MATHEMATICS

Class-IX

Topic-7

TRIANGLES

CH-07 TRIANGLES

A. CONGRUENT TRIANGLES

(a) Congruent figures

The figures are called congruent if they have same shape and same size. In other words, two figures are called congruent if they are having equal length, width and height.

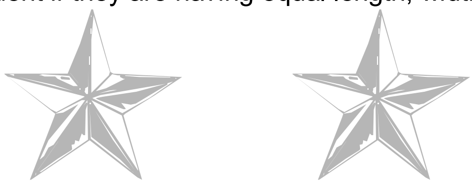
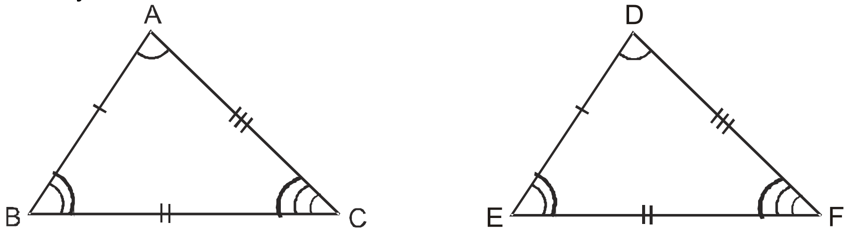


Fig.(i) Fig.(ii)

In the above figures {fig.(i) and fig.(ii)} both are equal in length, width and height, so these are congruent figures.

(b) Congruent Triangles

Two triangles are congruent if and only if one of them can be made to superimposed on the other, so as to cover it exactly.



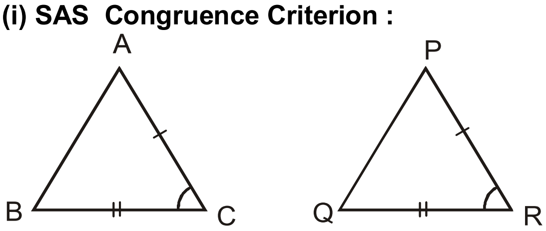
If two triangles ΔABC and ΔDEF are congruent then $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and $AB = DE$, $BC = EF$, $AC = DF$.

If two ΔABC & ΔDEF are congruent then we write $\Delta ABC \cong \Delta DEF$, we can not write as $\Delta ABC \cong \Delta DFE$ or $\Delta ABC \cong \Delta EDF$.

Hence, we can say that "two triangles are congruent if and only if there exists a one-one correspondence between their vertices such that the corresponding sides and the corresponding angles of the two triangles are equal.

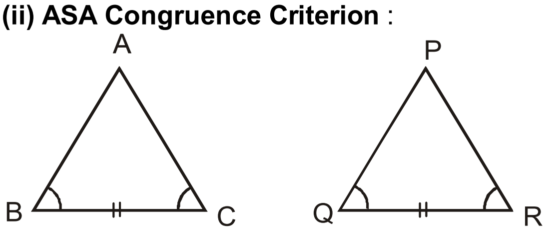
(c) Sufficient Conditions for Congruence of two Triangles :

(i) SAS Congruence Criterion :



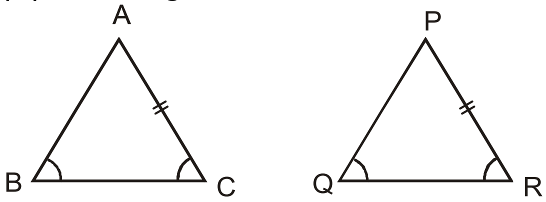
Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle.

(ii) ASA Congruence Criterion :



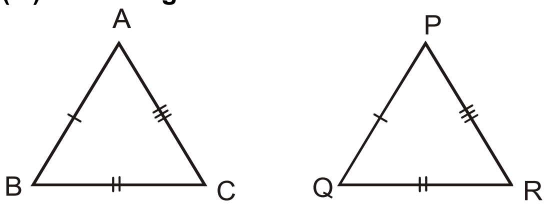
Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

(iii) AAS Congruence Criterion :



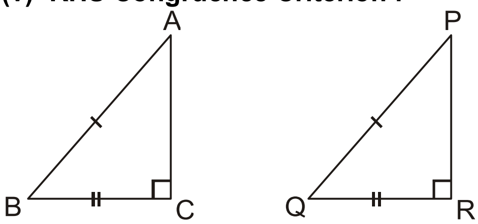
If any two angles and a non included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.

(iv) SSS Congruence Criterion :



Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.

(v) RHS Congruence Criterion :

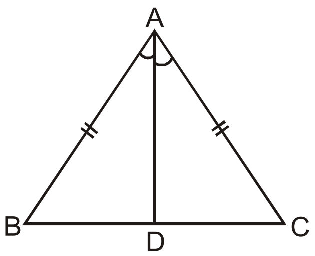


Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.

NOTE :

If two triangles are congruent then their corresponding sides and angles are also congruent by **CPCT (corresponding parts of congruent triangles are also congruent).**

- **Theorem** : Angles opposite to equal sides of an isosceles triangle are equal.
  Given : ΔABC in which AB = AC.
  To Prove : $\angle B = \angle C$.
  Construction : We draw the bisector AD of $\angle A$ which meets BC in D.



Proof : In ΔABD and ΔACD we have

AB = AC [Given]

$\angle BAD = \angle CAD$ [$\because$ AD is bisector of $\angle A$]

And, AD = AD [Common side]

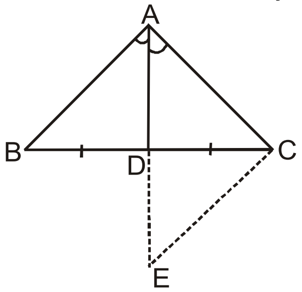
By SAS criterion of congruence, we have

$\Delta ABD \cong \Delta ACD$

$\angle B = \angle C$ [By CPCT] **Hence Proved.**

- **Converse :** If two angles of a triangle are equal, then sides opposite to them are also equal.
- **Theorem :** If the bisector of the vertical angle bisects the base of the triangle, then the triangle is isosceles.

