MATHEMATICS

Class-X

Topic-6

CIRCLES

INDEX		
S. No.	**Topic**	**Page No.**

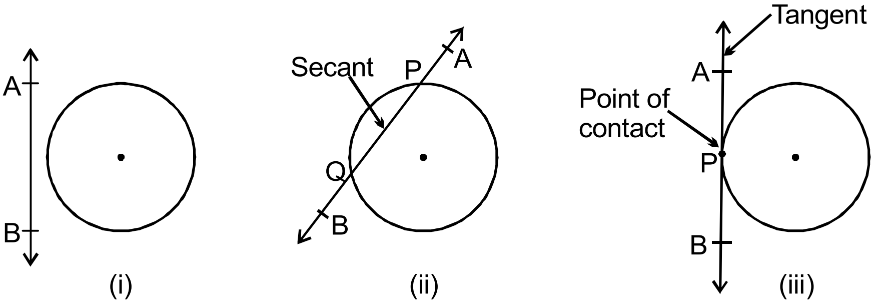
CH-6
CIRCLES

A. CIRCLES

(a) Secant and Tangent

A **circle** is the locus of a point which moves in plane in such a way that its distance from a fixed point remains constant.

Let us consider a circle and a line AB. There can be three different situation as shown in figure.



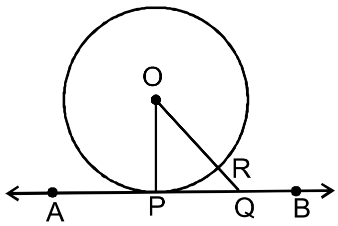
(i) (ii) (iii)

In figure (i) the line AB and the circle have no common point.
In figure (ii) the line AB intersects the circle at point P and Q.
Therefore the line AB and the circle have two common points P and Q. The line AB is called secant of the circle.
In figure (iii) the line touches the circle at point P. Therefore the line AB and the circle have only one common point P. The line AB is called tangent of the circle.

Secant : A line which intersects a circle at two distinct points is called the secant of the circle.

Tangent : A line which meets a circle at only one point is called the tangent to the circle. The point a which the line meets the circle is called the point of contact.
There is only one tangent passing through a point lying on the circle.

Theorem 1
Statement : A tangent to a circle is perpendicular to the radius through the point of contact.



Given : A circle C (O, r) and a tangent AB at a point P.

To prove : $OP \perp AB$.

Construction : Take any point Q, other than P on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

Proof : Among all line segments joining the point O to a point on AB, the shortest one is perpendicular to AB. So, to prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly, OP = OR (Radius)
Now, $OQ = OR + RQ \Rightarrow OQ > OR \Rightarrow OQ > OP$ $(\because OP = OR)$
Thus, OP is shorter than any other segment joining O to any point of AB. Hence, $OP \perp AB$.

Theorem 2 (Convers of Theorem 1)

Statement : A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.

Given : OP is a radius of a circle with center O. AB is a line through P and OP ⊥ AB.

To prove : AB is tangent to the circle at point P.

Construction : Take a point R other than P on AB. Join OR which intersect the circle at Q.

Proof : OP ⊥ AB (given)

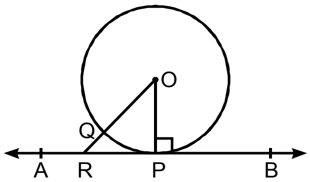
∴ OP is the shortest line segment drawn from point O to AB.

⇒ OP < OR

⇒ OR > OP

∴ R lies outside the circle.

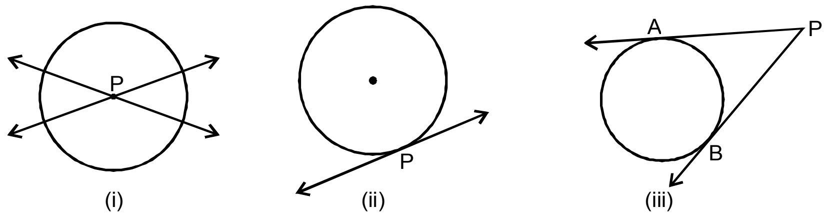
Thus every point on AB, other then P, lies outside the circle.



This shows that AB meets the circle only at point P.

Hence, AB is the tangent to the circle at point P.

(b) Number of tangents from a point on a circle



(a) If a point is inside the circle then it is not possible to draw any tangent to the circel through this point as shown in figure (i).

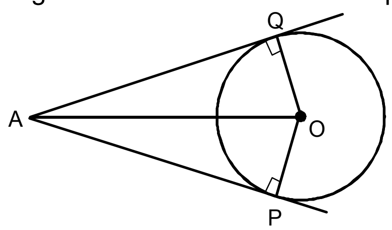
(b) If a point is on the circle then only one tangent to the circle through this point can be drawn as shown in figure (ii).

(c) If a point is outside the circle then exactly two tangents can be drawn to the circle through this point as shown in figure (iii).

PA and PB are the lengths of tangents drawn from P to the circle.

Theorem 3

Statement : Lengths of two tangents drawn from an external point to a circle are equal.



Given : AP and AQ are two tangents drawn from a point A to a circle C (O, r).

To prove : AP = AQ.

Construction : Join OP, OQ and OA.

Proof : In ΔAOQ and ΔAPO

∠OQA = ∠OPA

[Tangent at any point of a circle is perp. to radius through the point of contact]

AO = AO [Common]

OQ = OP [Radius]

So, by R.H.S. criterion of congruency ΔAOQ ≅ ΔAOP

∴ AQ = AP [By CPCT]. **Hence Proved.**

RESULTS :

(i) If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre $\angle$**AOQ =** $\angle$**AOP** [By CPCT]

(ii) If two tangents are drawn to a circle from an external point, they are equally inclined to the segment, joining the centre to that point $\angle$**OAQ =** $\angle$**OAP** [By CPCT]

(c) Common tangents to two circles

Definition : A line which touches the two given circles is **called common tangent** to the two circles. Let **C(O_1, r_1), C(O_2, r_2)** be two given circles. Let the distance between centres **O_1** and **O_2** be **d** i.e., **O_1O_2 = d**.

