

(i) $2x + 3y = 9$ and $3x + 4y = 5$ **(ii)** $mx - ny = m^2 + n^2$ and $x - y = 2n$

A-5. Solve each of the following pair of equations

(i) $\frac{7}{3^x} - \frac{6}{2^y} = 15$ and $\frac{8}{3^x} = \frac{9}{2^y}$.

(ii) $\frac{1}{3x} + \frac{1}{5y} = 1$ and $\frac{1}{5x} + \frac{1}{3y} = 1\frac{2}{15}$.

(iii) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ and $(a + b)(x + y) = a^2 + b^2$

(iv) $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$ and $x + y = 2ab$

A-6. Solve graphically and find the points where the given lines meets the y – axis :
$2x + y - 11 = 0$ and $x - y - 1 = 0$.

A-7. Use single graph paper & draw the graph of the following equations. Obtain the vertices of the triangles so obtained : $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

A-8. Draw the graph of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Calculate, the area bounded by these lines and x – axis.

Section (B) : Condition for solvability

B-1. Find the number of solutions of the following pair of linear equations :
$x + 2y - 8 = 0$ and $2x + 4y = 16$.

B-2. Find the value of k for which the given system of equations has a unique solution.
$3x + 5y = 12$ and $4x - 7y = k$.

B-3. Find the value of k for which the following system of linear equation becomes infinitely many solution or represent the coincident lines $6x + 3y = k - 3$ and $2kx + 6y = k$.

B-4. Find the value of k or which the following systems of equations be Inconsistent.
$2x + ky + k + 2 = 0$ and $kx + 8y + 3k = 0$.

B-5. Find the value of m for which the pair of linear equations.
$2x + 3y - 7 = 0$ and $(m - 1)x + (m + 1)y = (3m - 1)$ has infinitely many solutions.

Section (C) : Word problems

C-1. In a cyclic quadrilateral ABCD, $\angle A = (2x + 4)^\circ$, $\angle B = (y + 3)^\circ$, $\angle C = (2y + 10)^\circ$ and $\angle D = (4x - 5)^\circ$ then find out the angles of quadrilateral.

C-2. The sum of the numerator and the denominator of fraction is 4 more than twice the numerator. If 3 is added to each of the numerator and denominator, their ratio becomes 2 : 3. Find the fraction.

C-3. If a certain number is divided by the sum of its two digits , the quotient is 6 and remainder is 3. If the digits are interchanged and the resulting number is divided by the sum of the digits, then the quotient is 4 and the remainder is 9. Find the number

C-4. 2 men and 3 boys together can do a piece of work in 8 days. The same work is done in 6 days by 3 men and 2 boys together. How long would 1 boy alone or 1 man alone take to complete the work.

C-5. A man sold a chair and a table together for Rs. 1520 thereby making a profit of 25% on chair and 10% on table. By selling them together for Rs. 1535 he would have made a profit of 10% on the chair and 25% on the table. Find cost price of each.

C-6. Abdul travelled 300 km by train and 200 km by taxi taking 5 hours 30 minutes. But, if he travels 260 km by train and 240 km by taxi, he takes 6 minutes longer. Find the speed of the train and that of the taxi.

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Introduction to Linear equation in two variables and method of solving

A-1. If $x = y$, $3x - y = 4$ and $x + y + z = 6$ then the value of z is

(A) 1 (B) 2 (C) 3 (D) 4

A-2. If $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$, then z will be

(A) $y - x$ (B) $x - y$ (C) $\frac{y-x}{xy}$ (D) $\frac{xy}{y-x}$

A-3. The graphs of $2x + 3y - 6 = 0$, $4x - 3y - 6 = 0$, $x = 2$ and $y = \frac{2}{3}$ intersects in

(A) Four points (B) one point (C) two point (D) infinite no. of points

A-4. If $x = a$, $y = b$ is the solutions $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) – 1 and – 3

A-5. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(A) parallel (B) intersecting at (b, a) (C) coincident (D) intersecting at (a, b)

Section (B) : Condition for solvability

B-1. The system of linear equation $ax + by = 6$, $cx + dy = 8$ has no solution if

(A) $ad - bc > 0$ (B) $ad - bc < 0$ (C) $ad + bc = 0$ (D) $ad - bc = 0$

B-2. If the system $2x + 3y - 5 = 0$, $4x + ky - 10 = 0$ has an infinite number of solutions then