Given : A ΔABC in which AD is the bisector of $\angle A$ meeting BC in D such that BD = CD.
To Prove : ΔABC is an isosceles triangle.
Construction : We produce AD to E such that AD = DE and join EC.



Proof : In ΔADB and ΔEDC we have

$AD = DE$ [By construction]

$\angle ADB = \angle CDE$ [Vertically opposite angles]

$BD = DC$ [Given]

By SAS criterion of congruence, we get

$\Delta ADB \cong \Delta EDC$

$AB = EC$ and, $\angle BAD = \angle CED$ [By CPCT]

But, $\angle BAD = \angle CAD$

$\angle CAD = \angle CED$ [AD is the bisector of $\angle A$]

$AC = EC$ [Sides opposite to equal angles are equal]

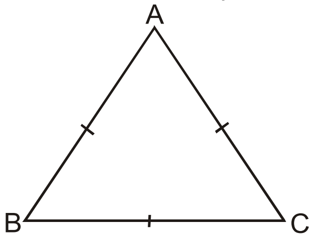
$AC = AB$. [By equation (i)] **Hence Proved.**

Solved Examples

Example. 1

Prove that measure of each angle of an equilateral triangle is 60º.

Sol. Let ΔABC be an equilateral triangle, then we have



$AB = BC = CA$ ... (i)

$\because$ $AB = BC$

$\therefore$ $\angle C = \angle A$ ... (ii) [Angles opposite to equal sides are equal]

Also, BC = CA

$\angle A = \angle B$ ... (iii) [Angles opposite to equal sides]

By (ii) & (iii) we get $\angle A = \angle B = \angle C$

Now in ΔABC

$\angle A + \angle B + \angle C = 180^0$

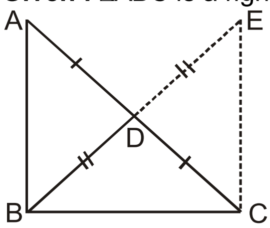
$3\angle A = 180^0$ [$\because$ $\angle A = \angle B = \angle C$]

$\angle A = 60^0 = \angle B = \angle C$. **Hence Proved.**

Example. 2

If D is the mid-point of the hypotenuse AC of a right triangle ABC, prove that $BD = \frac{1}{2}AC$.

Sol. **Given :** ΔABC is a right triangle such that $\angle B = 90^0$ and D is mid point of AC.



To prove : BD = $\frac{1}{2}$AC.

Construction : Produce BD to E such that BD = DE and join EC.

Proof :

In $\triangle$ADB and $\triangle$CDE

AD = DC [Given]

BD = DE [By construction]

And, $\angle$ADB = $\angle$CDE [Vertically opposite angles]

$\therefore$ By SAS criterion of congruence we have

$\triangle$ ADB $\cong$ Δ CDE

$\Rightarrow$ EC = AB and $\angle$CED = $\angle$ABD ... (i) [By CPCT]]

But $\angle$CED & $\angle$ABD are alternate interior angles

$\therefore$ CE || AB

$\Rightarrow$ $\angle$ABC + $\angle$ECB = 180^0 [Consecutive interior angles]

$\Rightarrow$ 90 + $\angle$ECB = 180^0

$\Rightarrow$ $\angle$ECB = 90^0.

Now, In $\triangle$ABC & $\triangle$ECB we have

AB = EC [By (i)]

BC = BC [Common]

And, $\angle$ABC = $\angle$ECB = 90^0

$\therefore$ By SAS criterion of congruence

$\triangle$ ABC $\cong$ Δ ECB

$\Rightarrow$ AC = EB [By CPCT]

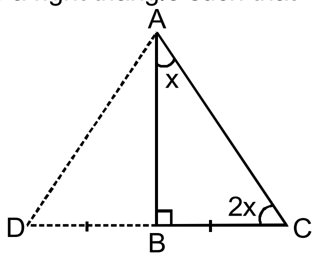
$\Rightarrow$ $\frac{1}{2}$AC = $\frac{1}{2}$EB

$\Rightarrow$ BD = $\frac{1}{2}$AC. **Hence Proved.**

Example. 3

In a right angled triangle, one acute angle is double the other. Prove that the hypotenuse is double the smallest side.

Sol. **Given** : $\triangle$ABC is a right triangle such that $\angle$B = 90^0 and $\angle$ACB = 2$\angle$CAB.



To Prove : AC = 2BC.

Construction : Produce CB to D such that BD = CB and join AD.

Proof : In $\triangle$ABD and $\triangle$ABC we have

BD = BC [By construction]

AB = AB [Common]

$\angle$ABD = $\angle$ABC = 90^0

$\therefore$ By SAS criterion of congruence we get

$\triangle$ ABD $\cong$ Δ ABC

$\Rightarrow$ AD = AC and $\angle$DAB = $\angle$CAB [By cpctc]

$\Rightarrow$ AD = AC and $\angle$DAB = x [$\because$$\angle$CAB = x]

Now, $\angle$DAC = $\angle$DAB + $\angle$CAB = x + x = 2x

$\therefore$ $\angle$DAC = $\angle$ACD

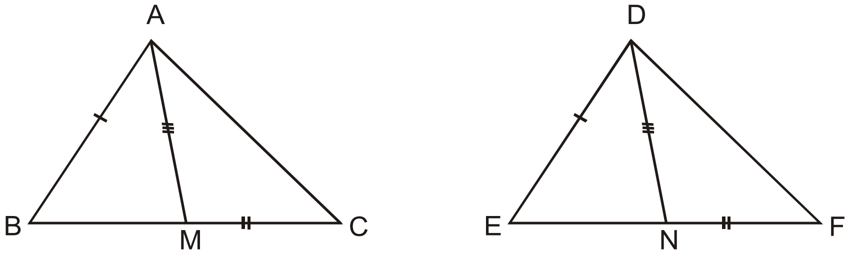
$\Rightarrow$ DC = AD [Side Opposite to equal angles]

$\Rightarrow$ 2BC = AD [$\because$DC = 2BC]

$\Rightarrow$ 2BC = AC [AD = AC] **Hence Proved.**

Example. 4

In figure, two sides AB and BC and the median AM of a ΔABC are respectively equal to sides DE and EF and the median DN of ΔDEF. Prove that $\Delta ABC \cong \Delta DEF$.



