MATHEMATICS

Class-IX

Topic-6

LINES AND ANGLES

INDEX

CH-06
LINES AND ANGLES

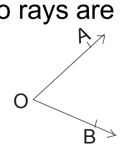
A. ANGLES AND THEIR TYPES

(a) Lines

A line has length but no width and no thickness.

(b) Angles

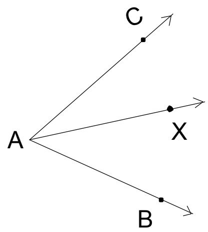
An angle is the union of two non-collinear rays with a common initial point. The common initial point is called the **'vertex'** of the angle and two rays are called the **'arms'** of the angles**.**



REMARK :

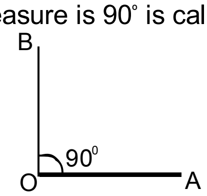
Every angle has a measure and unit of measurement is degree.
One right angle = 90°
1° = 60' (minutes)
1' = 60" (Seconds)

Angle addition axiom : If X is a point in the interior of ∠BAC, then m ∠BAC = m ∠BAX + m ∠XAC.

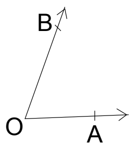


(c) Types of Angles :

(i) Right angle : An angle whose measure is 90° is called a **right angle**.

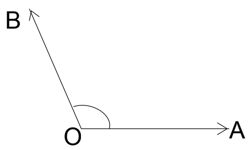


(ii) Acute angle : An angle whose measure is less than 90° is called an **acute angle**.



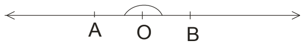
0° < ∠BOA < 90°

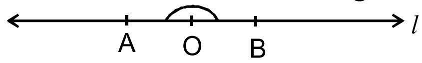
(iii) Obtuse angle : An angle whose measure is more than 90° but less than 180° is called an **obtuse angle**.



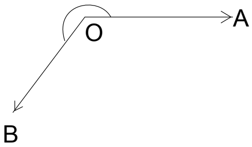
90° < ∠AOB < 180°.

(iv) Straight angle : An angle whose measure is 180º is called a **straight angle**.



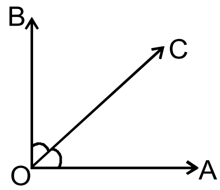


(v) Reflex angle : An angle whose measure is more than 180º is called a **reflex angle**.



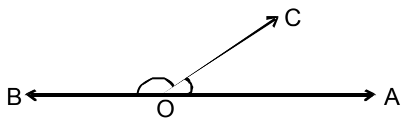
180º < ∠AOB < 360º.

(vi) Complementary angles : Two angles, the sum of whose measures is 90º are called **complementary angles**.



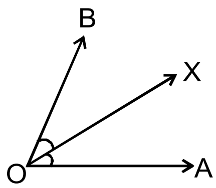
∠AOC & ∠BOC are complementary as their sum is 90°.

(vii) Supplementary angles : Two angles, the sum of whose measures is 180°, are called the **supplementary angles**.

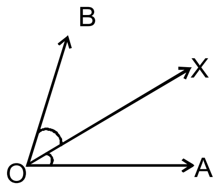


∠AOC & ∠BOC are supplementary as their sum is 180°.

(viii) Angle Bisectors : A ray OX is said to be the bisector of ∠AOB , if X is a point in the interior of ∠AOB, and ∠AOX = ∠BOX.

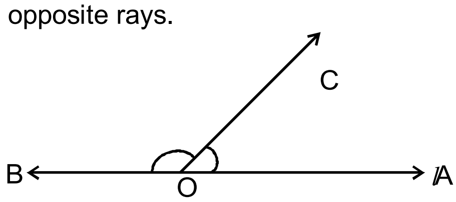


(ix) Adjacent angles : Two angles are called **adjacent angles**, if
(I) they have the same vertex,
(II) they have a common arm,
(III) non common arms are on either side of the common arm.



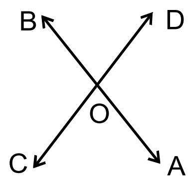
∠AOX and ∠BOX are adjacent angles, OX is common arm, OA and OB are non common arms and lies on either side of OX.

(x) Linear pair of angles : Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays.



∠AOC + ∠BOC = 180º.

(xi) Vertically opposite angles : Two angles are called a pair of **vertically opposite angles**, if their arms form two pairs of opposite rays.



$\angle$AOC & $\angle$BOD form a pair of vertically opposite angles. Also $\angle$AOD & $\angle$BOC form a pair of vertically opposite angles.
If two lines intersect, then the vertically opposite angles are equal i.e. $\angle$AOC = $\angle$BOD and $\angle$BOC = $\angle$AOD.

Solved Examples

Example.1

Two supplementary angles are in ratio 4 : 5, find the angles.

Sol. Let angles are 4x & 5x.
Angles are supplementary.
So, $4x + 5x = 180^o$
$9x = 180^o$
$x = \frac{180^o}{9} = 20^o$.
Angles are , 80^o & 100^o .

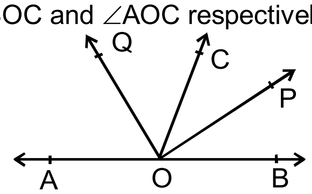
Example.2

If an angle differs from its complement by 10º, find the angle.

Sol. Let angle is x° then its complement is 90 – x⁰.
Now given,
$x^o - (90 - x^o) = 10^o$
$x^o - 90^o + x^o = 10^o$
$2x^o = 10^o + 90^o = 100^o$
$x^o = \frac{100^o}{2} = 50^o$.
Required angle is 50º.

Example.3

In figure, OP and OQ bisects $\angle$BOC and $\angle$AOC respectively. Prove that $\angle$POQ = 90º.



Sol. $\because$ OP bisects $\angle$BOC

$\therefore$ $\angle POC = \frac{1}{2}\angle BOC$ ...(i)

Also, OQ bisects $\angle$AOC

$\therefore$ $\angle COQ = \frac{1}{2}\angle AOC$ ...(ii)

$\because$ OC stands on AB

$\therefore$ $\angle AOC + \angle BOC = 180^o$ [Linear pair]

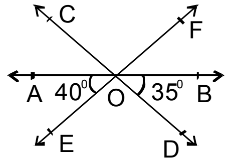
$\Rightarrow$ $\frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = \frac{1}{2} \times 180^o$

$\Rightarrow$ $\angle COQ + \angle POC = 90^o$ [Using (i) & (ii)]

$\Rightarrow$ $\angle POQ = 90^o$ [By angle sum property] **Hence Proved.**

Example.4

In figure, lines AB, CD and EF intersect at O. Find the measures of ∠AOC, ∠DOE and ∠BOF.



