SOLVED EXAMPLES

Ex.1 If $y = (1+x^{1/4})(1+x^{1/2})(1-x^{1/4})$, then dy/dx equals-

(A) –1 (B) 1 (C) x (D) $\sqrt{x}$

Sol. $y = (1+x^{1/2})(1-x^{1/2}) = 1-x$

$\therefore$ $dy/dx = -1$ **Ans.[A]**

Ex.2 If $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$, $\frac{dx}{d\theta}$ then dy/dx equals -

(A) $\tan\theta$ (B) $\cot\theta$

(C) $\tan\frac{1}{2}\theta$ (D) $\cot\frac{1}{2}\theta$

Sol. $\frac{dx}{d\theta} = a(1+\cos)$, $\frac{dx}{d\theta} = a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{1}{2}\theta$

Ans.[C]

Ex.3 If $y = \log\left(\frac{e^x}{e^x+1}\right)$, then dy/dx equals -

(A) $\frac{1}{e^x+1}$ (B) $\frac{1}{(e^x+1)^2}$

(C) $\frac{e^x-1}{e^x+1}$ (D) None of these

Sol. $y = \log e^x - \log(e^x+1)$

$= x - \log(e^x+1)$

$\therefore \frac{dy}{dx} = 1 - \frac{e^x}{e^x+1} = \frac{1}{e^x+1}$ **Ans.[A]**

Ex.4 If $y = \frac{1}{x^2-a^2}$, then $\frac{d^2y}{dx^2}$ equals -

(A) $\frac{3x^2+a^2}{(x^2-a^2)^3}$ (B) $\frac{3x^2+a^2}{(x^2-a^2)^4}$

(C) $\frac{2(3x^2+a^2)}{(x^2-a^2)^3}$ (D) $\frac{2(3x^2+a^2)}{(x^2-a^2)^4}$

Sol. $\frac{dy}{dx} = \frac{-2x}{(x^2-a^2)^2} \Rightarrow \frac{d^2y}{dx^2}$

$= -\frac{(x^2-a^2)^2.2 - 2x.2(x^2-a^2).2x}{(x^2-a^2)^4}$

$= \frac{2(3x^2+a^2)}{(x^2-a^2)^3}$ **Ans.[C]**

Ex.5 If $y = \frac{\sec x - \tan x}{\sec x + \tan x}$, then $\frac{dy}{dx}$ equals -

(A) $2\sec x(\sec x - \tan x)^2$

(B) $-2\sec x(\sec x - \tan x)^2$

(C) $2\sec x(\sec x + \tan x)^2$

(D) $-2\sec x(\sec x + \tan x)^2$

Sol. $y = \frac{\sec x - \tan x}{\sec x + \tan x} \cdot \frac{\sec x - \tan x}{\sec x - \tan x}$

$= (\sec x - \tan x)^2/1$

$\therefore \frac{dy}{dx} = 2(\sec x - \tan x)(\sec x\tan x - \sec^2 x)$

$= -2\sec x(\sec x - \tan x)^2$ **Ans.[B]**

Ex.6 If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx}$ equals -

(A) $\frac{1}{(1+x)^2}$ (B) $-\frac{1}{(1+x)^2}$

(C) $\frac{1}{1+x^2}$ (D) None of these

Sol. Let us first express y in terms of x because all alternatives are in terms of x. So

$x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 + x^2y - y^2x = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow x + y + xy = 0$ $(\because x \neq y)$

$\Rightarrow y = -\frac{x}{1-x}$

$\therefore \frac{dy}{dx} = -\frac{(1+x)1 - x.1}{(1+x)^2} = -\frac{1}{(1+x)^2}$ **Ans.[B]**

Ex.7 If $y = \sin^{-1}\sqrt{\sin x}$, then $\frac{dy}{dx}$ equals -

(A) $\frac{2\sqrt{\sin x}}{\sqrt{1+\sin x}}$ (B) $\frac{\sqrt{\sin x}}{\sqrt{1-\sin x}}$