Sol. $\because$ AM and DN are medians of ΔABC & ΔDEF respectively

$\therefore$ BM = MC & EN = NF

$\Rightarrow$ BM = $\frac{1}{2}$BC & EN = $\frac{1}{2}$EF

But, BC = EF $\therefore$ BM = EN ... (i)

In ΔABM & ΔDEN we have

AB = DE [Given]

AM = DN [Given]

BM = EN [By (i)]

$\therefore$ By SSS criterion of congruence we have

$\Delta ABM \cong \Delta DEN$

$\Rightarrow$ $\angle B = \angle E$ ... (ii) [By CPCT]

Now, In ΔABC & ΔDEF

AB = DE [Given]

$\angle B = \angle E$ [By (ii)]

BC = EF [Given]

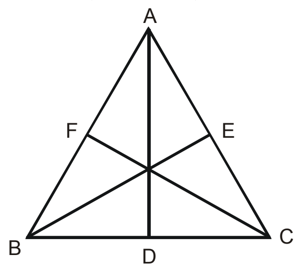
By SAS criterion of congruence we get

$\Delta ABC \cong \Delta DEF$ **Hence Proved.**

Example. 5

Prove that the medians of an equilateral triangle are equal.

Sol. **Given :** A ΔABC in which AB = BC = AC, and AD, BE and CF are its medians.



To prove : AD = BE = CF.

Prove : In ΔADC and BEA, we have :

DC = EA [BC = AC $\Rightarrow \frac{1}{2}$BC = $\frac{1}{2}$AC]

$\angle ACD = \angle BAE$ [Each equal to 60°]

AC = AB [Given]

$\therefore$ $\Delta ADC \cong \Delta BEA$ [SAS criteria]

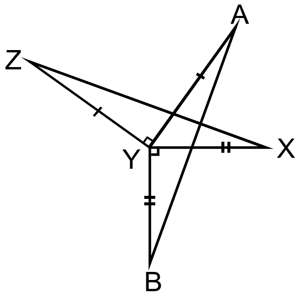
So, AD = BE [By CPCT]

Similarly, BE = CF

Hence, AD = BE = CF.

Example. 6

In the given figure, AY ⊥ ZY and BY ⊥ XY such that AY = ZY and BY = XY. Prove that AB = ZX.



Sol. ∠BYX = 90º and ∠AYZ = 90º.

∠BYX = ∠AYZ

⇒ ∠BYX + ∠AYX = ∠AYZ + ∠AYX

⇒ ∠AYB = ∠ZYX

Now, in ΔAYB and Δ ZYX

AY = ZY [Given]

BY = XY [given]

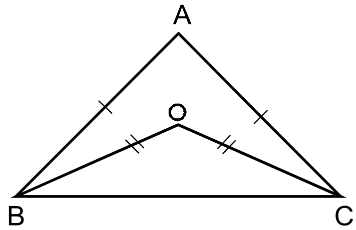
∠AYB = ∠ZYX [proved above]

∴ ΔAYB ≅ ΔZYX [By SAS congruency]

Hence, AB = ZX [By CPCT]

Example. 7

In the given figure, AB = AC and OB = OC. Prove that ∠ABO = ∠ACO.



Sol. AB = AC

So, ∠B = ∠C ...(i)

and OB = OC

⇒ ∠OBC = ∠OCB ....(ii)

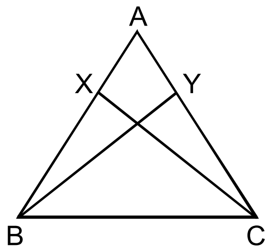
Subtract equation (ii) from (i)

∴ ∠B – ∠OBC = ∠C – ∠OCB

∠ABO = ∠ACO.

Example. 8

In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of ΔABC such that AX = AY. Prove that CX = BY.



Sol. In ΔAXC and ΔAYB

AX = AY [Given]

AC = AB [Given]

∠A = ∠A [Common]

So, by SAS congruency

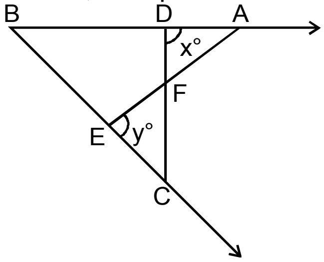
ΔAXC ≅ ΔAYB

So, by CPCT

CX = BY.

Example. 9

In the given figure, if x = y and AB = CB, then prove that AE = CD.



Sol. In ΔABE and ΔCDB

AB = CB [Given]

$\angle$ABE = $\angle$CBD [Common]

and $\angle$AEB = $\angle$CDB [$\because$ x° = y° $\Rightarrow$ 180° – x° = 180° – y° $\Rightarrow$ $\angle$CDB = $\angle$AEB]

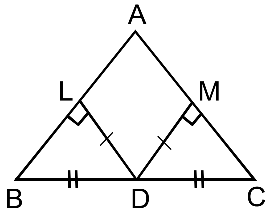
So, by AAS Congruency

$\Delta ABE \cong \Delta CBD$

$\therefore$ By CPCT, AE = CD.

Example. 10

In ΔABC, D is the midpoint of BC. If DL $\perp$ AB and DM $\perp$ AC such that DL = DM, prove that AB = AC.



