MATHEMATICS

Class-X

Topic-7

TRIGONOMETRY

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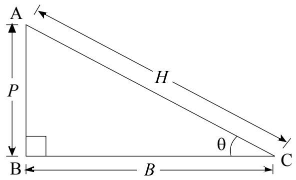
CH-07

TRIGONOMETRY

(A) INTRODUCTION TO TRIGONOMETRY & TRIGONOMETRIC RATIOS

Trigonometry means, the science which deals with the measurement of triangles. The word 'trigonometry' is derived from the Greek words 'trigon' and 'metron' and it means 'measuring the sides and angles of a triangle'.

(a) Trigonometric ratios



A right angled triangle is shown in **Figure**. ∠ **B** is of **90º**. Side opposite to ∠ **B** is called **hypotenuse**. There are two other angles i.e. ∠**A** and ∠**C**. If we consider ∠**C** as θ, then opposite side to this angle is called **perpendicular** and side adjacent to θ is called **base**.

(i) Six Trigonometric Ratios are

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{P}{H} = \frac{AB}{AC}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{H}{P} = \frac{AC}{AB}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{B}{H} = \frac{BC}{AC}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{H}{B} = \frac{AC}{BC}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{B} = \frac{AB}{BC}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{B}{P} = \frac{BC}{AB}$$

(ii) Interrelationship in Basic Trigonometric Ratios

$$\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$$

$$\sin\theta = \frac{1}{\text{cosec}\,\theta} \Rightarrow \text{cosec}\,\theta = \frac{1}{\sin\theta}$$

We also observe that

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Solved Examples

Example. 1

In the given triangle XY = 3 cm and XZ = 5 cm. Find all trigonometric ratios.

Sol. Using pythagoras theorem

$XZ^2 = XY^2 + YZ^2$

$\Rightarrow \quad 5^2 = 3^2 + P^2$

$\Rightarrow \quad P^2 = 16$

$\Rightarrow \quad P = 4$ cm

Here, P = 4 cm, B = 3 cm, H = 5 cm

$\therefore \quad \sin\theta = \frac{P}{H} = \frac{4}{5}, \qquad \cos\theta = \frac{B}{H} = \frac{3}{5} \qquad \tan\theta = \frac{P}{B} = \frac{4}{3},$

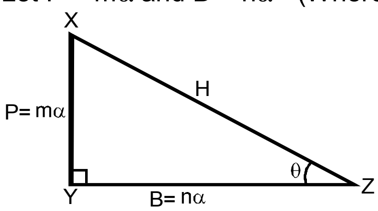
$\cot\theta = \frac{B}{P} = \frac{3}{4}, \qquad \sec\theta = \frac{H}{B} = \frac{5}{3}, \qquad \operatorname{cosec}\theta = \frac{H}{P} = \frac{5}{4}.$

Example. 2

If $\tan\theta = \frac{m}{n}$, then find $\sin\theta$.

Sol. $\therefore \quad \tan\theta = \frac{P}{B} = \frac{m}{n}$

Let P = mα and B = nα (Where a is some constant)



$H^2 = P^2 + B^2$

$H^2 = m^2\alpha^2 + n^2\alpha^2$

$H = \alpha\sqrt{m^2 + n^2}$

$\therefore \quad \sin\theta = \frac{P}{H} = \frac{m\alpha}{\alpha\sqrt{m^2 + n^2}}$

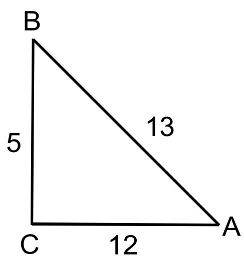
$\sin\theta = \frac{m}{\sqrt{m^2 + n^2}}.$

Example. 3

If $\operatorname{cosec} A = \frac{13}{5}$, then prove that $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$.

Sol. We have $\operatorname{cosec} A = \frac{13}{5} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$.

So, we draw a right triangle ABC, right angled at C such that Hypotenuse AB = 13 units and perpendicular BC = 5 units



By pythagoras theorem,

$AB^2 = BC^2 + AC^2 \quad \Rightarrow \quad (13)^2 = (5)^2 + AC^2$

$AC^2 = 169 - 25 = 144$

$AC = \sqrt{144} = 12$ units

$\tan A = \frac{BC}{AC} = \frac{5}{12}$ and $\sin A = \frac{BC}{AB} = \frac{5}{13}$ and $\sec A = \frac{AB}{AC} = \frac{13}{12}$

L.H.S $\tan^2 A - \sin^2 A = \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{25}{144} - \frac{25}{169} = \frac{25(169-144)}{144 \times 169} = \frac{25 \times 25}{144 \times 169}$.

R.H.S. $\sin^4 A \times \sec^2 A = \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2 = \frac{5^4 \times 13^2}{13^4 \times 12^2} = \frac{5^4}{13^2 \times 12^2} = \frac{25 \times 25}{144 \times 169}$

So, L.H.S = R.H.S. **Hence Proved.**

Check Your Level

1. If $\sin\theta = \frac{3}{5}$, find all the other ratios.

2. If $\tan\theta = \frac{12}{5}$, find the value of $\frac{\sin\theta + \cot\theta}{\cos\theta + \text{cosec}\theta}$

3. If cosec A = $\frac{5}{3}$, then prove that $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$.

4. In Δ ABC, right angled at B, AC + AB = 9 cm and BC = 3 cm. Determine the value of cot C, cosec C.

5. Find value of sin θ. sec θ .cosec θ. cos θ

Answers

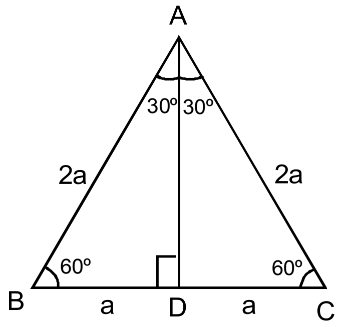
1. $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\cot\theta = \frac{4}{3}$, $\sec\theta = \frac{5}{4}$, $\text{cosec}\theta = \frac{5}{3}$ **2.** $\frac{209}{229}$