Sol. Given ∠ AOE = 40º & ∠ BOD = 35º

Clearly ∠AOC = ∠ BOD [Vertically opposite angles]

∠AOC = 35º

∠BOF = ∠AOE [Vertically opposite angles]

∠BOF = 40º

Now, ∠AOB = 180º [Straight angles]

∠AOC + ∠COF + ∠BOF = 180º [Angles sum property]

35º + ∠COF + 40º = 180º

∠COF = 180º – 75º = 105º

Now, ∠DOE = ∠COF [Vertically opposite angles]

∠DOE = 105º.

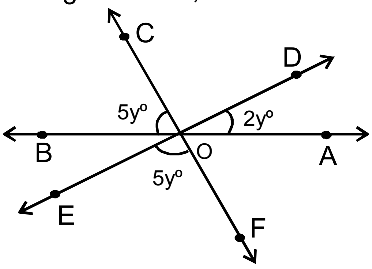
Check Your Level

1. Find the supplement of 100° 48′.

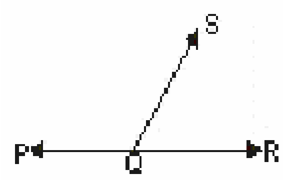
2. Find the angle such that an angle is equal to its supplement.

3. Find the angle such that an angle is equal to its complement.

4. In the given figure below, find the value of y.



5. In figure PQR is a straight line and ∠PQS : ∠SQR = 7 : 5 . Find ∠SQR



Answers

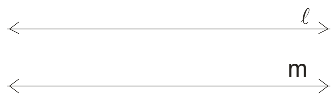
1. 79° 12′. **2.** 90° **3.** 45° **4.** 15° **5.** 75°

B. ANGLES MADE BY TRANSVERSAL

(a) Parallel Lines

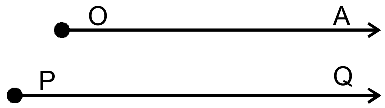
Parallel Lines : Two lines ℓ and m in the same plane are said to be parallel lines if they do not intersect when produced indefinitely in either direction and we write ℓ || m which is read as ℓ is parallel to m.

Clearly, when ℓ || m, we have m || ℓ.



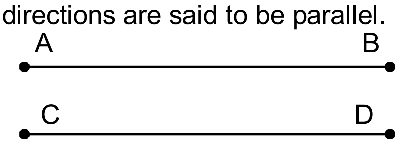
(b) Parallel Rays

Two rays are parallel if the corresponding lines determined by them are parallel. In other words, two rays in the same plane are parallel. If they do not intersect each other even if extended indefinitely beyond their initial points.
In fig. ray OA || ray PQ.



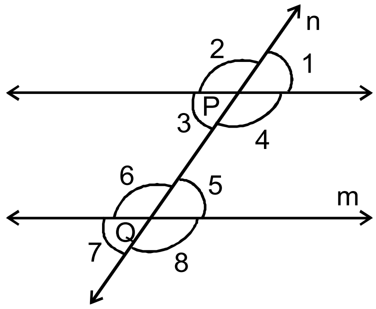
(c) Parallel segments :

Two segments are parallel if the corresponding lines determined by them are parallel.
In other words, two segments which are in the same plane and do not intersect each other even if extended indefinitely in both directions are said to be parallel.



(d) Angles made by a transversal with two lines

A line which intersects two or more given lines at distinct points is called a transversal to the given lines.



(i) Exterior angles : The angles whose arms do not include the line segment PQ are called **exterior angles**. In fig. angles 1, 2, 7 and 8 are exterior angles.

(ii) Interior angles : The angles whose arms include line segment PQ are called **interior angles**. In fig. angles 3, 4, 5 and 6 are interior angles.

(iii) Corresponding angles : A pair of angles in which one arm of both the angles is on the same side of the transversal and their other arms are directed in the same sense is called a pair of **corresponding angles**. In fig. $\angle 1$, $\angle 5$; $\angle 2$, $\angle 6$; $\angle 3$, $\angle 7$ and $\angle 4$, $\angle 8$ are four pairs of corresponding angles.

(iv) Alternate interior angles: A pair of angles in which one arm of each of the angles is on opposite side of the transversal and whose other arm include the segment PQ is called a pair of **alternate interior angles**. In fig $\angle 3$, $\angle 5$; $\angle 4$ and $\angle 6$ are alternate interior angles.

(v) Alternate exterior angles : A pair of angles in which one arm of each of the angles is on opposite sides of the transversal and whose other arms are directed in opposite direction and do not include segment PQ is called a pair of **alternate exterior angles**. In fig. $\angle 2$, $\angle 8$; $\angle 1$ and $\angle 7$ are alternate exterior angles.

NOTE :

Lines in a plane are parallel, if they do not intersect when produced indefinitely in either direction.

The distance between two intersecting lines is zero.

The distance between two parallel lines is the same everywhere and is equal to the perpendicular distance between them.

(e) Angles made by transversal to two parallel lines

If two parallel lines are intersected by a transversal, then

(i) Pairs of alternate (interior or exterior) angles are equal.

(ii) Pairs of corresponding angles are equal.

(iii) Interior angles on the same side of the transversal are supplementary.

If two non-parallel lines are intersected by transversal then none of (i), (ii) and (iii) hold true.

If two lines are intersected by a transversal, then they are parallel if anyone of the following is true:

(i) Pair of corresponding angles are equal.

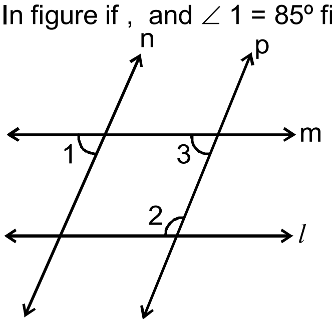
(ii) Pair of alternate interior angles are equal.

(iii) Pair of interior angles on the same side of the transversal are supplementary.

Solved Examples

Example.5

In figure if , and $\angle 1 = 85^\circ$ find $\angle 2$.



Sol. $\because$ and m is transversal.

$\therefore$ $\angle 1 = \angle 3 = 85^\circ$ [Corresponding angles]

Also & p is transversal.

