

MATHEMATICS

Class-X

Topic-4

QUADRATIC EQUATIONS



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CH-04

QUADRATIC EQUATIONS

(A) INTRODUCTION AND METHODS OF SOLVING

(a) Quadratic Equation

Any equation of the form $p(x) = 0$, where $p(x)$ is a polynomial of degree 2, is known as quadratic equation.

For Example : $3x^2 + 7x + 5 = 0$ is a quadratic equation. Many real life situation can be represented by quadratic equation.

An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real number and $a \neq 0$ is known as the standard form of a quadratic equation.

(b) Roots of a Quadratic Equation

A real number α is a root of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$. So any real number which satisfies a given quadratic equation is called the root of the quadratic equation.

In polynomial chapter, we learnt that if $x = \alpha$ satisfies a quadratic polynomial $p(x)$ i.e., $p(\alpha) = 0$, then α is the zero of $p(x)$.

This means that zeros of a quadratic polynomial $ax^2 + bx + c$ and roots of a quadratic equation $ax^2 + bx + c = 0$ are the same. A quadratic equation can have atmost two real roots.

NOTE : A quadratic equation is satisfied by exactly two values of 'x' which may be real or imaginary. The equation, $ax^2 + bx + c = 0$ is

A quadratic equation if $a \neq 0$. [Two roots]

A linear equation if $a = 0, b \neq 0$. [One root]

A contradiction if $a = b = 0, c \neq 0$. [No root]

An identity if $a = b = c = 0$. [Infinite roots]

A quadratic equation cannot have more than two roots.

It follows from the above statement that if a quadratic equation is satisfied by more than two values of x , then it is satisfied by every value of x and so it is an identity.

(c) Methods of Solving Quadratic Equation

(i) By Factorisation Method

ALGORITHM

Step-1 Factorise the constant term of the given quadratic equation.

Step-2 Express the coefficient of middle term as the sum or difference of the factors obtained in step 1. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step-3 Split the middle term in two parts obtained in step 2.

Step-4 Factorise the quadratic equation obtained in step 3.

(ii) By Completing the Square Method

ALGORITHM

Step-1 Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$.

Step-2 Make the coefficient of x^2 unity, if it is not unity. i.e., obtain $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Step-3 Shift the constant term $\frac{c}{a}$ on R.H.S. to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step-4 Add square of half of the coefficient of x . i.e. $\left(\frac{b}{2a}\right)^2$ on both sides to obtain

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Step-5 Write L.H.S. as the perfect square of a binomial expression and simplify R.H.S. to get

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step-6 Take square root of both sides to get $x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$

Step-7 Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on RHS.

(iii) By Using Quadratic Formula

Consider the quadratic equation : $ax^2 + bx + c = 0$. $a \neq 0$

Dividing throughout by a , we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \Rightarrow \quad x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\Rightarrow x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}, \text{ when } b^2 - 4ac \geq 0$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is known as **Quadratic formula** or **Shreedharachary's formula** for finding the roots of a quadratic equation.

Hence, if $b^2 - 4ac \geq 0$, then the root of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Discriminant : For the quadratic equation $ax^2 + bx + c = 0$, the expression $D = (b^2 - 4ac)$ is called its discriminant. In terms of discriminant D , the two roots are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

NOTE : If the discriminant $D = b^2 - 4ac < 0$, then the quadratic equation $ax^2 + bx + c = 0$ has no real roots.

Solved Examples

Example. 1

Which of the following are quadratic equations ?

(i) $x^2 - 6\sqrt{x} + 2 = 0$ (ii) $x + \frac{1}{x} = 1$

- Sol.** (i) $x^2 - 6\sqrt{x} + 2 = 0$ is not a quadratic equation as $x^2 - 6\sqrt{x} + 2$ is not a quadratic polynomial (the power of x is $1/2$)
- (ii) $x + \frac{1}{x} = 1$
 $\Rightarrow \frac{x^2 + 1}{x} = 1$
 $\Rightarrow x^2 + 1 = x$
 $\Rightarrow x^2 - x + 1 = 0$
 $\Rightarrow (x^2 - x + 1)$ is a quadratic polynomial
 $\therefore (x^2 - x + 1) = 0$ is a quadratic equation.

Example. 2

Determine whether the given values are solution of the given equation or not.

- (i) $3x^2 + 2x - 1 = 0$; $x = \frac{1}{3}$ (ii) $a^2x^2 - 3abx + 2b^2 = 0$; $x = \frac{a}{b}$, $x = \frac{b}{a}$

- Sol.** (i) $3x^2 + 2x - 1 = 0$

Putting $x = \frac{1}{3}$ in the LHS of the given equation, we get

$$\begin{aligned} \text{LHS} &= 3x^2 + 2x - 1 = 3 \times \left(\frac{1}{3}\right)^2 + 2 \times \frac{1}{3} - 1 = 3 \times \frac{1}{9} + \frac{2}{3} - 1 = \frac{1}{3} + \frac{2}{3} - 1 \\ &= \frac{1+2-3}{3} = 0 = \text{RHS} \end{aligned}$$

$\therefore x = \frac{1}{3}$ is a solution of $3x^2 + 2x - 1 = 0$.

- (ii) $a^2x^2 - 3abx + 2b^2 = 0$

We have $a^2x^2 - 3abx + 2b^2 = 0$

Putting $x = \frac{a}{b}$ in the LHS of the given equation.

$$\text{LHS} = a^2 \times \frac{a^2}{b^2} - 3ab \times \frac{a}{b} + 2b^2 = \frac{a^4}{b^2} - 3a^2 + 2b^2 \neq \text{RHS}$$

Putting $x = \frac{b}{a}$ in the LHS of the given equation

$$\text{LHS} = a^2 \times \frac{b^2}{a^2} - 3ab \times \frac{b}{a} + 2b^2 = b^2 - 3b^2 + 2b^2 = 0 = \text{RHS}.$$

hence $x = \frac{a}{b}$ is not the solution, but $x = \frac{b}{a}$ is a solution of the given equation.

Example. 3

Find the value of K for which the given value is a solution of the equation.

- (i) $x^2 + Kx + 3 = 0$, $x = 1$ (ii) $x^2 - x(a + b) + K = 0$, $x = a$

- Sol.** (i) Since $x = 1$ is a solution of the equation $x^2 + Kx + 3 = 0$

$$\therefore (1)^2 + K \times 1 + 3 = 0$$

$$\Rightarrow 1 + K + 3 = 0$$

$$\therefore K = -4.$$

- (ii) Since $x = a$ is a solution of equation

$$x^2 - x(a + b) + K = 0$$

$$\therefore (a)^2 - a(a + b) + K = 0$$

$$\Rightarrow a^2 - a^2 - ab + K = 0$$

$$\Rightarrow -ab + K = 0$$

$$\therefore K = ab.$$

Example. 4

If $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the value of a and b .

Sol. Since $x = \frac{2}{3}$ is a root of the equation $ax^2 + 7x + b = 0$

$$\therefore a\left(\frac{2}{3}\right)^2 + 7 \times \frac{2}{3} + b = 0$$

$$\Rightarrow 4a + 9b = -42 \quad \dots (i)$$

Again, $x = -3$ is a root of the equation $ax^2 + 7x + b = 0$

$$\therefore a(-3)^2 + 7(-3) + b = 0$$

$$\Rightarrow 9a - 21 + b = 0$$

$$\Rightarrow 9a + b = 21 \quad \dots (ii)$$

On solving (i) & (ii) $a = 3$, $b = -6$

Example. 5

Find the solutions of the quadratic equation $x^2 + 6x + 5 = 0$.

Sol. The quadratic polynomial $x^2 + 6x + 5$ can be factorised as follows

$$x^2 + 6x + 5$$

$$= x^2 + 5x + x + 5$$

$$= x(x + 5) + 1(x + 5)$$

$$= (x + 5)(x + 1)$$

Therefore the given quadratic equation becomes $(x + 5)(x + 1) = 0$

This gives $x = -5$ or $x = -1$.

Therefore, $x = -1, -5$ are the required solutions of the given equation.

