# MATHEMATICS

# Class-X

# Topic-5

## SIMILAR TRIANGLES

| INDEX | | |
|---|---|---|
| **S. No.** | **Topic** | **Page No.** |
| 1. | Theory | 1 – 19 |
| 2. | Exercise (Board Level) | 20 – 22 |
| 3. | Previous Year Problems | 22 – 24 |
| 4. | Exercise-1 | 24 – 32 |
| 5. | Exercise-2 | 32 – 34 |
| 6. | Exercise-3 | 34 – 39 |
| 7. | Answer Key | 40 – 41 |

# CH-05

# SIMILAR TRIANGLES

## A. INTRODUCTION TO SIMILAR TRIANGLES AND THALES THEOREM

### (a) Congruent figures

Two geometric figures which have the same shape and size are known as congruent figures. Congruent figures are alike in every respect.

### (b) Similar figures

Geometric figures which have the same shape but different sizes are known as similar figures. Two congruent figures are always similar but two similar figures need not be congruent.

**Examples**

**(i)** Any two line segments are similar.

**(ii)** Any two equiangular triangles are similar.

**(iii)** Any two squares are similar.

**(iv)** Any two circles are similar.

### (c) Similar polygons

Two polygons are said to be similar if

**(i)** their corresponding angles are equal and

**(ii)** the lengths of their corresponding sides are proportional.

If two polygons ABCDE and PQRST are similar we write, ABCDE ~ PQRST, where the symbol '~' stands for 'is similar to'.

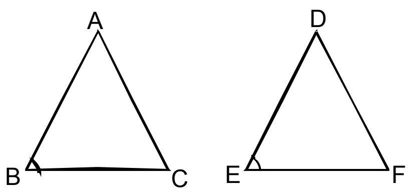
### (d) Equiangular triangles

Two triangles are said to be equiangular if their corresponding angles are equal.

### (e) Similar Triangles

Two triangles **ΔABC** and **ΔDEF** are said to be similar if their

**(i)** Corresponding angles are equal.
i.e. $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$
And




**(ii)** Corresponding sides are proportional.

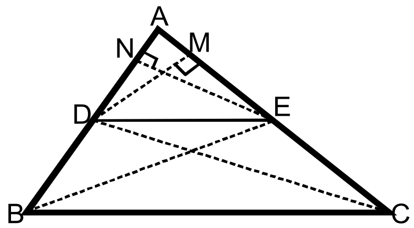
i.e. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

### (f) Basic Proportionality Theorem (BPT) or Thales Theorem

**Statement :** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

**Given :** A $\Delta ABC$ in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

**To Prove :** $\frac{AD}{DB} = \frac{AE}{EC}$.




**Construction :** Join BE and CD and draw DM $\perp$ AC and EN $\perp$ AB.

**Proof :** Area of $\Delta$ ADE $= \frac{1}{2}$ (base × height) $= \frac{1}{2}$ AD × EN.

Area of $\Delta$ ADE is denoted as ar(ADE).

So, ar(ADE) $= \frac{1}{2}$ AD × EN and ar(BDE) $= \frac{1}{2}$ DB × EN.

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ... (i)

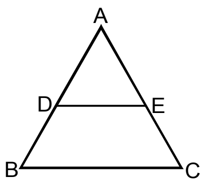
Similarly, ar(ADE) $= \frac{1}{2}$ AE × DM and ar(DEC) $= \frac{1}{2}$ EC × DM.

And $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ ... (ii)

Note that $\Delta$ BDE and $\Delta$ DEC are on the same base DE and between the two parallel lines BC & DE.
So, ar(BDE) = ar(DEC) ... (iii)
Therefore, from (i), (ii) and (iii), we have :

$\frac{AD}{DB} = \frac{AE}{EC}$ **Hence Proved.**

**Corollary :**
If in a $\Delta ABC$, a line DE || BC, intersects AB in D and AC in E, then




**(i)** $\frac{DB}{AD} = \frac{EC}{AE}$ **(ii)** $\frac{AB}{AD} = \frac{AC}{AE}$ **(iii)** $\frac{AD}{AB} = \frac{AE}{AC}$

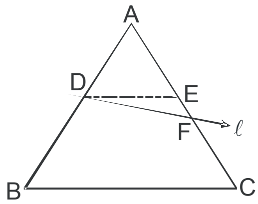
**(iv)** $\frac{AB}{DB} = \frac{AC}{EC}$ **(v)** $\frac{DB}{AB} = \frac{EC}{AC}$

## (g) Converse of Basic Proportionality Theorem (Thales Theorem)

**Statement :** If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

**Given :** A $\Delta$ ABC and a line intersecting AB at D and AC at E such that $\frac{AD}{DB} = \frac{AE}{EC}$.

**To prove :** DE II BC.




**Proof :** If possible let DE not be parallel to BC. Then there must be another line through D, which is parallel to BC. Let DF II BC

Then, by Thales' theorem, we have

$$\frac{AD}{DB} = \frac{AF}{FC} \qquad \text{....(i)}$$

But, $\frac{AD}{DB} = \frac{AE}{EC}$ (given) ....(ii)

From (i) and (ii) we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

$$\frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$$

$$\frac{AF + FC}{FC} = \frac{AE + EC}{EC}$$

$$\frac{AC}{FC} = \frac{AC}{EC}$$

FC = EC.

This is possible only when E and F coincide.

Hence , DE II BC.

### Some important results and theorems

**(i)** The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle. (called as **Angle Bisector Theorem**)

**(ii)** In a triangle **ABC**, if **D** is a point on **BC** such that **D** divides **BC** in the ratio **AB : AC**, then **AD** is the bisector of $\angle$**A.**

**(iii)** The external bisector of an angle of a triangle divides the opposite sides externally in the ratio of the sides containing the angle.

**(iv)** The line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side.

**(v)** The line joining the mid-points of two sides of a triangle is parallel to the third side.

**(vi)** The diagonals of a trapezium divide each other proportionally.

**(vii)** If the diagonals of a quadrilateral divide each other proportionally, then it is a trapezium.

**(viii)** Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

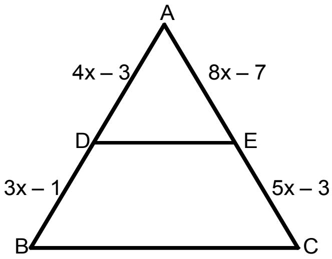
**(ix)** If three or more parallel lines are intersected by two transversals, then the intercepts made by them on the transversals are proportional.

## Solved Examples

**Example.1**

In a $\Delta$ABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x – 3, find the value of x.

**Sol.** In $\Delta ABC$, we have DE || BC




$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \quad \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$\Rightarrow \quad 20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

$\Rightarrow \quad 20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$\Rightarrow \quad 4x^2 - 2x - 2 = 0 \quad \Rightarrow \quad 2x^2 - x - 1 = 0$

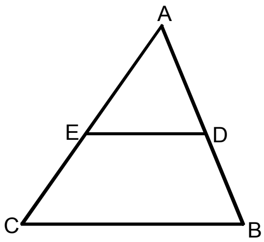
$\Rightarrow \quad (2x + 1)(x - 1) = 0 \quad \Rightarrow \quad x = 1 \text{ or } x = -\frac{1}{2}$

So, the required value of x is 1. [$x = -\frac{1}{2}$ is neglected as length can not be negative].

**Example.2**

D and E are respectively the points on the sides AB and AC of a $\Delta ABC$ such that AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm, show that DE || BC.

**Sol.** We have,




AB = 12 cm, AC = 18 cm, AD = 8 cm and AE = 12 cm.

$\therefore$ BD = AB – AD = (12 - 8) cm = 4 cm

CE = AC - AE = (18 – 12) cm = 6 cm

Now, $\frac{AD}{BD} = \frac{8}{4} = \frac{2}{1}$

And, $\frac{AE}{CE} = \frac{12}{6} = \frac{2}{1} \quad \Rightarrow \quad \therefore \frac{AD}{BD} = \frac{AE}{CE}$

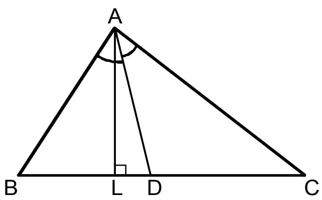
Thus, DE divides sides AB and AC of $\Delta ABC$ in the same ratio. Therefore, by the converse of basic proportionality theorem, we have DE || BC.

**Example.3**

In $\Delta ABC$, if AD is the bisector of $\angle A$, prove that $\frac{\text{Area }(\Delta ABD)}{\text{Area }(\Delta ACD)} = \frac{AB}{AC}$.

**Sol.** In $\Delta ABC$, AD is the bisector of $\angle A$.

$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$ ..(i) [By internal bisector theorem]



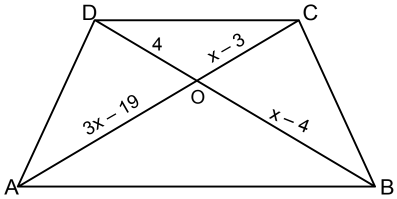
From A draw AL ⊥ BC

$$\therefore \frac{\text{Area } (\Delta ABD)}{\text{Area } (\Delta ACD)} = \frac{\frac{1}{2}BD.AL}{\frac{1}{2}DC.AL} = \frac{BD}{DC} = \frac{AB}{AC} \text{ [From (i)]}$$ **Hence Proved.**

**Example.4**

In the given figure, AB || CD. Find the value of x.

**Sol.** Since the diagonals of a trapezium divide each other proportionally.




$\therefore \quad \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \quad \frac{3x-19}{x-3} = \frac{x-4}{4}$

$\Rightarrow \quad 12x - 76 = x^2 - 4x - 3x + 12$

$\Rightarrow \quad x^2 - 19x + 88 = 0$

$\Rightarrow \quad x^2 - 11x - 8x + 88 = 0$

$\Rightarrow \quad (x - 8)(x - 11) = 0$

$\Rightarrow \quad x = 8$ or $x = 11$.

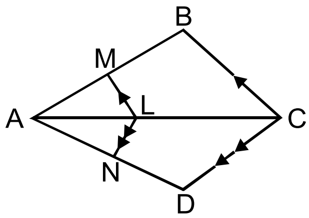
# Check Your Level

1. If three or more parallel lines are intersected by two transversal, prove that the intercepts made by them on the transversal are proportional.

2. Using converse of BPT prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side

3. If D,E are points on the sides AB and AC of ΔABC such that AD=6 cm, BD=9 cm, AE=8 cm,EC=12 cm. Prove that DE||BC.

4. In the given figure, if LM||CB and LN||CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$




5. ABCD is a trapezium in which AB is parallel to DC. If the diagonals intersect at O prove that AO.DO = BO.CO.

---

## (B) CRITERIA FOR SIMILARITY OF TWO TRIANGLES

### (a) AAA Similarity Criteria

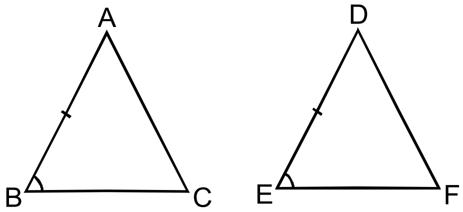
If two triangles are equiangular, then they are similar.