$\therefore$ $\angle 2 + \angle 3 = 180^\circ$ [Co - interior angles]

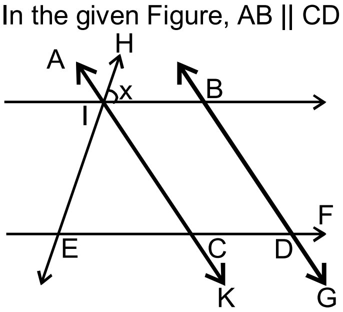
$\Rightarrow$ $\angle 2 + 85^\circ = 180^\circ$

$\Rightarrow$ $\angle 2 = 180^\circ - 85^\circ$

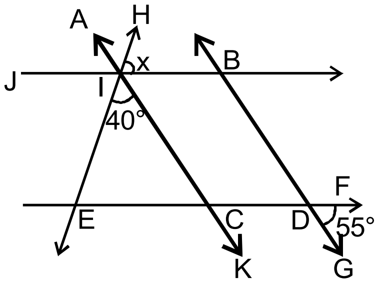
$\Rightarrow$ $\angle 2 = 95^\circ$.

Example.6

In the given Figure, AB || CD and AC || BD. If EIC = 40º, FDG = 55º, HIB = xº, then the value of x is



Sol.



(i) ∠AIH = ∠EIC (Vertically opp ∠s)

∠ AIH = 40°

(ii) ∠BDC = ∠FDG (Vertically opp ∠s)

∴ ∠BDC = 55°

∠ICE = ∠BDC (Corresponding ∠s)

∴ ∠ICE = 55°

∠AIJ = ∠ICE (Corresponding ∠s)

∴ ∠AIJ = 55°

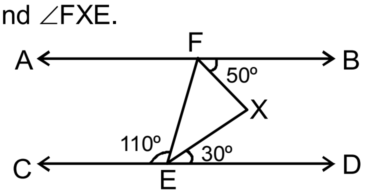
∠x = 180° – (∠AIJ + ∠HIA) {Linear pair}

∠x = 180° – (55° + 40°)

∠x = 180° – 95° = 85°

Example.7

In the given figure AB || CD. Find ∠FXE.



Sol. ∠BFE = ∠CEF = 110° [Alternate interior angles]

So, ∠XFE = ∠BFE – ∠BFX

= (110° – 50°) = 60°

∠CEF + ∠FEX + ∠XED = 180° ⇒ 110° + ∠FEX + 30° = 180°

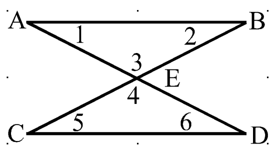
⇒ ∠FEX = 40°

Now, ∠XFE + ∠FEX + ∠FXE = 180° ⇒ 60° + 40° + ∠FXE = 180°

∴ ∠FXE = 80°.

Check Your Level

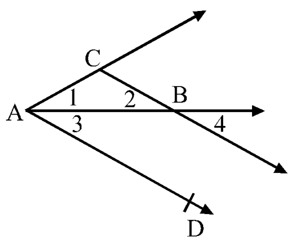
1. In the following diagram, list out



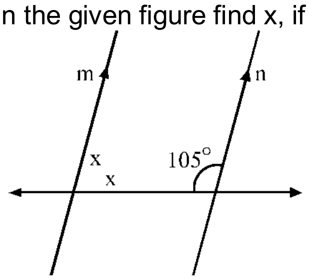
(i) Pairs of corresponding angles.

(ii) Pairs of alternate angles (Do not produce the segments).

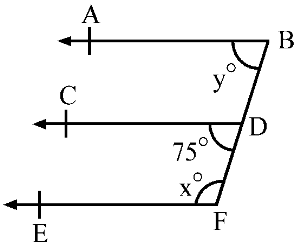
2. In the figure,∠4 = 4x, ∠3 = 2.5x + 24°, find the value of x, given that AD || CB.



3. In the given figure find x, if m || n.



4. In the figure, AB || CD || EF, find x and y.



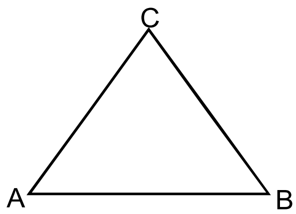
Answers

1. **(i)** No pair of corresponding angles **(ii)** ∠1 & ∠6 , ∠2 & ∠5 **2.** 16°

3. 37.5° **4.** x = 105°, y = 75°

C. TRIANGLES

A plane figure bounded by three lines in a plane is called a **triangle**. Every triangle have three sides and three angles. If ABC is any triangle then AB, BC & CA are three sides and ∠A, ∠B and ∠C are three angles.



(a) **Types of triangles :**

(i) On the basis of sides we have three types of triangle.

- **Scalene triangle** – A triangle in which no two sides are equal is called a **scalene triangle**.
- **Isosceles triangle** – A triangle having two sides equal is called an **isosceles triangle**.
- **Equilateral triangle** – A triangle in which all sides are equal is called an **equilateral triangle**.

(ii) **On the basis of angles we have three types :**

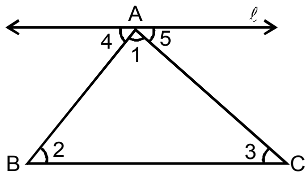
- **Right triangle** – A triangle in which any one angle is right angle is called **right triangle**.
- **Acute triangle** – A triangle in which all angles are acute is called an **acute triangle**.
- **Obtuse triangle** – A triangle in which any one angle is obtuse is called an **obtuse triangle**.

SOME IMPORTANT THEOREMS :

Theorem : The sum of interior angles of a triangle is 180º.
To Prove : ∠A + ∠B + ∠C = 180º or ∠1 + ∠2 + ∠3 = 180º.

Construction : Through A, draw a line ℓ parallel to BC.

Proof : Since ℓ || BC. Therefore,



$\angle 2 = \angle 4$ [Alternate interior angles]

$\angle 3 = \angle 5$ [Alternate interior angles]

$\therefore \angle 2 + \angle 3 = \angle 4 + \angle 5$

$\angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 4 + \angle 5$ [Adding $\angle 1$ both sides] $\angle 1 + \angle 2 + \angle 3 = 180^o$

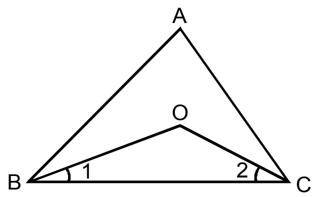
[Sum of angles at a point on a line is 180º]

$\angle A + \angle B + \angle C = 180^o$.

Theorem : if the bisectors of angles ∠ABC and ∠ACB of a triangle ABC meet at a point O, then $\angle BOC = 90^o + \frac{1}{2}\angle A$.

Given : A ΔABC such that the bisectors of ∠ABC and ∠ACB meet at a point O.

To Prove : $\angle BOC = 90^o + \frac{1}{2}\angle A$.