(A) $k = \frac{3}{2}$ (B) $k \neq \frac{3}{2}$ (C) $k \neq 6$ (D) $k = 6$

B-3. The value of k for which the pair of linear equations $4x + 6y - 1 = 0$ and $2x + ky - 7 = 0$ represents parallel lines is

(A) $k = 3$ (B) $k = 2$ (C) $k = 4$ (D) $k = -2$

B-4. The pair of linear equations $3x + 4y + 5 = 0$ and $12x + 16y + 15 = 0$ have

(A) unique solution (B) many solutions (C) no solution (D) exactly two solutions

B-5. One equation of a pair of dependent linear equations is – 5x + 7y = 2, the second equation will be
(A) 10x + 14y + 4 = 0 (B) –10x – 14y + 4 = 0
(C) –10x + 14y + 4 = 0 (D) 10x – 14y = – 4

B-6. Match the column

(i) $2x+5y=10$, $3x+4y=7$	(A) Unique solution
(ii) $2x+5y=10$, $6x+15y=20$	(B) Infinetly many solution
(iii) $5x+2y=10$, $10x+4y=20$	(C) No solution

(A) (i) – A, (ii) – B, (iii) – C (B) (i) – B, (ii) – C, (iii)– A
(C) (i) – A, (ii) – C, (iii) – B (D) (i) – C, (ii) – A, (iii) – B

Section (C) : Word problems

C-1. The sum of two numbers is 20, their product is 40. The sum of their reciprocal is
(A) $\frac{1}{2}$ (B) 2 (C) 4 (D) $\frac{1}{10}$

C-2. If Rs. 50 is distributed among 150 children giving 50 p to each boy and 25 p to each girl. Then the number of boys is
(A) 25 (B) 40 (C) 36 (D) 50

C-3. If in a fraction 1 less from two times of numerator (x) & 1 add in denominator (y) then new fraction will be
(A) $2\left(\frac{x-1}{y+1}\right)$ (B) $\frac{2(x+1)}{y+1}$ (C) $2\left(\frac{x}{y}\right)$ (D) $\frac{2x-1}{y+1}$

C-4. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively
(A) 4 and 24 (B) 5 and 30 (C) 6 and 36 (D) 3 and 24

C-5. The difference between a two digit number and the number obtained by the interchaging the digits is 27. What is the difference between the two digits of the number ?
(A) 9 (B) 6 (C) 12 (D) 3

Exercise-2

OBJECTIVE QUESTIONS

1. Two non negative real numbers 'x' and 'y' are such that 2x + y = 5. The sum of the maximum and minimum values of
(x + y) is :
(A) 2 (B) 5 (C) 8 (D) 7.5

2. If the system of equations px + qy = 8, 3x – qy= 38 has the solution (x, y) = (2, – 4), then p is equal to :
(A) 20 (B) 8 (C) 40 (D) 21.5

3. Real numbers a and b satisfy the equations $3^a = 81^{b+2}$ and $125^b = 5^{a-3}$. The value of ab, is :
(A) 17 (B) 9 (C) 12 (D) 60

4. If $29x + 37y = 103$, $37x + 29y = 95$, then :

(A) $x = 1, y = 2$ (B) $x = 2, y = 1$ (C) $x = 2, y = 3$ (D) $x = 3, y = 2$

5. On solving $\frac{25}{x+y} - \frac{3}{x-y} = 1$, $\frac{40}{x+y} + \frac{2}{x-y} = 5$ we get :

(A) $x = 8, y = 6$ (B) $x = 4, y = 6$ (C) $x = 6, y = 4$ (D) None of these

6. $\frac{2x}{a} + \frac{y}{b} = 2$, $\frac{x}{a} - \frac{y}{b} = 4$:

(A) $\frac{2}{a}, \frac{2}{b}$ (B) $2a, -2b$ (C) $-2a, 2b$ (D) $\frac{a}{2}, -\frac{b}{2}$

7. If $x = 2$ and $x = 3$ are roots of the equation $3x^2 - 2kx + 2m = 0$ then $(k, m) =$:

(A) $(\frac{15}{2}, 9)$ (B) $(9, \frac{15}{2})$ (C) $(\frac{9}{2}, 15)$ (D) $(15, 8)$