(C) $\frac{1}{2}\sqrt{1+\operatorname{cosec} x}$ (D) $\frac{1}{2}\sqrt{1-\operatorname{cosec} x}$

Sol. $\frac{dy}{dx} = \frac{1}{\sqrt{1-\sin x}}.\frac{1}{2\sqrt{\sin x}}.\cos x$

$= \frac{\sqrt{1+\sin x}}{2\sqrt{\sin x}} = \frac{1}{2}\sqrt{1+\operatorname{cosec} x}$ **Ans.[C]**

Ex.8 If $y = \log_x 10$, then the value of dy/dx equals-

(A) 1/x (B) 10/x

(C) $-\frac{(\log_x 10)^2}{x\log_e 10}$ (D) $\frac{1}{(x\log_e 10)}$

Sol. $y = \log_x 10 = \frac{\log_e 10}{\log_e x}$

$\therefore \frac{dy}{dx} = \log_e 10 \left\{-\frac{1}{(\log_e x)^2}.\frac{1}{x}\right\}$

$= -\frac{1}{x\log_e 10}.\frac{(\log_e 10)^2}{(\log_e x)^2}$

$= -\frac{(\log_e 10)^2}{x\log_e 10}$ **Ans.[C]**

Ex.9 If cos (xy) = x ,then $\frac{dy}{dx}$ is equal to -

(A) $\frac{y+\operatorname{cosec}(xy)}{x}$ (B) $\frac{y+\sin(xy)}{x}$

(C) $\frac{y+\cos(xy)}{x}$ (D) $-\frac{y+\operatorname{cosec}(xy)}{x}$

Sol. $\because \cos(xy) - x = 0$

$\therefore \frac{dy}{dx} = -\frac{-y\sin(xy)-1}{-x\sin(xy)} = -\frac{y+\operatorname{cosec}(xy)}{x}$

Ans.[D]

Ex.10 If $x^2 e^y + 2xye^x + 13 = 0$, then dy/dx equals -

(A) $-\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$ (B) $\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$

(C) $-\frac{2xe^{x-y}+2y(x+1)}{x(xe^{x-y}+2)}$ (D) None of these

Sol. Let $f(x,y) = x^2e^y + 2xye^x + 13$

$\therefore \frac{dy}{dx} = -\frac{\partial f}{\partial x}/\frac{\partial f}{\partial y}$

$= -\frac{2xe^y+2ye^x+2xye^x}{x^2e^y+2xe^x}$

Dividing Num^r and Den^r by e^x

$\frac{dy}{dx} = -\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$ **Ans.[A]**

Ex.11 If $x^y y^x = 1$, then $\frac{dy}{dx}$ equals -

(A) $\frac{x(y+x\log y)}{y(x+y\log x)}$

(B) $-\frac{x(x+y\log y)}{y(y+x\log x)}$

(C) $\frac{y(y+x\log y)}{x(x+y\log x)}$

(D) $-\frac{y(y+x\log y)}{x(x+y\log x)}$

Sol. Taking log on both sides, we have

y log x + x log y = 0

Now using partial derivatives, we have

$\frac{dy}{dx} = -\frac{y/x+\log y}{\log x + x/y} = -\frac{y(y+x\log y)}{x(x+y\log x)}$ **Ans [D]**

Ex.12 If $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$, then dy/dx equals-

(A) $x\,[1+\tan(\log x)] + \sec^2(\log x)$

(B) $2x\,[1+\tan(\log x)] + x\sec^2(\log x)$

(C) $2x\,[1+\tan(\log x)] + x\sec(\log x)$

(D) None of these

Sol. $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$

Taking logarithm of both the sides, we get

$\log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$

$\Rightarrow y = x^2 + x^2 \tan(\log x)$

$dy/dx = 2x + 2x\tan(\log x) + x^2 \sec^2(\log x).\ \frac{1}{x}$

$= 2x\,[1 + \tan(\log x)] + x\sec^2(\log x)$.**Ans.[B]**

Ex.13 If $y = \tan^{-1}\frac{3x - x^3}{1 - 3x^2}$, then dy/dx equals-

(A) 3x (B) tan 3x

(C) $\frac{3}{1+x^2}$ (D) $3\tan^{-1} x$

Sol. $y = \tan^{-1}\frac{3x - x^3}{1 - 3x^2} = 3\tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{3}{1+x^2}$ **Ans.[C]**