4. $\cot C = \frac{3}{4}$, $\text{cosec } C = \frac{5}{4}$ **5.** 1

(B) TRIGONOMETRIC ANGLES

(a) Trigonometric Ratios of 30° and 60°

Consider an equilateral triangle ABC with each side of length 2a. Since each angle of an equilateral triangle is of 60°. Therefore, each angle of ΔABC is of 60°. Let AD be perpendicular from A on BC. Since the triangle is equilateral. Therefore, AD is the bisector of ∠A and D is the mid-point of BC.



$\therefore$ BD = DC = a and $\angle BAD = 30^\circ$.

Thus, in ΔABD, $\angle D$ is a right angle, hypotenuse AB = 2a and BD = a.

So, by Pythagoras theorem, we have

$AB^2 = AD^2 + BD^2$

$\Rightarrow$ $(2a)^2 = AD^2 + a^2$

$\Rightarrow$ $AD^2 = 4a^2 - a^2$

$\Rightarrow$ $AD = \sqrt{3}\, a$

(i) Trigonometric ratios of 30º

In right triangle ADB, we have

Base = AD = $\sqrt{3}$ a, Perpendicular = BD = a, Hypotenuse = AB = 2a and $\angle DAB = 30^\circ$.

$$\therefore \quad \sin 30^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

$$\text{cosec } 30^\circ = \frac{1}{\sin 30^\circ} = 2.$$

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$$

$$\text{and,} \quad \cot 30^\circ = \frac{2}{\tan 30^\circ} = \sqrt{3}$$

(ii) Trigonometric ratios of 60º

In right triangle ADB, we have

Base = BD = a, Perpendicular = AD = $\sqrt{3}$ a, Hypotenuse = AB = 2a and $\angle ABD = 60^\circ$.

$$\therefore \quad \sin 60^\circ = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{AD}{BD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

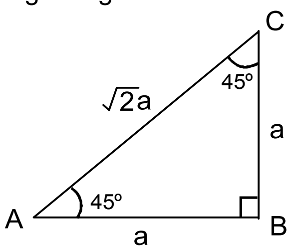
$$\text{cosec } 60^\circ = \frac{1}{\sin 60^\circ} = \frac{2}{\sqrt{3}}$$

$$\sec 60^\circ = \frac{1}{\cos 60^\circ} = 2$$

$$\text{and} \quad \cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}.$$

(b) Trigonometric Ratio of 45º

Consider a right triangle ABC with right angle at B such that $\angle A = 45^\circ$. Then,



$\angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow$ $45^\circ + 90^\circ + \angle C$
$\Rightarrow$ $\angle C = 45^\circ$
$\therefore$ $\angle A = \angle C$
$\Rightarrow$ $AB = BC$

Let AB = BC = a. Then, by Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$
$\Rightarrow$ $AC^2 = a^2 + a^2$
$\Rightarrow$ $AC^2 = 2a^2$
$\Rightarrow$ $AC = \sqrt{2}\,a$

Thus, in ΔABC, we have

$\angle A = 45^\circ$, Base = AB = a, Perpendicular = BC = a, and Hypotenuse = AC = $\sqrt{2}$ a.

$\therefore$ $\sin 45^\circ = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

$\cos 45^\circ = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

$\tan 45^\circ = \frac{BC}{AB} = \frac{a}{a} = 1$

$\text{cosec } 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}$.

$\sec 45^\circ = \frac{1}{\cos 45^\circ} = \sqrt{2}$ and $\cot 45^\circ = \frac{1}{\tan 45^\circ} = \frac{1}{1} = 1$.

(c) Trigonometric Table

$\theta \rightarrow$	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Solved Examples

Example. 4

Given that cos (A – B) = cos A cos B + sin A sin B, find the value of cos15°.

Sol. Putting A = 45° and B = 30°

We get

$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$\Rightarrow$ $\cos 15^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $\Rightarrow$ $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$.

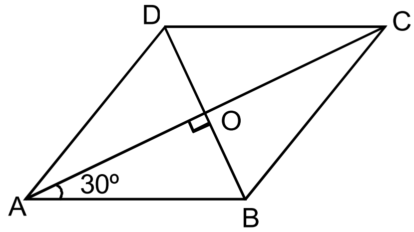
Example. 5

A Rhombus of side of 10 cm has two angles of 60º each. Find the length of diagonals and also find its area.

Sol. Let ABCD be a rhombus of side 10 cm and $\angle BAD = \angle BCD = 60^\circ$. Diagonals of parallelogram bisect each other.

So, AO = OC and BO = OD

In right triangle AOB



$\sin 30^\circ = \frac{OB}{AB} \Rightarrow \frac{1}{2} = \frac{OB}{10}$

$\Rightarrow$ OB = 5 cm

$\therefore$ BD = 2(OB) $\Rightarrow$ BD = 2 (5)

$\Rightarrow$ BD = 10 cm

$\cos 30^\circ = \frac{OA}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OA}{10}$

$\Rightarrow$ OA = $5\sqrt{3}$

$\therefore$ AC = 2(OA) $\Rightarrow$ AC = $2(5\sqrt{3})$

$\Rightarrow$ AC = $10\sqrt{3}$ cm

So, the length of diagonals AC = $10\sqrt{3}$ cm & BD = 10 cm.