Case 1

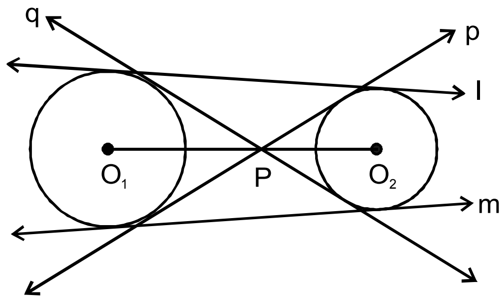


Fig.(i)

In fig. (i) **d > r_1 + r_2** i.e. two circles do not intersect.

In this case, **four common tangents** are possible.

The tangent lines **l** and **m** are called **direct common tangents** and the tangent lines **p** and **q** are called **indirect (transverse) common tangents**.

Case 2

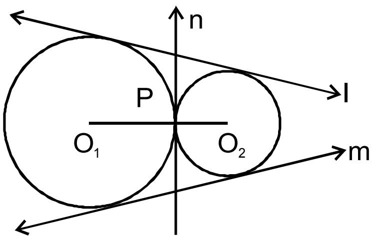


Fig.(ii)

In fig. (ii), **d = r_1 + r_2.** In this case, two circles touch externally and there are three common tangents.

Case 3

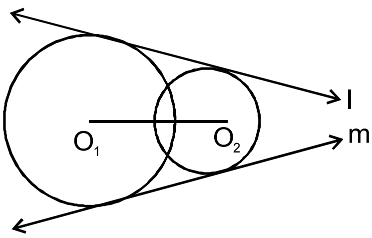


Fig.(iii)

In fig.(iii) **d < r_1 + r_2.** In this case two circles intersect in two distinct points and there are only two common tangents.

Case 4

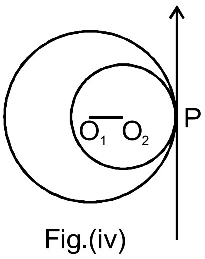


Fig.(iv)

In fig. (iv), **$d = r_1 - r_2$ ($r_1 > r_2$),** in this case, two circles touch internally and there is only one common tangent.

Case 5

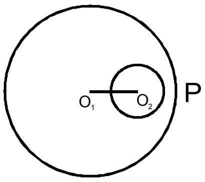


Fig.(v)

In fig. (v), the circle **$C(O_2, r_2)$** lies wholly in the circle **$C(O_1, r_1)$** and there is no common tangent.

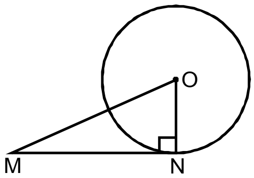
Solved Examples

Example. 1

A point M is 26 cm away from the centre of a circle and the length of tangent drawn from M to the circle is 24 cm. Find the radius of the circle.

Sol. **Given:** OM = 26 cm, MN = 24 cm

We know that the tangent at any point to a circle is perpendicular to the radius through point of contact.



$\therefore$ ONM is a right angled Δ

$OM^2 = ON^2 + MN^2$ (By Pythagoras theorem)

$\Rightarrow (26)^2 = ON^2 + (24)^2$

$\Rightarrow ON^2 = (26)^2 - (24)^2$

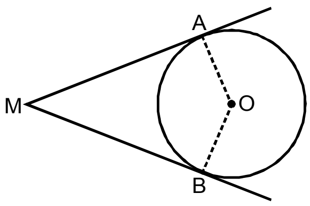
$\Rightarrow ON^2 = 676 - 576 = 100$

$\therefore ON = 10$ cm.

Hence, radius of circle is 10 cm.

Example. 2

In figure AM and BM are the tangents to a circle with centre O. Show that the point O, A, M, B are concyclic.



Sol. **Given :** AM and BM are the tangents to a circle with centre O.

To prove : O, A, M and B are concyclic, i.e., AOBM is a cyclic quadrilateral.

Proof : OA $\perp$ AM and OB $\perp$ BM ($\because$ A tangent is perpendicular to radius at the point of contact).

i.e., $\angle$OAM = 90° and $\angle$OBM = 90°

In quadrilateral AOBM

$\angle$OAM + $\angle$OBM = 90° + 90° = 180°

Thus, the sum of opposite angles of quadrilateral AOBM is 180°.

$\therefore$ AOBM is a cyclic quadrilateral.

Example. 3

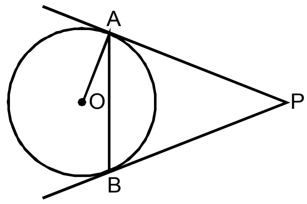
Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle$APB = 2$\angle$OAB.

Sol. **Given** : PA and PB are two tangents drawn from external point P to a circle with centre O.

To prove : $\angle$APB = 2$\angle$OAB.

Proof : OA $\perp$ PA ($\therefore$ A tangent to a circle is perpendicular to radius at the point of contact).

$\therefore$ $\angle$OAP = 90°



In ΔPAB

PA = PB ($\because$ Tangents drawn from an external point to a circle are equal)

$\Rightarrow$ $\angle$PAB = $\angle$PBA (In a Δ, angles opposite to equal sides are equal)

$\angle$PAB + $\angle$PBA + $\angle$APB = 180° (Angle sum property)

$\angle$PAB + $\angle$PAB + $\angle$APB = 180° ($\because$ $\angle$PAB = $\angle$PBA)

2$\angle$PAB = 180° – $\angle$APB.

$\therefore$ $\angle PAB = 90° - \frac{1}{2}\angle APB$ ...(i)

Now $\angle$OAP = 90°

$\Rightarrow$ $\angle$OAB + $\angle$PAB = 90°

$\Rightarrow$ $\angle OAB + 90° - \frac{1}{2}\angle APB = 90°$ (using (i))

$\Rightarrow$ $\angle OAB = \frac{1}{2}\angle APB$ $\therefore$ $\angle$APB = 2$\angle$OAB.

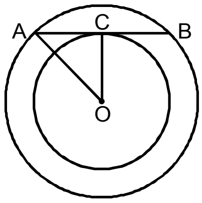
Example. 4

If the radii of the two concentric circles are 15 cm and 17 cm, show that the length of the chord of one circle which is tangent to other circle is 16 cm.

Sol. **Given :** OA = 17 cm, OC = 15 cm (given)

AB is a chord of larger circle which is the tangent for smaller circle at point C.