Given : ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

To prove : ΔABC ~ ΔDEF

Proof :

**Case -1** : If AB = DE




In ΔABC and ΔDEF

$\angle A = \angle D$

AB = DE

$\angle B = \angle E$

$\therefore$ ΔABC $\cong$ ΔDEF [By ASA congruence rule]

By CPCT,

BC = EF, AC = DF

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$

Hence, ΔABC ~ ΔDEF

**Case -2** **:** If AB > DE

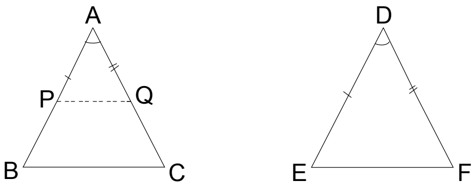
Construction : Mark points P and Q on AB and AC respectively such that AP = DE and AQ = DF

Proof : ΔAPQ and ΔDEF

AP = DE

$\angle A = \angle D$

AQ = DF




ΔAPQ $\cong$ ΔDEF [By SAS congruence rule]

By CPCT, $\angle P = \angle E$ and $\angle Q = \angle F$

But $\angle B = \angle E$ and $\angle C = \angle F$

$\therefore \angle P = \angle B$ and $\angle Q = \angle C$

$\Rightarrow$ PQ || BC [$\therefore$ corresponding angles are equal ]

By basic proportionality theorem in ΔABC, $\frac{AP}{AB} = \frac{AQ}{AC}$

$\frac{DE}{AB} = \frac{DF}{AC}$ ...(i) [By construction]

Similarly, we can prove that $\frac{DE}{AB} = \frac{EF}{BC}$ ...(ii)

From (i) & (ii) $\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$

$\therefore$ ΔABC ~ ΔDEF

**Case -3**: If AB < DE

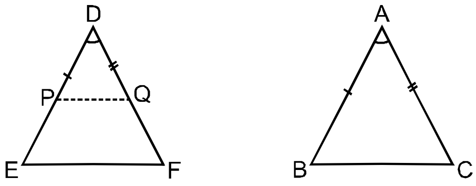
Construction : Mark points P and Q on DE and DF respectively such that DP = AB and DQ = AC

Proof : ΔDPQ and ΔABC

DP = AB

$\angle D = \angle A$

DQ = AC




$\Delta DPQ \cong \Delta ABC$ [By SAS congruence rule]

By CPCT, $\angle P = \angle B$ and $\angle Q = \angle C$

But $\angle E = \angle B$ and $\angle F = \angle C$

$\therefore \angle P = \angle E$ and $\angle Q = \angle F$ $\Rightarrow$ PQ || EF [$\because$ corresponding angles are equal ]

By basic proportionality theorem in $\Delta DEF$, $\frac{DP}{DE} = \frac{DQ}{DF}$

$\frac{AB}{DE} = \frac{AC}{DF}$ ...(i) [By construction]

Similarly, we can prove that $\frac{AB}{DE} = \frac{BC}{EF}$ ...(ii)

From (i) & (ii) $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

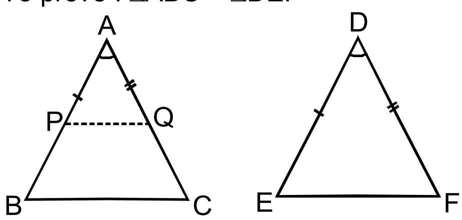
$\therefore \Delta ABC \sim \Delta DEF$

**(b) SSS Similarity Criteria**

If the corresponding sides of two triangles are proportional, then they are similar.

Given : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

To prove : $\Delta ABC \sim \Delta DEF$




Construction : Let AB > DE, now mark two points P and Q on AB and AC respectively such that AP = DE and AQ = DF.

Proof : Given $\frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{AB}{AP} = \frac{AC}{AQ}$

$\therefore$ By converse of BPT in $\Delta ABC$, PQ II BC.

$\Rightarrow \angle P = \angle B$ and $\angle Q = \angle C$ [Corresponding angles]

$\Delta APQ \sim \Delta ABC$ [By AA similarity]

$\frac{AB}{AP} = \frac{BC}{PQ}$ and $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{PQ}$ and $\frac{AB}{DE} = \frac{BC}{EF}$ [since AP = DE]

$\therefore \frac{BC}{PQ} = \frac{BC}{EF} \Rightarrow PQ = EF$

Now, in $\Delta APQ$ and $\Delta DEF$

AP = DE

PQ = EF

AQ = DF

$\Delta APQ \cong \Delta DEF$ [By SSS congruence rule]

But $\Delta APQ \sim \Delta ABC$

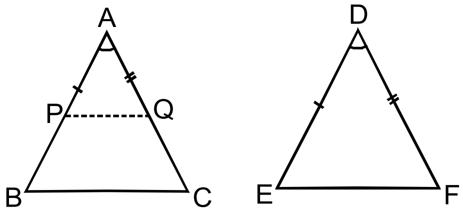
$\therefore \Delta DEF \sim \Delta ABC$.

**(c) SAS Similarity Criteria**

If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

Given : $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$

To prove : $\Delta ABC \sim \Delta DEF$




Construction : Let AB > DE, now mark two points and P and Q on AB and AC.

Proof : In $\Delta APQ$ and $\Delta DEF$

$AP = DE$

$\angle A = \angle D$

$AQ = DF$

$\therefore \Delta APQ \cong \Delta DEF$ [By SAS congruence rule]

Given, $\frac{AB}{DE} = \frac{AC}{DF}$

$\Rightarrow \frac{AB}{AP} = \frac{AC}{AQ}$ [By construction]

$\therefore PQ \parallel BC$ [By converse of Basic Proportionality Theorem]

$\therefore \angle P = \angle B, \ \angle Q = \angle C$

$\Rightarrow \Delta APQ \sim \Delta ABC$ [By AA similarity]

Hence, $\Delta DEF \sim \Delta ABC$

**(d) Results based upon characteristic properties of Similar Triangles**

**(i)** The ratio of the perimeters of two similar triangles is equal to the ratio of their corresponding sides.

**(ii)** If two triangles are equiangular, then the ratio of the corresponding sides is the same as the ratio of the corresponding medians.

**(iii)** If two triangles are equiangular, then the ratio of the corresponding sides is same as the ratio of the corresponding angle bisector segments.

**(iv)** If two triangles are equiangular then the ratio of the corresponding sides is same as the ratio of the corresponding altitudes.

**(v)** If one angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite side in the same ratio, then the triangles are similar.

**(vi)** If two sides and a median bisecting the third side of a triangle are respectively proportional to the corresponding sides and the median of another triangle, then two triangles are similar.

**(vii)** If two sides and a median bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding median of another triangle, then the triangles are similar.

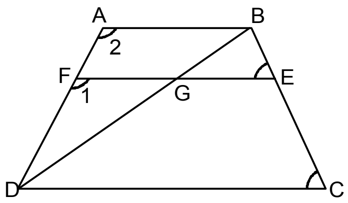
# Solved Examples

**Example.5**

In a trapezium ABCD, AB || DC and DC = 2AB. EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7FE = 10AB.

**Sol.** In $\Delta DFG$ and $\Delta DAB$,

$\angle 1 = \angle 2$ [Corresponding $\angle$ s $\because$ AB || FG]




$\angle FDG = \angle ADB$ [Common]

$\therefore$ $\Delta DFG \sim \Delta DAB$ [By AA rule of similarity]

$\therefore$ $\frac{DF}{DA} = \frac{FG}{AB}$ … (i)

Again in trapezium ABCD

EF || AB || DC

$\therefore$ $\frac{AF}{DF} = \frac{BE}{EC}$

$\Rightarrow$ $\frac{AF}{DF} = \frac{3}{4}$ $\left[\frac{BE}{EC} = \frac{3}{4}(\text{given})\right]$

$\Rightarrow$ $\frac{AF}{DF} + 1 = \frac{3}{4} + 1$

$\Rightarrow$ $\frac{AF + DF}{DF} = \frac{7}{4}$

$\Rightarrow$ $\frac{AD}{DF} = \frac{7}{4}$

$\Rightarrow$ $\frac{DF}{AD} = \frac{4}{7}$ … (ii)

From (i) and (ii), we get

$\frac{FG}{AB} = \frac{4}{7}$ i.e., $FG = \frac{4}{7}AB$ … (iii)

In $\Delta BEG$ and $\Delta BCD$, we have

$\angle BEG = \angle BCD$ [Corresponding angle EG || CD]

$\angle GBE = \angle DBC$ [Common]

$\therefore$ $\Delta BEG \sim \Delta BCD$ [By AA rule of similarity]

$\therefore$ $\frac{BE}{BC} = \frac{EG}{CD}$

$\therefore$ $\frac{3}{7} = \frac{EG}{CD}$ $\left[\frac{BE}{BC} = \frac{3}{7} \text{i.e.,} \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{EC + BE}{BE} = \frac{4+3}{3} \Rightarrow \frac{BC}{BE} = \frac{7}{3}\right]$

$\therefore$ $EG = \frac{3}{7}$ CD = $\frac{3}{7}$ (2 AB) [$\because$ CD = 2AB (given) ]

$\therefore$ $EG = \frac{6}{7}AB$ ... (iv)

Adding (iii) and (iv), we get

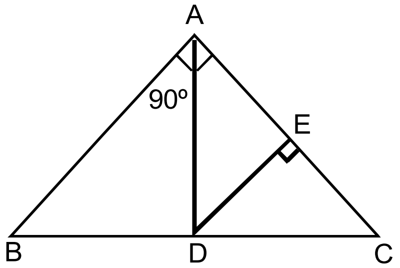
$FG + EG = \frac{4}{7}AB + \frac{6}{7}AB = \frac{10}{7}AB$

$\Rightarrow$ $EF = \frac{10}{7}AB$ i.e., 7EF = 10AB. **Hence proved.**

**Example.6**

$\angle BAC = 90^\circ$, AD is its bisector. If $DE \perp AC$, prove that $DE \times (AB + AC) = AB \times AC$.

**Sol.** It is given that AD is the bisector of $\angle A$ of $\Delta$ ABC.




$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \quad \frac{AB}{AC} + 1 = \frac{BD}{DC} + 1$ [Adding 1 on both sides]

$\Rightarrow \quad \frac{AB + AC}{AC} = \frac{BD + DC}{DC}$

$\Rightarrow \quad \frac{AB + AC}{AC} = \frac{BC}{DC}$ ...(i)

In $\Delta$'s CDE and CBA, we have

$\angle DCE = \angle BCA$ [Common]

$\angle DEC = \angle BAC$ [Each equal to 90º]

So, by AA-criterion of similarity

$\Delta\ CDE \sim \Delta\ CBA$

$\Rightarrow \quad \frac{CD}{CB} = \frac{DE}{BA}$

$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{DC}$ ....(ii)

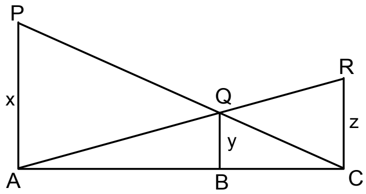
From (i) and (ii), we have

$\Rightarrow \quad \frac{AB + AC}{AC} = \frac{AB}{DE}$

$\Rightarrow \quad DE \times (AB + AC) = AB \times AC.$

**Example.7**

In the given figure, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.