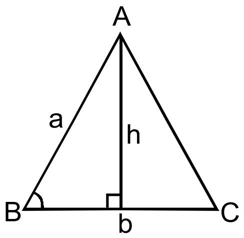
$$\text{Area of Rhombus} = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 10\sqrt{3} \times 10 = 50\sqrt{3}\ \text{cm}^2.$$

Example. 6

If triangle having adjacent sides a,b units and included angle is θ then prove that area of triangle is $\frac{1}{2}$ ab sinθ.

Sol. In triangle ABC



AD $\perp$ BC

AB = a , BC = b and $\angle ABC = \theta$

In triangle ABD

$\sin\theta = \frac{h}{a} \Rightarrow h = a\sin\theta$

area of triangle $= \frac{1}{2} BC \times AD = \frac{1}{2} b \times h = \frac{1}{2} b \times a \sin\theta$.

Check Your Level

1. Evaluate $\sin^2 45° \,.\, \cos^2 45° + \sin^2 60° \,.\, \cos^2 60°$.

2. Find the value of x from the equation, $x \tan 45° \,.\, \sec 60° = \cot^2 30°$.

3. If A = 60° and B = 30°, verify that sin A cos B + cos A sin B = sin (A + B)

4. If θ is an acute angle and sin θ = cos θ, find the value of $2 \tan^2 \theta + \sin^2 \theta - 1$.

5. Evaluate
   (a) $\sin^2 60° + \cos^2 45°$
   (b) $3 \cos^2 30° + \tan^2 60°$
   (c) $4 \sin^2 45° + 3 \tan^2 30° + 8 \sin 45°. \cos 45°$
   (d) $2 \sin^2 30° - 3 \cos^2 60° + \cot^2 30°$
   (e) $\dfrac{\sin 30°.\cos 60°}{\tan 45°.\cot 45°}$
   (f) $\dfrac{\sin 60°.\tan 60°}{\cos 30°.\cot 30°}$
   (g) $\dfrac{1-\tan^2 30°}{1+\tan^2 30°}$
   (h) $4(\sin^4 30° + \cos^4 60°) - 3(\sin^2 45° - \sin 90°)$

Answers

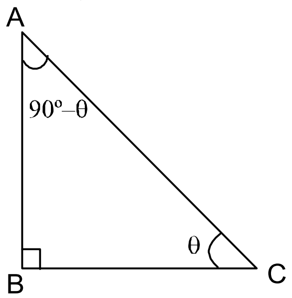
1. $\dfrac{7}{16}$ 2. $x = \dfrac{3}{2}$ 4. $\dfrac{3}{2}$

5. (a) $\dfrac{5}{4}$ (b) $\dfrac{21}{4}$ (c) 7 (d) $\dfrac{11}{4}$ (e) $\dfrac{1}{4}$ (f) 1

   (g) $\dfrac{1}{2}$ (h) 2

(C) COMPLEMENTARY ANGLES

We know that two angles are said to be complementary, if their sum is 90º. Thus, angles θ and (90º – θ) are complementary. We shall now express the trigonometric ratios of the complementary angle (90º – θ) of a given angle θ in terms of the trigonometric ratios of θ, where θ is an acute angle. We consider a right-angled triangle ABC, where ∠ABC = 90º, ∠ACB = θ, so that ∠BAC = 90º – θ.



Clearly, ∠BAC and ∠ACB are complementary angles. We shall now express the trigonometric ratios of (90º – θ) in terms of those of θ.
Now, with reference to the angle θ, BC is the base, AB is the perpendicular and AC is the hypotenuse.

Hence, we have

$\sin\theta = \frac{AB}{AC}$, $\cos\theta = \frac{BC}{AC}$, $\tan\theta = \frac{AB}{BC}$, $\cot\theta = \frac{BC}{AB}$, $\sec\theta = \frac{AC}{BC}$ and $\text{cosec}\,\theta = \frac{AC}{AB}$.

Again, with reference to angle BAC = 90º – θ, AB is the base, BC is the perpendicular and AC is the hypotenuse.

Hence,

$\sin(90^\circ - \theta) = \frac{BC}{AC} = \cos\theta$

$\cos(90^\circ - \theta) = \frac{AB}{AC} = \sin\theta$

$\tan(90^\circ - \theta) = \frac{BC}{AB} = \cot\theta$

$\cot(90^\circ - \theta) = \frac{AB}{BC} = \tan\theta$

$\sec(90^\circ - \theta) = \frac{AC}{AB} = \text{cosec}\,\theta$ and $\text{cosec}(90^\circ - \theta) = \frac{AC}{BC} = \sec\theta$.