$\therefore$ OC $\perp$ AB ($\because$ A tangent is perpendicular to radius at the point of contact).



In ΔAOC

$OA^2 = OC^2 + AC^2$

$\Rightarrow \quad 17^2 = 15^2 + AC^2$

$\Rightarrow \quad AC^2 = 17^2 - 15^2$

$\Rightarrow \quad AC^2 = 289 - 225 = 64$

$\therefore \quad AC = 8$ cm.

We know that perpendicular drawn from centre of a chord bisects the chord.

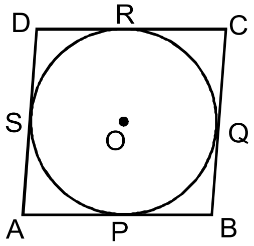
So, $AB = 2AC = 2 \times 8 = 16$ cm. **Ans.**

Example. 5

If all the sides of a parallelogram touches a circle, show that the parallelogram is a rhombus.

Sol. **Given :** Sides AB, BC, CD and DA of a $||^{gm}$ ABCD touch a circle at P, Q, R and S respectively.

To prove : $||^{gm}$ ABCD is a rhombus.



Proof : AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

[Tangents drawn from an external point to a circle are equal]

Adding (i), (ii), (iii) and (iv), we get

$\Rightarrow \quad AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow \quad (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow \quad AB + CD = AD + BC$

$\Rightarrow \quad AB + AB = AD + AD$

[In a $||^{gm}$ ABCD, opposite sides are equal]

$\Rightarrow \quad 2AB = 2AD$ or $AB = AD$

But $\quad AB = CD$ and $AD = BC$ [Opposite sides of a || gm]

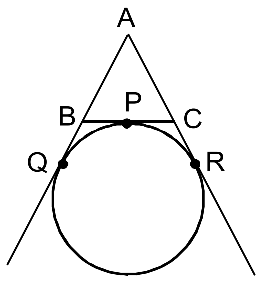
$\therefore \quad AB = BC = CD = DA$

Hence, $||^{gm}$ ABCD is a rhombus.

Example. 6

A circle touches the side BC of a Δ ABC at P and touches AB and AC when produced at Q and R respectively as shown in figure, Show that $AQ = \frac{1}{2}$ (Perimeter of Δ ABC).

Sol. **Given :** A circle is touching side BC of Δ ABC at P and touching AB and AC when produced at Q and R respectively.



To prove : $AQ = \frac{1}{2}$ (perimeter of Δ ABC)

Proof : AQ = AR ....(i)

BQ = BP ....(ii)

CP = CR ....(iii)

[Tangents drawn from an external point to a circle are equal]

Now, perimeter of Δ ABC = AB + BC + CA

= AB + BP + PC + CA

= (AB + BQ) + (CR + CA) [From (ii) and (iii)]

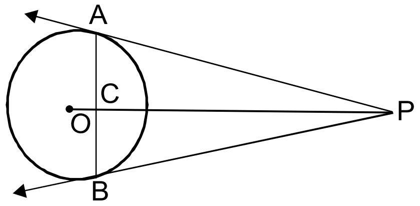
= AQ + AR = AQ + AQ [From(i)]

AQ = $\frac{1}{2}$ (perimeter of Δ ABC).

Example. 7

Prove that the tangents at the extremities of any chord make equal angles with the chord.

Sol. Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose, the tangents meet at point P. Join OP. Suppose OP meets AB at C.



We have to prove that

$\angle PAC = \angle PBC$

In triangles PCA and PCB

PA = PB [$\therefore$ Tangent from an external point are equal]

$\angle APC = \angle BPC$ [$\because$ PA and PB are equally inclined to OP]

And PC= PC [Common]

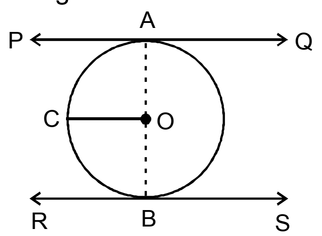
So, by SAS criteria of congruence

$\Delta PAC \cong \Delta PBC$ $\Rightarrow$ $\angle PAC = \angle PBC$ [By CPCT]

Example. 8

Prove that the segment joining the points of contact of two parallel tangents passes through the centre.

Sol. Let PAQ and RBS be two parallel tangents to a circle with centre O. Join OA and OB. Draw OC||PQ



Now, PA || CO

$\Rightarrow$ $\angle PAO + \angle COA = 180^o$ [Sum of co-interior angle is 180º]

$\Rightarrow$ $90^o + \angle COA = 180^o$ [$\therefore$ $\angle PAO = 90$]

$\Rightarrow$ $\angle COA = 90^o$

Similarly, $\angle COB = 90^o$

$\therefore$ $\angle COA + \angle COB = 90^o + 90^o = 180^o$

Hence, AOB is a straight line passing through O.

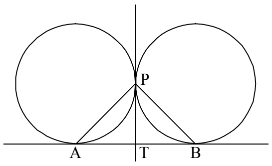
Example. 9

Two circles touch externally at P and a common tangent touches them at A and B. Prove that

(i) the common tangent at P bisects AB.

(ii) AB subtends a right angle at P.

Sol. Let PT be the common tangent at any point P. Since the tangent to a circle from an external point are equal,



∴ TA = TP, TB = TP

∴ TA = TB

i.e. PT bisects AB at T

TA = TP gives ∠ TAP = ∠ TPA (from ΔPAT)

TB = TP gives ∠ TBP = ∠ TPB [from ΔPBT]

∴ ∠ TAP + ∠ TBP = ∠TPA + ∠TPB = ∠APB

⇒ ∠ TAP + ∠ TBP + ∠APB = 2 ∠ APB

⇒ 2 ∠ APB = 180º [sum of ∠s of a Δ = 180º]

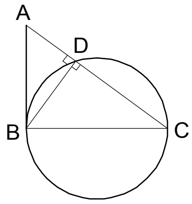
⇒ ∠ APB = 90º. **Hence proved**

Example. 10

In a right triangle ABC, the perpendicular BD on the hypotenuse AC is drawn. Prove that

(i) $AC \times AD = AB^2$ **(ii)** $AC \times CD = BC^2$

Sol. We draw a circle with BC as diameter. Since ∠ BDC = 90º.