Ex.14 If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ equals -

(A) $\frac{2x}{1+x^2}$ (B) $\frac{2}{1+x^2}$

(C) $-\frac{2x}{1+x^2}$ (D) $-\frac{2}{1+x^2}$

Sol. $y = 2\tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$ **Ans.[B]**

Ex.15 If $y = \tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$, then $\frac{dy}{dx}$ equals -

(A) $-\frac{1}{2\sqrt{1-x^2}}$ (B) $-\frac{1}{\sqrt{1-x^4}}$

(C) $-\frac{x}{\sqrt{1-x^4}}$ (D) $-\frac{x}{2\sqrt{1-x^4}}$

Sol. $y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right)$, where

$x^2 = \cos\theta$

$= \tan^{-1}\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right)$

$= \tan^{-1}\left(\frac{1+\tan\theta/2}{1-\tan\theta/2}\right)$

$= \tan^{-1}[\tan(\pi/4 + \theta/2)] = \pi/4 + \theta/2$

$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

$\therefore \frac{dy}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1-x^4}}\,.\,2x = -\frac{x}{\sqrt{1-x^4}}$ **Ans.[C]**

Ex.16 If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, then the value of dy/dx is -

(A) $\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (B) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(C) $-\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (D) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Sol. Substituting $x = \sin\theta$ and $y = \sin\phi$ in the given equation, we get

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow 2\cos\frac{\theta+\phi}{2}.\cos\frac{\theta-\phi}{2} = 2a\cos\frac{\theta+\phi}{2}.\sin\frac{\theta-\phi}{2}$

$\Rightarrow \cot\frac{\theta-\phi}{2} = a \Rightarrow \theta - \phi = 2\cot^{-1} a$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

Differentiating with respect to x, we get

$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ **Ans.[B]**

Ex.17 If $y = \sin^{-1}\frac{2x}{1+x^2}$, $z = \tan^{-1} x$, then the value of dy/dz is -

(A) $\frac{1}{1+x^2}$ (B) $\frac{2}{1+x^2}$

(C) 2 (D) None of these

Sol. $y = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1} x$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{1+x^2}$

and $z = \tan^{-1} x \Rightarrow \frac{dz}{dx} = \frac{1}{1+x^2}$

$$\therefore \frac{dz}{dx} = \frac{dy/dx}{dz/dx} = \frac{2}{1+x^2}.\frac{1+x^2}{1} \quad 2$$ **Ans.[C]**

Ex.18 If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$, then dy/dx equals -

(A) $\frac{\sin x}{2y+1}$ (B) $\frac{\cos x}{2y-1}$

(C) $\frac{\cos x}{2y+1}$ (D) None of these

Sol. Here $y = \sqrt{\sin x + y}$ $\Rightarrow y^2 = \sin x + y$

$$\therefore 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$$ **Ans.[B]**

Ex.19 If $y = \cfrac{x}{a + \cfrac{x}{b + \cfrac{x}{a + \cfrac{x}{b + ...\infty}}}}$ ....∞, then equals -

(A) $\frac{b}{a(b+2y)}$ (B) $\frac{a}{b(a+2y)}$

(C) $\frac{a}{b(b+2y)}$ (D) None of these

Sol. Here $y = \cfrac{x}{a + \cfrac{x}{b+y}} = \frac{x(b+y)}{a(b+y)+x}$

$\Rightarrow aby + ay^2 + xy = bx + xy$

$\Rightarrow ay^2 + aby = bx$

$\Rightarrow 2ay\frac{dy}{dx} + ab\frac{dy}{dx} = b$

$\Rightarrow \frac{dy}{dx} = \frac{b}{a(b+2y)}$ **Ans.[A]**

Ex.20 If $e^{x+e^{x+e^{x+......\infty}}}$, then dy/dx is -

(A) $\frac{y}{1+y}$ (B) $\frac{y}{y-1}$

(C) $\frac{y}{1-y}$ (D) None of these

Sol. $y = e^{x+y}$

$\Rightarrow \log y = x + y \quad \Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{1-y}$ **Ans.[C]**