Solved Examples

Example. 7

Find the value of $\frac{\sin 59^\circ}{\cos 31^\circ} + \frac{\cos 20^\circ}{\sin 70^\circ} - 2\frac{\cos 31^\circ}{\sin 59^\circ}$

Sol. $\frac{\sin 59^\circ}{\cos 31^\circ} + \frac{\cos 20^\circ}{\sin 70^\circ} - 2\frac{\cos 31^\circ}{\sin 59^\circ}$

$= \frac{\sin(90^\circ - 31^\circ)}{\cos 31^\circ} + \frac{\cos(90^\circ - 70^\circ)}{\sin 70^\circ} - 2\left(\frac{\cos(90^\circ - 59^\circ)}{\sin 59^\circ}\right)$

$= \frac{\cos 31^\circ}{\cos 31^\circ} + \frac{\sin 70^\circ}{\sin 70^\circ} - 2\left(\frac{\sin 59^\circ}{\sin 59^\circ}\right) = 1 + 1 - 2(1) = 2 - 2 = 0.$

Example. 8

Prove that : cosec (65º + θ) – sec (25º – θ) – tan (55º – θ) + cot (35º + θ) = 0

Sol. cosec (65º + θ) = cosec {90º – (25º – θ)}

= sec (25º – θ) .. (i)

cot (35º + θ) = cot {90º – (55º – θ)}

= tan (55º – θ) ... (ii)

∴ **L.H.S.**

cosec (65º + θ) – sec (25º – θ) – tan (55º – θ) + cot (35º + θ)

= sec (25º – θ) – sec (25º – θ) – tan (55º – θ) + tan (55º – θ) = 0 [using (i) & (ii)] **R.H.S.**

Example. 9

If sec 3A = cosec (A – 10º) where 4A is an acute angle, find the value of A.

Sol. sec 3A = Cosec (A – 10º)

cosec (90º–3A) = Cosec (A – 10º)

90º – 3A= A – 10º

4A = 100º

A = 25º

Example. 10

If A, B, C are the interior angles of a triangle ABC, then prove that $\tan\left(\frac{A+B}{2}\right) = \cot\left(\frac{C}{2}\right)$.

Sol. In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle B = 180^\circ - \angle C$

$$\frac{\angle A + \angle B}{2} = 90^\circ - \frac{\angle C}{2}$$

$$\tan\left(\frac{\angle A + \angle B}{2}\right) = \tan\left(90^\circ - \frac{\angle C}{2}\right)$$

$$\tan\left(\frac{\angle A + \angle B}{2}\right) = \cot\left(\frac{\angle C}{2}\right).$$

Check Your Level

1. Simplify $\frac{\cot 70^\circ}{\tan 20^\circ} + \frac{\cos 52^\circ}{\sin 38^\circ}$

2. Prove that $\cos 72^\circ \, . \cos 18^\circ - \sin 72^\circ \, . \sin 18^\circ = 0$

3. If $\sin A = \cos B$, what is $A + B$ equal to?

4. Evaluate : $\cos(40^\circ + \theta) - \sin(50^\circ - \theta)$

5. Evaluate : $\tan 13^\circ . \tan 21^\circ . \tan 69^\circ . \tan 77^\circ$

Answers

1. 2 **3.** 90° **4.** 0 **5.** 1

(D) TRIGONOMETRIC IDENTITIES

(i) $\mathbf{\sin^2\theta + \cos^2\theta = 1}$

(A) $\sin^2\theta = 1 - \cos^2\theta$ **(B)** $\cos^2\theta = 1 - \sin^2\theta$

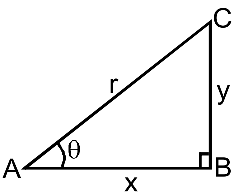
(ii) $\mathbf{1 + \tan^2\theta = \sec^2\theta}$ [where $\theta \neq 90^\circ$]

(A) $\sec^2\theta - 1 = \tan^2\theta$ **(B)** $\sec^2\theta - \tan^2\theta = 1$ **(C)** $\tan^2\theta - \sec^2\theta = -1$

(iii) $\mathbf{1 + \cot^2\theta = cosec^2\theta}$ [where $\theta \neq 0^\circ$]

(A) $cosec^2\theta - 1 = \cot^2\theta$ **(B)** $cosec^2\theta - \cot^2\theta = 1$ **(C)** $\cot^2\theta - cosec^2\theta = -1$

Proof : Consider a right-angled ΔABC in which $\angle B = 90^\circ$ and $\angle A = \theta^\circ$. Let AB = x units, BC = y units and AC = r units.



Then, by Pythagoras theorem, we have
Now,

(i) $\sin^2\theta + \cos^2\theta = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = \left(\frac{y^2}{r^2} + \frac{x^2}{r^2}\right) = \frac{(x^2+y^2)}{r^2} = \frac{r^2}{r^2} = 1 \quad [\because x^2 + y^2 = r^2]$

$\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = 1 + \left(\frac{y}{x}\right)^2 = \frac{x^2+y^2}{x^2} = \frac{r^2}{x^2} = \left(\frac{r}{x}\right)^2 = \sec^2\theta$

$\therefore \quad 1 + \tan^2\theta = \sec^2\theta.$

(iii) $1 + \cot^2\theta = 1 + \left(\frac{x}{y}\right)^2 = \frac{y^2+x^2}{y^2} = \frac{r^2}{y^2} = \left(\frac{r}{y}\right)^2 = \text{cosec}^2\theta$

$\therefore \quad 1 + \cot^2\theta = \text{cosec}^2\theta.$

Solved Examples

Example. 11

Prove that : $\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$.

Sol. **L.H.S.** $\cot\theta - \tan\theta = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \quad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta} \quad [\because \sin^2\theta = 1 - \cos^2\theta]$

$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta} = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$ **R.H.S. Hence Proved.**

Example. 12

Prove that : (cosec A – sin A) (sec A – cos A) (tan A + cot A) = 1.

Sol. L.H.S. (cosec A – sin A) (sec A – cos A) (tan A + cot A)

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right) = \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{1}{\sin A\cos A}\right) \quad [\because \sin^2 A + \cos^2 A = 1]$

= 1 **R.H.S** **Hence Proved.**

Example. 13

If $\sin\theta + \cos\theta = m$ and $\sec\theta + \text{cosec}\,\theta = n$, then prove that $n(m^2 - 1) = 2m$.