∴ The circle on BC as diameter will pass through D. Again

∴ BC is a diameter and AB ⊥ BC.

∴ AB is a tangent to the circle at B.

Since AB is a tangent and ADC is a secant to the circle.

∴ $AC \times AD = AB^2$ This proves (i)

Again $AC \times CD = AC \times (AC - AD) = AC^2 - AC \times AD$

$= AC^2 - AB^2$ [Using (i)]

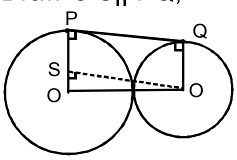
$= BC^2$ [ΔABC is a right triangle]

Hence, $AC \times CD = BC^2$. This proves (ii).

Example. 11

Two circles of radii R and r touch each other externally and PQ is the direct common tangent. Then show that $PQ^2 = 4rR$

Sol. Draw O'S|| PQ,



∴O'SPQ is rectangle

O'S=PQ, PS=QO'=r

OO'=R+r

OS=OP–PS=R–r

In Δ O'OS

$(O'S)^2 = (OO')^2 - (OS)^2$

$PQ^2 = (R+r)^2 - (R-r)^2$

$PQ^2 = R^2 + r^2 + 2Rr - (R^2 + r^2 - 2Rr)$

$PQ^2 = R^2 + r^2 + 2Rr - R^2 - r^2 + 2Rr$

$PQ^2 = 4rR$.

Check Your Level

1. A quadrilateral PQRS is drawn to circumscribe a circle. If PQ = 12 cm, QR = 15 cm and RS = 14 cm then find length of PS.

2. O is the centre of the circle. PA and PB are tangent. If $\angle PAB = 70^\circ$, then find $\angle APB$.

3. Find the maximum number of common tangents that can be drawn to two circles that do not intersect each other.

4. A tangent is drawn to a circle from a point 17 cm away from its centre. If the length of the tangent is 15 cm then find the radius of the circle.

5. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord to the larger circle which is a tangent to the smaller circle.

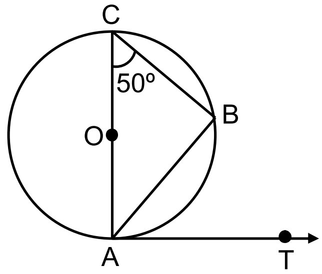
Answers

1.	11 cm	**2.**	40°	**3.**	4	**4.**	8 cm	**5.**	8 cm

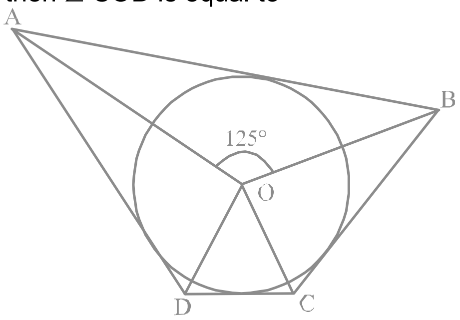
Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : **[01 MARK EACH]**

1. In the figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to

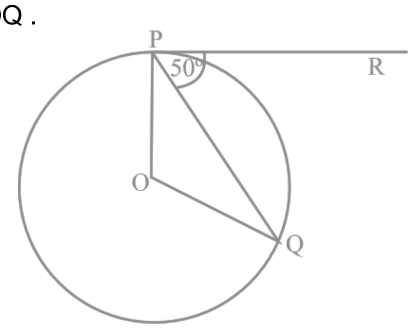


2. In Figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to



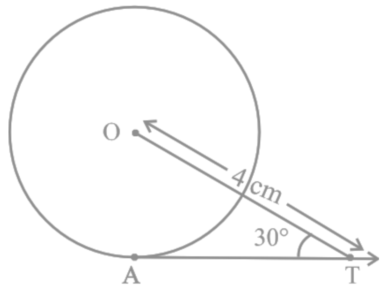
3. If radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

4. In the figure, if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then find $\angle POQ$.

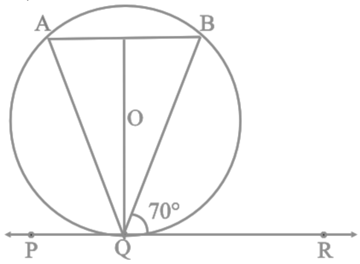


5. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

6. In the figure, AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle OTA = 30°$, then find AT.



7. In the figure, if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70°$, then find $\angle AQB$.



TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

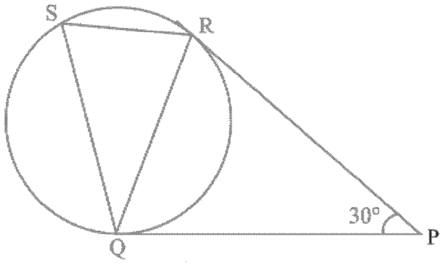
8. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

9. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

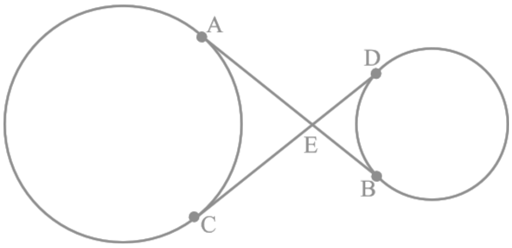
10. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC.

11. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

12. In the figure, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find the $\angle RQS$.



13. In figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.



14. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

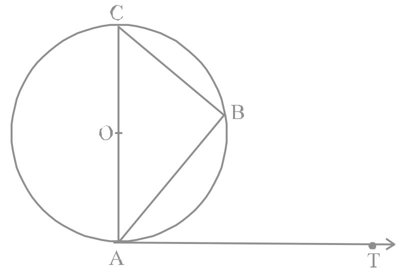
15. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: [03 MARK EACH]

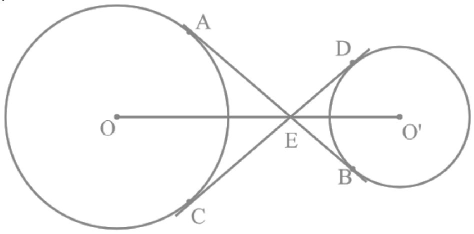
16. Two circles with centres O and O ' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

17. If a hexagon ABCDEF circumscribe a circle, prove that AB + CD + EF = BC + DE + FA.

18. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in figure. Prove that $\angle BAT = \angle ACB$.