Example. 6

Solve : $25x^2 - 30x + 9 = 0$

Sol. $25x^2 - 30x + 9 = 0$

$$(5x)^2 - 2(5x) \times 3 + (3)^2 = 0$$

$$(5x - 3)^2 = 0$$

This gives $x = \frac{3}{5}, \frac{3}{5}$ or simply $x = \frac{3}{5}$ as the required solution.

Example. 7

Solve $64x^2 - 625 = 0$

Sol. We have $64x^2 - 625 = 0$

$$\text{or } (8x)^2 - (25)^2 = 0$$

$$\text{or } (8x + 25)(8x - 25) = 0$$

i.e., $8x + 25 = 0$ or $8x - 25 = 0$.

This gives $x = -\frac{25}{8}$ or $\frac{25}{8}$.

Thus, $x = -\frac{25}{8}, \frac{25}{8}$ are solutions of the given equation.

Example. 8

Solve the quadratic equation : $16x^2 - 24x = 0$.

Sol. The given equation may be written as $8x(2x - 3) = 0$

This gives $x = 0$ or $x = \frac{3}{2}$.

$x = 0, \frac{3}{2}$ are the required solutions.

Example. 9

Solve the following quadratic equation by factorisation method : $x^2 - 2ax + a^2 - b^2 = 0$.

- Sol.** Here, factors of constant term ($a^2 - b^2$) are $(a - b)$ and $(a + b)$.
 Also, Coefficient of the middle term = $-2a = -[(a - b) + (a + b)]$
 $\therefore x^2 - 2ax + a^2 - b^2 = 0$
 $\Rightarrow x^2 - \{(a - b) + (a + b)\}x + (a - b)(a + b) = 0$
 $\Rightarrow x^2 - (a - b)x - (a + b)x + (a - b)(a + b) = 0$
 $\Rightarrow x\{x - (a - b)\} - (a + b)\{x - (a - b)\} = 0$
 $\Rightarrow \{x - (a - b)\}\{x - (a + b)\} = 0$
 $\Rightarrow x - (a - b) = 0$ or $x - (a + b) = 0$
 $\Rightarrow x = a - b$ or $x = a + b$

Example. 10

Solve : $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$.

- Sol.** Obviously, the given equation is valid if $x - 3 \neq 0$ and $2x + 3 \neq 0$.
 Multiplying throughout by $(x - 3)(2x + 3)$, we get
 $2x(2x + 3) + 1(x - 3) + 3x + 9 = 0 \quad \Rightarrow \quad 4x^2 + 10x + 6 = 0$
 $\Rightarrow \quad 2x^2 + 5x + 3 = 0 \quad \Rightarrow \quad (2x + 3)(x + 1) = 0$
 But $2x + 3 \neq 0$, so we get $x + 1 = 0$. This gives $x = -1$ as the only solution of the given equation.

Example. 11

Solve:- $x^2 + 6x + 8 = 0$

- Sol.** We have
 $x^2 + 6x + 8 = 0$
 $x^2 + 6x = -8$
 add square of half of the coefficient of x on both sides
 $x^2 + 6x + (3)^2 = -8 + (3)^2$
 $(x + 3)^2 = -8 + 9$
 $(x + 3)^2 = 1$
 $x + 3 = \pm 1$.
 $x + 3 = 1$ or $x + 3 = -1$
 $\therefore x = -2$ or -4 are the solutions.

Example. 12

By using the method of completing the square, show that the equation $4x^2 + 3x + 5 = 0$ has no real roots.

- Sol.** We have, $4x^2 + 3x + 5 = 0$..(i)
 divide (i) by 4 we get
 $\Rightarrow x^2 + \frac{3}{4}x = -\frac{5}{4}$
 add square of half of the coefficient of x on both sides
 $\Rightarrow x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = \left(\frac{3}{8}\right)^2 - \frac{5}{4}$
 $\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{9}{64} - \frac{5}{4}$
 $\Rightarrow \left(x + \frac{3}{8}\right)^2 = -\frac{71}{64}$
 Clearly, RHS is negative.
 But, $\left(x + \frac{3}{8}\right)^2$ cannot be negative for any real value of x .
 Hence, the given equation has no real roots.

Example. 13

Solve the quadratic equation $x^2 - 7x - 5 = 0$.

Sol. Comparing the given equation with $ax^2 + bx + c = 0$, $a = 1$, $b = -7$ and $c = -5$.

Therefore, $D = (-7)^2 - 4 \times 1 \times (-5) = 49 + 20 = 69 > 0$

Since D is positive, the equation has two roots given by

$$x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a} = \frac{7 + \sqrt{69}}{2}, \frac{7 - \sqrt{69}}{2}.$$

$$\Rightarrow x = \frac{7 + \sqrt{69}}{2}, \frac{7 - \sqrt{69}}{2} \text{ are the required solutions.}$$

Example. 14

Solve the quadratic equation $9x^2 - 9(a + b)x + 2a^2 + 5ab + 2b^2 = 0$.

Sol. $9x^2 - 9(a + b)x + 2a^2 + 5ab + 2b^2 = 0$

compare the given equation $Ax^2 + Bx + C = 0$

$$\therefore A = 9$$

$$B = -9(a + b)$$

$$C = 2a^2 + 5ab + 2b^2$$

$$\therefore x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{9(a + b) \pm \sqrt{9^2(a + b)^2 - 4 \times 9(2a^2 + 5ab + 2b^2)}}{18}$$

$$x = \frac{9(a + b) \pm \sqrt{81a^2 + 81b^2 + 162ab - 72a^2 - 72b^2 - 180ab}}{18}$$

$$x = \frac{9(a + b) \pm \sqrt{9a^2 + 9b^2 - 18ab}}{18}$$

$$x = \frac{9(a + b) \pm \sqrt{9(a - b)^2}}{18}$$

$$x = \frac{9(a + b) \pm 3(a - b)}{18}$$

Taking positive value

$$x = \frac{9a + 9b + 3a - 3b}{18} = \frac{2a + b}{3}$$

Taking negative value

$$x = \frac{9a + 9b - 3a + 3b}{18} = \frac{a + 2b}{3}$$

$$x = \frac{2a + b}{3}, \frac{a + 2b}{3}.$$

Example. 15

Solve for x : $9^{x+2} - 6 \times 3^{x+1} + 1 = 0$

Sol. We have $9^{x+2} - 6 \times 3^{x+1} + 1 = 0$

$$\Rightarrow 9^x \cdot 9^2 - 6 \cdot 3^x \cdot 3 + 1 = 0$$

$$\Rightarrow 81(3^x)^2 - 18 \times 3^x + 1 = 0 \quad \dots (i)$$

Let $3^x = y$, then equation (i) becomes

$$\Rightarrow 81y^2 - 18y + 1 = 0$$

$$\Rightarrow 81y^2 - 9y - 9y + 1 = 0$$

$$\begin{aligned} \Rightarrow 9y(9y - 1) - 1(9y - 1) &= 0 \\ \Rightarrow (9y - 1)(9y - 1) &= 0 \\ \Rightarrow y = \frac{1}{9} \text{ or } y = \frac{1}{9} &\quad \dots \text{ (ii)} \end{aligned}$$

Putting $y = 3^x$ in (ii), we get

$$\begin{aligned} 3^x &= \frac{1}{9} = 3^{-2} \\ x &= -2 \text{ and } -2. \end{aligned}$$

Example. 16

Find the value of $\sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$.

Sol. Let $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$
 $x = \sqrt{2 + x}$
 by squaring on both sides
 $\Rightarrow x^2 = 2 + x$
 $\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$
 $\Rightarrow x = -1, 2.$

Example. 17

Solve for $x : 2x^4 + x^3 - 6x^2 + x + 2 = 0$

Sol. $2x^4 + x^3 - 6x^2 + x + 2 = 0$
 divide the equation by x^2 we get
 $2(x^2 + \frac{1}{x^2}) + (x + \frac{1}{x}) - 6 = 0 \quad \dots \text{(i)}$
 $x + \frac{1}{x} = y$ then $x^2 + \frac{1}{x^2} = y^2 - 2$ put these value in (i)
 $\Rightarrow 2(y^2 - 2) + y - 6 = 0$
 $\Rightarrow 2y^2 - 4 + y - 6 = 0$
 $\Rightarrow 2y^2 + y - 10 = 0$
 $\Rightarrow 2y^2 + 5y - 4y - 10 = 0$
 $\Rightarrow y(2y + 5) - 2(2y + 5) = 0$
 $\Rightarrow (y - 2)(2y + 5) = 0$
 $\Rightarrow y = 2, -\frac{5}{2}$
 $x + \frac{1}{x} = y = 2 \quad \text{or} \quad x + \frac{1}{x} = y = -\frac{5}{2}$
 $\Rightarrow x^2 - 2x + 1 = 0 \quad \frac{x^2 + 1}{x} = \frac{-5}{2}$
 $\Rightarrow (x - 1)^2 = 0$
 $\Rightarrow x = 1, 1 \quad \Rightarrow 2x^2 + 2 + 5x = 0$
 $\Rightarrow (x + 2)(2x + 1) = 0$
 $\Rightarrow x = -2, -\frac{1}{2}.$

Example. 18

Find the value of $2 + \frac{1}{2 + \frac{1}{2 + \dots}}$.