Sol. **L.H.S.** $n(m^2 - 1) = (\sec\theta + \text{cosec}\,\theta)[(\sin\theta + \cos\theta)^2 - 1]$

$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1) = \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)(1 + 2\sin\theta\cos\theta - 1)$

$= \frac{(\cos\theta + \sin\theta)}{\sin\theta\cos\theta}(2\sin\theta\cos\theta) = 2(\sin\theta + \cos\theta) = 2m$ **R.H.S** **Hence Proved.**

Example. 14

If $\sec\theta = x + \frac{1}{4x}$, then prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$.

Sol. $\sec\theta = x + \frac{1}{4x}$ ....(i)

$\therefore \quad 1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \quad \tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \quad \tan^2\theta = \left(x + \frac{1}{4x}\right)^2 - 1$

$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} + 2 \times x \times \frac{1}{4x} - 1$

$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$

$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$

$\Rightarrow \quad \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right)$

so, $\quad \tan\theta = x - \frac{1}{4x}$ .....(ii)

or $\quad \tan\theta = -\left(x - \frac{1}{4x}\right)$ .....(iii)

Adding equation (i) and (ii)

$\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x}$

$\sec\theta + \tan\theta = 2x$

Adding equation (i) and (iii)

$\sec\theta + \tan\theta = x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$

Hence, $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$.

Example. 15

If θ is an acute angle and $\tan\theta + \cot\theta = 2$, find the value of $\tan^9\theta + \cot^9\theta$

Sol. We have, $\tan\theta + \cot\theta = 2$

$\Rightarrow \quad \tan\theta + \frac{1}{\tan\theta} = 2 \qquad \Rightarrow \quad \frac{\tan^2\theta + 1}{\tan\theta} = 2$

$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta \qquad \Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \quad (\tan\theta - 1)^2 = 0 \qquad \Rightarrow \quad \tan\theta - 1 = 0$

$\Rightarrow \quad \tan\theta = 1 \qquad \Rightarrow \quad \tan\theta = \tan 45^\circ$

$\Rightarrow \quad \theta = 45^\circ$

$\therefore \quad \tan^9\theta + \cot^9\theta$

$= \quad \tan^9 45^\circ + \cot^9 45^\circ$

$= \quad (\tan 45)^9 + (\cot 45)^9 = (1)^9 + (1)^9 = 2.$

Example. 16

Evaluate : $\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ} + 2\sin^2 38^\circ \sec^2 52^\circ - \sin^2 45^\circ + \frac{2}{\sqrt{3}}\tan 17^\circ \tan 60^\circ \tan 73^\circ.$

Sol. $\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{cosec}^2 57^\circ - \tan^2 33^\circ} + 2\sin^2 38^\circ \sec^2 52^\circ - \sin^2 45^\circ + \frac{2}{\sqrt{3}}\tan 17^\circ \tan 60^\circ \tan 73^\circ$

$= \frac{\sec^2(90^\circ - 36^\circ) - \cot^2 36^\circ}{\operatorname{cosec}^2(90^\circ - 33^\circ) - \tan^2 33^\circ} + 2\sin^2 38^\circ \sec^2(90^\circ - 38^\circ) - \sin^2 45^\circ + \frac{2}{\sqrt{3}}\tan(90^\circ - 73^\circ)\tan 73^\circ \tan 60^\circ$

$$= \frac{\operatorname{cosec}^2 36^\circ - \cot^2 36^\circ}{\sec^2 33^\circ - \tan^2 33^\circ} + 2\sin^2 38^\circ \operatorname{cosec}^2 38^\circ - \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{2}{\sqrt{3}} \cot 73^\circ \tan 73^\circ \times \sqrt{3}$$

$$= \frac{1}{1} + 2\sin^2 38^\circ \times \frac{1}{\sin^2 38^\circ} - \frac{1}{2} + \frac{2}{\sqrt{3}} \times \frac{1}{\tan 73^\circ} \times \tan 73^\circ \times \sqrt{3}$$

$[\operatorname{cosec}^2 \theta - \cot^2 \theta = 1, \sec^2\theta - \tan^2 \theta = 1]$

$$= 1 + 2 - \frac{1}{2} + 2 = 5 - \frac{1}{2} = \frac{9}{2}.$$

Check Your Level

1. Express cos θ in terms of sin θ.

2. Express tan θ in terms of cos θ.

3. Prove that, $(\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$

4. Prove that $\cot^2 \theta + \operatorname{cosec}^2 \theta = \operatorname{cosec}^4 \theta - \cot^4 \theta$

5. Prove that $\frac{\tan\theta}{\sec\theta - 1} + \frac{\tan\theta}{\sec\theta + 1} = 2\operatorname{cosec}\theta$

6. If $x = a \cos \theta$, $y = a \sin \theta$, what is the value of $x^2 + y^2$?

7. If $x = \operatorname{cosec} \theta - \cot \theta$, $y = \operatorname{cosec} \theta + \cot \theta$, find xy.

Answers

1. $\cos\theta = \sqrt{1 - \sin^2\theta}$ **2.** $\tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$ **6.** a^2 **7.** $xy = 1$

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : **[01 MARK EACH]**

1. Find the value of $\frac{\tan 30^\circ}{\cot 60^\circ}$.

2. If sin A = $\frac{1}{2}$, then find the value of cot A.

3. Find the value of the expression [cosec (75° + θ) – sec (15° – θ) – tan (55° + θ) + cot (35° – θ)] is

4. Given that sin θ = $\frac{a}{b}$, then find cos θ.

5. If Δ ABC is right angled at C, then find the value of cos (A + B).

6. Given that sin α = $\frac{1}{2}$ and cosβ = $\frac{1}{2}$, then the value of (α + β) is

7. Find the value of the expression $\left[\frac{\sin^2 22^\circ + \sin^2 68^\circ}{\cos^2 22^\circ + \cos^2 68^\circ} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ\right]$.