19. In the figure, the common tangent, AB and CD to two circles with centres O and O' intersect at E. Prove that the points O, E, O' are collinear.

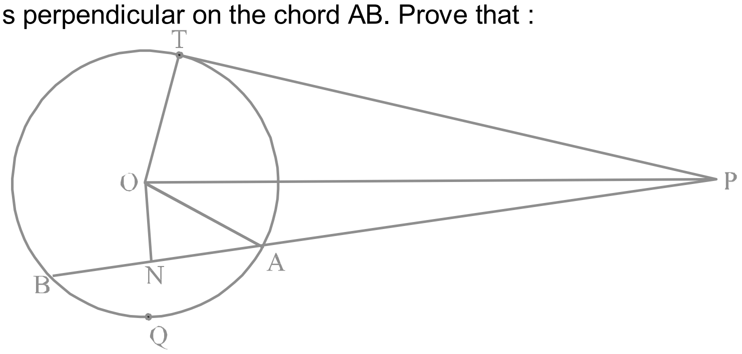


TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[04 MARK EACH]**

20. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC

21. If an isosceles triangle ABC, in which AB = AC = 6 cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.

22. In the figure, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that :

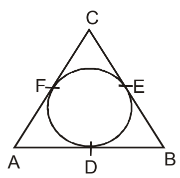


(i) $PA.\ PB = PN^2 - AN^2$ **(ii)** $PN^2 - AN^2 = OP^2 - OT^2$
(iii) $PA.PB = PT^2$

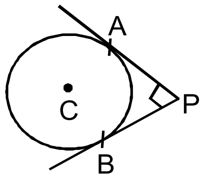
Previous Year Problems

1. Prove that the parallelogram circumscribing a circle is a rhombus.
[2 MARKS/CBSE 10TH BOARD: 2013]

2. In Figure, a circle inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10cm, then find the lengths of AD, BE and CF.
[2 MARKS/CBSE 10TH BOARD: 2013]

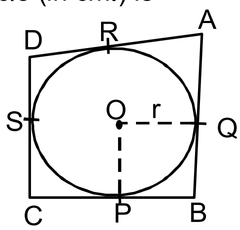


3. In Figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4cm. If PA ⊥ PB, then the length of each tangent is :
[1 MARK /CBSE 10TH BOARD: 2013]



(A) 3 cm (B) 4 cm (C) 5 cm (D) 6cm

4. In Figure a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively, If AB = 29 cm, AD = 23 cm, ∠ B = 90° and DS = 5cm, then the radius of the circle (in cm.) is **[1 MARK /CBSE 10TH BOARD: 2013]**



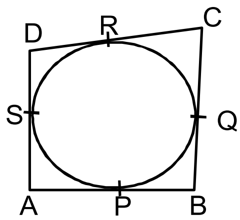
(A) 11 (B) 18 (C) 6 (D) 15

5. In Figure, l and m are two parallel tangents to a circle with centre O, touching the circle at A and B respectively. Another tangent at C intersects the line l at D and m at E. Prove that ∠ DOE = 90°
[4 MARKS/CBSE 10TH BOARD: 2013, 2016]

6. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

7. Two concentric circles are of radii 5 cm and 3 cm. Length of the chord of the larger circle (in cm), which touches the smaller circle is **[1 MARK /CBSE 10TH BOARD: 2013]**
(A) 4 (B) 5 (C) 8 (D) 10

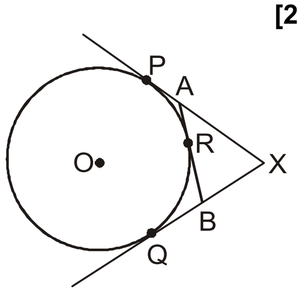
8. In Figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7 cm, CR = 3 cm and AS = 5 cm, find x. **[1 MARK /CBSE 10TH BOARD: 2013]**



(A) 10 (B) 9 (C) 8 (D) 7

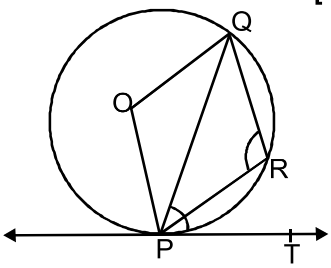
9. A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal.
[4 MARKS/CBSE 10TH BOARD: 2013, 2015, 2017]

10. In Figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA + AR = XB + BR.
[2 MARKS/CBSE 10TH BOARD: 2014]



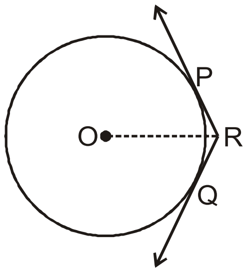
11. In Figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠ QPT = 60°, find ∠ PRQ.

[1 MARKS/CBSE 10TH BOARD: 2014]



12. In Figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If ∠ PRQ = 120°, then prove that OR = PR + RQ.

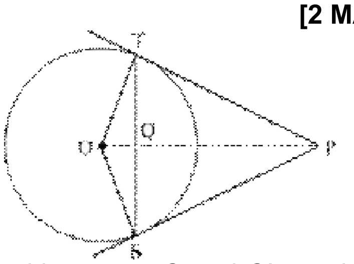
[2 MARKS/CBSE 10TH BOARD: 2014]



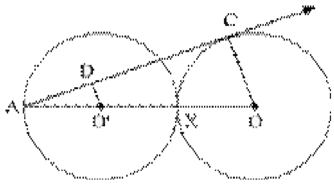
13. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. **[2 MARKS/ CBSE 10TH BOARD: 2015]**

14. In Figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that ∠ OTS = ∠ OST = 30°.

[2 MARKS/ CBSE 10TH BOARD: 2015]



15. In the figure, two equal circles, with centres O and O', touch each other at X.OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$. **[4 MARKS/ CBSE 10TH BOARD: 2015]**



16. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

[4 MARKS/CBSE 10TH BOARD: 2015, 2016, 2017]

17. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP. **[1 MARK/CBSE 10TH BOARD: 2017]**

18. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. **[2 MARKS/CBSE 10TH BOARD: 2017]**

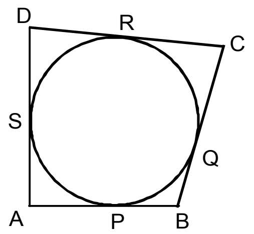
Exercise-1

SUBJECTIVE QUESTIONS

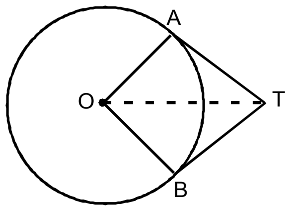
Subjective Easy, only learning value problems

Section (A) : Definition of Current, Current Densities, Drift

A-1. In figure, a circle touches all the four sides of a quadrilateral ABCD with AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.