Sol. Let $x = 2 + \frac{1}{2 + \frac{1}{2 + \dots \infty}}$

$$\Rightarrow x = 2 + \frac{1}{x} \Rightarrow x^2 - 2x - 1 = 0$$

$$\Rightarrow \text{by quadratic formula } x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{8}}{2}$$

$$\Rightarrow x = 1 + \sqrt{2}, 1 - \sqrt{2}$$

$1 - \sqrt{2}$ is neglected

so $x = 1 + \sqrt{2}$.

Check Your Level

1. Solve by factorization: $9x^2 + 15x - 14 = 0$
2. Solve by completing the square method: $x^2 - 2bx - 1 = -b^2$
3. Solve using Quadratic formula: $(a - b)^2 x^2 - (a - b)^2 x - ab = 0$
4. Solve for x : $7^{x+1} + 7^{1-x} = 50$
5. Solve for x : $8\sqrt{\frac{x}{x+3}} + \sqrt{\frac{x+3}{x}} = 6$

Answers

- | | | |
|------------------------------------|---------------------------|--|
| 1. $x = -\frac{7}{3}, \frac{2}{3}$ | 2. $x = (b - 1), (b + 1)$ | 3. $x = \frac{a}{a - b}, \frac{-b}{a - b}$ |
| 4. $x = 1, -1$ | 5. $x = 1/5$. | |

(B) RELATION BETWEEN ROOTS AND COEFFICIENTS

(i) The solutions of quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(ii) The expression $b^2 - 4ac = D$ is called discriminant of the quadratic equation $ax^2 + bx + c = 0$.

If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

(a) Sum of the roots = $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

$$\alpha + \beta = -\frac{\text{constant term}}{\text{coefficient of } x^2}$$

(b) Product of the roots = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$$\alpha \beta = \frac{c}{a}$$

(iii) A quadratic equation whose roots are α and β is $(x - \alpha)(x - \beta) = 0$

i.e. $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$.

Solved Examples

Example. 19

Find the quadratic equation having roots 3 and -4.

Sol. Quadratic equation whose roots are given is $x^2 - (\text{sum of roots})x + \text{Product of roots} = 0$.

Roots = 3 and -4

\therefore Equation is $x^2 - (3 - 4)x + 3(-4) = 0$

$$x^2 + x - 12 = 0$$

Example. 20

If α, β are the roots of the equation $4x^2 + 3x + 7 = 0$ then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Sol. $4x^2 + 3x + 7 = 0$

$$\text{Sum of roots} = \alpha + \beta = \frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{3}{4}$$

$$\text{product of roots} = \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{7}{4}. \text{ Then, } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -\frac{3/4}{7/4} = -\frac{3}{7}.$$

Example. 21

The quadratic equation $ax^2 + bx + c = 0$ has real roots α and β . If a, b, c real and of the same sign, then determine the sign of its roots.

Sol. Case - I : Let a, b, c all are positive

$$\alpha + \beta = \frac{-b}{a} \text{ i.e. -ive no.} \quad \dots (i)$$

$$\alpha\beta = \frac{c}{a} \text{ i.e. + ive no.} \quad \dots (ii)$$

from (i) & (ii) we say that two number α and β whose sum is negative and product is positive it is possible only when both root are negative.

Case - II : Let a, b, c all are - ive

$$\alpha + \beta = \frac{-b}{a} \text{ i.e. - ive no.} \quad \dots(iii)$$

$$\alpha\beta = \frac{c}{a} \text{ i.e. + ive no.} \quad \dots(iv)$$

from (iii) & (iv) we say that two no α and β whose sum is negative and product is +ive it is possible both roots are - ive .

so, the quadratic equation $ax^2 + bx + c = 0$ have both roots negative if a, b, c are real and have same sign.

Check Your Level

- Find the sum of the roots of the equation $2x^2 + 5x + 3 = 0$
- Find the value of 'a' if the roots of $x^2 + (2a+3)x + 7 = 0$ are equal in magnitude but opposite in sign
- Find the value of a if the roots of the equation $(a+1)x^2 + 2x + 3a + 7 = 0$ are reciprocal of each other.
- Form a quadratic equation whose roots are 3, -1.
- if are the roots of $3x - 5x + 7 = 0$ then find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Answers

1. $-\frac{5}{2}$ 2. $-\frac{3}{2}$ 3. $a = -3$ 4. $x^2 - 2x - 3 = 0$
5. $-\frac{17}{21}$

(C) NATURE OF ROOTS

Consider the quadratic equation, $ax^2 + bx + c = 0$ having α and β as its roots and $b^2 - 4ac$ is called **discriminant** of roots of quadratic equation. It is denoted by **D** or Δ .

Roots of the given quadratic equation may be

- (i) Real and unequal (ii) Real and equal (iii) Unreal

Let the roots of the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$, $b, c \in \mathbf{R}$) be α and β then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \dots \text{(i)}$$

and
$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \dots \text{(ii)}$$

The nature of roots depends upon the value of expression ' $b^2 - 4ac$ ' with in the square root sign. This is known as **discriminant** of the given quadratic equation.

Consider the Following Cases

Case-1 When $b^2 - 4ac > 0$, ($D > 0$)

In this case roots of the given equation are real and distinct and are as follows

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

(i) When $a(\neq 0)$, $b, c \in \mathbf{Q}$ and $b^2 - 4ac$ is a perfect square

In this case both the roots are rational and distinct.

(ii) When $a(\neq 0)$, $b, c \in \mathbf{Q}$ and $b^2 - 4ac$ is not a perfect square

In this case both the roots are irrational and distinct.

Case-2 When $b^2 - 4ac = 0$, ($D = 0$)

In this case both the roots are real and equal to $-\frac{b}{2a}$.

Case-3 When $b^2 - 4ac < 0$, ($D < 0$)

In this case $b^2 - 4ac < 0$, then $4ac - b^2 > 0$

$$\therefore \alpha = \frac{-b + \sqrt{-(4ac - b^2)}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{-(4ac - b^2)}}{2a}$$

$$\text{or} \quad \alpha = \frac{-b + i\sqrt{4ac - b^2}}{2a} \quad \text{and} \quad \beta = \frac{-b - i\sqrt{4ac - b^2}}{2a} \quad [\because \sqrt{-1} = i]$$

i.e. in this case both the roots are imaginary and distinct.

NOTE : If $a, b, c \in \mathbf{Q}$ and $b^2 - 4ac$ is positive ($D > 0$) but not a perfect square, then the roots are irrational and they always occur in conjugate pairs like $2 +$ and $2 -$. However, if a, b, c are irrational numbers and $b^2 - 4ac$ is positive but not a perfect square, then the roots may not occur in conjugate pairs.

Example. 22

Find the nature of the roots of the quadratic equation $4x^2 - 5x + 3 = 0$.

Sol. The given equation is $4x^2 - 5x + 3 = 0$. This is of the form $ax^2 + bx + c = 0$, where $a=4, b=-5$ and $c=3$
 $D = b^2 - 4ac = (-5)^2 - 4(4)(3) = -23 < 0$
 Hence, the given equation has no real roots.

Example. 23

For what value of k , $(4 - k)x^2 + (2k + 4)x + (8k + 1)$ is a perfect square.