8. If 4 tan θ = 3, then find $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$.

9. If $\sqrt{3}$ tan θ = 1, then find the value of $\sin^2\theta - \cos^2\theta$.

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : **[02 MARKS EACH]**

10. Prove that $\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cos^2\theta = 1$

11. If sinθ + cosθ = $\sqrt{3}$, then prove that tanθ + cotθ = 1

12. Prove that : $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$ = 2cosecθ

13. Prove that : (sinα + cosα) (tanα + cotα) = secα + cosecα

14. Prove that : 1 + $\frac{\cot^2\alpha}{1+\text{cosec}\alpha}$ = cosecα

15. Prove that : tanθ + tan (90° – θ) = sec θ sec (90° – θ)

16. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ.

17. Show that $\frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\ \tan(30^\circ-\theta)}$ = 1.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[03 MARK EACH]**

18. If cosecθ + cotθ = p, then prove that cosθ = $\frac{p^2-1}{p^2+1}$.

19. Given that sinθ + 2cosθ = 1, then prove that 2sinθ – cosθ = 2.

20. If a sinθ + b cosθ = c, then prove that a cosθ – b sinθ = $\sqrt{a^2+b^2-c^2}$.

21. If 1 + $\sin^2\theta$ = 3sinθ cosθ , then prove that tanθ = 1 or $\frac{1}{2}$.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[04 MARK EACH]**

22. Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$

23. If sinθ + cosθ = p and secθ + cosecθ = q, then prove that q (p^2 – 1) = 2p.

24. If tanθ + secθ = *l*, then prove that secθ = $\frac{l^2+1}{2l}$.

Previous Year Problems

1. If 0° < x < 90° and $2\sin^2 x$ = $\frac{1}{2}$, then the value of x is **[1 MARK/ CBSE 10TH BOARD: 2013]**

(A) 90° (B) 30° (C) 15° (D) 60°

2. If tanθ = $\frac{1}{\sqrt{7}}$, then the value of $\frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}$ is **[1 MARK/ CBSE 10TH BOARD: 2013]**

(A) $\frac{1}{5}$ (B) $\frac{3}{4}$ (C) $\frac{6}{4}$ (D) $\frac{4}{\sqrt{7}}$

3. If cotθ + $\frac{1}{\cot\theta}$ = 2, then the value of $\cot^2\theta$ + $\frac{1}{\cot^2\theta}$ is **[1 MARK/ CBSE 10TH BOARD: 2013]**

(A) – 1 (B) 1 (C) 2 (D) – 2

4. The value of $\text{cosec}^2 30°\ \sin^2 45° - \sec^2 60°$ is **[1 MARK/ CBSE 10TH BOARD: 2013]**

5. Without using trigonometric tables, find the value of $\frac{\cos 70°}{\sin 20°}$ + cos57°cosec33° – 2 cos60°

[2 MARKS/ CBSE 10TH BOARD: 2013]

(A) – 1 (B) 1 (C) – 2 (D) 2

OR

If A, B, C are interior angles of ΔABC, then show that cos $\left(\frac{B+C}{2}\right)$ = sin $\frac{A}{2}$.

6. $\frac{\cos\theta}{\text{cosec}\theta+1}+\frac{\cos\theta}{\text{cosec}\theta-1}$ = 2tanθ **[3 MARKS /CBSE 10TH BOARD: 2013]**

7. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ = 2 secθ **[3 MARKS/CBSE 10TH BOARD: 2013M, 2014]**

8. Without using trigonometric tables, evaluate the following :

$\frac{\sec 37°}{\text{cosec}53°}$ + 2 cot 15° cot25° cot45° cot75° cot65° – 3 ($\sin^2 18°$ + $\sin^2 72°$)

[CBSE 10TH BOARD: 2013]

OR

Prove that : $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$ = 1 + secθ cosecθ **[CBSE 10TH BOARD: 2013]**

9. If x = $3\sec^2\theta$ – 1, y = $\tan^2\theta$ – 2 then x – 3y is equal to **[2 MARKSCBSE 10TH BOARD: 2014]**

(A) 3 (B) 4 (C) 5 (D) 8

10. If $\tan\theta = \frac{2}{3}$, then the value of $\frac{(2+2\sec\theta)(1-\sec\theta)}{(2+2\operatorname{cosec}\theta)(1-\operatorname{cosec}\theta)}$ is

[2 MARKS/CBSE 10TH BOARD: 2014]

(A) $\frac{81}{16}$ (B) $\frac{16}{81}$ (C) $\frac{75}{16}$ (D) $\frac{77}{16}$

11. If $\sin\theta + \sin^2\theta = 1$, then the value of $\cos^2\theta + \cos^4\theta$ is

[2 MARKSCBSE 10TH BOARD: 2014]

(A) 2 (B) 1 (C) – 2 (D) – 1

12. Prove that : $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$. **[CBSE 10TH BOARD: 2014]**

OR

Find acute angles A and B, if

$\sin(A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$, $A > B$ **[CBSE 10TH BOARD: 2014]**

13. **Prove that :** $\tan^2A + \cot^2A = \sec^2A \operatorname{cosec}^2A - 2$ **[CBSE 10TH BOARD: 2014]**