Ex.21 If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ equals -

(A) $\tan^2\theta$ (B) $\sec^2\theta$

(C) $\sec\theta$ (D) $|\sec\theta|$

Sol. $\frac{dy}{dx} = \left(\frac{dy}{d\theta}\right)/\left(\frac{dx}{d\theta}\right)$

$= \frac{3a\sin^2\theta.\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$

$\therefore$ exp. $= \sqrt{1+\tan^2\theta} = \sec\theta$ **Ans.[D]**

Ex.22 If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then $(1-x^2)\frac{dy}{dx}$ equals -

(A) x + y (B) 1 + xy

(C) 1 – xy (D) xy –2

Sol. From the given equation, we have

$y^2(1-x^2) = (\sin^{-1}x)^2$

$\Rightarrow (1-x^2)\, 2y\frac{dy}{dx} - 2xy^2 = 2\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$\Rightarrow 2(1-x^2)\, y\frac{dy}{dx} - 2xy^2 = 2y$

$\Rightarrow (1-x^2)\frac{dy}{dx} = 1 + xy$ **Ans.[B]**

Ex.23 If $(a+bx)e^{y/x} = x$, then the value of $x^3\frac{d^2y}{dx^2}$ is -

(A) $\left(y\frac{dy}{dx} - x\right)^2$ (B) $\left(x\frac{dy}{dx} - y\right)^2$

(C) $x\frac{dy}{dx} - y$ (D) None of these

Sol. Taking logarithm of both the sides

$\log(a+bx) + y/x = \log x$

Now differentiating with respect to x, we get

$$\frac{b}{a+bx} + \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x}$$

$$\Rightarrow x\ \frac{dy}{dx} - y = x^2\left(\frac{a+bx-bx}{x(a+bx)}\right) = \frac{ax}{(a+bx)}$$

Again differentiating with respect to x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(a+bx)a-ax(b)}{(a+bx)^2}$$

$$x^3\ \frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2 = \left(x\frac{dy}{dx} - y\right)^2$$ **Ans.[B]**

Ex.24 $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$ equals -

(A) $\frac{x^2-1}{x^2-4}$ (B) 1

(C) $\frac{x^2+1}{x^2-4}$ (D) $e^x\frac{x^2-1}{x^2-4}$

Sol. Derivative

$$= \frac{d}{dx}\left[\log e^x + \frac{3}{4}\{\log(x-2) - \log(x+2)\}\right]$$

$$= \frac{d}{dx}\left[x + \frac{3}{4}\{\log(x-2) - \log(x+2)\}\right]$$

$$= 1 + \frac{3}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)$$

$$= 1 + \frac{3}{4}\frac{4}{x^2-4} = \frac{x^2-1}{x^2-4}$$ **Ans. [A]**

Ex.25 If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin^2 x$, then dy/dx equals -

(A) $\frac{2(1+x-x^2)}{(1+x^2)^2}\ \sin^2\left(\frac{2x-1}{x^2+1}\right)$

(B) $\frac{2(1+x-x^2)}{(1+x^2)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(C) $\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(D) $\sin\left(\frac{2x-1}{x^2+1}\right)^2$

Sol. $\frac{dy}{dx} = f'\left(\frac{2x-1}{x^2+1}\right)\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$

$$= \sin^2\left(\frac{2x-1}{x^2+1}\right).\frac{(x^2+1)2-(2x-1)2x}{(x^2+1)^2}$$ **Ans.[A]**

Ex.26 If f(x) = |x–2| and g(x) = f[f(x)], then for x > 20, g'(x) is equal to -

(A) 1 (B) –1

(C) 0 (D) None of these

Sol. $\because$ g(x) = f [f(x)]