**Sol.** In $\Delta$PAC, we have BQ || AP

$\Rightarrow \quad \frac{BQ}{AP} = \frac{CB}{CA}$ [$\therefore\ \Delta CBQ \sim \Delta CAP$]

$\Rightarrow \quad \frac{y}{x} = \frac{CB}{CA}$ …(i)

In $\Delta$ACR, we have BQ || CR

$\Rightarrow \quad \frac{BQ}{CR} = \frac{AB}{AC}$ [$\therefore\ \Delta\ ABQ \sim\ \Delta ACR$ ]

$\Rightarrow \quad \frac{y}{z} = \frac{AB}{AC}$ …(ii)

Adding (i) and (ii), we get

$$\frac{y}{x}+\frac{y}{z}=\frac{CB}{AC}+\frac{AB}{AC} \quad \Rightarrow \quad \frac{y}{x}+\frac{y}{z}=\frac{AB+BC}{AC}$$

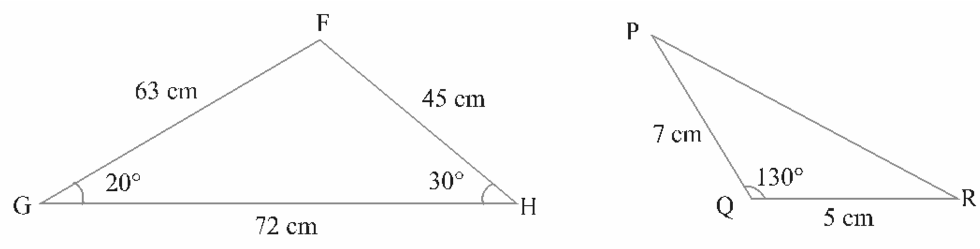
$$\Rightarrow \quad \frac{y}{x}+\frac{y}{z}=\frac{AC}{AC} \quad \Rightarrow \quad \frac{y}{x}+\frac{y}{z}=1$$

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{z}=\frac{1}{y}.$$

**Hence Proved.**

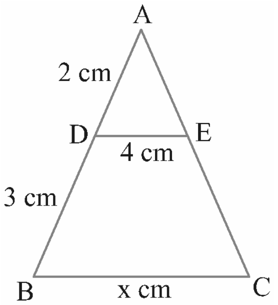
# Check Your Level

**1.** In the adjoining figure, check whether similar. If yes identify the similarity criterion two triangles FGH and QPR are




**2.** $\Delta ABC \sim \Delta PQR$. If AB = 6 cm, BC = 4 cm, AC = 8 cm, PR = 6 cm then find PQ + QR =

**3.** In the figure, if DE|| BC, then find x.




**4.** ABC is a triangle and DE is drawn parallel to BC such that AD : DB = 2 : 3. If DE = 5 cm, find the length of BC.

**5.** D is a point on the side QR of triangle PQR such that angles PDR and QPR are equal. Prove that $QR.DR = PR^2$ .

**Answers**

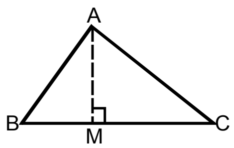
**1.** Yes, SAS similarity **2.** 7.5 **3.** 10 **4.** 12.5 cm

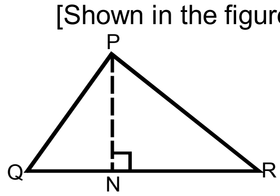
## (C) AREAS OF SIMILAR TRIANGLES

**Statement :** The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

**Given :** Two triangles ABC and PQR such that

$\Delta ABC \sim \Delta PQR$ [Shown in the figure]





**To prove :** $\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$.

**Construction :** Draw altitudes AM and PN of the triangle ABC and PQR.

**Proof :** $ar(ABC) = \frac{1}{2} BC \times AM$ and $ar(PQR) = \frac{1}{2} QR \times PN$

So, $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2}BC \times AM}{\frac{1}{2}QR \times PN} = \frac{BC \times AM}{QR \times PN}$ ... (i)

Now, in $\Delta$ ABM and $\Delta$ PQN,

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle M = \angle N$ [ 90° each ]

So, $\Delta ABM \sim \Delta PQN$ [AA similarity criterion]

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ ... (ii)

Also, $\Delta ABC \sim \Delta PQR$ [Given]

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ... (iii)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{AB}{PQ}$ [From (i) and (ii)]

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (iii)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (iii), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2.$$

**(a) Properties of areas of similar triangles**

**(i)** The areas of two similar triangles are in the ratio of the squares of corresponding altitudes.

**(ii)** The areas of two similar triangles are in the ratio of the squares of the corresponding medians.

**(iii)** The area of two similar triangles are in the ratio of the squares of the corresponding angle bisector segments.

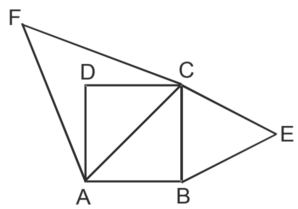
## Solved Examples

**Example 8.**

Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

**Solution.**

**Given :** A square ABCD. Equilateral triangles $\Delta BCE$ and $\Delta ACF$ have been described on side BC and diagonal AC respectively.




[ABCD is a square
Diagonal = $\sqrt{2}$ (side)
AC = $\sqrt{2}$ BC]

**To Prove :** Area (ΔBCE) = $\frac{1}{2}$. Area (ΔACF)

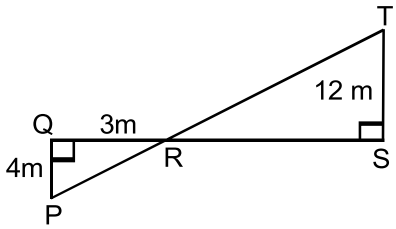
**Proof :** Since ΔBCE and ΔACF are equilateral. Therefore, they are equiangular ( each angle being equal to 60º) and hence ΔBCE ~ ΔACF.

$$\Rightarrow \quad \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \quad \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{\left(\sqrt{2}BC\right)^2} = \frac{1}{2} \Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{1}{2}.$$ **Hence Proved.**

**Example 9.**

Area of triangle RST is




(A) 6 m² (B) 72 m² (C) 54 m² (D) None of these

**Sol.** In ΔRQP and ΔRST

∠Q = ∠S = 90º

∠PRQ = ∠SRT (V.O.A.)

By AA

ΔRQP ~ ΔRST

$$\frac{RQ}{RS} = \frac{QP}{ST}$$

$$\frac{3}{RS} = \frac{4}{12}$$

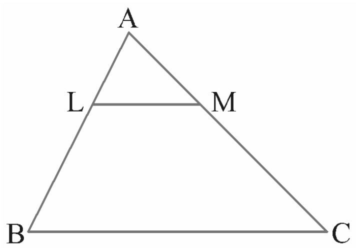
∴ RS = 9

Area of ΔRST = $\frac{1}{2} \times 9 \times 12 = 54$ m²

# Check Your Level

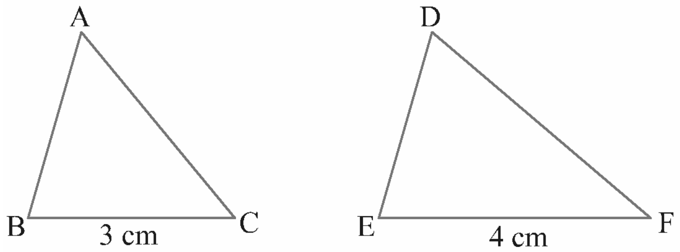
**1.** If ΔABC ~ ΔDEF such that area of ΔABC is 9 cm² and the area of ΔDEF is 16 cm² and BC = 2.1 cm find the length of EF is ?

**2.** In the diagram, LM is parallel to BC and AL = 1 cm, LB = 3 cm, MC = 4.5 cm and BC = 8 cm. Find the length of LM. If the area of triangle ALM is 18 sq cm, what is the area of triangle ABC?

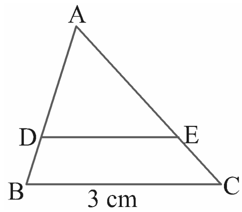



**3.** D and E are points on AB and AC respectively of triangle ABC such that DE is parallel to BC. If AD = 3 cm, DB = 2 cm, area of ΔABC is 10 sq cm, find the area of ΔADE.

**4.** In the given figure, $\Delta ABC$ and $\Delta DEF$ are similar BC = 3 cm, EF = 4 cm and area of $\Delta$ ABC = 54 $cm^2$. Find the area of $\Delta DEF$.




**5.** In the adjacent figure, DE || BC and AD : DB = 5 : 4, then find the value of $\frac{ar(\Delta ADE)}{ar(\Delta CED)}$




**Answers**

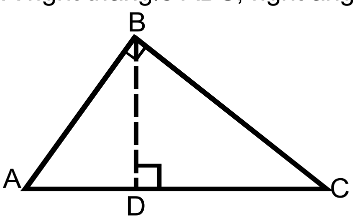
| | | | | | |
|---|---|---|---|---|---|
| **1.** | 2.8 cm | **2.** | LM = 2 cm & Area = 288 sq. cm | **3.** | 3.6 sq. cm |
| **4.** | 96 sq. cm | **Ans.** | 5 : 4 | | |

---

## (D) PYTHAGORAS THEOREM

**Statement :** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**Given :** A right triangle ABC, right angled at B.




**To prove :** $AC^2 = AB^2 + BC^2$

**Construction :** $BD \perp AC$

**Proof :** $\Delta$ ADB & $\Delta$ ABC

$\angle DAB = \angle CAB$ [Common]

$\angle BDA = \angle CBA$ [90° each]

So, $\Delta ADB \sim \Delta ABC$ [By AA similarity]

$\frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional]

or, $AD . AC = AB^2$ ... (i)

Similarly $\Delta BDC \sim \Delta ABC$

So, $\frac{CD}{BC} = \frac{BC}{AC}$

or $CD . AC = BC^2$ ... (ii)

Adding (i) and (ii),

$AD . AC + CD . AC = AB^2 + BC^2$

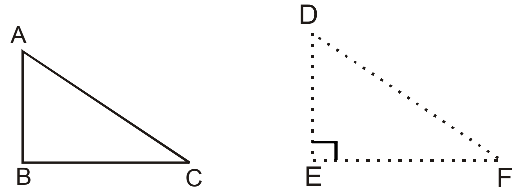
or, $AC (AD + CD) = AB^2 + BC^2$

or, $AC. AC = AB^2 + BC^2$

or, $\mathbf{AC^2 = AB^2 + BC^2}$ **Hence Proved.**

## (a) Converse of pythagoras theorem

**Statement :** In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.




**Given :** A triangle ABC such that $AC^2 = AB^2 + BC^2$.

**Construction :** Construct a triangle DEF such that DE = AB, EF = BC and $\angle E = 90^o$.

**Proof :** In order to prove that $\angle B = 90^o$, it is sufficient to show $\Delta ABC \sim \Delta DEF$. For this we proceed as follows.

Since $\Delta$ DEF is a right-angled triangle with right angle at E. Therefore, by Pythagoras theorem, we have

$DF^2 = DE^2 + EF^2$

$\Rightarrow DF^2 = AB^2 + BC^2$ [DE = AB and EF = BC (By construction)]

$\Rightarrow DF^2 = AC^2$ [$AB^2 + BC^2 = AC^2$ (Given)]

$\Rightarrow DF = AC$ ....(i)

Thus, in $\Delta$ ABC and $\Delta$ DEF, we have

AB = DE, BC = EF [By construction]

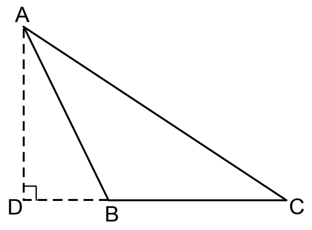
And AC = DF [From equation (i)]

$\therefore \Delta ABC \cong \Delta DEF$ [By SSS criteria of congruency]

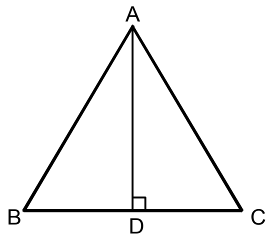
$\Rightarrow \angle B = \angle E = 90^o$. Hence, $\Delta$ ABC is a right triangle, right angled at B.

## (b) Some results deduced from pythagoras theorem

**(i)** In the given figure **ΔABC** is an obtuse triangle, obtuse angled at **B**. If **AD** $\perp$ **CB**, then $\mathbf{AC^2 = AB^2 + BC^2 + 2BC. BD}$




**(ii)** In the given figure, if $\angle$ **B** of **ΔABC** is an acute angle and **AD** $\perp$ **BC**, then $\mathbf{AC^2 = AB^2 + BC^2 - 2BC . BD}$



**(iii)** In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side (called as **Apollonius Theorem**)

**(iv)** Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

## Solved Examples

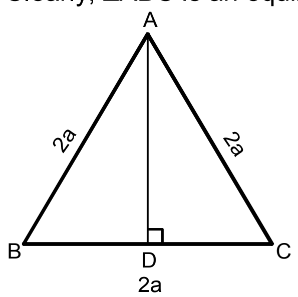
**Example 10**

In a $\Delta ABC$, $AB = BC = CA = 2a$ and $AD \perp BC$. Prove that

**(i)** $AD = a\sqrt{3}$ **(ii)** area $(\Delta ABC) = \sqrt{3}\, a^2$

**Sol.** **(i)** Here, $AD \perp BC$.

Clearly, $\Delta ABC$ is an equilateral triangle.