Sol. The given equation is a perfect square, if its discriminant is zero.

$$(2k + 4)^2 - 4(4 - k)(8k + 1) = 0$$

$$\Rightarrow 4(k + 2)^2 - 4(4 - k)(8k + 1) = 0$$

$$\begin{aligned} \Rightarrow & 4[(k+2)^2 - (4-k)(8k+1)] = 0 \\ \Rightarrow & [(k^2 + 4k + 4) - (-8k^2 + 31k + 4)] = 0 \\ \Rightarrow & 9k^2 - 27k = 0 \\ \Rightarrow & 9k(k-3) = 0 \Rightarrow k = 0 \text{ or } k = 3 \end{aligned}$$

Hence, the given equation is a perfect square, if $k = 0$ or $k = 3$.

Example. 24

If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, show that $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$.

Sol. Since the roots of the given equations are equal, so discriminant will be equal to zero.

$$\begin{aligned} \Rightarrow & b^2(c-a)^2 - 4a(b-c)c(a-b) = 0 \\ \Rightarrow & b^2(c^2 + a^2 - 2ac) - 4ac(ba - ca - b^2 + bc) = 0, \\ \Rightarrow & a^2b^2 + b^2c^2 + 4a^2c^2 + 2b^2ac - 4a^2bc - 4abc^2 = 0 \\ \Rightarrow & (ab + bc - 2ac)^2 = 0 \\ \Rightarrow & ab + bc - 2ac = 0 \\ \Rightarrow & ab + bc = 2ac \\ \Rightarrow & \frac{1}{c} + \frac{1}{a} = \frac{2}{b} \\ \Rightarrow & \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \end{aligned}$$

Example. 25

If the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, then prove that $2b = a + c$.

Sol.

If the roots of the given equation are equal, then discriminant is zero i.e

$$\begin{aligned} & (c-a)^2 - 4(b-c)(a-b) = 0 \\ \Rightarrow & c^2 + a^2 - 2ac + 4b^2 - 4ab + 4ac - 4bc = 0 \\ \Rightarrow & c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0 \\ \Rightarrow & (c+a-2b)^2 = 0 \\ \Rightarrow & c+a = 2b. \end{aligned}$$

Check Your Level

Discuss the nature of roots (01 to 03)

1. $7x^2 - 11x + 4 = 0$
2. $6x^2 - 13x - 15 = 0$
3. $x^2 - 7x + 8 = 0$
4. For what value of k will $18x^2 - kx + 2 = 0$ have equal roots ?
5. Show that $3x^2 - 7x + 8 = 0$ cannot be satisfied by any real value of x .

Answers

1. Real, Unequal 2. Real, Unequal 3. Real, Unequal
4. $k = \pm 12$

(D) WORD PROBLEMS
ALGORITHM :

The method of problem solving consists of the following three steps :

Step (i) Translating the word problem into symbolic language (mathematical statement) which means identifying relationships existing in the problem and then forming the quadratic equation.

Step (ii) Solving the quadratic equation thus formed.

Step (iii) Interpreting the solution of the equation, which means translating the result of mathematical statement into verbal language.

Solved Examples

Example. 26

The length of a hall is 5 m more than its breadth. If the area of the floor of the hall is 84 m², what are the length and the breadth of the hall?

Sol. Let the breadth of the hall be x metres.
Then the length of the hall is $(x + 5)$ metres.
The area of the floor = $x(x + 5)$ m²
Therefore, $x(x + 5) = 84$
or $x^2 + 5x - 84 = 0$
or $(x + 12)(x - 7) = 0$
This gives $x = 7$ or $x = -12$.

Since, the breadth of the hall cannot be negative, we reject $x = -12$ and take $x = 7$ only.
Thus, breadth of the hall = 7 metres, and length of the hall = $(7 + 5)$, i.e., 12 metres.

Example. 27

Out of a group of swans, $\frac{7}{2}$ times the square root of the total number are playing on the shore of a tank. The two remaining ones are playing, in deep water. What is the total number of swans?

Sol. Let us denote the number of swans by x .

Then, the number of swans playing on the shore of the tank = $\frac{7}{2}\sqrt{x}$. There are two remaining swans.

Therefore,

$$x = \frac{7}{2}\sqrt{x} + 2$$

$$\text{or } x - 2 = \frac{7}{2}\sqrt{x}$$

$$\text{or } 4(x^2 - 4x + 4) = 49x$$

$$\text{or } 4x^2 - 65x + 16 = 0$$

$$\text{or } 4x(x - 16) - 1(x - 16) = 0$$

$$\text{This gives } x = 16 \text{ or } x = \frac{1}{4}.$$

$$\text{or } (x - 2)^2 = \left(\frac{7}{2}\right)^2 x$$

$$\text{or } 4x^2 - 64x - x + 16 = 0$$

$$\text{or } (x - 16)(4x - 1) = 0$$

$$\text{We reject } x = \frac{1}{4} \text{ and take } x = 16.$$

Hence, the total number of swans is 16.

Example. 28

The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol. Let the length of the shorter side be x cm. Then, the length of the longer side = $(x + 5)$ cm.
Since the triangle is right-angled, the sum of the squares of the sides must be equal to the square of the hypotenuse (pythagoras Theorem).

$$x^2 + (x + 5)^2 = 25^2$$

or $x^2 + x^2 + 10x + 25 = 625$

or $2x^2 + 10x - 600 = 0$

or $x^2 + 5x - 300 = 0$

or $(x + 20)(x - 15) = 0$

This gives $x = 15$ or $x = -20$.

We reject $x = -20$ and take $x = 15$.

Thus, length of shorter side = 15 cm.

Length of longer side = $(15 + 5)$ cm, i.e., 20 cm.

Example. 29

Swati can row her boat at a speed of 5 km/h in still water. If it takes her 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the stream.

Sol. Let the speed of the stream be x km/h.

\therefore Speed of the boat in upstream = $(5 - x)$ km/h .

Speed of the boat in downstream = $(5 + x)$ km/h.

Time, say t_1 (in hours), for going 5.25 km upstream = $\frac{5.25}{5 - x}$

Time, say t_2 (in hours), for returning 5.25 km downstream = $\frac{5.25}{5 + x}$

Obviously $t_1 > t_2$.

Therefore, according to the given condition of the problem,

$$t_1 = t_2 + 1$$

i.e., $\frac{5.25}{5 - x} = \frac{5.25}{5 + x} + 1$

or $\frac{21}{4} \left(\frac{1}{5 - x} - \frac{1}{5 + x} \right) = 1$

or $21 \left(\frac{5 + x - 5 + x}{25 - x^2} \right) = 4$

or $42x = 100 - 4x^2$

or $4x^2 + 42x - 100 = 0$

or $2x^2 + 21x - 50 = 0$

or $(2x + 25)(x - 2) = 0$

This gives $x = 2$, since we reject $x = \frac{-25}{2}$.

Thus, the speed of the stream is 2 km/h.

Example. 30

The sum of the squares of two positive integers is 208. If the square of the larger number is 18 times the smaller number, find the numbers.

Sol. Let x be the smaller number.

Then, square of the larger number will be $18x$.

Therefore,

$$x^2 + 18x = 208$$

or $x^2 + 18x - 208 = 0$

or $(x - 8)(x + 26) = 0$

This gives $x = 8$ or $x = -26$

Since the numbers are positive integers, we reject $x = -26$ and take $x = 8$.

Therefore, square of larger number = $18 \times 8 = 144$.

So, larger number = $\sqrt{144} = 12$.

Hence, the larger number is 12 and the smaller is 8.

Example. 31

The sum 'S' of first n natural numbers is given by the relation $S = \frac{n(n+1)}{2}$. Find n, if the sum is 276.

Sol. We have

$$S = \frac{n(n+1)}{2} = 276$$

$$\text{or } n^2 + n - 552 = 0$$

$$\text{This gives } n = \frac{-1 + \sqrt{1 + 2208}}{2}, \frac{-1 - \sqrt{1 + 2208}}{2}$$

$$\text{or } n = \frac{-1 + \sqrt{2209}}{2}, \frac{-1 - \sqrt{2209}}{2}$$

$$\text{or } n = \frac{-1 + 47}{2}, \frac{-1 - 47}{2}$$

$$\text{or } n = 23, -24$$

We reject $n = -24$, since -24 is not a natural number.