Proof : In ΔBOC, we have

$\angle 1 + \angle 2 + \angle BOC = 180^o$

$\angle 1 + \angle 2 = 180^o - \angle BOC$ .... (i)

In ΔABC, we have

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 2\angle 1 + 2\angle 2 = 180^o$

$\frac{\angle A}{2} + \angle 1 + \angle 2 = 90^o$

$\angle 1 + \angle 2 = 90^o - \frac{1}{2}\angle A$ ...(ii)

From (i) and (ii)

$180^o - \angle BOC = 90^o - \frac{1}{2}\angle A$

$\angle BOC = 90^o + \frac{1}{2}\angle A$. **Hence Proved**

Exterior Angle of a Triangle :

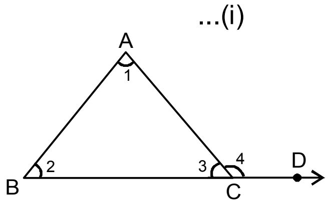
If the side of the triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.

Given : A triangle ABC. D is a point on BC produced, forming exterior angle ∠4.

To Prove : $\angle 4 = \angle 1 + \angle 2$ i.e. $\angle ACD = \angle CAB + \angle CBA$.

Proof : In triangle ABC, we have

$\angle 1 + \angle 2 + \angle 3 = 180^o$ ...(i)



Also, $\angle 3 + \angle 4 = 180^\circ$ [$\because$ $\angle 3$ and $\angle 4$ form a linear pair]..(ii)

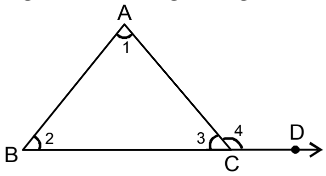
From (i) and (ii), we have

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

$\Rightarrow$ $\angle 1 + \angle 2 = \angle 4$

Hence, $\angle ACD = \angle CAB + \angle CBA$.

Corollary : An exterior angle of a triangle is greater than either of the interior opposite angles.



Proof : Let ABC be a triangle whose side BC is produced to form exterior angle $\angle A$.

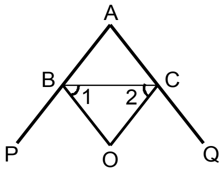
Then, $\angle 1 + \angle 2 = \angle 4$ $\Rightarrow$ $\angle 4 > \angle 1$ and $\angle 4 > \angle 2$

i.e., $\angle ACD > \angle CAB$ and $\angle ACD > \angle CBA$

Theorem : The sides AB and AC of a ΔABC are produced to P and Q respectively. If the bisectors of $\angle PBC$ and $\angle QCB$ intersect at O, then $\angle BOC = 90^\circ - \frac{1}{2}\angle A$.

Given : A ΔABC in which sides AB and AC are produced to P and Q respectively.

The bisectors of $\angle PBC$ and $\angle QCB$ intersect at O.



To Prove : $\angle BOC = 90^\circ - \frac{1}{2}\angle A$.

Proof : Since $\angle ABC$ and $\angle CBP$ form a linear pair.

$\therefore$ $\angle ABC + CBP = 180^\circ$

$\Rightarrow$ $\angle B + 2\angle 1 = 180^\circ$

[BO is the bisector of $\angle CBP$ $\because$ $\angle CBP = 2\angle 1$]

$\Rightarrow$ $2\angle 1 = 180^\circ - \angle B$

$\Rightarrow$ $\angle 1 = 90^\circ - \frac{1}{2}\angle B$ ...(i)

Again, $\angle ACB$ and $\angle QCB$ form a linear pair.

$\therefore$ $\angle ACB + \angle QCB = 180^\circ$

$\Rightarrow$ $\angle C + 2\angle 2 = 180^\circ$

[$\because$ OC is the bisector of $\angle QCB$ $\because$ $\angle QCB = 2\angle 2$]

$\Rightarrow$ $2\angle 2 = 180^\circ - \angle C$

$\Rightarrow$ $\angle 2 = 90^\circ - \frac{1}{2}\angle C$ ...(ii)

In ΔBOC, we have

$\angle 1 + \angle 2 + \angle BOC = 180^\circ$

$\Rightarrow$ $90^\circ - \frac{1}{2}\angle B + 90^\circ - \frac{1}{2}\angle C + \angle BOC = 180^\circ$ [Using (i) and (ii)]

$\Rightarrow$ $180^\circ - \frac{1}{2}(\angle B + \angle C) + \angle BOC = 180^\circ$

$\Rightarrow$ $\angle BOC = \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow$ $\angle BOC = \frac{1}{2}(180^\circ - \angle A)$ [$\because$ $\angle A + \angle B + \angle C = 180^\circ$ $\therefore$ $\angle B + \angle C = 180^\circ - \angle A$]

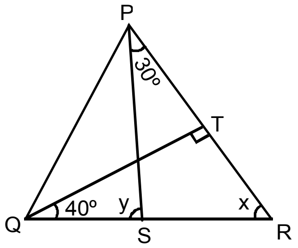
$\Rightarrow$ $\angle BOC = 90^\circ - \frac{1}{2}\angle A$.

Solved Examples

Example.8

In figure , If QT ⊥ PR, ∠TQR = 40º and ∠SPR = 30º, find x and y.

Sol.



In ΔTQR

∠TQR + ∠QTR + ∠TRQ = 180º

⇒ 40º + 90º + ∠TRQ = 180º ⇒ ∠TRQ = 180º – 130º = 50º

⇒ x = 50º

In ΔPSR, using exterior angle property, we have

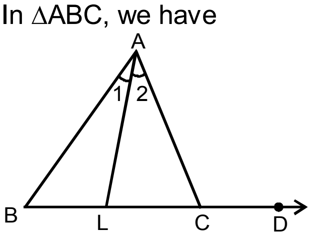
∠PSQ = ∠PRS + ∠RPS

⇒ y = x + 30º ⇒ y = 50º + 30º = 80º.

Example.9

The side BC of a ΔABC is produced, such that D is on ray BC. The bisector of ∠A meets BC in L as shown in figure. Prove that ∠ABC + ∠ACD = 2∠ALC.

Sol. In ΔABC, we have



ext. ∠ACD = ∠B + ∠A

⇒ ext. ∠ACD = ∠B + 2∠1 ...(i) [∵ AL is the bisector of ∠A ∵ ∠A = 2∠1]

⇒ ∠ACD = ∠B + 2∠1

In ΔABL, we have

ext. ∠ALC = ∠B + ∠BAL

⇒ ext. ∠ALC = ∠B +∠1

⇒ 2∠ALC = 2∠B + 2∠1 ...(ii) [Multiplying both sides by 2]

Subtracting (i) from (ii), we get

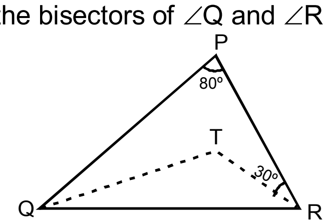
2∠ALC – ∠ACD = ∠B

⇒ ∠ACD + ∠B = 2∠ALC ⇒ ∠ACD + ∠ABC = 2 ∠ALC.