Thus, in $\Delta ABD$ and $\Delta ACD$

$AD = AD$ [Common]

$\angle ADB = \angle ADC$ [90° each]

And $AB = AC$

$\therefore$ By RHS congruency condition

$\Delta ABD \cong \Delta ACD \Rightarrow BD = DC = a$

Now, $\Delta ABD$ is a right angled triangle

$\therefore \quad AD = \sqrt{AB^2 - BD^2}$ [Using Pythagoras Theorem]

$AD = \sqrt{4a^2 - a^2} = \sqrt{3}\,a$ or $a\sqrt{3}$.

**(ii)** Area $(\Delta ABC) = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 2a \times a\sqrt{3} = a^2\sqrt{3}$.

**Example 11.**

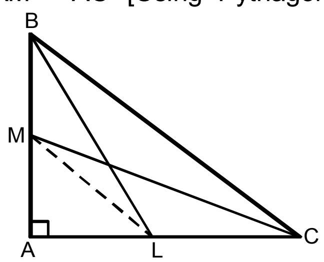
BL and CM are medians of $\Delta ABC$ right angled at A. Prove that $4(BL^2 + CM^2) = 5\,BC^2$.

**Sol.** In $\Delta BAL$

$BL^2 = AL^2 + AB^2$ [Using Pythagoras theorem] … (i)

and, In $\Delta CAM$

$CM^2 = AM^2 + AC^2$ [Using Pythagoras theorem] … (ii)




Adding (i) and (ii) and then multiplying by 4, we get

$4(BL^2 + CM^2) = 4(AL^2 + AB^2 + AM^2 + AC^2)$

$= 4\{AL^2 + AM^2 + (AB^2 + AC^2)\}$ [$\therefore \Delta$ ABC is a right triangle]

$= 4(AL^2 + AM^2 + BC^2)$

$= 4(ML^2 + BC^2)$ [$\therefore \Delta LAM$ is a right triangle ]

$= 4ML^2 + 4\,BC^2$

[A line joining mid-points of two sides is parallel to third side and is equal to half of it, ML =BC/2]

$= BC^2 + 4BC^2 = 5BC^2$. **Hence proved.**

**Example 12.**

O is any point inside a rectangle ABCD. Prove that $OB^2 + OD^2 = OA^2 + OC^2$.

**Sol.** Through O, draw PQ || BC so that P lies on AB and Q lies on DC.

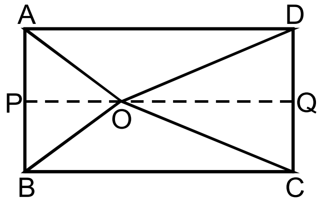
Now, PQ || BC

**Therefore,**

$PQ \perp AB$ and $PQ \perp DC$ [$\angle B = 90^\circ$ and $\angle C = 90^\circ$]

So, $\angle BPQ = 90^\circ$ and $\angle CQP = 90^\circ$

Therefore, BPQC and APQD are both rectangles.




Now, from $\Delta$ OPB,

$OB^2 = BP^2 + OP^2$ ... (i)

Similarly, from $\Delta$ ODQ,

$OD^2 = OQ^2 + DQ^2$ ... (ii)

From $\Delta$ OQC, we have

$OC^2 = OQ^2 + CQ^2$ ... (iii)

And from $\Delta$ OAP, we have

$OA^2 = AP^2 + OP^2$ ... (iv)

Adding (i) and (ii)

$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$

$= CQ^2 + OP^2 + OQ^2 + AP^2$ [As BP = CQ and DQ = AP]

$= CQ^2 + OQ^2 + OP^2 + AP^2$

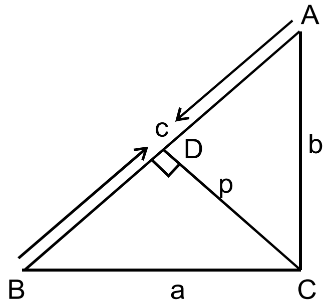
$= OC^2 + OA^2$ [From (iii) and (iv)] **Hence Proved.**

**Example 13.**

ABC is a right triangle, right-angled at C. Let BC = a, CA = b, AB = c and let p be the length of perpendicular form C on AB, prove that

**(i)** cp = ab **(ii)** $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

**Sol.** **(i)** Let $CD \perp AB$. Then, CD = p




$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ (Base × height) $= \frac{1}{2}$ (AB × CD) $= \frac{1}{2}$cp

Also, Area of $\Delta$ ABC $= \frac{1}{2}$ (BC × AC) $= \frac{1}{2}$ab

$\therefore$ $\frac{1}{2}cp = \frac{1}{2}ab$ $\Rightarrow$ cp = ab.

**(ii)** Since $\Delta$ ABC is a right triangle, right angled at C.

$\therefore \quad AB^2 = BC^2 + AC^2$

$\Rightarrow \quad c^2 = a^2 + b^2$

$\Rightarrow \quad \left(\frac{ab}{p}\right)^2 = a^2 + b^2 \qquad \left[\because cp = ab \Rightarrow c = \frac{ab}{p}\right]$

$\Rightarrow \quad \frac{a^2b^2}{p^2} = a^2 + b^2$

$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$

$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

**Example 14.**

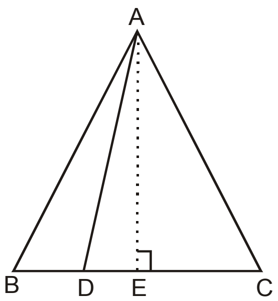
In an equilateral triangle ABC, the side BC is trisected at D. Prove that $9\,AD^2 = 7AB^2$.

**Sol.** ABC be an equilateral triangle and D be point on BC such that $BD = \frac{1}{3}BC$ (Given)

Draw AE $\perp$ BC, Join AD.

BE = EC ( Altitude drawn from any vertex of an equilateral triangle bisects the opposite side)

So, $\quad BE = EC = \frac{BC}{2}$




In $\Delta$ ABC

$AB^2 = AE^2 + EB^2 \qquad$ ...(i)

$AD^2 = AE^2 + ED^2 \qquad$ ...(ii)

From (i) and (ii)

$AB^2 = AD^2 - ED^2 + EB^2$

$\Rightarrow \quad AB^2 = AD^2 - \frac{BC^2}{36} + \frac{BC^2}{4} \qquad \left[\because BD + DE = \frac{BC}{2} \Rightarrow \frac{BC}{3} + DE = \frac{BC}{2} \Rightarrow DE = \frac{BC}{6}\right]$

$\Rightarrow \quad AB^2 + \frac{BC^2}{36} - \frac{BC^2}{4} = AD^2 \qquad \left[\therefore EB = \frac{BC}{2}\right]$

$\Rightarrow \quad AB^2 + \frac{AB^2}{36} - \frac{AB^2}{4} = AD^2 \qquad [\therefore AB = BC]$

$\Rightarrow \quad \frac{36AB^2 + AB^2 - 9AB^2}{36} = AD^2$

$\Rightarrow \quad \frac{28AB^2}{36} = AD^2$

$\Rightarrow \quad 7AB^2 = 9AD^2$.

# Check Your Level

1. A man of height 1.8 m, standing 5 m away from a lamp post observes that the length of his shadow is 1.5 m. What is the height of the lamp post ?

2. A rope from the top of a mast on a sailboat is attached to a point 2 metres from the base of the mast. The rope is 8 metres long. How high is the mast ?

3. Identify the triangle as acute angled, obtuse angled, right angled whose sides are given below.
   **(i)** a = 12, b = 15, c = 20 **(ii)** a = 15, b = 8, c = 17
   **(iii)** a = 12, b = 5, c = 17 **(iv)** a = 8, b = 9, c = 12

4. In a triangle ABC, $\angle A = 90°$. If $AD \perp BC$ prove that $AB^2 - BD^2 = AC^2 - CD^2$.

5. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time another tower casts a shadow 40 m long on the ground. Find the height of the tower.

**Answers**

**1.** 7.8 m **2.** $2\sqrt{15}$ m

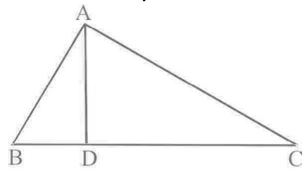
**3.** **(i)** obtuse **(ii)** Right **(iii)** obtuse **(iv)** acute **5.** 60 m

# Exercise Board Level

**TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]**

**1.** The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is

**2 .** In Figure, $\angle BAC = 90°$ and $AD \perp BC$. Then prove that $BD . CD = AD^2$



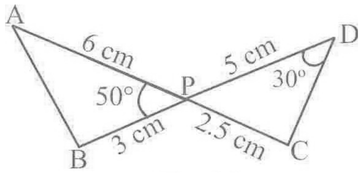

**3.** If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then show that ED . PR = FE. RQ

**4.** In triangles $\Delta ABC$ and $\Delta DEF$, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, show that the two triangles are similar but not congruent

**5.** If $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{3}$, then $\frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to

**TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]**

**6.** In Figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, $\angle PBA$ is equal to




**7.** If $\Delta ABC \sim \Delta QRP$, $\frac{ar(ABC)}{ar(PQR)} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then PR is equal to

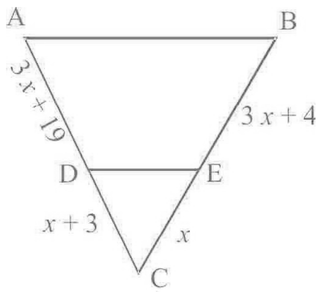
**8.** If S is a point on side PQ of a $\Delta PQR$ such that PS = QS = RS, then prove that $PR^2 + QR^2 = PQ^2$.

**9.** If $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, AB = 5cm, AC = 8cm and DF = 7.5 cm. Then, DE and $\angle F$.

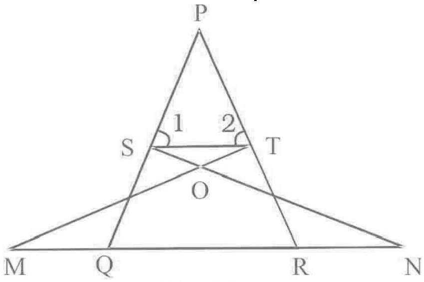
**10.** Corresponding sides of two similar triangles are in the ratio of 2 : 3. If the area of the smaller triangle is 48 cm², find the area of the larger triangle.

**TYPE (III) : LONG ANSWER TYPE QUESTIONS: [04 MARK EACH]**

**11.** Find the value of x for which DE || AB in Figure



**12.** In Figure, if ∠ 1 = ∠ 2 and Δ NSQ ≅ Δ MTR, then prove that Δ PTS ~ Δ PRQ.




**13.** Find the altitude of an equilateral triangle of side 8 cm.

**14.** In a Δ PQR, $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that QM ⊥ PR.Prove that $QM^2 = PM \times MR$.

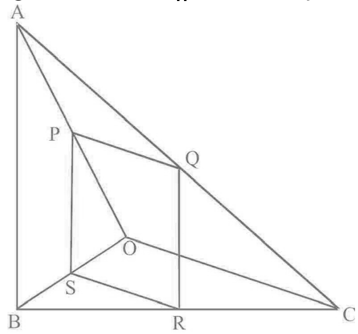
**15.** ABCD is a trapezium in which AB || DC and P and Q are points on AD and BC, respectively such that PQ ||DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.

**16.** Hypotenuse of a right triangle is 25 cm and out of the remaining two sides, one is longer than the other by 5 cm. Find the lengths of the other two sides.

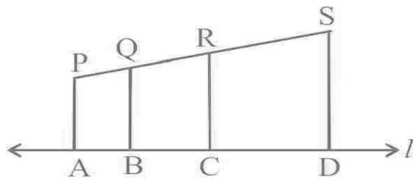
**17.** A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3m, find how far she is away from the base of the pole.

**TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS [05 MARK EACH]**

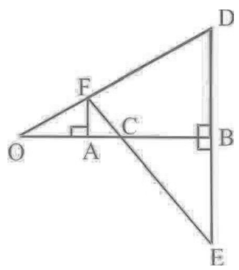
**18 .** In Figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR.