Sol. In ΔBDL and ΔDMC

DL = DM [Given]

$\angle$DLB = $\angle$DMC [Each 90°]

BD = DC [D is the midpoint of BC]

$\therefore$ $\Delta DLB \cong \Delta DMC$ [By RHS congruency]

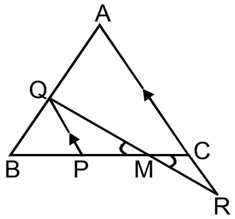
So, by CPCT

$\angle$B = $\angle$C

Hence, AB = AC.

Example. 11

In the given figure, ABC is an equilateral triangle; PQ || AC and AC is produced to R such that CR = BP. Prove that QR bisects PC.



Sol. Let QR intersect PC at M.

$\therefore$ $\angle$BPQ = $\angle$BCA [Alternate interior angles $\because$PQ ||AR]

So, $\angle$B = $\angle$BPQ = $\angle$BQP = 60°

$\therefore$ ΔBQP is an equilateral triangle.

So, PQ = BP = CR.

In ΔPMQ and ΔCMR

$\angle$QMP = $\angle$RMC [Vertically opposite angles]

$\angle$MQP = $\angle$MRC [Alternate angles]

PQ = CR [CR = BP and BP = PQ]

So, $\Delta PMQ \cong \Delta CMR$ [By AAS congruency]

$\therefore$ By CPCT

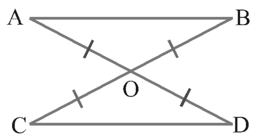
PM = MC.

Hence, QR bisects PC.

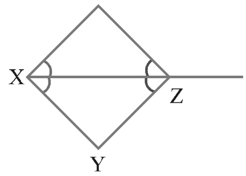
Check Your Level

1. In the following example the data is marked on the diagrams. Write criterion of congruence that supports your conclusion. If more than one criterion is involved state all.

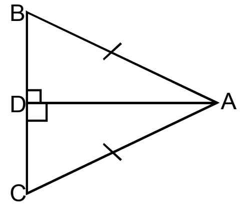
(i)



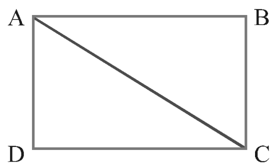
(ii)



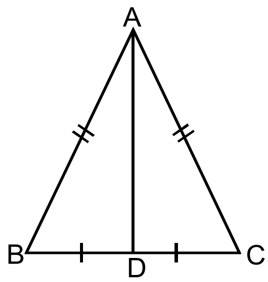
(iii)



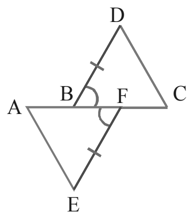
2. Given AB = 4 cm, DC = 4 cm, AD = BC. Prove $\Delta\ ABC \cong \Delta\ ADC$.



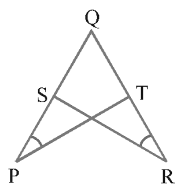
3. Given AD bisects BC and AB = AC. Prove $\Delta\ ADB \cong \Delta\ ADC$.



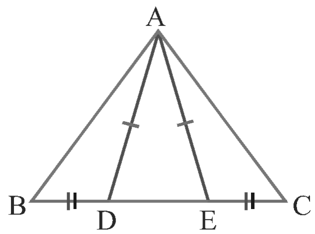
4. Given AB = CF, EF = BD and Prove $\Delta AFE \cong \Delta CBD$.



5. Given, PQ = RQ. Prove $\Delta\ PQT \cong \Delta\ RQS$

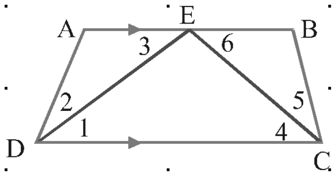


6. Given AB = AC, AD = AE, BD = CE. Prove $\Delta\ BAE \cong \Delta\ CAD$



7. Through any point on the bisector of an angle, a straight line is drawn parallel to either arm of an angle. Prove that the triangle so formed is isosceles

8. In a quadrilateral ABCD, AB || DC. The bisectors of angles D and C meet at E. Prove that AB = AD + BC.

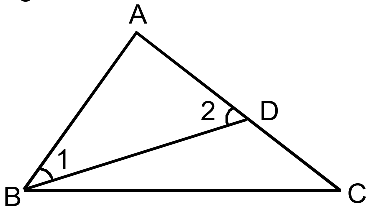


(B) SOME INEQUALITY RELATIONS IN A TRIANGLE

Theorem : If two sides of a triangle are unequal, the longer side has greater angle opposite to it.
Given : A ΔABC in which AC > AB.
To Prove : $\angle ABC > \angle ACB$.
Construction : Mark a point D on AC such that AB = AD. Join BD.
Proof : In ΔABD, we have

AB = AD [By construction]
$\Rightarrow$ $\angle 1 = \angle 2$ ...(i)
[Angle opposite to equal sides are equal]

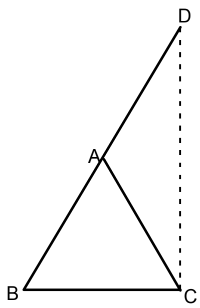
Now, consider ΔBCD. We find that $\angle 2$, is the exterior angle of ΔBCD and an exterior angle is always greater than interior opposite angle. Therefore,



$\angle 2 > \angle DCB$
$\Rightarrow$ $\angle 2 > \angle ACB$ ....(ii) [$\because \angle ACB = \angle DCB$]

From (i) and (ii), we have
$\angle 1 = \angle 2$ and $\angle 2 > \angle ACB$
$\Rightarrow$ $\angle 1 > \angle ACB$ ....(iii)
But, $\angle 1$ is a part of $\angle ABC$.
$\therefore$ $\angle ABC > \angle 1$ ....(iv)
From (iii) and (iv), we get
$\angle ABC > \angle ACB$.

Theorem : The sum of any two sides of a triangle is greater than the third side.
Given : A ΔABC.
To Prove : AB + AC > BC, AB + BC > AC and BC + AC > AB.
Construction : Produce side BA to D such that AD = AC. Join CD.