= f {|x–2|}

= || x –2| –2|

But x > 20 $\Rightarrow$ |x–2| = x – 2

$\Rightarrow$ g(x) = |x –2–2| = x – 4

$\therefore$ g'(x) = 1 **Ans.[A]**

Ex.27 f(x) is a function such that f '' (x) = – f(x) and f '(x) = g(x) and h(x) is a function such that h(x) = $[f(x)]^2 + [g(x)]^2$ and h(5) = 11, then the value of h (10) is -

(A) 0 (B) 1

(C) 10 (D) None of these

Sol. h' (x) = 2f(x) f ' (x) + 2g(x) g'(x)

= 2f(x) g(x) + 2g (x) f '' (x)

= 2f (x) g(x) – 2f (x) g(x)

= 0 [$\because$ f '' (x) = –f(x)]

$\Rightarrow$ h(x) = c

$\Rightarrow$ h(10) =h (5) = 11 **Ans.[D]**

Ex.28 If x = (sec θ – cos θ) and y = $\sec^n\theta - \cos^n\theta$, then $\left(\frac{dy}{dx}\right)^2$ equals -

(A) $\frac{y^2+4}{n^2(x^2+4)}$ (B) $\frac{y^2+4}{n(x^2+4)}$

(C) $\frac{n^2(y^2+4)}{x^2+4}$ (D) None of these

Sol. Here $\frac{dx}{d\theta}$ = sec θ tan θ + sin θ

= tan θ (sec θ + cos θ)

= tan θ $\sqrt{(\sec\theta - \cos\theta)^2 + 4}$

= tan θ $\sqrt{x^2 + 4}$

and $\frac{dy}{d\theta}$ = n $\sec^n$ θ tan θ + n $\cos^{n-1}$ θ sin θ

= n tan θ ($\sec^n$ θ + $\cos^n$ θ)

= n tan θ $\sqrt{(\sec^n\theta - \cos^n\theta)^2 + 4}$

= n tan θ $\sqrt{y^2 + 4}$

$\therefore \frac{dy}{dx} = \frac{n\tan\theta\sqrt{y^2+4}}{\tan\theta\sqrt{x^2+4}}$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2+4)}{x^2+4}$ **Ans.[C]**

Ex.29 The value of the derivative of |x–1| + |x–3| at x = 2 is -

(A) –2 (B) 0

(C) 2 (D) Not defined

Sol. When 1 < x ≤ 3,

f(x) = (x–1) – (x–3) = 2

$\Rightarrow$ f ' (2 – 0) = 0, f ' (2 + 0) = 0

$\therefore$ f ' (2) = 0 **Ans.[B]**

Ex.30 If f(x) = $\log_x$ (ℓn x), then at x = e, f '(x) equals-

(A) 0 (B) 1

(C) e (D) 1/e

Sol. $\because$ ℓ n x = $\log_e$ x, so

f(x) = $\log_x$ ($\log_e$x) = $\frac{\log(\log x)}{\log x}$

$\Rightarrow f'(x) = \frac{\log x\left(\frac{1}{x\log x}\right) - \log(\log x)\frac{1}{x}}{(\log x)^2}$

$\therefore f'(e) = \frac{1/e - 0}{(1)^2} = \frac{1}{e}$ **Ans.[D]**

Ex.31 The first derivative of the function (sin 2x cos 2x cos 3x+$\log_2 2^{x+3}$) w.r.t. x at x = π is -

(A) 2 (B) –1

(C) –2 + 2π $\log_e$ 2 (D) –2+ $\log_e$ 2

Sol. Let y = sin 2x cos 2x cos 3x + $\log_2 2^{x+3}$

= $\frac{1}{2}$ sin 4x cos 3x + (x+3) $\log_2$2

= $\frac{1}{4}$ [sin 7x + sin x] + x+ 3

$\therefore \frac{dy}{dx} = \frac{1}{4}$ [7 cos 7x + cos x] + 1

$\therefore \left(\frac{dy}{dx}\right)_{x=\pi} = \frac{1}{4}$ [7 cos 7 π + cos π] + 1

= $\frac{1}{4}$ [– 8] + 1 = – 1 **Ans.[B]**