**19.** In Figure, PA, QB, RC and SD are all perpendiculars to a line l, AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS




**20.** In Figure, OB is the perpendicular bisector of the line segment DE, FA ⊥ OB and FE intersects OB at the point C. Prove that $\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$.



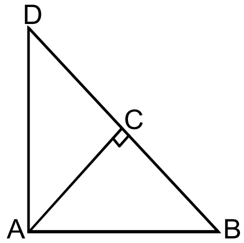
**21.** Legs (sides other than the hypotenuse) of a right triangle are of lengths 16cm and 8 cm. Find the length of the side of the largest square that can be inscribed in the triangle.

---

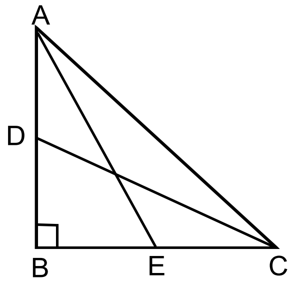
# Previous Year Problems

**1.** In figure, $\Delta ABD$ is a right triangle, right-angled at A and $AC \perp BD$. Prove that $AB^2 = BC \cdot BD$

**[2 MARKS/CBSE 10TH BOARD: 2013]**




**2.** In the figure, ABC is a triangle with $\angle B = 90°$, Medians AE and CD of respective lengths $\sqrt{40}$ cm and 5 cm are drawn. Find the length of the hypotenuse AC. **[3 MARKS/CBSE 10TH BOARD: 2013]**



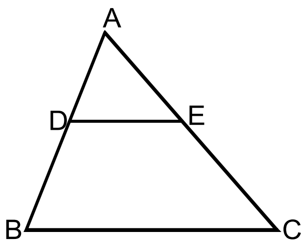

**3.** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

**OR**

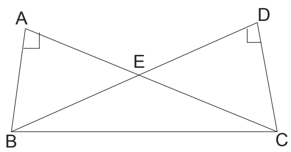
Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

**[4 MARKS/CBSE 10TH BOARD: 2013, 2014, 2015]**

**4.** In $\Delta$ ABC, D and E are points on the sides AB and AC respectively such that DE || BC. If $\frac{AD}{DB} = \frac{2}{3}$ and AE = 18 cm, then find the AC. **[CBSE 10TH BOARD: 2014]**

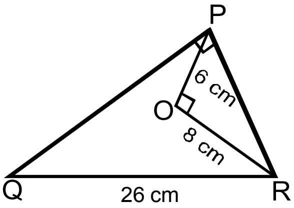



**5.** In figure, two triangles ABC and DBC are on the same base BC in which $\angle A = \angle D = 90°$. If CA and BD meet each other at E, show that AE × CE = BE × ED. **[CBSE 10TH BOARD: 2014]**



6. A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.
**[CBSE 10TH BOARD: 2014]**

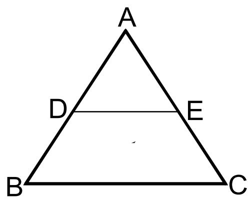
7. Calculate area (ΔPQR) from figure **[CBSE 10TH BOARD: 2014]**




8. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
**OR**
Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides. **[CBSE 10TH BOARD: 2014]**

9. Δ ABC and Δ PQR are similar triangles such that ∠ A = 32° and ∠ R = 65°, then ∠ B is
**[CBSE 10TH BOARD: 2015]**
(A) 83° (B) 33° (C) 63° (D) 93°

10. In Figure, DE||BC and BD = CE. Prove that Δ ABC is an isosceles triangle.
**[CBSE 10TH BOARD: 2015]**

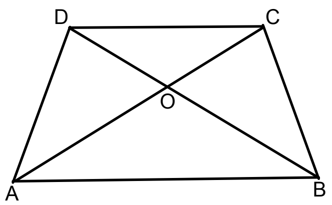



11. In a Δ ABC, P and Q are points on sides AB and AC respectively, such that PQ || BC. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm, find AB and PQ. **[CBSE 10TH BOARD: 2015]**

12. Find the length of an altitude of an equilateral triangle of side 2 cm.
**[CBSE 10TH BOARD: 2015]**

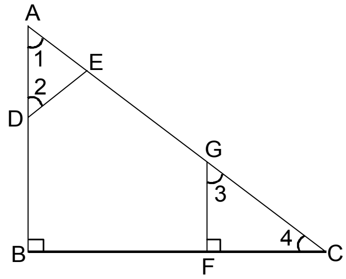
13. If ABC is an equilateral triangle with AD ⊥ BC, then prove $AD^2 = 3DC^2$.
**[CBSE 10TH BOARD: 2016]**

14. The diagonals of a trapezium ABCD with AB||DC intersect each other at point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD **[CBSE 10TH BOARD: 2016]**



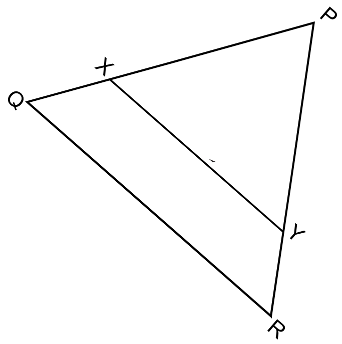

15. If Δ ABC ≅ Δ RQP, ∠ A = 80° and ∠ B = 60°, the value of ∠P is **[CBSE 10TH BOARD: 2017]**
(A) 80° (B) 30° (C) 40° (D) 50°

**16.** In figure, AB ⊥ BC , DE ⊥ AC and GF ⊥ BC, Prove that Δ ADE ~ Δ GCF **[CBSE 10TH BOARD: 2017]**




**17.** In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC. **[CBSE 10TH BOARD: 2017]**

**18.** If Figure , XY || QR, $\frac{PQ}{XQ} = \frac{7}{3}$ and PR = 6.3 cm. Find YR. **[CBSE 10TH BOARD: 2017]**



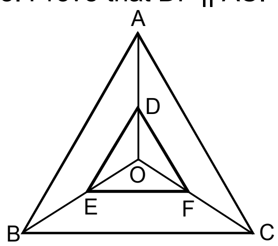

# Exercise-1

## SUBJECTIVE QUESTIONS

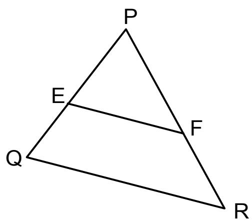
**Subjective Easy, only learning value problems**

### Section (A) : Introduction to Similar Triangles and Thales Theorem

**A-1.** Any point O, inside ΔABC, is joined to its vertices. From a point D on AO, DE is drawn so that DE || AB and EF || BC as shown in figure. Prove that DF || AC.

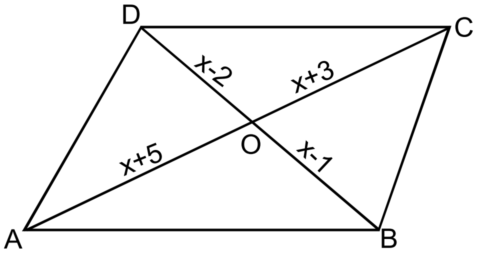



**A-2.** Kitchen garden of Ms. Sanjana is in the form of a triangle as shown. She wants to divide it in two parts; one triangle and one trapezium.

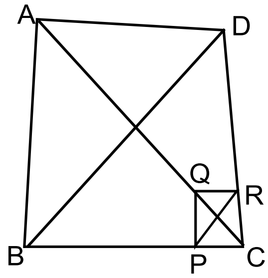



She takes PE = 4m, QE = 4.5 m, PF = 8m and RF = 9m. Is EF || QR ? Justify your answer.

**A-3.** In given figure AB || DC. Find the value of x.



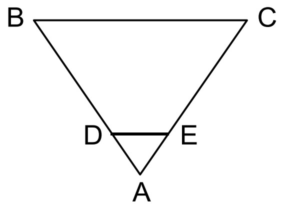

**A-4.** In figure, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that PQ || BA and PR || BD. Prove that QR || AD.



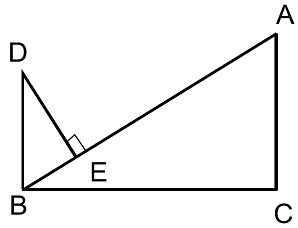

## Section (B) : Criteria for Similarity of triangles

**B-1.** In $\Delta$LMN, $\angle$L = 50º and $\angle$N = 60º. If $\Delta$LMN ~ $\Delta$PQR, then find $\angle$Q.

**B-2.** In figure, DE || BC in $\Delta$ABC such that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.

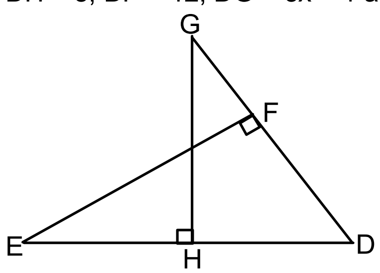



**B-3.** In figure , DB $\perp$ BC, DE $\perp$ AB and AC $\perp$ BC. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



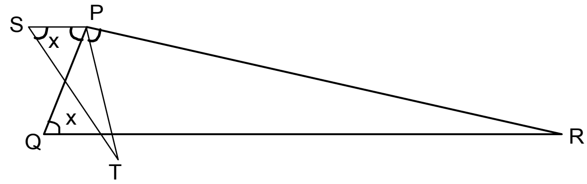

**B-4.** In $\Delta$ ABC, D and E are points on AB and AC respectively such that DE || BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE.

**B-5.** Given : $\angle$GHE = $\angle$DFE = 90º, DH = 8, DF = 12, DG = 3x – 1 and DE = 4x + 2.

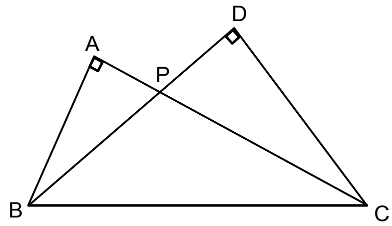



Find the lengths of segments DG and DE.

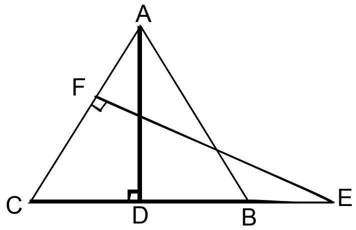
**B-6.** In figure, ∠QPS = ∠RPT and ∠PST = ∠PQR. Prove that ΔPST ~ ΔPQR and hence find the ratio ST : PT, if PR : QR = 4 : 5.




**B-7.** In figure, ABC and DBC are two right triangles with the common hypotenuse BC and with their sides AC and DB intersecting at P. Prove that AP × PC = DP × PB.

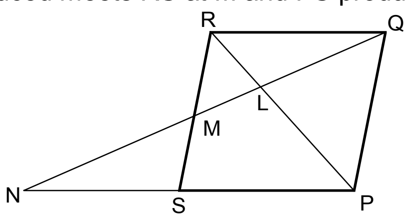



**B-8.** In figure, ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, such that FE ⊥ AC. If AD ⊥ CB, prove that : AB × FE = AD × EC.



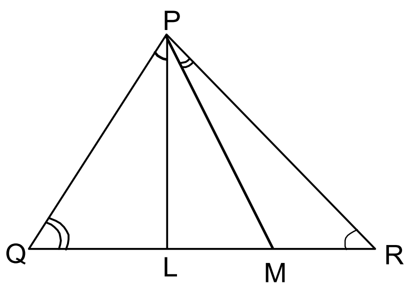

**B-9.** A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

**B-10.** In the figure, PQRS is a parallelogram with PQ = 16 cm and QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.



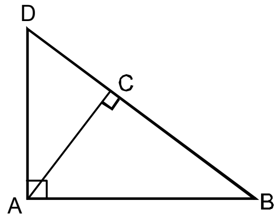

Find the lengths of PN and RM.