Therefore, $n = 23$.

Check Your Level

1. The sum of the squares of two consecutive natural numbers is 145. Find the numbers.
2. The length of a rectangle is greater than twice its breadth by 2 cm. The length of its diagonal is 13 cm. Find the length and breadth of the rectangle.
3. The sum of the reciprocals of two consecutive odd natural numbers is $\frac{12}{35}$. Find the numbers.
4. A car covers a distance 300 km with same speed, it will cover the same distance in one hour less if its speed is increased by 10 km per hour. Find the speed of the car.
5. A cistern can be filled by two pipes in $33\frac{1}{3}$ minutes; if the larger pipe takes 15 minutes less than the smaller to fill the cistern, find in what time it will be filled by each pipe separately.

Answers

1. 8, 9 2. $l = 12, b = 5$ 3. 5, 7 4. 50 km/hr
5. Slower pipe time = 60 minute , faster pipe = 75 minutes

Exercise Board Level
TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS :
[01 MARK EACH]

1. Check whether the following are quadratic equation ?

- | | |
|--|--|
| (i) $x^2 + 2x + 1 = (4 - x)^2 + 3$ | (ii) $-2x^2 = (5 - x) \left(2x - \frac{2}{5} \right)$ |
| (iii) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k = -1$ | (iv) $x^3 - x^2 = (x - 1)^3$ |
| (v) $2(x - 1)^2 = 4x^2 - 2x + 1$ | (vi) $2x - x^2 = x^2 + 5$ |
| (vii) $(\sqrt{2}x + \sqrt{3})^2 = 3x^2 - 5x$ | (viii) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$ |

2. Check whether the following equations has 2 as a root ?

- | | |
|---------------------------|--------------------------|
| (i) $x^2 - 4x + 5 = 0$ | (ii) $x^2 + 3x - 12 = 0$ |
| (iii) $2x^2 - 7x + 6 = 0$ | (iv) $3x^2 - 6x - 2 = 0$ |

 3. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then find the value of k

4. Check whether the following equations has the sum of its roots as 3 ?

- | | |
|---|--------------------------|
| (i) $2x^2 - 3x + 6 = 0$ | (ii) $-x^2 + 3x - 3 = 0$ |
| (iii) $\sqrt{2}x^2 + \frac{3}{\sqrt{2}}x + 1 = 0$ | (iv) $3x^2 - 3x + 3 = 0$ |

 5. Find the values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

 6. Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square ?

7. Find the nature of the roots of the following quadratic equations

- | | |
|----------------------------------|--|
| (i) $2x^2 - \sqrt{5}x + 1 = 0$ | (ii) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ |
| (iii) $x^2 + 3x + 2\sqrt{2} = 0$ | (iv) $x^2 + x - 5 = 0$ |

TYPE (II) : SHORT ANSWER TYPE QUESTIONS :
[02 MARKS EACH]

8. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

- | | | |
|-------------------------|----------------------------|---|
| (i) $2x^2 - 3x - 5 = 0$ | (ii) $-3x^2 + 5x + 12 = 0$ | (iii) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$ |
|-------------------------|----------------------------|---|

9. Find the roots of the following quadratic equations by the factorisation method :

- | | |
|-----------------------------------|---|
| (i) $2x^2 + \frac{5}{3}x - 2 = 0$ | (ii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ |
|-----------------------------------|---|

10. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number

11. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

12. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

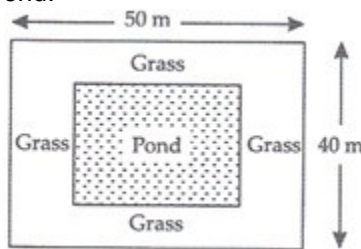
13. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

TYPE (III) : LONG ANSWER TYPE QUESTIONS:
[04 MARK EACH]

14. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.
15. Check whether the equation $6x^2 - 7x + 2 = 0$ has real roots, and if it has, find them by the method of completing the squares.

TYPE (IV) : VERY LONG ANSWER TYPE QUESTIONS
[05 MARK EACH]

16. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m² [see Fig. 4.1]. Find the length and breadth of the pond.



17. At t minutes past 2 pm, the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t .

Previous Year Problems

1. Solve the following quadratic equation for x : **[2 MARKS/CBSE 10TH BOARD: 2010]**
 $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$
2. For what value of k , the roots of the quadratic equation $kx(x - 2\sqrt{5}) + 10 = 0$ are equal ? **[3 MARKS/CBSE 10TH BOARD: 2011]**
3. Sum of the areas of two squares is 400 cm². If the difference of their perimeters is 16cm, find the sides of the two squares. **[4 MARKS/CBSE 10TH BOARD: 2012]**
4. Solve the following for x : **[4 MARKS/ CBSE 10TH BOARD: 2013]**

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$
5. Solve for x : **[2 MARKS/ CBSE 10TH BOARD: 2013]**
 $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$
6. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k . **[3 marks/CBSE 10TH BOARD: 2014]**

7. The sum of the squares of two consecutive even numbers is 340. Find the numbers.
[3 MARKS/CBSE 10TH BOARD: 2014]
8. Solve for x :
 $3\left(\frac{3x-1}{2x+3}\right) - 2\left(\frac{2x+3}{3x-1}\right) = 5$; $x \neq \frac{1}{3}, -\frac{3}{2}$ [4 MARKS/CBSE 10TH BOARD: 2014]
9. If the quadratic equation $px^2 - 2\sqrt{5} px + 15 = 0$ has two equal roots, then find the value of p.
[1 MARK / CBSE 10TH BOARD: 2014]
10. Solve the following quadratic equation for x : [2 MARKS / CBSE 10TH BOARD: 2014]
 $4x^2 + 4bx - (a^2 - b^2) = 0$
11. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field.
[4 MARKS / CBSE 10TH BOARD: 2014]
12. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ?
[4 MARKS/CBSE 10TH BOARD: 2015]
13. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.
[2 MARKS/CBSE 10TH BOARD: 2015]
14. Solve for x : $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$, $x \neq 1, 2, 3$
[3 MARKS/CBSE 10TH BOARD: 2015]
15. Solve for x : $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$, $x \neq -1, -2, -4$ [4 MARKS/CBSE 10TH BOARD: 2015]
16. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.
[4 MARKS/ CBSE 10TH BOARD: 2016]
17. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.
[2 MARKS/CBSE 10TH BOARD: 2017]
18. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.
[3 MARKS/CBSE 10TH BOARD: 2017]
19. Solve for x [4 MARKS/CBSE 10TH BOARD: 2017]
 $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}$, $x \neq -1, -\frac{1}{5}, -4$
20. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ?
[4 MARKS/CBSE 10TH BOARD: 2017]

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction and Method of solving

A-1. Which of the following are quadratic equations ?

(i) $x - \frac{3}{x} = x^2$ (ii) $(2x + 1)(3x + 2) = 6(x - 1)(x - 2)$

A-2. Determine whether the given values are solution of the given equation or not.

(i) $2x^2 - 5x - 3 = 0$; $x = 3$ (ii) $x + \frac{1}{x} = \frac{13}{6}$; $x = \frac{5}{6}$, $x = \frac{4}{3}$

A-3. Find the value of K for which the given value is a solution of the equation.

(i) $3x^2 + 2x - K = 0$, $x = \frac{1}{3}$ (ii) $Kx^2 - 3abx + 2b^2 = 0$, $x = \frac{b}{a}$

A-4. Is $x = -4$, a solution of the equation $2x^2 + 5x - 12 = 0$?

A-5. If $x = -2$ and $x = \frac{1}{5}$ are solutions of the equations $5x^2 + kx + \lambda = 0$. Find the value of k and λ .

A-6. Solve each of the following quadratic equation by factorization method :

(i) $\left(x - \frac{1}{2}\right)^2 = 4$
 (ii) $x^2 - 2bx + b^2 - a^2 = 0$

A-7. Find the roots of the equations by method of completing the squares.