Proof : In ΔACD, we have
AC = AD [By construction]
$\Rightarrow$ $\angle ADC = \angle ACD$ [Angles opposite to equal sides are equal]
$\Rightarrow$ $\angle ACD = \angle ADC$

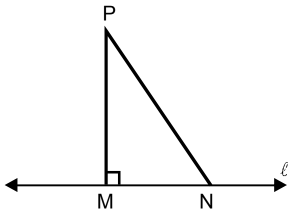
⇒ $\angle BCA + \angle ACD > \angle ADC$ [∵ $\angle BCA + \angle ACD > \angle ACD$]
⇒ $\angle BCD > \angle ADC$
⇒ $\angle BCD > \angle BDC$ [∵ $\angle ADC = \angle BDC$]
⇒ $BD > BC$ [∵ Side opposite to greater angle is larger]
⇒ $BA + AD > BC$
⇒ $BA + AC > BC$ [∵ AC = AD (By construction)]
⇒ $AB + AC > BC$

Thus, $AB + AC > BC$

Similarly, $AB + BC > AC$ and $BC + AC > AB$.

Theorem : Of all the line segments that can be drawn to a given line, from a point, not lying on it, the perpendicular line segment is the shortest.

Given : A straight line ℓ and a point P not lying on ℓ. $PM \perp \ell$ and N is any point on ℓ other than M.



To Prove : PM < PN.

Proof : In ΔPMN, we have

$\angle M = 90^o$
⇒ $\angle N < 90^o$ [∵ $\angle M = 90^o \Rightarrow \angle MPN + \angle PNM = 90^o$
⇒ $\angle P + \angle N = 90^o$ ⇒ $\angle N < 90^o$]
⇒ $\angle N < \angle M$
⇒ $PM < PN$ [Side opposite to greater angle is larger]

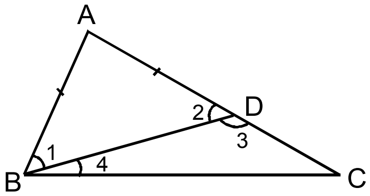
Thus, PM < PN.

Hence, PM is the shortest of all line segments from P to ℓ.

Theorem : Prove that the difference between any two sides of a triangle is less than its third side.

Given : A ΔABC.

To Prove :



(i) $AC - AB < BC$

(ii) $BC - AC < AB$

(iii) $BC - AB < AC$

Construction : Let AC > AB. Then, along AC, set off AD = AB. Join BD.

Proof : AB = AD ⇒ $\angle 1 = \angle 2$.

Side CD of ΔBCD has been produced to A.

∴ $\angle 2 > \angle 4$ [∵ exterior angle > each interior opposite angle]

Again, side AD of ΔABD has been produced to C.

∴ $\angle 3 > \angle 1$ [∵ exterior angle > each interior opposite angle]

Consequently, $\angle 3 > \angle 2$ [∵ $\angle 1 = \angle 2$]

Now, $\angle 3 > \angle 2$ and $\angle 2 > \angle 4 \Rightarrow \angle 3 > \angle 4$.

∴ BC > CD [side opposite to greater angle is longer]
⇒ $CD < BC$
⇒ $AC - AD < BC$
⇒ $AC - AB < BC$ [∵ AD = AB]

Hence, $AC - AB < BC$.

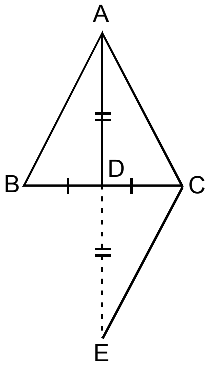
Similarly, $BC - AC < AB$ and $BC - AB < AC$.

Solved Examples

Example. 12

Prove that any two sides of the triangle are together greater than twice the median drawn to the third side.

Sol. **Given :** $\triangle ABC$ and AD is the median.



To prove : AB + AC > 2AD

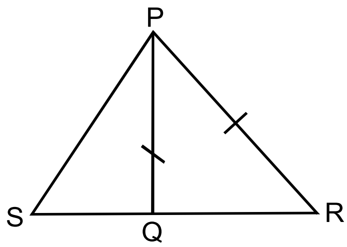
Construction : Produce AD to E such that AD = DE. Join EC.

Proof : In $\triangle ADB$ and $\triangle CDE$

AD = DE	[By construction]
BD = DC	[AD is the median]
$\angle$ ADB = $\angle$ CDE	[Vertically opposite angles]
$\triangle ADB \cong \triangle CDE$	[By SAS congruency]
So, by CPCT	
AB = EC	
In $\triangle AEC$	
AC + EC > 2AD.	[Sum of two sides of a triangle is always greater than the third side]
So, AC + AB > 2AD.	[As EC = AB].

Example. 13

In figure, PQ = PR, show that PS > PQ.



Sol. In $\triangle PQR$

$\therefore$ PQ = PR

$\Rightarrow \angle PRQ = \angle PQR$ ... (i)

In $\triangle PSQ$, SQ is produced to R

$\therefore$ Ext. $\angle PQR > \angle PSQ$ ... (ii)

$\Rightarrow$ $\angle PRQ > \angle PSQ$ [From equation (i) and (ii)]

$\Rightarrow$ $\angle PRS > \angle PSR$

$\Rightarrow$ PS > PR [Side opposite to greater angle is larger]

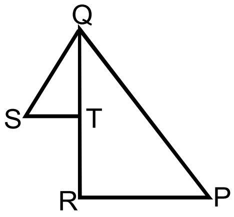
But, PR = PQ

$\therefore$ PS > PQ **Hence Proved.**

Example. 14

In figure, T is a point on side QR of ΔPQR and S is a point such that RT = ST. Prove that PQ + PR > QS

Sol. In ΔPQR we have



PQ + PR > QR

$\Rightarrow$ PQ + PR > QT + TR

$\Rightarrow$ PQ + PR > QT + ST ... (i) [$\because$ RT = ST]