**B-11.** In a triangle PQR, L and M are two points on the base QR, such that ∠LPQ = ∠QRP and ∠RPM = ∠RQP. Prove that

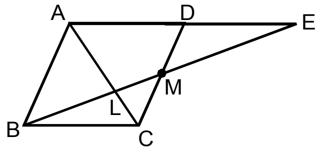



**(i)** ΔPQL ~ ΔRPM **(ii)** QL × RM = PL × PM **(iii)** $PQ^2 = QR \times QL$

**B-12.** In figure, $\Delta ABD$ is a right triangle, right-angled at A and $AC \perp BD$. Prove that $AB^2 = BC \cdot BD$.



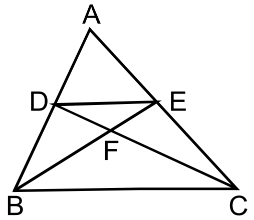

**B-13.** In figure, M is mid-point of side CD of a parallelogram ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL = 2BL.




## Section (C) : Areas of Similar Triangles

**C-1.** If the areas of two similar triangles are in the ratio 25 : 64, write the ratio of their corresponding sides.

**C-2.** In the given figure, DE is parallel to the base BC of triangle ABC and AD : DB = 5 : 3. Find the ratio:-




**(i)** $\frac{AD}{AB}$ **(ii)** $\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta CFB}$

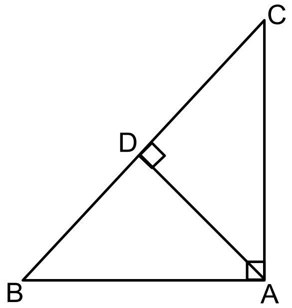
**C-3.** D, E and F are the mid-points of the sides AB, BC and CA respectively of $\Delta ABC$. Find $\frac{ar(\Delta DEF)}{ar(\Delta ABC)}$.

**C-4.** Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
Using the above, do the following :
The diagonals of a trapezium ABCD, with AB || DC, intersect each other at the point O. If AB = 2 CD, find the ratio of the area of $\Delta AOB$ to the area of $\Delta COD$.

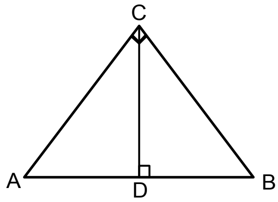
**C-5.** AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that area ($\Delta ADE$) : area ($\Delta ABC$) = 3 : 4.

## Section (D) : Pythagoras Theorem

**D-1.** In figure, $\angle BAC = 90^\circ$, $AD \perp BC$. Prove that $AB^2 = BD^2 + AC^2 - CD^2$.



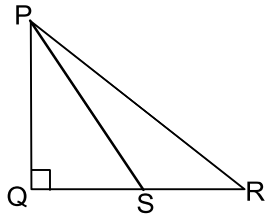
**D-2.** In figure, $\angle ACB = 90^\circ$, $CD \perp AB$, prove that $CD^2 = BD.AD$.




**D-3.** The perpendicular AD on the base BC of a $\Delta ABC$ meets BC at D so that 2DB = 3CD. Prove that $5AB^2 = 5AC^2 + BC^2$.

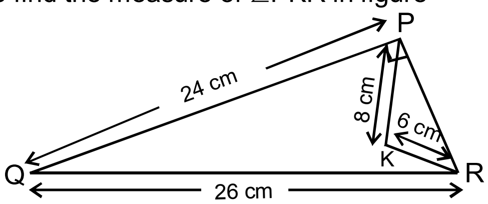
**D-4.** D and E are points on the sides CA and CB respectively of $\Delta ABC$ right-angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

**D-5.** In a right triangle, prove that the square on the hypotenuse is equal to sum of the squares on the other two sides. Using the above result, prove the following :
In figure PQR is a right triangle, right angled at Q. If QS = SR, show that $PR^2 = 4PS^2 - 3PQ^2$.




**D-6.** In $\Delta$ ABC, $\angle ABC = 135^\circ$. Prove that $AC^2 = AB^2 + BC^2 + 4ar(\Delta ABC)$.

**D-7.** In a triangle, if the square of one side is equal to the sum of the square of the other two sides, prove that the angle opposite to the first side is a right angle.
Use the above theorem to find the measure of $\angle PKR$ in figure



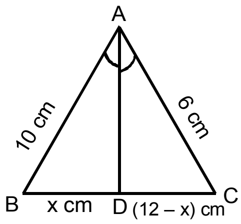

**D-8.** In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.
Using the above, do the following :
In an isosceles triangle PQR, PQ = QR and $PR^2 = 2PQ^2$. Prove that $\angle Q$ is a right angle.

## OBJECTIVE QUESTIONS

**Single Choice Objective, straight concept/formula oriented**

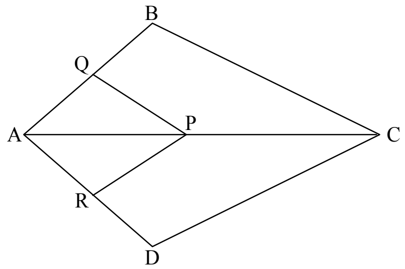
### Section (A) : Introduction to Similar Triangles and Thales Theorem

**A-1.** In a $\Delta ABC$, AD is the bisector of $\angle A$, meeting side BC at D. If AB = 10 cm, AC = 6 cm, BC = 12 cm, find BD.



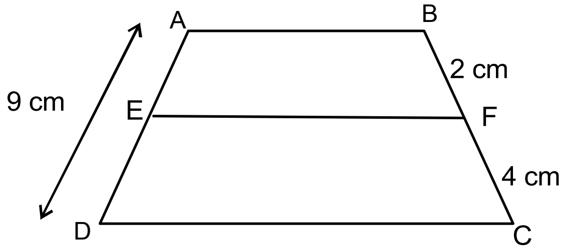

(A) 3.3 (B) 18 (C) 7.5 (D) 1.33

**A-2.** In figure, if PQ | | BC and PR | | CD, then



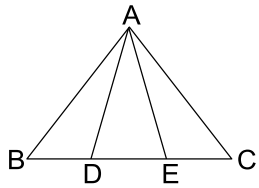

(A) $\frac{AR}{AD} = \frac{AD}{AQ}$ (B) $\frac{AR}{AD} = \frac{AQ}{AB}$ (C) $\frac{AP}{PC} = \frac{AD}{AR}$ (D) None of these

**A-3.** In figure, ABCD is a trapezium in which AB || EF || DC. The length of AE is




(A) 2 cm (B) 3 cm (C) 4 cm (D) 7 cm

**A-4.** If AD and AE are angle bisectors of $\angle$BAE and $\angle$DAC respectively, then




(A) $\frac{BD}{EC} = \frac{AB}{AC}$ (B) $\frac{BD}{DE} = \frac{AD}{AC}$ (C) $\frac{BD}{DE} = \frac{DE}{EC}$ (D) $\frac{BD}{EC} = \frac{(AB)(AD)}{(AE)(AC)}$

## Section (B) : Criteria for Similarity of triangles

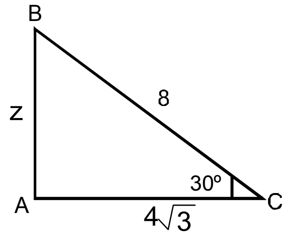
**B-1.** The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, then the corresponding side of the other triangle is :
(A) 6.2 cm (B) 3.4 cm (C) 5.4 cm (D) 8.4 cm

**B-2.** Two triangles ABC and PQR are similar, if BC : CA : AB = 2 : 3 : 4, then $\frac{QR}{PR}$ is :
(A) $\frac{2}{5}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{2}{3}$

**B-3.** $\Delta$ABC and $\Delta$PQR are similar triangles such that $\angle A = 32^\circ$ and $\angle R = 65^\circ$, then $\angle B$ is :
(A) 83° (B) 32° (C) 65° (D) 97°

**B-4.** The perimeters of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, length of AB is :
(A) 10 cm (B) 8 cm (C) 5 cm (D) 6 cm

**B-5.** In figure, $\Delta ABC \sim \Delta PQR$, then y + z is :




(A) $8\sqrt{3}$ cm (B) $4 + 3\sqrt{3}$ cm (C) $5 + 4\sqrt{3}$ cm (D) $6 + 3\sqrt{3}$ cm

**B-6.** If in $\Delta ABC$ and $\Delta DEF$, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar if :

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ (C) $\angle B = \angle D$ (D) $\angle A = \angle F$

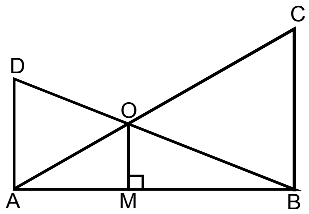
**B-7.** A vertical stick 30 m long casts a shadow 15 m long on the ground. At the same time, a tower casts a shadow 75 m long on the ground. The height of the tower is :

(A)150 m (B) 100 m (C) 25 m (D) 200 m

**B-8.** If the ratio of the corresponding sides of two similar triangles is 2 : 3, then the ratio of their corresponding altitude is :

(A) 3 : 2 (B) 16 : 81 (C) 4 : 9 (D) 2 : 3

**B-9.** In the given figure $DA \perp AB$ , $CB \perp AB$ and $OM \perp AB$ . If AO = 5.4 cm, OC = 7.2 cm and BO = 6 cm, then the length of DO is:




(A) 4.5 cm (B) 4 cm (C) 5 cm (D) 6.5 cm

## Section (C) : Areas of Similar Triangles

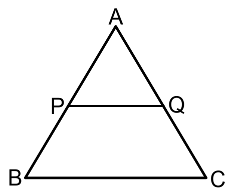
**C-1.** In a triangle ABC, a straight line parallel to BC intersects AB and AC at point D and E respectively. If the area of ADE is one-fifth of the area of ABC and BC = 10 cm, then DE equals :

(A) 2 cm (B) $2\sqrt{5}$ cm (C) 4 cm (D) 4 cm

**C-2.** $\Delta ABC \sim \Delta PQR$. M is the mid point of BC and N is the mid point of QR. If the area of $\Delta ABC$ = 100 sq. cm. and the area of $\Delta PQR$ = 144 sq. cm. If AM = 4 cm, then PN is :

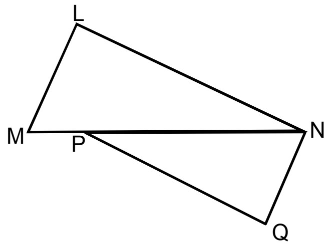
(A) 4.8 cm (B) 12 cm (C) 4 cm (D) 5.6 cm

**C-3.** In the figure, PQ || BC and AP : PB = 1 : 2. Find $\frac{ar(\Delta APQ)}{ar(\Delta ABC)}$.



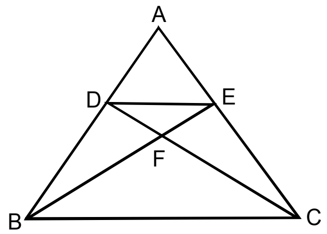

(A) 1 : 4 (B) 4 : 1 (C) 1 : 9 (D) 2 : 9

**C-4.** In the given figure, LM || NQ and LN || PQ. If MP = $\frac{1}{3}$MN, find the ratio of the areas of $\Delta$LMN and $\Delta$QNP.