(i) $8x^2 + 2x - 3 = 0$ (ii) $x^2 - 4ax + 4a^2 - b^2 = 0$

A-8. Find the roots of the quadratic equations by using the quadratic formula in each of the following

(i) $x^2 + 2\sqrt{2}x - 6 = 0$ (ii) $5x^2 - 5x - 3 = 0$

Section (B) : Relation between roots and coefficients

B-1. The product of the root of the equation $x^2 - 8x + k = 0$ is 12. Find the value of k

B-2. Show that the condition that one root is twice the other root of the quadratic equation $x^2 + px + q = 0$ is $2p^2 = 9q$

B-3. Find the value of k such that the equation $x^2 + (k-1)x + 1 = 0$ has such two roots that are equal in magnitude but opposite in signs.

B-4. The roots of the equation $x^2 + Ax + B = 0$ are 5 and 4. The roots of $x^2 + Cx + D = 0$ are 2 and 9 then find the root of $x^2 + Ax + D = 0$?

B-5. If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal, then show that $b^2 = ac$

Section (C) : Nature of roots

- C-1.** Find the discriminant of the quadratic equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.
- C-2.** For what value of K the equation $9x^2 + 3kx + 4 = 0$ has equal roots?
- C-3.** If (-5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.
- C-4.** Find the value of k for which quadratic equation $(k - 2)x^2 + 2(2k - 3)x + 5k - 6 = 0$ has equal roots.
- C-6.** Show that the roots of the equation $x^2 + ax - 4 = 0$ (where $a \in \mathbb{R}$) are real and distinct

Section (D) : Word problems

- D-1.** A man is five times as old as his son and the sum of the squares of their ages is 2106. Find their ages.
- D-2.** The lengths of the sides of a right triangle are $5x + 2$, $5x$ and $3x - 1$. If $x > 0$ find the length of each sides.
- D-3.** The sum of two numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$.
- D-4.** The numerator of a fraction is 1 less than its denominator. If 3 is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction.
- D-5.** A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented
Section (A) : Introduction and Method of solving

- A-1.** Which of the following equations has -1 as a root ?
 (A) $x^2 + 3x - 10 = 0$ (B) $x^2 - x - 12 = 0$ (C) $3x^2 - 2x - 5 = 0$ (D) $9x^2 + 24x + 16 = 0$
- A-2.** The value of k for which 3 is a root of the equation $kx^2 - 7x + 3 = 0$ is :
 (A) 2 (B) -2 (C) 3 (D) -3
- A-3.** The roots of the equation $x^2 - 9x + 20 = 0$ are :
 (A) 4 and 5 (B) -4 and 5 (C) 4 and -5 (D) -4 and -5
- A-4.** Solve the quadratic equation : $\frac{x-1}{x-2} - \frac{x-2}{x-3} = \frac{x-5}{x-6} - \frac{x-6}{x-7}$.
 (A) $\frac{9}{2}$ (B) $\frac{3}{2}$ (C) $\frac{7}{2}$ (D) $\frac{1}{2}$
- A-5.** Solve for x : $3^{x+2} + 3^{-x} = 10$.
 (A) $-2, 0$ (B) $-1, 0$ (C) $-2, -1$ (D) None of these

Section (B) : Relation between roots and coefficients

- B-1.** If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other then $k =$
 (A) 0 (B) 5 (C) $\frac{1}{6}$ (D) 6
- B-2.** Which of the following equations has the sum of its roots as 2.
 (A) $5x^2 - \frac{2}{5}x - 3 = 0$ (B) $x^2 - 2x - 5 = 0$ (C) $2x^2 - 2x + 1 = 0$ (D) $3x^2 - 2x + 7 = 0$
- B-3.** The product of the solutions of the equation $\frac{3}{x} = \frac{8}{x-3} - 1$, is :
 (A) -6 (B) -9 (C) 9 (D) 10
- B-4.** The sum of the reciprocals of the roots of the equation, $\frac{2009}{2010}x + 1 + \frac{1}{x} = 0$, is :
 (A) $-\frac{2010}{2009}$ (B) -1 (C) $\frac{2009}{2010}$ (D) 1
- B-5.** A quadratic equation $ax^2 - 2ax + b = 0$ has two real solutions. Their average is :
 (A) 1 (B) 2 (C) $\frac{b}{a}$ (D) $\frac{2b}{a}$

Section (C) : Nature of roots

- C-1.** The roots of the equation $x^2 - x - 3 = 0$ are :
 (A) Imaginary (B) Rational (C) Irrational (D) None of these
- C-2.** If one of the roots of the quadratic equation having rational coefficients is $2 + \sqrt{3}$ then find the quadratic equation.
 (A) $x^2 - (2 + \sqrt{3})x + 1 = 0$ (B) $x^2 + (2 + \sqrt{3})x + 1 = 0$
 (C) $x^2 - 4x + 1 = 0$ (D) $x^2 + 4x - 1 = 0$
- C-3.** Which of the following equations has two equal real roots ?
 (A) $3x^2 + 14x - 5 = 0$ (B) $4x^2 + 2x - 1 = 0$ (C) $9x^2 - 6x + 1 = 0$ (D) $x^2 - 5x + 4 = 0$
- C-4.** If the equation $16x^2 + 6kx + 4 = 0$ has equal roots, then the value of k is :
 (A) ± 8 (B) $\pm \frac{8}{3}$ (C) $\pm \frac{3}{8}$ (D) 0
- C-5.** The quadratic equations $x^2 - 5x + 3 = 0$ has :
 (A) no real roots (B) two distinct real roots
 (C) two equal real roots (D) more than two real roots

Section (D) : Word problems

- D-1.** The difference between two numbers is 5 and difference in their squares is 65. The larger number is :
 (A) 9 (B) 10 (C) 11 (D) 12
- D-2.** The sum of ages of a father and son is 45 years. Five years ago, the product of their ages was 4 times the age of the father at that time. The present age of the father is :
 (A) 30 yrs (B) 31 yrs (C) 36 yrs (D) 41 yrs

- D-3.** The sum of the squares of two consecutive positive integers is 545. Find the sum of these integers.
 (A) 33 (B) 34 (C) 35 (D) 36
- D-4.** The sides (in cm) of a right triangle containing the right angle are $5x$ and $3x - 1$. If the area of the triangle is 60 cm^2 . Find its perimeter.
 (A) 30 (B) 35 (C) 40 (D) 45
- D-5.** A two digit number is four times the sum and three times the product of its digits, find the number.
 (A) 24 (B) 42 (C) 36 (D) 63

Exercise-2

OBJECTIVE QUESTIONS

- The number of roots of the equation $x - \frac{2}{(x-1)} = 1 - \frac{2}{(x-1)}$ is :
 (A) 0 (B) 1 (C) 2 (D) infinite
- The set of all real values of p for which the equation $x + 1 = \sqrt{px}$ has exactly one root, is :
 (A) $\{0\}$ (B) $\{4\}$ (C) $\{0, 4\}$ (D) $\{0, 2\}$
- Solve for x : $2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$.
 (A) $\frac{1}{2}, 1, 2$ (B) $\frac{1}{2}, 1, -2$ (C) $\frac{1}{2}, 1, 4$ (D) $\frac{1}{2}, 1, -4$
- If $\sin a$ and $\cos a$ are the roots of the equation $4x^2 - kx - 1 = 0$, ($k > 0$) then the value of k is :
 (A) $2\sqrt{2}$ (B) 4 (C) 2 (D) $4\sqrt{2}$
- The roots of equation $x^2 + px + q = 0$ are 1 and 2. The roots of the equation $qx^2 - px + 1 = 0$ must be
 (A) $-1, -\frac{1}{2}$ (B) $\frac{1}{2}, 1$ (C) $-\frac{1}{2}, 1$ (D) $-1, \frac{1}{2}$
- If the roots of the equation $px^2 + qx + r = 0$ are in the ratio $\ell : m$ then :
 (A) $(\ell + m)^2 pq = \ell mr^2$ (B) $(\ell + m)^2 pr = \ell mq$ (C) $(\ell + m)^2 pr = \ell mq^2$ (D) None of these
- If $2 - \sqrt{3}$ is a root of the quadratic equation $x^2 + 2(\sqrt{3} - 1)x + 3 - 2\sqrt{3} = 0$, then the second root is
 (A) $\sqrt{3} - 2$ (B) $\sqrt{3}$ (C) $2 + \sqrt{3}$ (D) $-\sqrt{3}$
- $-2 + \sqrt{3}$ is a solution to which of the following equations ?
 (A) $3x^2 + 12x + 3 = 0$ (B) $x^2 + 2x + 3 = 0$ (C) $x^2 + 4x + 2 = 0$ (D) $3x^2 + 2x - 1 = 0$
- The value (s) of k for which the quadratic equation $kx^2 - kx + 1 = 0$ has equal roots is :
 (A) $k = 0$ only (B) $k = 4$ only (C) $k = 0, 4$ (D) $k = -4$
- An aeroplane left 30 minutes later than its scheduled time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/h from its usual speed. Determine its usual speed.
 (A) 700 km/h (B) 750 km/h (C) 770 km/h (D) 800 km/h

11. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
 (A) 25, 15 hr (B) 15, 5 hr (C) 35, 25 hr (D) 10, 20 hr
12. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present age of her sister.
 (A) 12 yr (B) 6 yr (C) 8 yr (D) 9 yr
13. One of the two students, while solving a quadratic equation in x , copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The correct roots are :
 (A) 3, -2 (B) $-3, 2$ (C) $-6, -1$ (D) 6, -1
14. Out of a number of saras birds, one fourth of the number are moving about in lots, $\frac{1}{9}$ th along with $\frac{1}{4}$ th as well as 7 times the square root of the number move on a hill, 56 birds remains in vakula trees. What is the total number of birds ?
 (A) 576 (B) 500 (C) 676 (D) 400
15. If n is an integer such that $n \times n = n + n$, then the number of such number n , is :
 (A) 0 (B) 1 (C) 2 (D) 3
16. A positive number whose reciprocal equals one less than the number, is :
 (A) $\frac{(1+\sqrt{2})}{2}$ (B) $\frac{(\sqrt{2}-1)}{4}$ (C) $\frac{(1+\sqrt{5})}{2}$ (D) $\frac{(\sqrt{2}+\sqrt{5})}{2}$
17. If α is a root, repeated twice, of the quadratic equation $(a-d)x^2 + ax + (a+d) = 0$ then $\frac{d^2}{a^2}$ has the value equal to :
 (A) $\sin^2 90^\circ$ (B) $\cos^2 60^\circ$ (C) $\sin^2 45^\circ$ (D) $\cos^2 30^\circ$
18. Find the quadratic equation whose roots are half of the reciprocal of the roots of the equation $ax^2 + bx + c = 0$.
 (A) $4ax^2 + 2bx + c = 0$ (B) $4cx^2 + 2bx + a = 0$
 (C) $2cx^2 + bx + a = 0$ (D) $2ax^2 + bx + c = 0$
19. $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}} = ?$
 (A) 3 (B) 6 (C) 9 (D) ∞

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. The value of x in the equation $\frac{3x+4}{2x+1} = \frac{x-8}{2x-5}$ is: **[Raj. NTSE Stage-1 2006]**
 (A) -1 (B) -3 (C) -4 (D) -5

2. If one root is $3 + \sqrt{5}$, then quadratic equation will be : **[Raj. NTSE Stage-1 2006]**
 (A) $x^2 + 6x - 4 = 0$ (B) $x^2 + 6x + 4 = 0$ (C) $x^2 - 6x + 4 = 0$ (D) $x^2 - 6x - 4 = 0$
3. The value of x in the equation $\frac{x-1}{x+1} = \frac{x+5}{2x+5}$ is: **[Raj. NTSE Stage-1 2007]**
 (A) -1 (B) -5 (C) 1 (D) 5
4. If one root of a quadratic equation is $\frac{1}{3-\sqrt{2}}$, then the equation will be : **[Raj. NTSE Stage-1 2007]**
 (A) $7x^2 - 6x + 1 = 0$ (B) $6x^2 - 7x + 1 = 0$ (C) $x^2 - 6x + 7 = 0$ (D) $x^2 - 7x + 6 = 0$
5. If the sum of the two roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero, then the product of the two roots is —. **[Orissa NTSE Stage-1 2012]**
 (A) 0 (B) $\frac{a^2 + b^2}{2}$ (C) $\frac{a+b}{2}$ (D) $-\frac{(a^2 + b^2)}{2}$
6. Roots of $ax^2 + b = 0$ are real and distinct if : **[Orissa NTSE Stage-1 2013]**
 (A) $ab > 0$ (B) $a > 0, b > 0$ (C) $ab = 0$ (D) $ab < 0$
7. If the roots of the equation $\alpha x^2 + \beta x + \gamma = 0$ are 1 and 2, then one of the roots of the equation $\beta x^2 + \alpha x + \gamma = 0$ is : **[Orissa NTSE Stage-1 2013]**
 (A) 1 (B) 0 (C) -2 (D) 2
8. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to product of their reciprocal then, **[MP NTSE Stage-1 2013]**
 (A) $a^2 + bc = 0$ (B) $b^2 + ca = 0$ (C) $c^2 + ab = 0$ (D) $b + c = 0$
9. If one root of $x^2 - 4x + k = 0$ is 6 then the value of k is : **[MP NTSE Stage-1 2013]**
 (A) -12 (B) 2 (C) -2 (D) 12
10. Which of the following quadratic equation has sum of their roots 4 and sum of the cubes of their roots as 28 ? **[Maharashtra NTSE Stage-1 2013]**
 (A) $x^2 - 4x + 3 = 0$ (B) $x^2 - 4x - 5 = 0$ (C) $x^2 - 3x + 4 = 0$ (D) $x^2 + 4x + 3 = 0$
11. A bus takes 5 hours more than a train to cover the distance of 900 km from Vardha to Pune. If speed of the train is 15 km/hr more than that of the bus, then what is the speed of bus per hour ? **[Maharashtra NTSE Stage-1 2013]**
 (A) 60 km (B) 75 km (C) 55 km (D) 45 km
12. If $b^2 - 4ac \geq 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ is : **[Raj. NTSE Stage-1 2013]**
 (A) $\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ (B) $-\frac{b}{2a} \pm \frac{\sqrt{b^2 + 4ac}}{2a}$ (C) $\frac{b}{2a} \pm \frac{\sqrt{b^2 + 4ac}}{2a}$ (D) $-\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
13. Graph drawn from the equation $y = x^2 - 3x - 4$ will be : **[Raj. NTSE Stage-1 2013]**
 (A) Circle (B) Parabola (C) Straight line (D) Hyperbola
14. If $\alpha + \beta = 3$ and $\alpha^3 + \beta^3 = 9$, find the quadratic equation whose roots are α and β : **[Maharashtra NTSE Stage-1 2014]**
 (A) $x(x - 2) = 3$ (B) $x + \frac{2}{x} + 3 = 0$ (C) $x^2 - 2x + 3 = 0$ (D) $x + \frac{2}{x} = 3$