In ΔQST

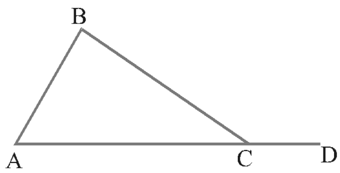
QT + ST > SQ ... (ii)

From (i) & (ii)

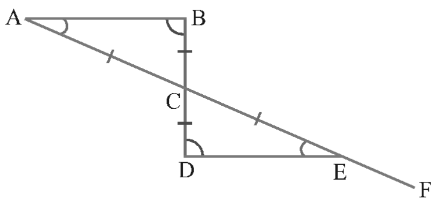
$\therefore$ PQ + PR > SQ. **Hence Proved.**

Check Your Level

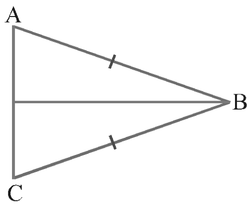
1. In the figure, compare $\angle BCD$ with $\angle A$



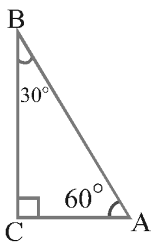
2. If Δ ABC $\cong$ Δ CDE, compare $\angle FED$ with $\angle B$



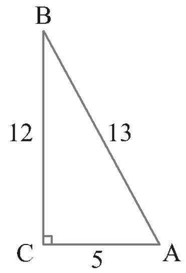
3. If AB = BC and $\angle B = \frac{\angle A}{2}$, prove that AB > AC?



4. List all sides of ΔABC in the increasing order of their length.



5. List the angles of the triangle ABC in the increasing order of their magnitude.



Answers

1. ∠BCD > A (Exterior angle of a triangle is greater than either of the two interior opposite angles).

2. ∠FED > ∠B **4.** CA < BC < AB **5.** ∠B < ∠A < ∠C

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]

1. In Δ ABC, AB = AC and $\angle$ B = 50°. Then find $\angle$ C.

2. In ΔPQR, $\angle$ R = $\angle$ P and QR = 4cm and PR = 5 cm. Find the length of PQ.

3. It is given that $\Delta ABC \cong \Delta FDE$ and AB = 5 cm, $\angle$B = 40° and $\angle$ A = 80°. Then find DF ?

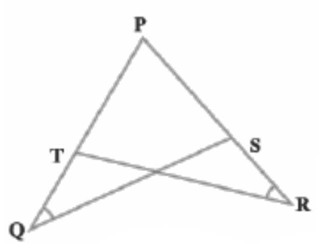
4. In Δ PQR, if $\angle$ R > $\angle$ Q, then which is longer between PQ and PR ?

5. In triangles ABC and DEF, AB = FD and $\angle$ A = $\angle$ D. The two triangles will be congruent by SAS axiom then find the side corresponding to AC.

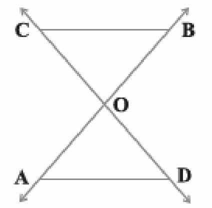
TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

6. In triangles ABC and PQR, $\angle$A = $\angle$Q and $\angle$B = $\angle$R. Which side of ΔPQR should be equal to side AB of ΔABC so that the two triangles are congruent? Give reason for your answer.

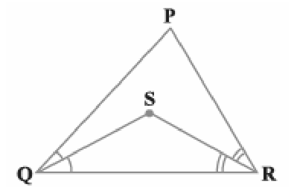
7. In Figure, PQ = PR and $\angle$Q = $\angle$R. Prove that $\Delta PQS \cong \Delta PRT$.



8. In Figure, two lines AB and CD intersect each other at the point O such that BC || DA and BC = DA. Show that O is the mid- point of both the line-segments AB and CD.

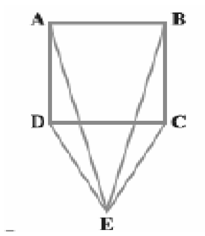


9. In Figure , PQ > PR and QS and RS are the bisectors of $\angle$ Q and $\angle$ R, respectively. Show that SQ > SR.



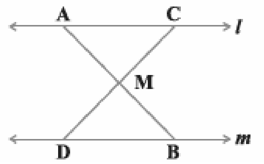
10. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.

11. CDE is an equilateral triangle formed on a side CD of a square ABCD (according to figure). Show that $\Delta ADE \cong \Delta BCE$.

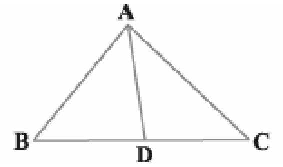


12. D is any point on side AC of a Δ ABC with AB = AC. Show that CD < BD.

13. In Figure, $l \parallel m$ and M is the mid-point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its end points on l and m, respectively.

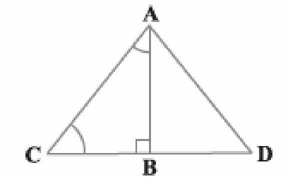


14. In Figure, AD is the bisector of ∠BAC. Prove that AB > BD.



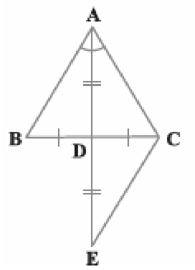
TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[04 MARK EACH]**

15. In Figure, ABC is a right triangle and right angled at B such that ∠BCA = 2 ∠BAC. Show that hypotenuse AC = 2 BC.



16. Prove that if in two triangles two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent.

17. If the bisector of an angle of a triangle also bisects the opposite side, prove that the triangle is isosceles.



18. S is any point in the interior of Δ PQR. Show that SQ + SR < PQ + PR.

19. ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.

20. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle.

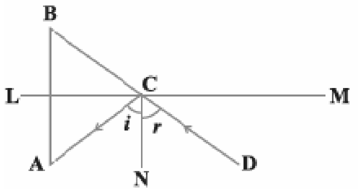
21. ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD.