8. The solution of the equations $\frac{3x-y+1}{3} = \frac{2x+y+2}{5} = \frac{3x+2y+1}{6}$ given by :

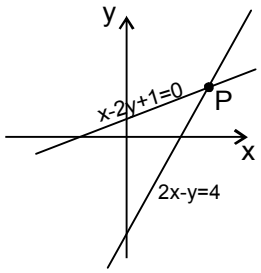
(A) $x = 2, y = 1$ (B) $x = 1, y = 1$ (C) $x = -1, y = -1$ (D) $x = 1, y = 2$

9. The solution of the equations $\frac{m}{x} + \frac{n}{y} = a$, $\frac{n}{x} + \frac{m}{y} = b$ is given by :

(A) $x = \frac{n^2+m^2}{am-bn}, y = \frac{m^2-n^2}{bm-an}$ (B) $x = \frac{m^2-n^2}{am-bn}, y = \frac{n^2-m^2}{bm-an}$

(C) $x = \frac{n^2+m^2}{am-bn}, y = \frac{m^2-n^2}{bm-an}$ (D) $x = \frac{n^2-m^2}{am-bn}, y = \frac{n^2-m^2}{bm-an}$

10. In the graph, co-ordinates of the point P are :



(A) (4, 4) (B) (5, 3) (C) (3, 2) (D) (2, 3)

11. Given $3x - 4y = 7$ and $x + cy = 13$, for what value of "c" will the two equation not have a solution ?

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) -4 (D) $\frac{-4}{3}$

12. For what value of k, the system of equations $kx + 2y = 2$ and $3x + y = 1$ will be coincident ?

(A) 2 (B) 3 (C) 5 (D) 6

13. The equations $2x - 3y + 5 = 0$ and $6y - 4x = 10$, when solved simultaneously , have :

(A) only one solution (B) no solution

(C) only two solutions (D) infinite number of solutions

14. If $2a = b$, the pair of equations $ax + by = 2a^2 - 3b^2$, $x + 2y = 2a - 6b$ possess :

(A) no solution (B) only one solution

(C) only two solutions (D) an infinite number of solutions

15. If the system of equations $3x + 4y = 12$ and $(a + b)x + 2(a - b)y = 5a - 1$ has infinitely many solutions then a & b satisfy the equation
(A) $a - 5b = 0$ (B) $5a - b = 0$ (C) $a + 5b = 0$ (D) $5a + b = 0$

16. Let $\frac{p}{q}$ be a fraction expressed in the lowest form. If the numerator is increased by 2 and the denominator is increased by 1, the resulting fraction equals $\frac{1}{2}$. If, however, the numerator is increased by 1 and the denominator is decreased by 2, the resulting fraction equals . The value of (p + q) equals (p, q are natural number)
(A) 7 (B) 9 (C) 12 (D) 13

17. The digit in the ten's place of a two-digit number is three times that in the one's place. If the digits are reversed the new number will be 36 less than the original number. Find the number.
(A) 64 (B) 52 (C) 62 (D) 42

18. The sum of the present ages of father and his son is 60 years. 6 years ago, father's age was five times the age of the son. After six years son's age will be :
(A) 20 years (B) 14 years (C) 12 years (D) 18 years

19. After covering a distance of 30 km with a uniform speed there is some defect in a train engine and therefore, its speed is reduced to 4/5 of its original speed. Consequently, the train reaches its destination late by 45 minutes. Had it happened after covering 18 kilometers more, the train would have reached 9 minutes earlier. Find the speed of the train and the distance of journey.
(A) Speed = 40 km/hr, Distance = 160 km (B) Speed = 20 km/hr, Distance = 100 km
(C) Speed = 30 km/hr, Distance = 120 km (D) Speed = 35 km/hr, Distance = 140 km

20. A wizard having powers of mystic in conditions and magical medicines seeing a cock, fight going on, spoke privately to both the owners of cocks. To one he said; if your bird wins, than you give me your stake-money, but if you do not win, I shall give you two third of that'. Going to the other, he promised in the same way to give three fourths. From both of them his gain would be only 12 gold coins. Find the stake of money each of the cock-owners have.
(A) 27 gold coins & 30 gold coins respectively. (B) 12 gold coins & 20 gold coins respectively.
(C) 33 gold coins & 30 gold coins respectively. (D) 42 gold coins & 40 gold coins respectively.