27. If α and β are the roots of the quadratic equation $x^2 - 12x + 35 = 0$, then find the value of $2 \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \right)$:
[Maharashtra NTSE Stage-1 2015]
 (A) $\frac{74}{35}$ (B) $\frac{148}{35}$ (C) $\frac{12}{35}$ (D) $\frac{34}{35}$
28. The sum of the squares of two consecutive natural numbers is 313. Then the numbers will be
[MP NTSE Stage-1 2015]
 (A) 12, 13 (B) 13, -12 (C) 12, -13 (D) -12, -13
29. If the difference of two numbers is 5 and difference of their squares is 300, then sum of the numbers is
[Raj. NTSE Stage-1 2016]
 (A) 1500 (B) 6 (C) 12 (D) 60
30. If the equation $ax^2 + 2x - 2 = 0$ has real and distinct roots, then the value of a is
[Raj. NTSE Stage-1 2016]
 (A) $a > \frac{-1}{2}$ (B) $a \leq \frac{-1}{2}$ (C) $a \geq \frac{-1}{2}$ (D) $a = \frac{-1}{2}$
31. If the sum of two numbers is 22 and sum of their squares is 404 then the product of the number is
[Bihar NTSE Stage-1 2016]
 (A) 40 (B) 44 (C) 80 (D) 88
32. If $\sin 87^\circ$ and $\cos 87^\circ$ are the roots of the equation $x^2 - bx + c = 0$, then the value of b^2 is :
[Haryana NTSE Stage-1 2016]
 (A) c (B) $1 + 2c$ (C) $1 - c^2$ (D) $1 + c^2$
33. If $\frac{x^2 - bx}{ax - c} = \frac{m - 1}{m + 1}$, has roots which are numerically equal but of opposite signs. the value of m must be
[Delhi NTSE Stage-1 2016]
 (A) $(a - b) / (a + b)$ (B) $(a + b) / (a - b)$ (C) c (D) $\frac{1}{c}$
34. Find the quadratic equation whose one root is $2 + \sqrt{5}$ **[Maharashtra NTSE Stage-1 2016]**
 (A) $x^2 - 4x + 1 = 0$ (B) $x^2 - 4x - 1 = 0$
 (C) $x^2 - 4x + 3 = 0$ (D) $x^2 - 4x - 3 = 0$
35. Out of a group of Swans, $\frac{7}{2}$ times the square root of number of Swans are playing on the shore of the tank. Remaining two are quarrelling in the water. Calculate the total number of Swans. Find the number of Swans playing on the shore of the tank. **[Maharashtra NTSE Stage-1 2016]**
 (A) 14, 16 (B) 16, 12 (C) 14, 12 (D) 16, 14
36. The roots of the equation: $3x^2 - 4\sqrt{3}x + 4 = 0$ are : **[MP NTSE Stage-1 2016]**
 (A) Real & unequal (B) Real & equal (C) Imaginary (D) Real & Imaginary both
37. The perimeter of a rectangular field is 82 m and area is 400 m². Then the breadth of the field is : **[MP NTSE Stage-1 2016]**
 (A) 9 m (B) 12 m (C) 20 m (D) 25 m

38. The sum of squares of the two consecutive natural numbers is 421, the numbers are : **[MP NTSE Stage-1 2016]**
 (A) 14 , 15 (B) 21 , 22 (C) 9 , 10 (D) 17 , 18
39. The difference between the two roots of a quadratic equation is 2 and the difference between the cubes of the roots is 98, then which of the following is that quadratic equation? **[Maharashtra NTSE Stage-1 2017]**
 (A) $x^2 - 8x + 15 = 0$ (B) $x^2 + 8x - 15 = 0$ (C) $x^2 + 5x + 15 = 0$ (D) $x^2 - 5x - 15 = 0$
40. One of the root of a quadratic equation is $(3 - \sqrt{2})$, then which of the following is that equation? **[Maharashtra NTSE Stage-1 2017]**
 (A) $(x^2 - 6x - 7) = 0$ (B) $(x^2 + 6x - 7) = 0$ (C) $(x^2 + 6x + 7) = 0$ (D) $(x^2 - 6x + 7) = 0$
41. For what value of k , the equation $3x^2 + 2x + k = 0$ will have real roots : **[MP NTSE Stage-1 2017]**
 (A) $k \leq \frac{1}{3}$ (B) $k \geq \frac{2}{3}$ (C) $k = \frac{2}{3}$ only (D) None of these
42. The product of Meera's age 5 years ago and her age 8 years later is 30. Her present age is - **[MP NTSE Stage-1 2017]**
 (A) 11 years (B) 9 years (C) 7 years (D) 5 Years
43. The sum of the roots of quadratic equation $2x + \frac{4}{x} = 9$ is **[UP NTSE Stage-1 2017]**
 (A) $\frac{7}{2}$ (B) $\frac{9}{2}$ (C) 3 (D) $-\frac{9}{2}$

Answer Key
Exercise Board Level
TYPE (I)

- | | | | | | |
|----|-------------------|---------------------------|---------------------|------------------------------|---------|
| 1. | (i) No | (ii) No | (iii) No | (iv) Yes | (v) Yes |
| | (vi) Yes | (vii) Yes | (viii) No | | |
| 2. | (i) No | (ii) No | (iii) Yes | (iv) No | |
| 3. | 2 | | | | |
| 4. | (i) No | (ii) Yes | (iii) No | (iv) No | |
| 5. | 0, 8 | 6. | $\frac{1}{64}$ | | |
| 7. | (i) no real roots | (ii) real and equal roots | (iii) no real roots | (iv) real and distinct roots | |

TYPE (II)

- | | | | |
|-------|---------------------------------|--------------------------------------|--------------------------------------|
| 8. | (i) $\frac{5}{2}, -1$ | (ii) $\frac{-4}{3}, 3$ | (iii) $3 + \sqrt{11}, \sqrt{11} - 3$ |
| 9. | (i) $\frac{2}{3}, -\frac{3}{2}$ | (ii) $\frac{-\sqrt{2}}{6}, \sqrt{2}$ | 10. 12 |
| 11. 8 | | 12. 45 km/h | 13. 14 years |

TYPE (III)

- | | |
|--|---|
| 14. Nisha's age = 5 years, Asha's age = 27 years | 15. Yes, $x = \frac{2}{3}, \frac{1}{2}$ |
|--|---|

TYPE (IV)

- | | |
|-----------------------------------|-------------|
| 16. Length = 34 m, Breadth = 24 m | 17. 14 min. |
|-----------------------------------|-------------|

Previous Year Problems

- | | | | |
|---|--|--|--------------------------------|
| 1. $x = \frac{-2}{\sqrt{3}}$ and $\frac{\sqrt{3}}{4}$ | 2. $k = 2$ | 3. 16 & 12 | 4. $x = -a$ & $-b/2$ |
| 5. $x = \sqrt{6}$ & $-\sqrt{2/3}$ | 6. $k = 1$ | 7. 12 & 14 | 8. $x = 0$ & -7 |
| 9. $p = 3$ | 10. $\frac{a-b}{2}$ & $\frac{-a-b}{2}$ | 12. 36 km/hr | 13. $k = 7/4$ |
| 11. Length of Sides = 10 & 24 | | 15. $x = 2(\sqrt{3} + 1)$ & $2(-\sqrt{3} + 1)$ | |
| 14. $x = 0$ & 4 | | 17. $p = 3$ | 19. $x = 1$ & $\frac{-11}{17}$ |
| 16. Speed of stream = 8 km/hr | | 20. 5 hrs & 8 Hrs | |

Exercise-1
SUBJECTIVE QUESTIONS
Subjective Easy, only learning value problems
Section (A)

- A-1.** (i) No (ii) No **A-2.** (i) Yes (ii) No
A-3. (i) $k = 1$ (ii) $k = a^2$ **A-4.** yes **A-5.** $\lambda = -2$ and $k = 9$.
A-6. (i) $x = 5/2$ or $-3/2$ (ii) $x = b-a$ or $b+a$
A-7. (i) $x = \frac{1}{2}, \frac{-3}{4}$ (ii) $x = b + 2a$ and $x = 2a - b$
A-8. (i) $\sqrt{2}$ and $-3\sqrt{2}$ (ii) $\frac{5 \pm \sqrt{85}}{10}$

Section (B)

- B-1.** $k = 12$ **B-3.** $k = 1$ **B-4.** 6 and 3

Section (C)

- C-1.** 64 **C-2.** $k = \pm 4$ **C-3.** p and k are 7 and $\frac{7}{4}$ respectively
C-4. $k = 1$ or $k = 3$.

Section (D)

- D-1.** Age of son = 9 years, Age of father = 45 years **D-2.** 17, 15 and 8
D-3. 3 and 5. **D-4.** $\frac{3}{4}$ **D-5.** 6 km/hr.

OBJECTIVE QUESTIONS
Section (A)

- A-1.** (C) **A-2.** (A) **A-3.** (A) **A-4.** (A) **A-5.** (A)

Section (B)

- B-1.** (B) **B-2.** (B) **B-3.** (B) **B-4.** (B) **B-5.** (A)

Section (C)

- C-1.** (C) **C-2.** (C) **C-3.** (C) **C-4.** (B) **C-5.** (B)

Section (D)

- D-1.** (A) **D-2.** (C) **D-3.** (A) **D-4.** (C) **D-5.** (A)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	A	C	A	A	A	C	D	A	B	B	A	B	D	A	C	C	D	B	A

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	D	A	D	D	A	A	A	A	D	D	B	D	B	B	C	C	C	B
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	C	C	D	C	C	C	B	A	D	A	A	B	A	B	D	B	D	A	A	D
Ques.	41	42	43																	
Ans.	A	C	B																	