22. In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse

23. ABCD is quadrilateral such that AB = AD and CB = CD. Prove that AC is the perpendicular bisector of BD.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[05 MARK EACH]**

24. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



25. Show that in a quadrilateral ABCD, AB + BC + CD + DA < 2 (BD + AC).

26. Line segment joining the midpoints M and N of parallel sides AB and DC, respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.

27. ABC is a right triangle such that AB = AC and bisector of angle C intersects the side AB at D. Prove that AC + AD = BC.

28. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.

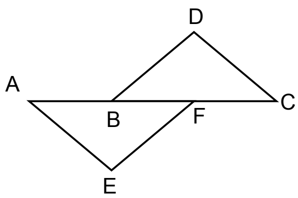
Exercise-1

SUBJECTIVE QUESTIONS

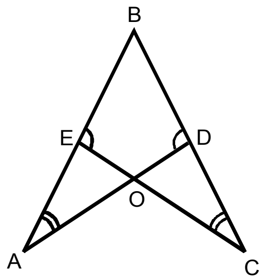
Subjective Easy, only learning value problems

Section (A) : Congruent triangles

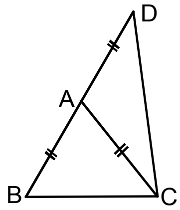
A-1 In figure, it is given that AB = CF, EF = BD and $\angle AFE = \angle CBD$. Prove that $\Delta AFE \cong \Delta CBD$.



A-2 In figure, it is given that $\angle A = \angle C$ and AB = BC. Prove that $\Delta ABD \cong \Delta CBE$.

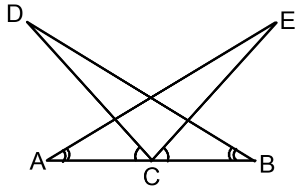


A-3 In the figure, AB = AC = AD, prove that ∠BCD is a right angle.

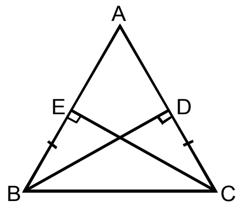


A-4 ABC is a triangle and D is the mid point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

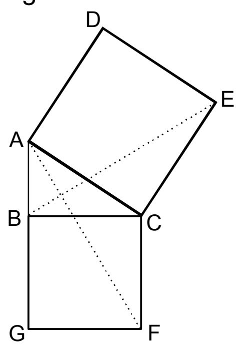
A-5 In figure AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC. Prove that triangles DBC and EAC are congruent, and hence DC = EC and BD = AE.



A-6 In figure, BD and CE are altitudes. If BE = CD, prove that BD = CE.



A-7 In figure, Δ ABC is a right angled triangle at B. ADEC and BCFG are squares. Prove that AF = BE.



A-8 ΔABC and ΔDBC are two isosceles triangle on the same base BC, and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that

(i) ΔABD ≅ ΔACD **(ii)** ΔABP ≅ ΔACP

(iii) AP bisects ∠A as well as ∠D **(iv)** AP is the perpendicular bisector of BC.

Section (B) : Some Inequalities in triangles

B-1. If D is any point on the base BC produced, of an isosceles triangle ABC, prove that AD > AB.

B-2. In ΔPQR, S is any point on the side QR. Show that PQ + QR + RP > 2 PS.

B-3. Prove that the perimeter of a triangle is greater than the sum of its three medians.

B-4. O is any point in the interior of a triangle ABC. Prove that :

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + OB + OC

(iii) $OA + OB + OC > \frac{1}{2}(AB + BC + AC)$.

B-5. PQRS is a quadrilateral in which diagonal PR and QS intersect in O. Show that :

(i) PQ + QR + RS + SP > PR + QS

(ii) PQ + QR + RS + SP < 2(PR + QS)

B-6. Prove that the sum of three altitudes of a triangle is less than the sum of three sides of the triangle.

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Congruent Triangles

A-1. If the three altitudes of a Δ are equal then triangle is :

(A) isosceles (B) equilateral (C) right angled (D none

A-2. ABCD is a square and P, Q, R are points on AB, BC and CD respectively such that AP = BQ = CR and ∠PQR = 90º, then ∠QPR :

(A) 45º (B) 50º (C) 60º (D) 70º

A-3. In a ΔXYZ, LM || YZ and bisectors YN and ZN of ∠Y & ∠Z respectively meet at N on LM. Then YL + ZM =

(A) YZ (B) XY (C) XZ (D) LM

A-4. In ΔAOC and ΔXYZ, ∠A = ∠X, AO = XZ, AC = XY, then by which congruence rule is ΔAOC ≅ ΔXZY

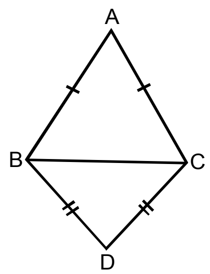
(A) SAS (B) ASA (C) SSS (D) RHS

A-5. Two equilateral triangles are congruent when :

(A) their angles are equal (B) their sides are equal

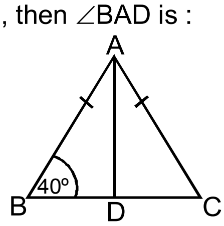
(C) their sides are proportional (D) their areas are proportional

A-6. In the given figure, if AB = AC and BD = DC. Δ ABD and Δ ACD are congruent by which criterion.