21. Two candles of equal length start burning at the same instant. One of the candles burns in 5 hrs. and the other in 4 hrs. By the time one candle is 2 times the length of the other. The candles have already burnt for :
(A) $2\frac{1}{2}$ hrs. (B) $3\frac{1}{2}$ hrs. (C) $3\frac{1}{9}$ hrs. (D) $3\frac{1}{3}$ hrs.

22. The sum of digits of a two-digit number is 7 and the ten's place digit is 25% less than the unit's place digit. What is the number ?
(A) 25 (B) 43 (C) 16 (D) 34

23. At a certain fast food restaurant, Amit can buy 3 burgers, 7 shakes and one order of fries for Rs.120. At the same place, it would cost Rs.164.50 for 4 burgers, 10 shakes and one order of fries. How much would it cost for an ordinary meal of one burger, one shake and one order of fries ?
(A) Rs. 31 (B) Rs. 41 (C) Rs. 21 (D) Cannot be determined

24. Five years ago, A was three times as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B (in years) ?
(A) 45, 15 (B) 30, 40 (C) 50, 30 (D) 50, 20

25. Shyam visited Ram during his brief vacation. In the mornings they both would go for yoga. In the evenings they would play tennis. To have more fun, they indulge only on one activity per day, i.e. either they went for yoga or played tennis each day. There were days when they were lazy and stayed home all day long. There were 24 mornings when they did nothing, 14 evenings when they stayed at home and a total of 22 days when they did yoga or played tennis. For how many days Shyam stayed with Ram ?
(A) 20 (B) 25 (C) 30 (D) 40

26. The sum of two numbers is 8. If their sum is four times their difference, find the numbers.
(A) 6, 2 (B) 7, 1 (C) 5, 3 (D) 6, 3

27. If three times the larger of two number is divided by the smaller, the quotient and the remainder, each is equal to 6. If five times the smaller is divided by the larger, the quotient is 2 and the remainder is 3. The smaller number is :
(A) 6 (B) 7 (C) 8 (D) 9

28. The ratio of the number of boys and girls in a school is 3 : 2. If 20% of the boys and 30% of the girls are scholarship holders, the percentage of the students who are not scholarship holders is :
(A) 50 (B) 72 (C) 75 (D) 76

29. The area of a rectangle gets reduced by 9 m^2 if its length is reduced by 5 m and breadth increased by 3 m. If we increase the length by 3 m and breadth by 2 m, the area is increased by 67 m^2. The length of the rectangle is :
(A) 9 m (B) 15.6 m (C) 17 m (D) 18.5 m

30. Four years ago father's age was 6 times that of his son. Twelve years from now, father's age will be twice that of the son. What is the ratio of father and son's present ages ?
(A) 6 : 1 (B) 7 : 1 (C) 8 : 2 (D) 7 : 2

31. A boat travels with a speed of 15 km/hr in still water. If a river flowing at 5 km/hr, the boat travels some distance downstream and then returns. The ratio of average speed to the speed in still water is
(A) 8 : 3 (B) 3 : 8 (C) 8 : 9 (D) 9 : 8

32. Shubham travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if he travels 240 km by train and the rest by car. Find the speed of the train and the car respectively (in km/hr.).
(A) 40, 80 (B) 60, 120 (C) 80, 100 (D) 100, 120

33. The sum and product of two numbers is 20 and 40 respectively. The sum of the reciprocal of the numbers will be
(A) $\frac{1}{2}$ (B) $\frac{1}{10}$ (C) 4 (D) 2

34. A change making machine contains 1 rupee, 2 rupee and 5 rupee coins. The total number of coins is 300. The amount is Rs. 960. If the number of 1 rupee coins and the number of 2 rupee coins are interchanged, the value comes down by Rs. 40. The total number of 5 rupee coins is
(A) 100 (B) 140 (C) 60 (D) 150

35. $2x + 3y = 10$. How many integral values of x and y are possible ?
(A) 1 (B) 3 (C) 4 (D) 2

36. The course of an enemy submarine as plotted on a set of rectangular axes gives the equation $2x + 3y = 5$. On the same axes, the course of destroyer is indicated by $x - y = 10$. The point (x, y) at which the submarine can be destroyed is :
(A) (–3, 7) (B) (7, –3) (C) (–7, 3) (D) (3, –7)