(A) 9 : 4 (B) 1 : 9 (C)1 : 3 (D)3 : 1

**C-5.** In the given figure, DE ||BC and AD : DB = 5 : 4. Find $\frac{ar(\Delta DEF)}{ar(\Delta CFB)}$.



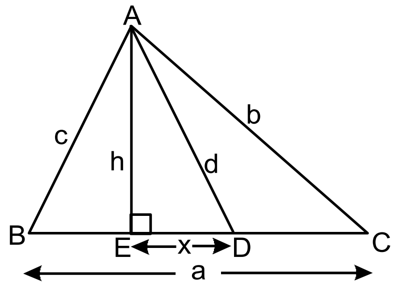

(A) 25 : 16 (B) 25 : 81 (C)5 : 4 (D) 4:5

## Section (D) : Pythagoras Theorem

**D-1.** In a triangle ABC, if angle B = 90° and D is the point in BC such that BD = 2 DC, then
(A) $AC^2 = AD^2 + 3\ CD^2$ (B) $AC^2 = AD^2 + 5\ CD^2$
(C) $AC^2 = AD^2 + 7\ CD^2$ (D) $AC^2 = AB^2 + 5\ BD^2$

**D-2.** In an isosceles $\Delta$ABC, if AC = BC and $AB^2 = 2\ AC^2$, then $\angle$C is equal to :
(A) 45° (B) 60° (C) 30° (D) 90°

**D-3.** In the following figure, AE $\perp$ BC, D is the mid point of BC, then x is equal to :



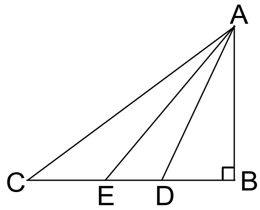

(A) $\frac{1}{a}\left[b^2 - d^2 - \frac{a^2}{4}\right]$ (B) $\frac{h+d}{3}$ (C) $\frac{c+d-h}{2}$ (D) $\frac{a^2+b^2+d^2-c^2}{4}$

**D-4.** P and Q are the mid points of the sides AB and BC respectively of the triangle ABC, right-angled at B, then

(A) $AQ^2 + CP^2 = AC^2$ (B) $AQ^2 + CP^2 = \frac{4}{5}\ AC^2$

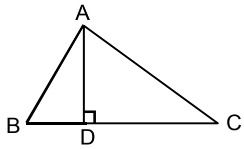
(C) $AQ^2 + CP^2 = \frac{5}{4}AC^2$ (D) $AQ^2 + CP^2 = \frac{3}{5}AC^2$

**D-5.** In figure, D and E trisect BC and $K(AE^2) = 3AC^2 + 5AD^2$. Find constant K.




(A) 5 (B) 6 (C) 8 (D) None of these

**D-6.** In figure $AD \perp BC$ and $BD = \frac{1}{3}CD$ and $K(CA^2 - AB^2) = BC^2$. Find constant K.

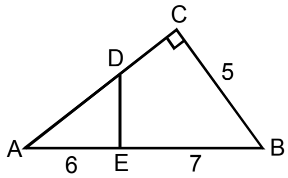



(A) 0 (B) 1 (C) 2 (D) 3

# Exercise-2

## OBJECTIVE QUESTIONS

**1.** In the figure C is a right angle, $DE \perp AB$, AE = 6, EB = 7 and BC = 5. The area of the quadrilateral EBCD is



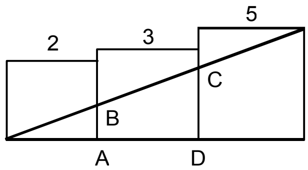

(A) 27.5 (B) 25 (C) 22.5 (D) 20

**2.** The median AD of $\Delta ABC$ meets BC at D. The internal bisectors of $\angle ADB$ and $\angle ADC$ meet AB and AC at E and F respectively. Then EF :

(A) is perpendicular to AD (B) is parallel to BC

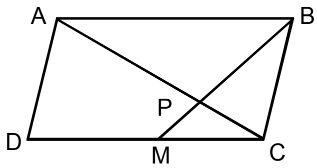
(C) divides AD in the ratio of AB : AC (D) none of these

**3.** Three squares have the dimensions indicated in the diagram. The area of the quadrilateral ABCD, is




(A) $\frac{21}{4}$ (B) $\frac{15}{4}$ (C) $\frac{42}{4}$ (D) data not sufficient

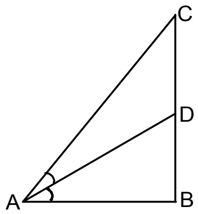
**4.** ABCD is a parallelogram, M is the midpoint of DC. If AP = 65 and PM = 30 then the largest possible integral value of AB is :




(A) 124 (B) 120 (C) 119 (D) 118

**5.** ABCD is a parallelogram, P is a point on AB such that AP : PB = 3 : 2. Q is a point on CD such that CQ : QD = 7 : 3. If PQ meets AC at R, then AR : AC is :
(A) 5 : 11 (B) 6 : 13 (C) 4 : 7 (D) 2 : 5

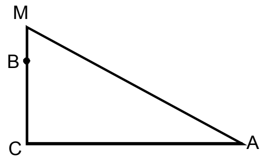
**6.** If CD = 15, DB = 9, AD bisects $\angle A$, $\angle ABC = 90^\circ$, then AB has length :




(A) 32 (B) 18 (C) 7 (D) 24

**7.** In a right triangle with sides a and b, and hypotenuse c, the altitude drawn on the hypotenuse is x. Then which one of the following is correct ?
(A) $ab = x^2$ (B) $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$ (C) $a^2 + b^2 = 2x^2$ (D) $\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{b^2}$

**8.** In the right triangle shown the sum of the distances BM and MA is equal to the distances BC and CA. If MB = x, CB = h and CA = d, then x equals.

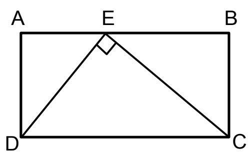



(A) $\frac{hd}{2h+d}$ (B) $d - h$ (C) $h + d - \sqrt{2d}$ (D) $\sqrt{h^2 + d^2} - h$

**9.** A rhombus is inscribed in triangle ABC in such a way that one of its vertices is A and two its sides lie along AB and AC and fourth vertex lies on BC , where AC = 6, AB = 12 and BC = 8, the side of the rhombus, is :
(A) 2 (B) 3 (C) 4 (D) 5

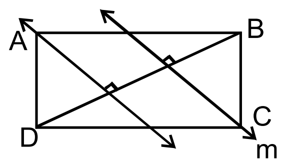
**10.** ABCD (in order) is a rectangle with AB = CD = $\frac{12}{5}$ and BC = DA = 5. Point P is taken on AD such that $\angle BPC = 90^\circ$. The value of (BP + PC) is equal to :
(A) 5 (B) 6 (C) 7 (D) 8

**11.** In the diagram, ABCD is a rectangle and point E lies on AB. Triangle DEC has $\angle DEC = 90^\circ$, DE = 3 and EC = 4. The length of AD is :



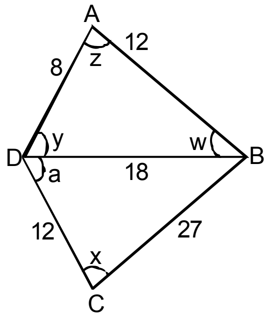

(A) 2.4 (B) 2.8 (C) 1.8 (D) 3.2

**12.** In the figure, DB is diagonal of rectangle ABCD and line l through A and line m through C divide DB in three equal parts each of length 1 cm and are perpendicular to DB. Area (in cm²) of rectangle ABCD is : **[Harayana NTSE Stage-1 2014]**




(A) $2\sqrt{2}$ (B) $2\sqrt{3}$ (C) $3\sqrt{2}$ (D) $3\sqrt{3}$

**13.** In the quadrilateral ABCD : **[Harayana NTSE Stage-1 2015]**




(A) x = y, a = z (B) x = z, a = y (C) x = z, a =y (D) x = y, a = w

**14.** 'O' is any point inside the rectangle PQRS, then **[West Bengal NTSE Stage-1 2016]**

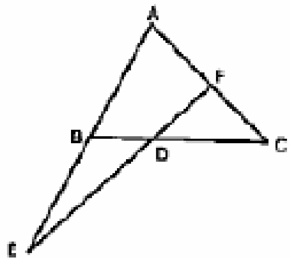
(A) $OP^2 + OR^2 = OQ^2 + OS^2$ (B) $OP^2 + OQ^2 = OR^2 + OS^2$

(C) $OP^2 + OS^2 = OQ^2 + QR^2$ (D) None of the above

# Exercise-3

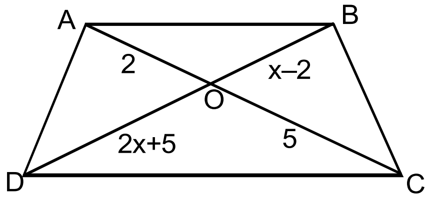
## NTSE PROBLEMS (PREVIOUS YEARS)

**1.** In the given figure $\frac{BD}{CD} = \frac{3}{4}$ and AE = 6BE, then $\frac{CF}{AF}$ = ______ **[Orissa NTSE Stage-1 2012]**




(A) 2/9 (B) 4/6 (C) 3/8 (D) 5/9

**2.** In a given figure in trapezium ABCD if AB || CD then value of x is : **[Raj. NTSE Stage-1 2013]**




(A) $\frac{29}{8}$ (B) $\frac{8}{29}$ (C) 20 (D) $\frac{1}{20}$

**3.** $\Delta ABC \sim \Delta PQR$ and $\frac{\text{area } \Delta \text{ ABC}}{\text{area } \Delta \text{ PQR}} = \frac{16}{9}$. If PQ = 18 cm and BC = 12 cm, then AB and QR are respectively: **[Delhi NTSE Stage-1 2013]**

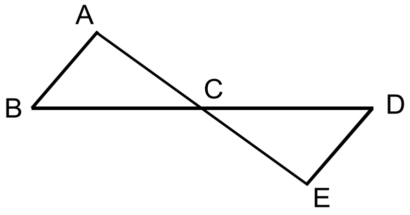
(A) 9 cm, 24 cm (B) 24 cm, 9 cm (C) 32 cm, 675 cm (D) 135 cm, 16 cm

**4.** E and F are respectively, the mid points of the sides AB and AC of $\Delta ABC$ and the area of the quadrilateral BEFC is k times the area of $\Delta ABC$. The value of k is : **[Delhi NTSE Stage-1 2013]**

(A) $\frac{1}{2}$ (B) 3 (C) $\frac{3}{4}$ (D) 4

**5.** The ratio of the areas of two similar triangles is equal to : **[M.P. NTSE Stage-1 2013]**
(A) The ratio of corresponding medians
(B) The ratio of corresponding sides
(C) The ratio of the squares of corresponding sides
(D) None of these

**6.** In the figure, ΔABC is similar to ΔEDC. If we have AB = 4cm, ED = 3 cm, CE = 4.2 cm and CD = 4.8 cm, then the values of CA and CB respectively are : **[M.P. NTSE Stage-1 2013]**

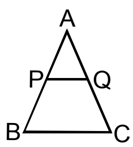



(A) 6 cm, 6.6 cm (B) 4.8 cm, 6.6 cm (C) 6.4 cm, 5.6 cm (D) 5.6 cm, 6.4 cm

**7.** ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is **[Raj. NTSE Stage-1 2014]**
(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

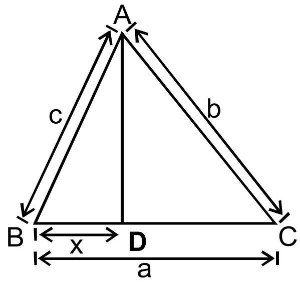
**8.** In a Δ PRS, ∠PRS = 120°.A point Q is taken on PR such that PQ = QS and QR = RS then ∠QPS = .......
**[Bihar NTSE Stage-1 2014]**
(A) 15° (B) 30° (C) 45° (D) 12°

**9.** In below figure PQ || BC and AP : PB = 1 : 2. Then the ratio of area of ΔAPQ and ΔABC will be :
**[Chattisgarh NTSE Stage-1 2014]**




(A) 1 : 2 (B) 1 : 4 (C) 1 : 9 (D) 4 : 1

**10.** In ΔABC, segment AD⊥BC,

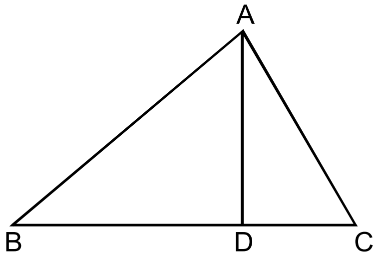



If BD = x units, then x is : **[Delhi NTSE Stage-1 2014]**

(A) $x = \frac{c^2 + a^2 - b^2}{2a}$ (B) $\frac{a^2 + b^2 - c^2}{2c}$ (C) $\frac{b^2 + c^2 - a^2}{2b}$ (D) $\frac{b^2 + c^2 - a^2}{2abc}$