Example.10

In figure, TQ and TR are the bisectors of ∠Q and ∠R respectively. If ∠QPR = 80º and ∠PRT = 30º, determine ∠TQR and ∠QTR.

Sol. Since the bisectors of ∠Q and ∠R meet at T.



∴ $\angle QTR = 90^\circ + \frac{1}{2}\angle QPR$

⇒ $\angle QTR = 90^\circ + \frac{1}{2}(80^\circ)$

⇒ $\angle QTR = 90^o + 40^o = 130^o$

In ΔQTR, we have

$\angle TQR + \angle QTR + \angle TRQ = 180^o$

⇒ $TQR + 130^o + 30^o = 180^o$ [$\because \angle TRQ = \angle PRT = 30^o$]

⇒ $\angle TQR = 20^o$

Thus, $\angle TQR = 20^o$ and $\angle QTR = 130^o$.

Check Your Level

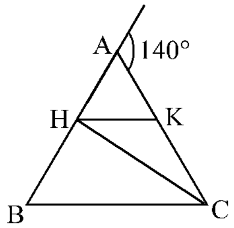
1. The sum of the acute angles of an obtuse triangle is 70° and their difference is 10°. Find the bigger acute angle.

2. If one angle of a triangle is equal to the sum of the other two, then determine the type of triangle

3. In ΔABC, 2A = 3B, 5B = 2C then determine the angles of the triangle

4. If one angle of a triangle is equal to half the sum of the other two equal angles, then determine type of the triangle.

5. In the figure if AB = AC, CH = CB and HK || BC then find $\angle HCK$.



Answers

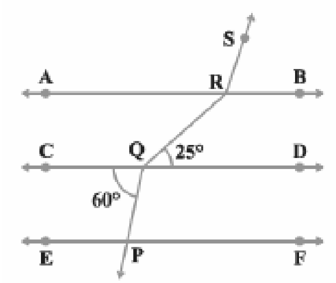
1.	40°	**2.**	right angle triangle	**3.**	A=54°,B=36°,C=90°
4.	Equilateral triangle	**5.**	30°		

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : **[01 MARK EACH]**

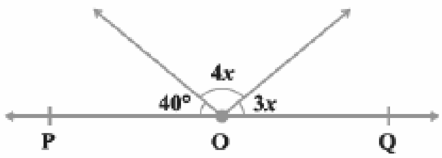
1. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 2 : 3, then the greater of the two angles ?

2. In Figure, if AB || CD || EF, PQ || RS, ∠RQD = 25° and ∠CQP = 60°, then ∠QRS = ?



3. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. What is the measure of each of these equal angle.

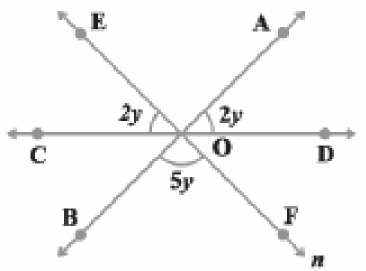
4. In Figure, POQ is a line. Find the value of x is



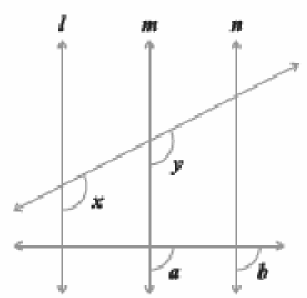
5. Angles of a triangle are in the ratio 2 : 4 : 3. Find the smallest angle of the triangle ?

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : **[02 MARKS EACH]**

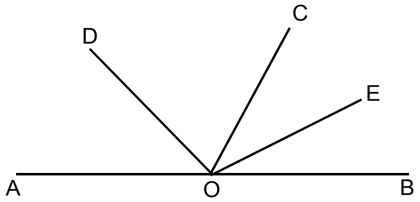
6. In Figure, AB, CD and EF are three lines concurrent at O. Find the value of y.



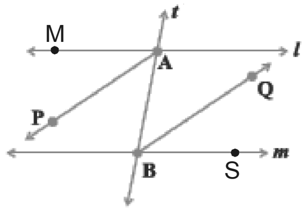
7. In Figure, x = y and a = b. Prove that l || n.



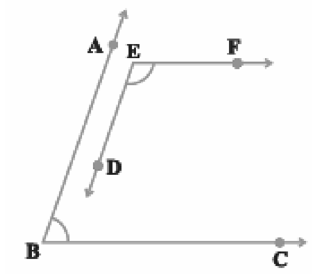
8. In Figure, OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that the points A, O and B are collinear.



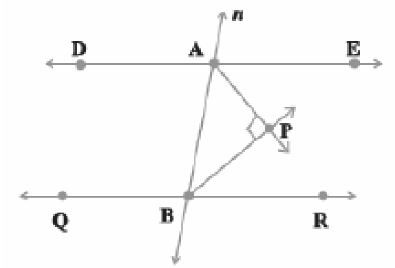
9. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (see Figure). Show that AP || BQ.



10. In Figure, BA || ED and BC || EF. Show that ∠ABC + ∠DEF = 180°.



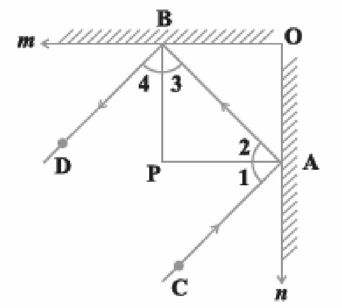
11. In Figure, DE || QR and AP and BP are bisectors of ∠EAB and ∠RBA, respectively. Prove that ∠APB = 90°.



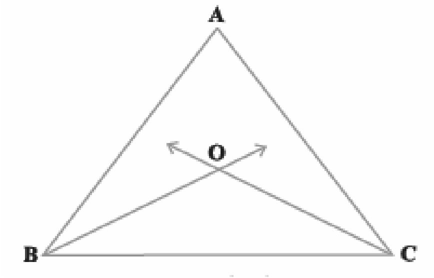
12. The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles of the triangle.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[04 MARK EACH]**

13. In Figure, m and n are two plane mirrors perpendicular to each other. Show that incident ray CA is parallel to reflected ray BD.



14. Bisectors of angles B and C of a triangle ABC intersect each other at the point O. Prove that $\angle BOC = 90° + \frac{1}{2}\angle A$.



15. Bisectors of interior $\angle B$ and exterior $\angle ACD$ of a Δ ABC intersect at the point T. Prove that $\angle BTC = \frac{1}{2}\angle BAC$.