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. If $\frac{15}{x}+\frac{2}{y}=17$ and x = 3, then value of y is : **[Raj. NTSE Stage-1 2006]**

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $-\frac{1}{6}$ (D) $-\frac{1}{5}$

2. For which values of 'a' and 'b' does the following pair of linear equations have an infinite number of solutions 2x + 3y = 7, (a – b) x + (a + b) y = 3a + b – 2. **[Raj. NTSE Stage-1 2013]**

(A) a = 5 , b = 1 (B) a = 4, b = 2 (C) a = 1, b = 5 (D) a = 2, b = 4

3. If 5x – 2y = k one of answer is (2, – 2), then k = ... **[Gujarat NTSE Stage-1 2013]**

(A) – 40 (B) 6 (C) 14 (D) 10

4. Age of Ramesh is x. Ramesh is 5 years older than Mahesh, but 3 years younger than suresh, then suresh's age is .....years. **[Gujarat NTSE Stage-1 2013]**

(A) x + 3 (B) x + 5 (C) x + 5 (D) x – 8

5. Solve equation : $\frac{2x}{5}+1=\frac{x}{3}+3$, then x = ... **[Gujarat NTSE Stage-1 2013]**

(A) 10 (B) 40 (C) 30 (D) 15

6. If $\frac{7}{2}x+\frac{5}{2}y=5$; 4x + 2y = 7, then what is the value of x – y ? **[Maharashtra NTSE Stage-1 2013]**

(A) 1 (B) 4 (C) 2 (D) –2

7. A farmer divides his herd of x cows among his 4 son's such that first son gets one-half of the herd, the second son gets one fourth, the third son gets one-fifth and the fourth son gets 7 cows, then the value of x is : **[MP_NTSE Stage-1_2013]**

(A) 100 (B) 140 (C) 160 (D) 180

8. If the system of equations kx + 3y – (k – 3) = 0, 12 x + ky – k = 0 has infinitely many solutions, then k = **[Raj. NTSE Stage-1 2014]**

(A) 6 (B) – 6 (C) 0 (D) None of these.

9. In three given numbers. the second number is twice than the first number and thrice than the third number. If the average of the three numbers. is 44, what is the largest number ? **[Gujarat NTSE Stage-1 2014]**

(A) 24 (B) 72 (C) 36 (D) 8

10. The difference between a two digit given number and the number obtained by interchanging the digits is 27. The sum of the two digits is ..... **[Jharkhand NTSE Stage-1 2014]**

(A) 3 (B) 5 (C) 7 (D) cannot be found

11. In a given fraction if the numerator is multiplied by 3 and denominator is subtracted by 3, the fraction becomes $\frac{18}{11}$ and if the numerator is increased by 8 and the denominator doubled the fraction becomes $\frac{2}{5}$. The sum of the numerator and the denominator of the given fraction is : **[Jharkhand NTSE Stage-1 2014]**

(A) 27 (B) 33 (C) 37 (D) 42

12. If after 5 years 'A' will be twice as old as 'B'. At present 'B' is 5 times older than 'C' whose age before 3 years was 3 year. The present age of 'A' is **[MP_NTSE Stage-1_2014]**

(A) 50 (B) 60 (C) 55 (D) 65

13. What type of graphs of the equation x + y = 5 and x – y = 1 will be ? **[Maharashtra NTSE Stage-1 2014]**

(A) Parallel lines (B) Intersecting lines (C) Lines coincide (D) Concurrent lines

14. A boatman can row downstream 30 km in 2 hours and upstream 6 km. in 2 hours. The speed of boatman's rowing and speed of water current are respectively. **[MP_NTSE Stage-1_2014]**

(A) 9, 6 (B) 6, 4 (C) 12, 9 (D) None of these

15. If $\frac{x}{3} + 7 = 15 - \frac{x}{5}$, then find the solution ? **[Gujarat NTSE Stage-1 2015]**

(A) x = 20 (B) x = 15 (C) x = 21 (D) x = 18

16. $3^{2x-y} = 3^{x+y} = \sqrt{27}$ then what will be the value of 3^{x-y} ? **[Delhi NTSE Stage-1 2015]**