14. **Prove that :** $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta$ **[3 MARKS/CBSE 10TH BOARD: 2014]**

15. Prove that $\frac{\cot\theta - 1 + \operatorname{cosec}\theta}{\cot\theta + 1 - \operatorname{cosec}\theta} = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$ **[3 MARKS/CBSE 10TH BOARD: 2014]**

OR

If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2)^2 = 16\,mn$

16. If $x = \cot A + \cos A$ and $y = \cot A - \cos A$. Show that $x^2 - y^2 = 4\sqrt{xy}$.

[3 MARKS/CBSE 10TH BOARD: 2014]

17. Two poles of height 10 m and 15 m stand vertically on a plane ground. If the distance between their feet is $5\sqrt{3}$, find the distance between their tops. **[3 MARKS/CBSE 10TH BOARD: 2014]**

18. Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$. **[3 MARKS/CBSE 10TH BOARD: 2014]**

19. If $\sin\alpha = \frac{1}{2}$, then the value of $4\cos^3\alpha - 3\cos\alpha$ is **[2 MARKS/CBSE 10TH BOARD: 2015]**

(A) 0 (B) 1 (C) – 1 (D) 2

20. If $\cos 2\theta = \sin(\theta - 12°)$, where 2θ and $(\theta - 12°)$ are both acute angles, then the value of θ is

[2 MARKS/CBSE 10TH BOARD: 2015]

(A) 24° (B) 28° (C) 32° (D) 34°

21. If $\tan 2A = \cot(A - 18°)$,Where 2A is an acute angle, then the value of A is

[2 MARKS/CBSE 10TH BOARD: 2015, 2017]

(A) 24° (B) 12° (C) 36° (D) 63°

22. If $\sin 5\theta = \cos 4\theta$, where 5θ and 4θ are acute angles, then the value of θ is

[2 MARKS/CBSE 10TH BOARD: 2015]

(A)10° (B)100° (C)12° (D) 15°

23. If $\tan\theta = \frac{12}{13}$, then the value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ is **[2 MARKS/CBSE 10TH BOARD: 2015]**

(A) $\frac{307}{25}$ (B) $\frac{312}{25}$ (C) $\frac{309}{25}$ (D) $\frac{316}{25}$

24. Prove that : $\frac{\sqrt{\operatorname{cosec}A-1}}{\sqrt{\operatorname{cosec}A+1}} + \frac{\sqrt{\operatorname{cosec}A+1}}{\sqrt{\operatorname{cosec}A-1}} = 2\sec A$ **[1 MARK /CBSE 10TH BOARD: 2015, 2016]**

25. Prove that : $\frac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec}A)^2$ **[1 MARK /CBSE 10TH BOARD: 2015]**

26. If $\theta = 45°$, the value of $\operatorname{cosec}^2\theta$ is **[2 MARKS/CBSE 10TH BOARD: 2016]**

(A) $\frac{1}{2}$ (B) 1 (C) $-\frac{1}{2}$ (D) 2

27. If $\sin(60° + \theta) - \cos(30° - \theta)$ is equal to **[2 MARKS/CBSE 10TH BOARD: 2016]**

(A) $2\cos\theta$ (B) $2\sin\theta$ (C) 1 (D) 0

28. The value of $[(\sec\theta + \tan\theta)(1 - \sin\theta)]$ is equal to **[2 MARKS/CBSE 10TH BOARD: 2016]**

(A) $\tan^2\theta$ (B) $\sin^2\theta$ (C) $\cos\theta$ (D) $\sin\theta$

29. If A = 45° and B = 30°, then the value of sin A cosB + cos A sin B is
[2 MARKS/CBSE 10TH BOARD: 2016]

(A) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (B) $\frac{\sqrt{3}+1}{2\sqrt{3}}$ (C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (D) $\frac{\sqrt{3}-1}{2\sqrt{3}}$

30. If $\cot\theta = \frac{7}{8}$, Find the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ **[3 MARKS/CBSE 10TH BOARD: 2016]**

31. $\frac{\sqrt{\operatorname{cosec}A-1}}{\sqrt{\operatorname{cosec}A+1}} + \frac{\sqrt{\operatorname{cosec}A+1}}{\sqrt{\operatorname{cosec}A-1}} = 2\sec A$ **[3 MARKS/CBSE 10TH BOARD: 2016]**

OR

If ABC is a right angle triangle, right-angles at C. If $\angle A = 30°$ and AB = 50 units, find the remaining two sides and $\angle B$ of ABC.

32. Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$ **[1 MARK /CBSE 10TH BOARD: 2016, 2107]**

33. Prove that : $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$ **[2 MARKS/CBSE 10TH BOARD: 2016, 2017]**

34. Prove that :

$\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B\cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$ **[2 MARKS/CBSE 10TH BOARD: 2016]**

35. If A = B = 60°. Verify **[3 MARKS/CBSE 10TH BOARD: 2016]**

(i) cos(A – B) = cos A cos B + sin A sin B

(ii) sin (A – B) = sin A cosB – cos A sin B

(iii) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$.