16. Prove that through a given point, we can draw only one perpendicular to a given line.

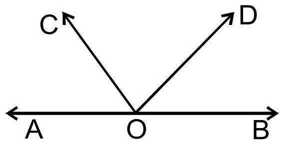
Exercise-1

SUBJECTIVE QUESTIONS

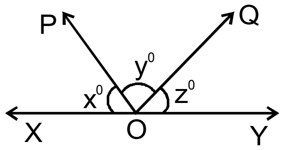
Subjective Easy, only learning value problems

Section (A) : Angles and their types

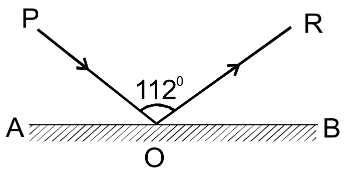
A-1. In figure, find $\angle COD$ when $\angle AOC + \angle BOD = 100^0$.



A-2. In figure, x : y : z = 5 : 4 : 6. If XOY is a straight line find the values of x, y and z.



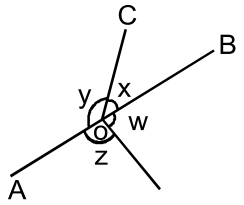
A-3. In the given figure, AB is a mirror, PO is the incident ray and OR, the reflected ray. If $\angle POR = 112^0$ find $\angle POA$.



A-4. The supplement of an angle is one third of itself. Determine the angle and its supplement.

A-5. If two complementary angles are in the ratio 13:5, then find the angles.

A-6. In figure, if x + y = w + z then prove that AOB is a straight line.



A-7. Two complementary angles are such that two times the measure of one is equal to three times measure of the other. Find the measure of the larger angle.

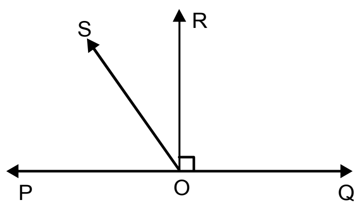
A-8. Find the complement of each of the following angles :

(i) $36^0\ 40'$ **(ii)** $42^0\ 25'\ 36''$

A-9. Write the supplementary angles of the following angles.

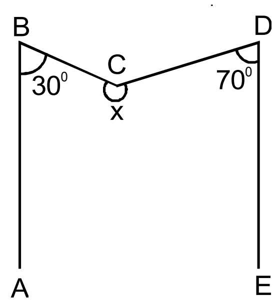
(i) $54^0\ 28'$ **(ii)** $98^0\ 35'\ 20''$

A-10. In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

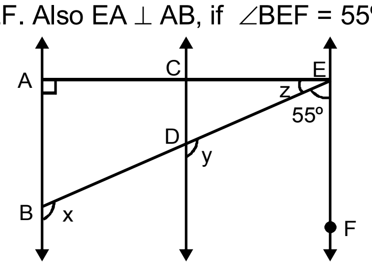


Section (B) : Angles made by transversal

B-1. In figure, and $\angle ABC = 30^0$, $\angle EDC = 70^0$ then find x^0



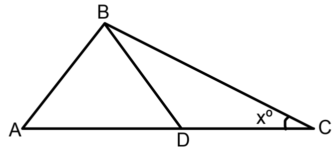
B-2. In figure, AB || CD and CD || EF. Also EA ⊥ AB, if $\angle BEF = 55^o$, find the values of x, y and z.



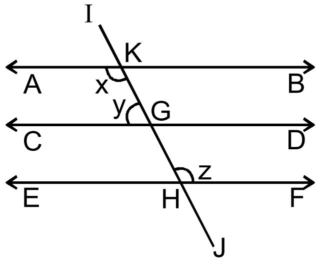
B-3. If two parallel lines are intersected by a transversal, prove that the bisectors of the interior angles on the same side of transversal intersect each other at right angles.

B-4. If two parallel lines are intersect by a transversal, prove that the bisectors of the two pairs of interior angle enclose a rectangle.

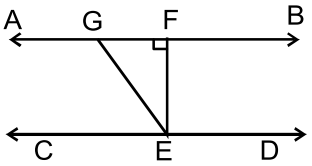
B-5. In the diagram, If AB = AD = BD = DC, then find x°.



B-6. In figure, if and y : z = 3 : 7, find x.



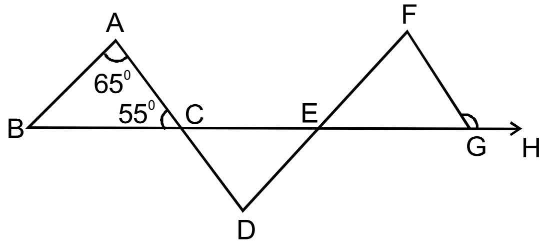
B-7. In figure, if AB ||CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.



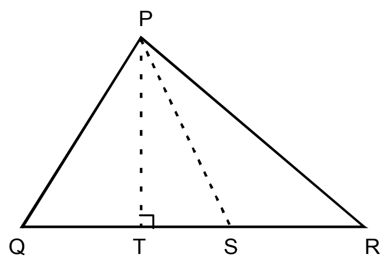
Section (C) : Triangles

C-1. Δ ABC is an isosceles triangle in which ∠B = ∠C and L & M are points on AB & AC respectively such that LM || BC.. If ∠A = 50° find ∠LMC.

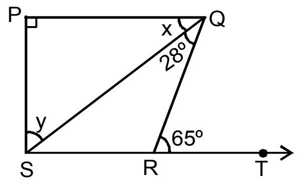
C-2. In figure if AB ∥ DF , AD ∥ FG , ∠BAC = 65°, ∠ACB = 55° Find ∠FGH.



C-3. In figure, PS is the bisector of ∠QPR and PT ⊥ QR. Show that $\angle TPS = \frac{1}{2}(\angle Q - \angle R)$.



C-4. In figure, if PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°, then find the value of x and y.



C-5. Prove that sum of all the angles of a triangle is 180°. Also, find the angle of a triangle , if they are in the ratio 5:6:7.