(A) $\frac{1}{\sqrt{27}}$ (B) 3 (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

17. 4 boys and 3 girls spent Rs.120 on the average, of which boys spend Rs.150 on the average, then the average amount spent by girls is **[Delhi NTSE Stage-1 2015]**

(A) Rs.80 (B) Rs.60 (C) Rs.90 (D) Rs.100

18. When my father was 31, I was 8. Now he is twice as old as I am. How old am I? **[Delhi NTSE Stage-1 2015]**

(A) 23 years (B) 46 years (C) 22 years (D) 24 years

19. Which of the following is the solution set for the simultaneous equation ?

$\frac{2x-1}{3} = \frac{y+6}{5} = \frac{3x-y}{7}$ **[Maharashtra NTSE Stage-1 2015]**

(A) x = 2; y = –1 (B) x = 3; y = –1 (C) x = –1; y = 2 (D) x = 4; y = 1

20. A boat. whose speed is 15 km/hr in still water. takes 4 hours 30 minutes to go 30 km In downstream' and to return upstream to the same spot. Find the speed of the stream per hour. **[Orissa NTSE Stage - 1_2015]**

(A) 3 km/hr (B) 5km/hr (C) 7 km/hr (D) 2 km/hr

21. The sum of two numbers is 100 and one number is two less than twice the other number. Then the numbers are..... **[MP_NTSE Stage-1_2015]**

(A) 34, 66 (B) 24,76 (C) 44,56 (D) 46,54

22. Ganesh has to pay Rs. 482 for 19 apples and 11 guavas. If he would have exchanged the number of apples and Guaves purchased, then he would have paid Rs. 64 less. Find how much more amount he has to pay to purchase apple than 1 guava ? **[Maharashtra NTSE Stage-1 2016]**

(A) Rs. 19 (B) Rs. 8 (C) Rs. 11 (D) Rs. 7

23. The present age difference between father and son is 14 years. The ratio of their age will be 4 : 3 after 11 years. How old is son now ? **[Bihar NTSE Stage-1 2016]**

(A) 25 years (B) 31 years (C) 30 years (D) 28 years

24. For which value of p the following pair of linear equations 3x + py = 7, px + 3y = 15 will have no solutions? **[Raj. NTSE Stage-1 2016]**

(A) ± 9 (B) ± 5 (C) ± 3 (D) ± 4

25. Line x + y = 2 passes through the ____ quadrants. **[Gujarat NTSE Stage-1 2016]**
(A) 1st and 3rd both (B) 2nd and 3rd both (C) 3rd and 4th both (D) 1st, 2nd, 4th all

26. 3 year ago the sum of ages of father and his son was 40 years. After 2 years , the sum of ages of the father and his son will be: **[Gujarat NTSE Stage-1 2016]**
(A) 40 (B) 46 (C) 50 (D) 60

27. In a two digit number, the number of ten's place is double of the number of unit's place. If we exchange the numbers mutually then the number decreases by 18,then the number is : **[MP_NTSE Stage-1_2016]**
(A) 24 (B) 36 (C) 39 (D) 42

28. The system of equations x + 2y = 6 , 3x + 6y = 18 **[MP_NTSE Stage-1_2016]**
(A) is inconsistent (B) has a unique solution
(C) Has an infinite number of solutions (D) None of these

29. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. Father's present age is **[UP_NTSE Stage-1_2017]**
(A) 24 years (B) 28 years (C) 30 years (D) 32 years

30. In the equations 3x + 2y = 13xy and 4x – 5y = 2xy, the values of x and y that satisfy the equations are **[Raj. NTSE Stage-1 2017]**
(A) (2,3) (B) (3,2) (C) $\left(\frac{1}{2},\frac{1}{3}\right)$ (D) $\left(\frac{1}{3},\frac{1}{2}\right)$

31. A boat takes 7 hours to travel 30 km upstream and 28 km downstream. It takes 5 hours to travel 21 km upstream and to return back. Find the speed of the boat in still water. **[Maharashtra NTSE Stage-1 2017]**
(A) 10 km/hr (B) 20 km/hr (C) 14 km/hr (D) 6 km/hr

32. The cost of 20 guavas and 5 apples is same as that of 12 guavas and 7 apples, then how many times the cost of an apple is to that of a guava? **[Maharashtra NTSE Stage-1 2017]**
(A) two times (B) half times (C) four times (D) five times