A-2. In figure, if $\angle ATO = 40^\circ$, find $\angle AOB$.

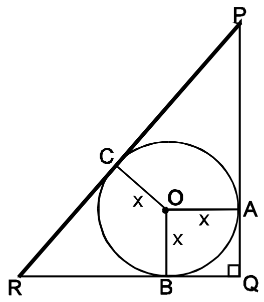


A-3. Find the length of tangent, drawn from a point 8 cm away from the centre of a circle of radius 6 cm.

A-4. Two circles touch each other externally, then find the number of common tangents to the circles.

A-5. ABCD is a quadrilateral such that $\angle D = 90^\circ$. A circle C (O, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find r.

A-6. PQR is a right angled triangle with PQ = 12 cm and QR = 5 cm. A circle with centre O and radius x is inscribed in PQR. Find the value of x.

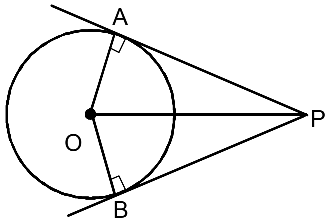


A-7. From an external point P, two tangents PA and PB are drawn to the circle with centre O. Prove that OP is the perpendicular bisector of AB.

A-8. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

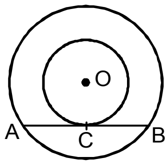
A-9. A circle touches the sides of a quadrilateral ABCD at P, Q, R, S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

A-10. In figure OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle.



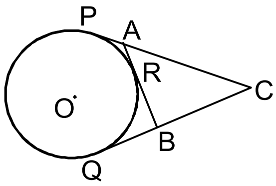
A-11. The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.

A-12. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following:



In figure, O is the centre of the two concentric circles. AB is a chord of the larger circle touching the smaller circle at C. Prove that AC = BC.

A-13. In figure, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11cm and BR = 4 cm, find the length of BC.



OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Circles

A-1. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

(A) $\sqrt{7}$ cm (B) $2\sqrt{7}$ cm (C) 10 cm (D) 5 cm

A-2. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 12 cm. Length of PQ is :

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

A-3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80º, then $\angle$POA is equal to :

(A) 50º (B) 60º (C) 70º (D) 80º

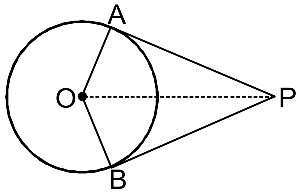
A-4. PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then $\angle$OQP is equal to :

(A) 30° (B) 45° (C) 60° (D) 90°

A-5. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is :

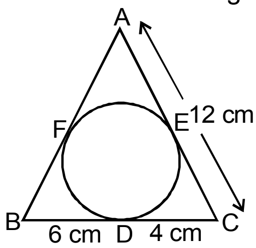
(A) 60 cm^2 (B) 65 cm^2 (C) 30 cm^2 (D) 32.5 cm^2

A-6. In figure, PA and PB are tangents from a point P to a circle with centre O. Then the quadrilateral OAPB must be a :



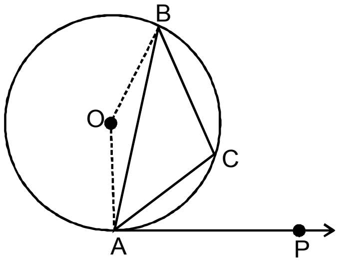
(A) square (B) rhombus (C) cyclic quadrilateral (D) parallelogram

A-7. In figure, ΔABC is circumscribing a circle. Then the length of AB is :



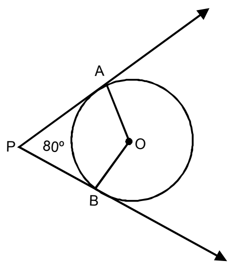
(A) 6 cm (B) 8 cm (C) 12 cm (D) 14 cm

A-8. In figure, AB is a chord of a circle with centre O and AP is the tangent at A such that BAP = 75°. Then ACB is equal to :



(A) 135° (B) 120° (C) 105° (D) 90°

A-9. In figure, if PA and PB are tangents to circle with centre O such that APB = 80°, then OAB is equal to

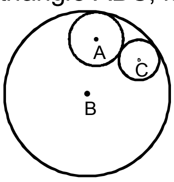


(A) 25° (B) 30° (C) 40° (D) 50°

Exercise-2

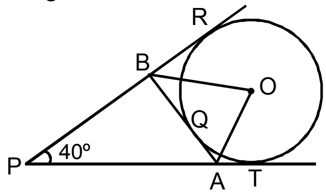
OBJECTIVE QUESTIONS

1. The three circles in the figure centered at A, B and C are tangent to one another and have radii 7, 21 and 6 respectively. The area of the triangle ABC, is



(A) 54 (B) 64 (C) 74 (D) 84

2. Triangle PAB is formed by three tangents to circle with centre O and $\angle APB = 40^o$, then angle AOB



(A) 45º (B) 50º (C) 60º (D) 70º

3. On a plane are two points A and B at a distance of 5 unit apart. The number of straight lines in this plane which are at distance of 2 units from A and 3 units from B, is:
(A) 1 (B) 2 (C) 3 (D) 4