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Angles and their types

A-1. Two parallel lines have :
(A) a common point (B) two common point
(C) no common point (D) infinite common points

A-2. An angle is 14^0 more than its complementary angle then angle is :
(A) 38^0 (B) 52^0 (C) 50^0 (D) none of these

A-3. The angle between the bisectors of two adjacent supplementary angles is :
(A) acute angle (B) right angle (C) obtuse angle (D) none of these

A-4. X lies in the interior of $\angle BAC$. If $\angle BAC = 70^0$ and $\angle BAX = 42^0$ then $\angle XAC$ =
(A) 28^0 (B) 29^0 (C) 27^0 (D) 30^0

A-5. If the supplement of an angle is three times its complement, then angle is :
(A) 40^0 (B) 35^0 (C) 50^0 (D) 45^0

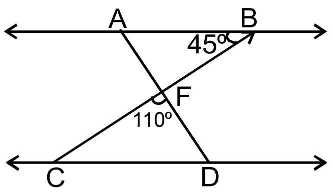
A-6. Two angles forms a linear pair whose measures are a & b are such that $2a - 3b = 60^0$ then $\frac{4a}{5b} = ?$
(A) 0 (B) $\frac{8}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

A-7. The complement of $(90^o - a)$ is :
(A) $-a^o$ (B) $90^o + a$ (C) $90^o - a$ (D) a^o

Section (B) : Angles made by transversal

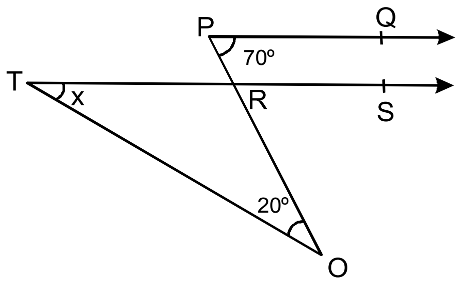
B-1. If two lines intersected by a transversal, then each pair of corresponding angles so formed is :
(A) Equal (B) Complementary (C) Supplementary (D) None of these

B-2. In the given figure, AB || CD, $\angle ABF = 45^o$ and $\angle CFD = 110^o$. Then, $\angle FDC$ is :



(A) 25^o (B) 45^o (C) 35^o (D) 30^o

B-3. In the given figure PQ II RS, $\angle QPR = 70^o$, $\angle ROT = 20^o$. Then, find the value of x.

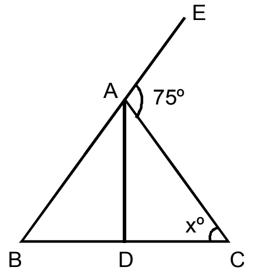


(A) 20^o (B) 70^o (C) 110^o (D) 50^o

Section (C) : Triangles

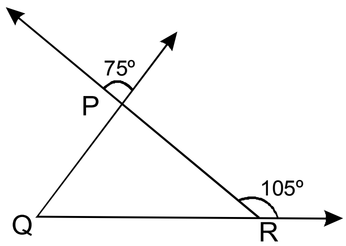
C-1. If one angle of triangle is equal to the sum of the other two then triangle is :
(A) acute a triangle (B) obtuse triangle (C) right triangle (D) none

C-2. In the adjoining figure, AD = BD = AC ; ∠CAE = 75º and ∠ACD = xº. Then the value of x is :



(A) 45º (B) 50º (C) 60º (D) $37\frac{1}{2}$ º

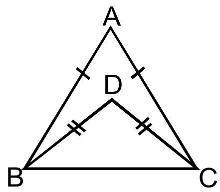
C-3. In the given figure, ∠PQR is :



(A) 40° (B) 50° (C) 30° (D) 105°

C-4. A triangle can have :
(A) Two right angles (B) Two obtuse angles
(C) All angles more than 60º (D) Two acute angles

C-5. In the given figure, the ratio ∠ABD : ∠ACD is :



(A) 1 : 1 (B) 2 : 1 (C) 1 : 2 (D) 3 : 1

Exercise-2

OBJECTIVE QUESTIONS

1. If one angle of a triangle is 72° and the difference of the other two angles is 12°, Find the ratio of other two angles.
(A) 4 : 5 (B) 2 : 1 (C) 3 : 4 (D) 5 : 3

2. An angle is 18° less than its complementary angle. The measure of this angle is
(A) 36° (B) 48° (C) 83° (D) 81°

3. In Δ ABC, ∠ A :∠ B : ∠ C = 2 : 3 : 5, then angle at B is
(A) 54° (B) 126° (C) 136° (D) 64°

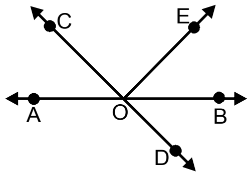
4. Which of the following statement is not false ?
(A) If two angles forming a linear pair, then each of these angle is 90°.
(B) Angles forming a linear pair can both be acute angles.
(C) Both of the angles forming a linear pair can be obtuse.
(D) Bisectors of the adjacent angles forming linear pair form a right angle.

5. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 5: 4, then the greatest angle is
(A) 54° (B) 100° (C) 120° (D) 136°

6. If angle with measure x and y form a complementary pair, then angles with which of the following measures will form a supplementary pair ?
(A) (x + 47°), (y + 43°) (B) (x – 23°), (y + 23°)
(C) (x – 43°), (y – 47°) (D) No such pair is possible

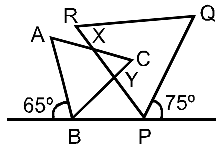
7. If one angle of a triangle is equal to the sum of the other two angles, then triangle is a/an
(A) Acute angled triangle (B) Obtuse angled triangle
(C) Right angled triangle (D) None of these

8. In figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 100° and ∠BOD = 60°, Find ∠BOE and reflex ∠COE respectively.



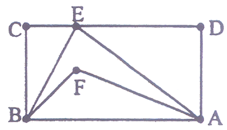
(A) 40°, 280° (B) 40° , 260° (C) 30 , 260° (D) 30° , 250°

9. In the diagram if ΔABC and ΔPQR are equilateral. The ∠CXY equals



(A) 35° (B) 40° (C) 45° (D) 50°

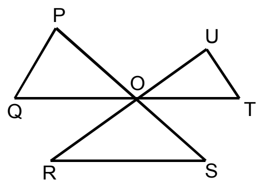
10. In the figure shown, ∠BEA = 100°. Point F is chosen inside DBEA so that line FA bisects ∠EAB and line FB bisects ∠EBA. The measure of ∠BFA, is :



(A) 140° (B) 145° (C) 150° (D) 155°

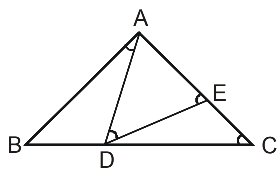
11. The altitudes of triangle are 12, 15 and 20 units. The largest angle in the triangle is :
(A) 75° (B) 90° (C) 120° (D) 135°

12. Lines PS, QT and RU intersect at a common point O, as shown P is joined to Q, R to S and T to U, to form triangles. The value of ∠P + ∠Q + ∠R + ∠S + ∠T + ∠U is :