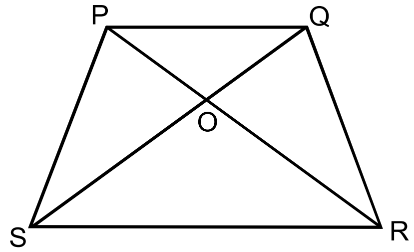
**11.** In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is **[Raj. NTSE Stage-1 2014]**
(A) 120° (B) 60° (C) 90° (D) 45°

**12.** In the figure given below, point D is on side BC of $\Delta ABC$ such that $\angle ABC = \angle DAC$. If AB = 9, AD = 4, AC = 5, then Ar($\Delta$ADC) : Ar($\Delta$BAC) = **[Maharashtra NTSE Stage-1 2014]**



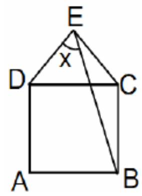

(A) 81 : 16 (B) 9 : 4 (C) 4 : 9 (D) 16 : 81

**13.** In trapezium PQRS, seg PQ || seg SR. Diagonal PR and diagonal QS intersects each other at point O. If PO = x + 4; QO = x + 2; RO = x + 10 and SO = x + 7, then PQ : RS = ? **[Maharashtra NTSE Stage-1 2014]**




(A) 3 : 2 (B) 2 : 3 (C) 4 : 9 (D) 9 : 4

**14.** In the figure given below, equilateral triangle EDC surmounts square ABCD. Find the angle DEB represented by x **[Delhi NTSE Stage-1 2015]**

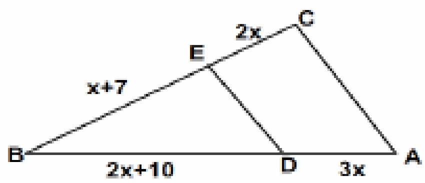



(A) 60° (B) 15° (C) 30° (D) 45°

**15.** The length of sides of triangle are integers and its perimeter is 14. How many such distinct triangles are possible? **[Bihar NTSE Stage-1 2015]**

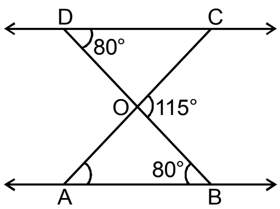
(A) 6 (B) 5 (C) 4 (D) 3

**16.** In the figure given below, DE||AC, find the value of x. **[Delhi NTSE Stage-1 2015]**



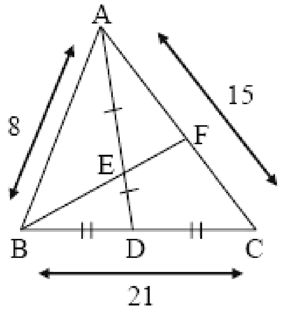

(A) 2 (B) 3 (C) 1 (D) 4

**17.** In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$ and $\angle CDO = 80°$. Then $\angle OAB$ is equal to **[Raj. NTSE Stage-1 2015]**



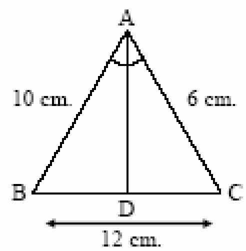

(A) 80° (B) 35° (C) 45° (D) 65°

**18.** In Δ ABC, AD is median and E is the mid-point of AD. If BE is extended, it meets AC in F. AB = 8 cm, BC = 21 cm and AC = 15 cm, then AF = ? **[Jharkhand NTSE Stage-1 2015]**



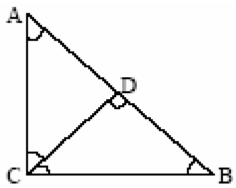

(A) 7cm. (B) 3 cm. (C) 12 cm. (D) 5 cm.

**19.** In the given figure, AD is the bisector of ∠BAC. If AB = 10 cm, AC = 6 and BC = 12 cm, find BD : **[Jharkhand NTSE Stage-1 2015]**




(A) 4.5 cm. (B) 9 cm. (C) 7.5 cm. (D) 3 cm.

**20.** In the given figure, ∠ACB = 90° and CD ⊥ AB, then **[Jharkhand NTSE Stage-1 2015]**




(A) $CD^2 = BD.AD$ (B) $BC^2 = AD.BD$ (C) $AC^2 = AD.BC$ (D) $AD^2 = CD.BD$

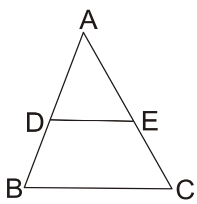
**21.** Δ ABC is a right triangle in which ∠ C = 90° and CD ⊥ AB. If BC = a, AC = b, AB= c and CD = p, then **[Jharkhand NTSE Stage-1 2015]**

(A) $p^2 = a^2 + b^2$ (B) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (C) $\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (D) None of these

**22.** The ratio of areas of two similar triangles is $2\frac{1}{4}$ : 1. If the perimeter of large triangle is 36 cm, then find the perimeter of smaller triangle. **[Maharashtra NTSE Stage-1 2015]**

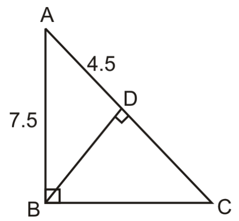
(A) 16 cm (B) 20 cm (C) 12 cm (D) 24 cm

**23.** In the following figure, seg DE || side BC in Δ ABC. If 3Ar(Δ ADE) = Ar ( DECB), then find the ratio BC : DE? **[Maharashtra NTSE Stage-1 2015]**




(A) 1 : 2 (B) 16 : 1 (C) 1 : 16 (D) 2 : 1

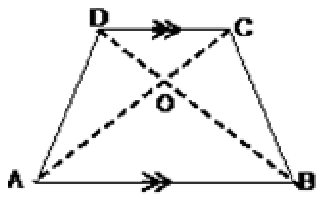
**24.**




In the above figure $\Delta ABC$, $m\angle B = 90^\circ$, $BD \perp AC$, $AD = 4.5$, $AB = 7.5$, then find Ar($\Delta$ BDC) ; Ar($\Delta$ ABC). **[Maharashtra NTSE Stage-1 2016]**

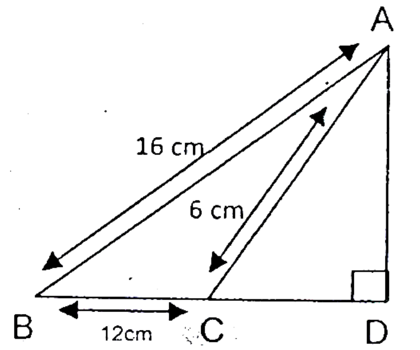
(A) 16 : 25 (B) 4 : 5 (C) 25 : 16 (D) 5 : 4

**25.** In the given figure ABCD is a trapezium in which AB||DC and AB : DC = 3 : 2, The ratio of the areas of $\Delta AOB$ and $\Delta COD$ is **[Raj. NTSE Stage-1 2016]**



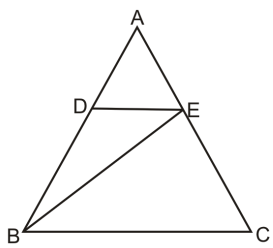

(A) 3 : 2 (B) 2 : 3 (C) 4 : 9 (D) 9 : 4

**26.** In the figure $\angle D = 90^\circ$, AB = 16 cm, BC = 12 cm and CA = 6 cm, then CD is: **[Delhi NTSE Stage-1 2016]**




(A) $\frac{13}{6}$cm (B) $\frac{17}{6}$cm (C) $\frac{19}{6}$cm (D) $\frac{18}{5}$cm

**27.** In the above figure $\Delta ABC$, DE || BC, Ar($\Delta ADE$) = 48 aq.cm. $\frac{AD}{DB} = \frac{4}{5}$ , Find the area of $\Delta BEC$. **[Maharashtra NTSE Stage-1 2016]**

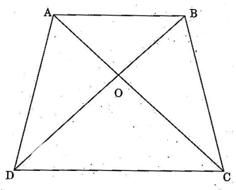



(A) 60 sq. cm (B) 95 sq. cm (C) 108 sq. cm (D) 135 sq. cm

**28.** In $\Delta ABC$, $m\angle B = 90^\circ$ , AB = $4\sqrt{5}$ , $BD \perp AC$, AD = 4, then ar ($\Delta ABC$) = ? **[Maharashtra NTSE Stage-1 2017]**

(A) 96 sq. units (B) 80 sq. units (C) 120 sq. units (D) 160 sq. units

**29.** In the following figure, seg AB || seg CD. Diagonals AC and BD intersect at point O. If AO : OC = 1 : 3, then $\frac{A(\Delta AOB)}{A(\Delta ABD)}$ = ? **[Maharashtra NTSE Stage-1 2017]**




(A) 1: 4 (B) 1 : 3 (C) 1 ; 2 (D) 1 ; 1

**30.** In ΔABC points P and Q trisect side AB, points T and U trisect side AC and points R and S trisect side BC. Then perimeter of hexagon PQRSTU is how many times of the perimeter of ΔABC? **[Maharashtra NTSE Stage-1 2017]**

(A) $\frac{1}{3}$ times (B) $\frac{2}{3}$ times (C) $\frac{1}{6}$ times (D) $\frac{1}{2}$ times

# Answer Key

## Exercise Board Level

**TYPE (I)**

**1.** 10 cm
**5.** 9

**TYPE (II)**

**6.** 100°
**7.** 10 cm
**9.** DE = 12 cm, ∠F = 100°
**10.** 108 cm²

**TYPE (III)**

**11.** 2
**13.** $4\sqrt{3}$ cm.
**15.** 60 cm
**16.** 15 cm and 20 cm
**17.** 9 m

**TYPE (IV)**

**19.** PQ = 8 cm, QR = 12 cm, RS = 16 cm
**21.** $\frac{16}{3}$ cm.

## Previous Year Problems

**2.** $\sqrt{52}$
**4.** 45
**6.** 4.6 cm
**7.** 120 cm²
**9.** (A)
**11.** AB = 6cm, PQ = 2.4cm
**12.** $\sqrt{3}$
**14.** 4 : 1
**15.** (C)
**17.** AD = 24 cm
**18.** YR = 2.7 cm

## Exercise-1

### SUBJECTIVE QUESTIONS

**Subjective Easy, only learning value problems**

### Section (A)

**A-2.** Yes
**A-3.** x = 7

### Section (B)

**B-1.** ∠Q is 70°.
**B-2.** 2 cm.
**B-4.** BC = 3.6 cm, CE = 4.8 cm
**B-5.** DG = 20 units, DE = 30 units
**B-6.** $\frac{5}{4}$
**B-9.** 1.6 m
**B-10.** PN = 15 cm and RM = 10.67 cm

### Section (C)

**C-1.** 5 : 8.
**C-2.** (i) $\frac{5}{8}$ (ii) $\frac{25}{64}$.
**C-3.** $\frac{1}{4}$
**C-4.** $\frac{4}{1}$.

### Section (D)

**D-7.** 90°.

## OBJECTIVE QUESTIONS

### Section (A)

**A-1.** (C) **A-2.** (B) **A-3.** (B) **A-4.** (D)

### Section (B)

**B-1.** (C) **B-2.** (D) **B-3.** (A) **B-4.** (A) **B-5.** (B)

**B-6.** (C) **B-7.** (A) **B-8.** (D) **B-9.** (A)

### Section (C)

**C-1.** (B) **C-2.** (A) **C-3.** (C) **C-4.** (A) **C-5.** (B)

### Section (D)

**D-1.** (B) **D-2.** (D) **D-3.** (A) **D-4.** (C) **D-5.** (C)

**D-6.** (C)

# Exercise-2

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | C | B | A | A | B | B | D | A | C | C | A | C | A | A |

# Exercise-3

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | A | C | B | C | C | D | C | A | C | A | C | D | B | D | C | C | B | D | C | A |
| Ques. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | | | | | | | | | | |
| Ans. | B | D | D | A | D | C | D | B | A | B | | | | | | | | | | |