Answer Key

Exercise Board Level

TYPE (I)

1. **(i)** Yes **(ii)** No **(iii)** No

2. **(i)** No **(ii)** Yes **(iii)** No

3. **(i)** No **(ii)** yes **(iii)** yes **(iv)** No

4. False

TYPE (II)

5. 2 **6.** $\frac{15}{4}$ **7.** no value **8.** – 6

9. 31 and $\frac{-5}{7}$. **10.** x = 1 , y = 4

TYPE (III)

11. 100 **12.** a = 5 , b = 2 **13.** x = 1, y = 4 and ratio = 4 : 1

14. 10 kmph, 4 kmph **15.** 83 **16.** Rs. 2500, Rs. 30

TYPE (IV)

17. 8 sq. units **18 .** Rs. 600, Rs. 400

19. Investment in Scheme A = Rs. 12000, Investment in Scheme A = Rs. 10000 **20.** 500

Previous Year Problems

1. x = 1, y = 2 **OR** x = 3, y = 2 **2.** A = 27, B = 28 **OR** x = 14/5 , y = 5/7

3. No. of Boys = 3 , No. of Girls = 7 **4.** (B) **5.** (A)

6. (C)

7. Cost of a Table = Rs. 1,000 , and Cost of a Chair = Rs. 500 **8.** x = 1 , y = – 4 **OR** 3/5

9. (2 , 4) , (5 , 0) , (– 1 , 0) **10.** (B) **11.** k = 6

12. x = – 92/351 , y = – 38/351 **OR** Speed of train = 80 km/hr , Speed of car = 100 km/hr.

13. (A) **14.** (B) **15.** x = 3 , y = 3 **16.** 70 & 50 **17.** 41

18. No. of Pen = 13 , No. of Pencil = 27

19. Speed of Car A = 60 km/hr, Speed of Car B = 40 km/hr **OR** x = 7/24 , y = 13/24

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-1. $\frac{-8}{3}$ **A-2.** **(i)** x = 27 and y = – 14 **(ii)** x = a and y = b

A-3. **(i)** x = 0.1 and y = 0.3 **(ii)** x = 9 and y = 6.

A-4. **(i)** x = – 21 and y = 17. **(ii)** x = m + n and y = m – n

A-5. **(i)** x = –2 and y = –3 **(ii)** $x = \frac{2}{3}$ and $y = \frac{2}{5}$.

(iii) $x = a + b$ and $y = -\frac{2ab}{a+b}$ **(iv)** x = ab and y = ab

A-6. Solution is x = 4 and y = 3 and lines meets y-axis at (0, 11) and (0, – 1).

A-7. (– 4, 2), (1, 3) and (2, 5). **A-8.** 7.5 sq. unit

Section (B)

B-1. Infinitely many solutions. **B-2.** k = any real number

B-3. k = 6 **B-4.** k = – 4 **B-5.** m = 5.

Section (C)

C-1. $\angle A = 70°, \angle B = 53°, \angle C = 110°, \angle D = 127°$ **C-2.** $\frac{5}{9}$

C-3. 75. **C-4.** Man - 20 days , Boy = 120 days

C-5. Cost price of chair = Rs. 600 and cost price of table = Rs. 700.

C-6. Speed of train = 100 km/hr, Speed of taxi = 80 km/hr.

OBJECTIVE QUESTIONS

Section (A)

A-1. (B) **A-2.** (D) **A-3.** (B) **A-4.** (C) **A-5.** (D)

Section (B)

B-1. (D) **B-2.** (D) **B-3.** (A) **B-4.** (C) **B-5.** (D)

B-6. (C)

Section (C)

C-1. (A) **C-2.** (D) **C-3.** (D) **C-4.** (C) **C-5.** (D)

Exercise-2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D	A	D	A	C	B	A	B	C	C	D	D	D	D	A	B	C	A	C	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**				
D	D	A	D	C	C	B	D	C	D	C	C	A	B	D	B				

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	C	A	C	B	B	A	B	D	C	D	B	A	B	D	A	A	A	B
Ques.	21	22	23	24	25	26	27	28	29	30	31	32								
Ans.	A	B	B	C	D	C	D	C	B	C	A	C								