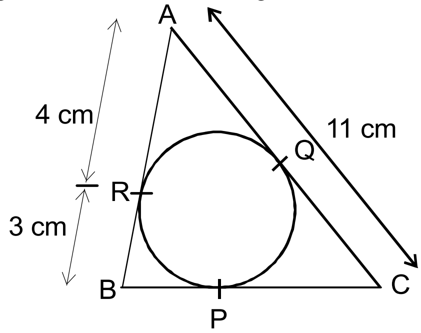
4. Let C be a circle with centre O. Let T be a point on the circle, and P a point outside the circle such that PT is tangent to C. Assume that the segment OP intersects C in a point Q. If PT = 12 and PQ = 8, the radius of C, is :
(A) r = 40 (B) r = 5 (C) $r = 4\sqrt{5}$ (D) $r = 4\sqrt{13}$

5. Three circles are mutually tangent externally. Their centres form a triangle whose sides are of lengths 3, 4 and 5. The total area of the three circles (in square units), is :
(A) 9π (B) 16π (C) 21π (D) 14π

6. Two circle touch each other externally at C and AB is a common tangent to the circles. Then, $\angle ACB$ =
(A) 60º (B) 45º (C) 30º (D) 90º

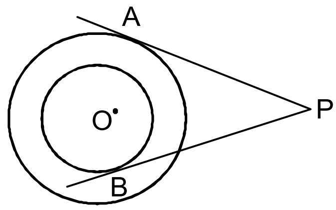
7. AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm, then AB is equal to :
(A) 4 cm (B) 6 cm (C) 8 cm (D) 12 cm

8. In figure, ΔABC is circumscribing a circle. Find the length of BC.



(A) 8 cm (B) 10 cm (C) 12 cm (D) 14 cm

9. In figure, there are two concentric circles with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.



(A) $4\sqrt{10}$ cm. (B) $2\sqrt{10}$ cm. (C) $\sqrt{10}$ cm (D) $3\sqrt{10}$ cm

10. Which of the following shapes of equal perimeter, the one having the largest area is :
(A) circle (B) equilateral triangle (C) square (D) regular pentagon

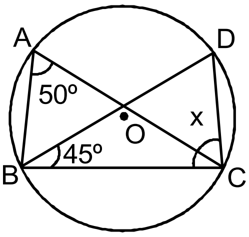
11. A triangle with side lengths in the ratio 3 : 4 : 5 is inscribed in a circle of radius 3. The area of the triangle is equal to :
(A) 8.64 (B) 12 (C) 6 (D) 10.28

12. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY at a distance 8 cm from A is :
(A) 4 cm (B) 5 cm (C) 6 cm (D) 8 cm

Exercise-3

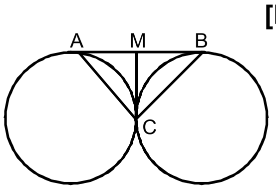
NTSE PROBLEMS (PREVIOUS YEARS)

1. In the following figure. O is the centre of the circle. The value of x is **[Raj. NTSE Stage-1 2007]**



(A) 45° (B) 65° (C) 85° (D) 95°

2. Two circles of equal radius touch each other externally at point C. AB is their common tangent. Value of $\angle CAB$ is : **[NTSE Stage-I/Rajasthan/2009]**



(A) 30° (B) 40° (C) 45° (D) 60°

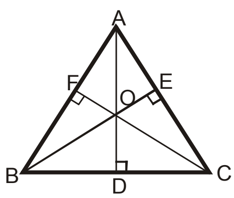
3. The chord of maximum length in a circle is called : **[Raj. NTSE Stage-1 2013]**
(A) Radius (B) Arc (C) Diameter (D) Point

4. One of the side of a triangle is divided into line segment of lengths 6 cm and 8 cm by the point of tangency of the incircle of the triangle. If the radius of the incircle is 4 cm, then the length (in cm) of the longer of the two remaining sides of the triangle is : **[Harayana NTSE Stage-1 2013]**
(A) 12 (B) 13 (C) 15 (D) 16

5. The circumference of the circumcircle of the triangle formed by x -axis, y-axis and graph of $3x + 4y = 12$ is: **[Harayana NTSE Stage-1 2013]**
(A) 3π units (B) 4π units (C) 5π units (D) 6.25 units

6. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of the centre and the distance between them is 17 cm, the radius of the circle is : **[Delhi NTSE Stage-1 2013]**
(A) 14 cm (B) 10 cm (C) 13 cm (D) 15 cm

7. In the figure given below, point O is orthocentre of ΔABC and points D, E and F are foot of the perpendiculars, then how many sets make the 4 cyclic points from the point O ? **[Maharashtra NTSE Stage-1 2013]**



(A) 4 (B) 3 (C) 2 (D) 6

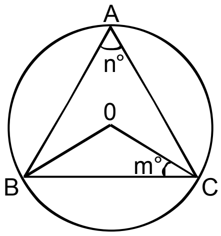
8. If two circles are such that one is not contained in the other and are non-intersecting, then number of common tangents are : **[UP NTSE Stage-1 2013]**

(A) One (B) Two (C) Three (D) Four

9. AB and AC are equal chord of a circle with centre O. Then by which angle OA bisects BC. **[MP NTSE Stage-1 2014]**

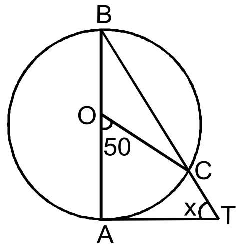
(A) 30° (B) 60° (C) 90° (D) 120°

10. In the following figure O is the centre of circle and $\angle BAC = n^o$, $\angle OCB = m^o$ then **[UP NTSE Stage-1 2014]**



(A) $m^o + n^o = 90^o$ (B) $m^o + n^o = 180^o$ (C) $m^o + n^o = 120^o$ (D) $m^o + n^o = 150^o$

11. In the below figure AB is a diameter of circle and AT is tangent line then value of x will be : **[Chattisgarh NTSE Stage-1 2014]**

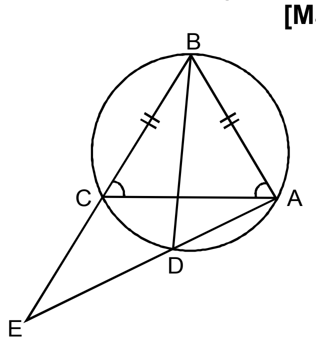


(A) 65° (B) 50° (C) 45° (D) 90°

12. The hypotenuse of a right triangle is 10 cm and radius of the inscribed circle is 2 cm. The perimeter of the triangle is : **[Delhi NTSE Stage-1 2014]**