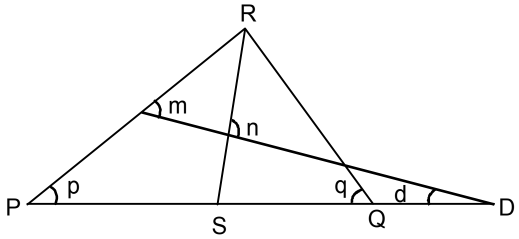
(A) 270° (B) 360° (C) 450° (D) 540°

13. Triangle ABC is isosceles with AB = AC. The measure of angle BAD is 30° and AD = AE. The measure of angle EDC, is :



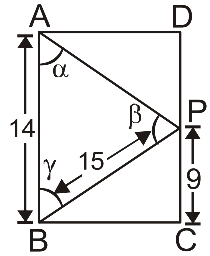
(A) 5° (B) 10° (C) 15° (D) 20°

14. Given triangle PQR with RS bisecting ∠R, PQ extended to D and ∠n a right angle, then :



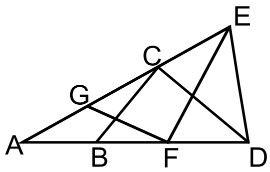
(A) $\angle m = \frac{1}{2}(\angle p - \angle q)$ (B) $\angle m = \frac{1}{2}(\angle p + \angle q)$ (C) $\angle d = \frac{1}{2}(\angle q + \angle p)$ (D) $\angle d = \frac{1}{2}\angle m$

15. In a rectangle ABCD, as shown in figure, a point P is taken on the side CD such that PC = 9, BP = 15 and AB = 14 then the correct relation between angles of ΔAPB is :



(A) $\alpha > \beta > \gamma$ (B) $\alpha > \gamma > \beta$ (C) $\beta > \gamma > \alpha$ (D) $\gamma > \alpha > \beta$

16. In the figure, AB = BC = CD = DE = EF = FG = GA, then ∠DAE is equal to :



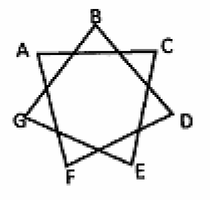
(A) 24° (B) 25° (C) 27° (D) $\frac{180°}{7}$

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. In the given figure m∠A + m∠B + m∠C + m∠D + m∠E + m∠F + m∠G = __________.

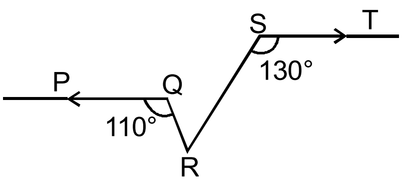
[U.P. NTSE Stage-1 2012]



(A) 360° (B) 500° (C) 520° (D) 540°

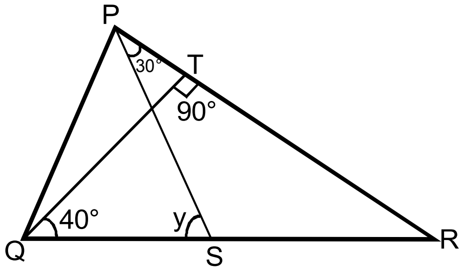
2. In a given figure PQ || ST, ∠PQR = 110°, ∠RST = 130° then value of ∠QRS is

[Raj. NTSE Stage-1 2013]



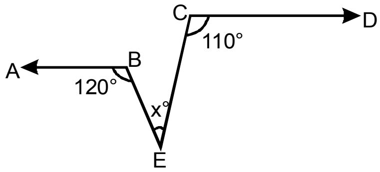
(A) 20° (B) 50° (C) 60° (D) 70°

3. In figure, if QT ⊥ PR, ∠TQR = 40° and ∠SPR = 30°, then y is **[Raj. NTSE Stage-1 2014]**



(A) 70° (B) 110° (C) 90° (D) 80°

4. In given figure AB || CD, ∠ABE = 120º ∠DCE = 110º and ∠BEC = xº then xº will be –



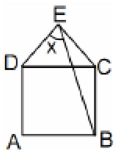
[U.P. NTSE Stage-1 2014]

(A) 60º (B) 50º (C) 40º (D) 70º

5. In a ∆ PRS, ∠PRS = 120°.A point Q is taken on PR such that PQ = QS and QR = RS then ∠QPS = ... **[Bihar NTSE Stage-1 2014]**

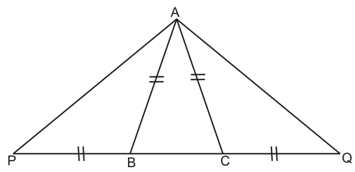
(A) 15° (B) 30° (C) 45° (D) 12°

6. In the figure given below, equilateral triangle EDC surmounts square ABCD. Find the angle DEB represented by x **[Delhi NTSE Stage-1 2015]**



(A) 60° (B) 15° (C) 30° (D) 45°

7.



[Maharashtra NTSE Stage-1 2016]

In the above figure ∆APQ, P – B – C - Q and AB = AC = PB = CQ . Find the angle congruent to ∠PAQ

(A) ∠ACP (B) ∠ABP (C) ∠APC (D) ∠BAQ

Answer Key

BOARD LEVEL EXERCISE

TYPE (I)

1. 108° **2.** 145° **3.** $\left(52\frac{1}{2}\right)^{\circ}$ **4.** 20° **5.** 40°

TYPE (II)

6. 20° **12.** 40° , 60° , 80°

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-1. ∠ COD = 80° . **A-2.** x = 60°, y = 48°, z = 72°. **A-3.** ∠ POA = 34°.

A-4. Angle = 135° and supplement = 45°. **A-5.** 65, 25

A-7. 54° **A-8.** **(i)** 53° 20′ **(ii)** 47° 34′ 24″

A-9. **(i)** 125° 32´ **(ii)** 81° 24° 40´´

Section (B)

B-1. 260° **B-2.** x = y = 125°, z = 35°. **B-5.** x = 30°.

B-6. x = 126°. **B-7.** ∠AGE = 126°, ∠GEF = 36°, ∠FGE = 54°

Section (C)

C-1. ∠ LMC = 115°. **C-2.** ∠FGH = 125° **C-4.** y = 53°, x = 37°

OBJECTIVE QUESTIONS

Section (A)

A-1. (C) **A-2.** (B) **A-3.** (B) **A-4.** (A) **A-5.** (D)

A-6. (B) **A-7.** (D)

Section (B)

B-1. (D) **B-2.** (A) **B-3.** (D)

Section (C)

C-1. (C) **C-2.** (B) **C-3.** (C) **C-4.** (D) **C-5.** (A)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	A	A	A	D	B	A	C	A	B	A	B	B	C	B	A	D

Exercise-3

Ques.	1	2	3	4	5	6	7
Ans.	D	C	D	B	A	D	B