(A) SSS (B) ASA (C) SAS (D) RHS

A-7. In the given figure, AD is the median, then ∠BAD is :



(A) 55º (B) 50º (C) 100º (D) 40º

A-8. Δ ABC ≅ ΔPQR. If AB = 5cm, ∠B = 40º and ∠A = 80º, then which of the following is true ?
(A) QP = 5 cm, ∠P = 60º
(B) QP = 5cm , ∠R = 60º
(C) QR = 5 cm, ∠R= 80º
(D) QR = 5cm, ∠Q = 40º

Section (B) : Some inequalities in triangles

B-1. In a ΔPQR, PS is bisector of ∠P and ∠Q = 70º ∠R = 30º, then :
(A) QS > PQ > PR (B) QS < PQ < PR (C) PQ > QS > SR (D) PQ < QS < SR

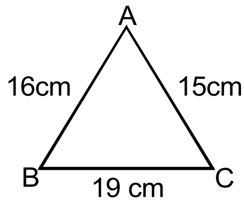
B-2. If D is any point on the side BC of a ΔABC, then :
(A) AB + BC + CA > 2AD
(B) AB + BC + CA < 2AD
(C) AB + BC + CA > 3 AD
(D) None

B-3. If length of the largest side of a triangle is 12 cm, then other two sides of triangle can be :
(A) 4.8 cm, 8.2 cm (B) 3.2 cm, 7.8 cm (C) 6.4 cm, 2.8 cm (D) 7.6 cm, 3.4 cm

B-4. It is not possible to construct a triangle when its sides are :
(A) 8.3 cm, 3.4 cm, 6.1 cm
(B) 5.4 cm, 2.3 cm, 3.1 cm
(C) 6 cm, 7 cm, 10 cm
(D) 3cm, 5 cm, 5cm

B-5. P is point on side BC of ΔABC such that AP bisects ∠BAC. Then :
(A) BP = CP (B) BA = BP (C) BP > BA (D) CP < CA

B-6. In the given figure, which of the following statement is true ?

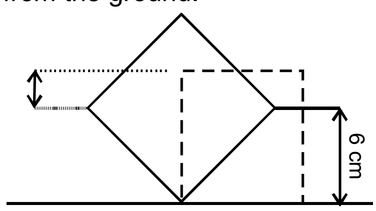


(A) ∠B = ∠C
(B) ∠B is the greatest angle in triangle
(C) ∠B is the smallest angle in triangle
(D) ∠A is the smallest angle in triangle

Exercise-2

OBJECTIVE QUESTIONS

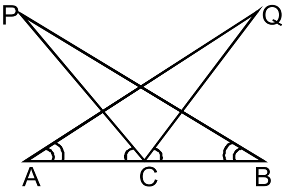
1. A square board side 10 centimeters, standing vertically, is tilted to the left so that the bottom-right corner is raised 6 centimeters from the ground.



By what distance is the top-left corner lowered from its original position ?
(A) 1 cm (B) 2 cm (C) 3 cm (D) 0.5 cm

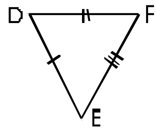
2. In a right angled triangle ABC, P is mid point of AC. Which one is true ?
(A) PA = $\frac{AC}{2}$ (B) PB = $\frac{AC}{2}$ (C) PA = PB (D) All of these

3. In the given figure AC = CB, ∠PCA = ∠QCB and ∠PBC = ∠QAC, then the true statement is :



(A) PB > QA (B) ∠CPB ≠ ∠CQA (C) PC ≠ QC (D) ΔPCB ≅ ΔQCA

4. For given figure, which one is correct :



(A) Δ ABC ≅ Δ DEF (B) Δ ABC ≅ ΔFED (C) Δ ABC ≅ Δ DFE (D) Δ ABC ≅ Δ EDF

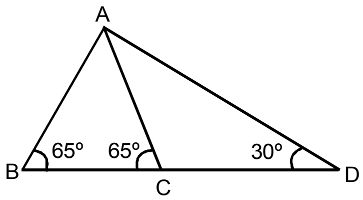
5. The sides of a triangle with positive area have lengths 4, 6 and x. The sides of a second triangle with positive area have length 4, 6 and y. The smallest positive number that is not the possible value of |x – y| is (x and y are integers) :
(A) 2 (B) 4 (C) 6 (D) 8

6. The sides of a triangle are in the ratio 4 : 6 : 11. Which of the following words best described the triangle?
(A) obtuse (B) isosceles (C) acute (D) impossible

7. The number of triangles with any three of the lengths 1, 4, 6 and 8 cm, are :
(A) 4 (B) 2 (C) 1 (D) 0

8. The perimeter of a triangle is :
(A) greater than the sum of its altitudes (B) less than the sum of its altitudes
(C) equal to the sum of its altitudes (D) none of these

9. In the given diagram ∠B = ∠C = 65º and ∠D = 30º, then the true statement is :



(A) BC = CA (B) CA > CD (C) BD > AD (D) AC = AD

10. In a ΔPQR, PS is bisector of ∠P, ∠Q = 70º and ∠R = 30º, then :
(A) QR < PR > PQ (B) QR > PR > PQ (C) QR = PR = PQ (D) QR < PR = PQ

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. In ΔABC and ΔDEF, AC =BC=DF=EF, length AB=2FH, where FH⊥DE. Which of the following statements is (are) true ?
I. ∠ACB and ∠DFE are complementary **II.** ∠ACB and ∠DFE are supplementary
III. Area of ΔABC = Area of ΔDEF **IV.** Area of ΔABC = 1.5x (Area of ΔDEF)
[Harayana NTSE Stage-1 2013]
(A) II only (B) III only (C) I and III only (D) II and III only

2. The length of sides of triangle are integers and its perimeter is 14. How many such distinct triangles are possible? **[Bihar NTSE Stage-1 2015]**
(A) 6 (B) 5 (C) 4 (D) 3

Answer Key

Exercise Board Level

TYPE (I)

1.	50°	**2.**	4 cm	**3.**	5 cm	**4.**	PQ > PR	**5.**	DE
6.	QR, ASA criterion								

Exercise-1

OBJECTIVE QUESTIONS

Section (A)

A-1.	(B)	**A-2.**	(A)	**A-3.**	(D)	**A-4.**	(A)	**A-5.**	(B)
A-6.	(A)	**A-7.**	(B)	**A-8.**	(B)				

Section (B)

B-1.	(B)	**B-2.**	(A)	**B-3.**	(A)	**B-4.**	(B)	**B-5.**	(D)
B-6.	(C)								

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	D	C	D	D	C	A	C	B

Exercise-3

Ques.	1	2
Ans.	D	C