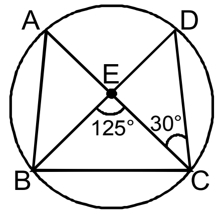
(A) 15 cm (B) 22 cm (C) 24 cm (D) 18 cm

13. In figure, for ΔABC, chord AB = chord BC, $\angle ABC = 72°$ and the angle bisector of $\angle ABC$ intersects the circle in Point D, then what is the measure of angle $\angle BEA$? **[Maharashtra NTSE Stage-1 2014]**



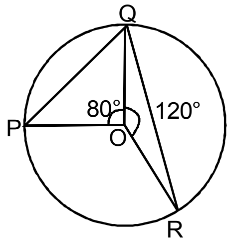
(A) 100° (B) 36° (C) 18° (D) 54°

14. In figure, A, B, C and D are four point on a cirlcle. AC and BD intersect at a point E such that $\angle BEC = 125°$ and $\angle ECD = 30°$. Then $\angle BAC$ = **[Raj. NTSE Stage-1 2014]**



(A) 95° (B) 110° (C) 85° (D) 105°

15. In the given figure find $\angle PQR$ (where O is centre of the circle) **[UP NTSE Stage-1 2014]**

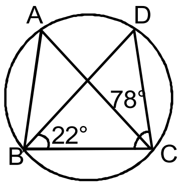


(A) 60° (B) 80° (C) 100° (D) 120°

16. If two equal circles of radius r passes through centre of the other then the length of their common chord is **[UP NTSE Stage-1 2014]**

(A) $\frac{r}{\sqrt{3}}$ (B) $r\sqrt{3}$ (C) $\frac{\sqrt{3}}{4}r$ (D) $r\sqrt{2}$

17. In the given figure, $\angle DBC = 22°$ and $\angle DCB = 78°$ then $\angle BAC$ is equal to **[Raj. NTSE Stage-1 2015]**



(A) 90° (B) 80° (C) 78° (D) 22°

18. The radii of two concentric circle with centre O are 5 cm and 13 cm respectively. The line drawn from the point A of outer circle touches the inner circle at point M and line AE intersects the inner circle at the points C and D. If AE = 25 cm, then find AD. (A–C–D–E) **[Maharashtra NTSE Stage-1 2015]**

(A) 16 cm (B) 10 cm (C) 12 cm (D) 8 cm

19. If two chords of a circle are equidistance from the centre of the circle, then they are............ **[MP NTSE Stage-1 2015]**

(A) Equal to each other (B) Not equal to each other.
(C) Intersect each other. (D) None of these

20. In the given figure O is the centre of a circle, XY, PQ, AB are tangents of the circle. If XY || PQ, then the value of $\angle AOB$ is **[Raj. NTSE Stage-1 2016]**

(A) 80° (B) 90° (C) 70° (D) 100°

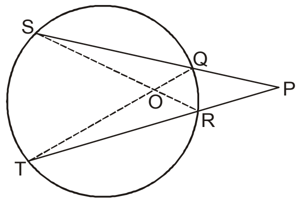
21. In ΔABC, $m\angle B = 140°$, 'P' is the centre of the circumcircle of ΔABC. Find $m\angle PBC$ **[Maharashtra NTSE Stage-1 2016]**

(A) 40° (B) 50° (C) 80° (D) 100°

22. The incircle of ΔABC touches the sides AB, BC and AC in the point P, Q and R respectively. If AP = 7 cm, BC = 13 cm, find the perimeter of **[Maharashtra NTSE Stage-1 2016]**

(A) 27 cm (B) 30 cm (C) 40 cm (D) 41 cm

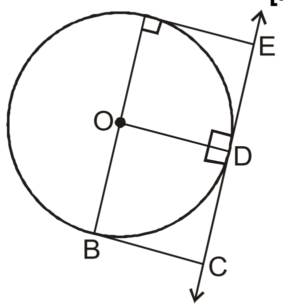
23. In the following figure secants QS and TR intersect each other at point P, which is outside the circle. O is the point of intersection of chords SR and TQ. If OS = 5 cm, OT = 10 cm, TR = 12 cm, PR = 8 cm, then find $\ell(PQ)$. **[Maharashtra NTSE Stage-1 2017]**



(A) 6 cm (B) 10 cm (C) 12 cm (D) 16 cm

24. Radius of a circle with centre 'O' is $4\sqrt{5}$ cm. AB is the diameter of the circle AE || BC and BC = 8 cm. Line EC is tangent to the circle at point D. Find the length of DE.

[Maharashtra NTSE Stage-1 2017]



(A) $4\sqrt{5}$ cm (B) $6\sqrt{5}$ cm (C) 8 cm (D) 10 cm

Answer Key

Exercise Board Level

TYPE (I)

1. 50° **2.** 55° **3.** 6 cm **4.** 100° **5.** $3\sqrt{3}$ cm

6. $2\sqrt{3}$ cm **7.** 40°

TYPE (II)

12. 30°

TYPE (III)

16. 4.8 cm

TYPE (IV)

20. 24 cm **21.** $8\sqrt{2}$ cm^2

Previous Year Problems

2. AD = 7 cm , BE = 5 cm , CF = 3 cm **3.** (B) **4.** (A) **7.** (C)

8. (B) **11.** 120° **15.** 1/3 **17.** OP = 2a

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-1. 3 cm **A-2.** 100º **A-3.** $\sqrt{28}$ cm **A-4.** 3 **A-5.** 14 cm.

A-6. x = 2 cm. **A-11.** 13 cm, 15 cm **A-13.** 7 cm

OBJECTIVE QUESTIONS

Section (A)

A-1. (B) **A-2.** (D) **A-3.** (A) **A-4.** (B) **A-5.** (A)

A-6. (C) **A-7.** (D) **A-8.** (C) **A-9.** (C)

Exercise-2

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	D	D	C	B	D	D	C	B	A	A	A	D

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	C	C	C	C	B	D	C	A	A	C	C	A	B	B	B	A	A	B
Ques.	21	22	23	24																
Ans.	B	C	B	D																