36. Prove that: (cosec A – sin A)(sec A – cos A) = $\frac{1}{\tan A + \cot A}$

[2 MARKS/CBSE 10TH BOARD: 2016]

37. If $\cos\theta + \cos^2\theta = 1$, the value of $\sin^2\theta + \sin^4\theta$ is **[1 MARKS/CBSE 10TH BOARD: 2017]**

(A) 0 (B) 1 (C) 2 (D) – 1

38. If sec x + tan x = p , then sec x is equal to **[2 MARKS/CBSE 10TH BOARD: 2017]**

(A) $\frac{P^2-1}{p}$ (B) $\frac{P^2+1}{p}$ (C) $\frac{P^2-1}{2p}$ (D) $\frac{P^2+1}{2p}$

39. If cosx = cos60°cos30° + sin60°sin30° , then the value of x is

[1 MARKS/CBSE 10TH BOARD: 2017]

(A) 90° (B) 45° (C) 30° (D) 60°

40. Prove that : $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$ **[3 MARKS/CBSE 10TH BOARD: 2017]**

OR

Prove that :

(i) tan20° tan35° tan45° tan55° tan70° = 1

(ii) sin48° sec42° + cosec 42° cos 48° = 2

(iii) $\frac{\sin 70°}{\cos 20°} + \frac{\operatorname{cosec} 20°}{\sec 70°}$ – 2 cos70° cosec 20° = 0 **[3 MARKS/CBSE 10TH BOARD: 2017]**

41. Prove that : $\frac{\tan 45°}{\operatorname{cosec} 30°} + \frac{\sec 60°}{\cot 45°} = \frac{5\sin 90°}{2\cos 0°}$ **[2 MARKS/CBSE 10TH BOARD: 2017]**

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction to Trigonometry and trigonometric ratios

A-1. In ΔABC, if AB + BC = 7 cm, AC = 5 cm and ∠B = 90°, then find cos A.

A-2. If sin A = $\frac{m^2}{n^2}$, find sec A.

A-3. If tan θ = $\frac{1}{\sqrt{2}}$, find sin θ.

A-4. If cot θ = $\frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \frac{1}{\sqrt{7}}$.

A-5. If cot θ = $\frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.

Section (B) : Trigonometric angles

B-1. Evaluate : cos1ºcos2ºcos3º---------cos180º

B-2. Find the value of tan60º, geometrically.

B-3. In a ΔABC, right-angled at C, AC = 6 cm and AB = 12 cm. Find $\angle A$.

B-4 Evaluate : $\frac{2}{3}(\cos^4 30^\circ - \sin^4 45^\circ) - 3(\sin^2 60^\circ - \sec^2 45^\circ) + \frac{1}{4}\cot^2 30^\circ$.

B-5 If cos x = cos 60º cos 30° + sin 60° sin 30°, then find the value of x.

Section (C) : Complementary angles

C-1. Evaluate :

$\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ . \tan 10^\circ . \tan 20^\circ . \tan 70^\circ . \tan 80^\circ . \tan 89^\circ$

C-2. If $\sin 3\theta = \cos(\theta - 6^\circ)$ and 3θ and $(\theta - 6^\circ)$ are acute angles, find the value of θ.

C-3. If A, B, C are the interior angles of a Δ ABC, show that :

(i) $\sin\frac{B+C}{2} = \cos\frac{A}{2}$ **(ii)** $\cos\frac{B+C}{2} = \sin\frac{A}{2}$

C-4. Find the value of $\sin(60^\circ + \theta) - \cos(30^\circ - \theta)$

Section (D) : Trigonometric identities

Prove the following (Q. D-1 to D-5)

D-1. $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

D-2. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \sec\theta\,\mathrm{cosec}\,\theta + 1$.

D-3. $(\sin\theta + \sec\theta)^2 + (\cos\theta + \mathrm{cosec}\,\theta)^2 = (1 + \sec\theta\,\mathrm{cosec}\,\theta)^2$

D-4. $(\sin^8\theta - \cos^8\theta) = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

D-5. $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$.

D-6. If $7\sin^2\theta + 3\cos^2\theta = 4$, show that $\tan\theta = \frac{1}{\sqrt{3}}$.

D-7. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, find the value of $25\left(x^2 - \frac{1}{x^2}\right)$.

D-8. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then Prove that : $x^2 + y^2 + z^2 = r^2$.

D-9. If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$, then prove that $(x^2y)^{2/3} - (xy^2)^{2/3} = 1$.

D-10. If sec A + tan A = p, then show that $\frac{p^2-1}{p^2+1} = \sin A$.

D-11. Evaluate :

(i) $\frac{\sin\theta\cos\theta\sin(90^\circ-\theta)}{\cos(90^\circ-\theta)} + \frac{\cos\theta\sin\theta\cos(90^\circ-\theta)}{\sin(90^\circ-\theta)} + \frac{\sin^2 27^\circ+\sin^2 63^\circ}{\cos^2 40^\circ+\cos^2 50^\circ}$

(ii) $\frac{\cos^2 20^\circ+\cos^2 70^\circ}{\sec^2 50^\circ-\cot^2 40^\circ}$ +2 cosec^2 58° – 2cot 58° tan 32° – 4 tan 13° tan 37° tan 45° tan 53° tan 77°

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Introduction to Trigonometry and trigonometric ratios

A-1. If 5 tan θ = 4, then value of $\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}$ is :

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{4}{5}$ (D) $\frac{2}{3}$

A-2. If 7 sin α = 24 cos α where 0° < α < 90°, then value of 14 tan α – 75 cos α – 7 sec α is equal to :

(A) 1 (B) 2 (C) 3 (D) 4

A-3. If tan θ = 4, then $\left(\frac{\tan\theta}{\frac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta}\right)$ is equal to :

(A) 0 (B) 2 $\sqrt{2}$ (C) $\sqrt{2}$ (D) 1

A-4. If A is an acute angle in a right ΔABC, right angled at B, then the value of sinA + cosA is :

(A) equal to one (B) greater than one (C) less than one (D) equal to two

Section (B) : Trigonometric angles

B-1. As x increases from 0° to 90°, the value of cos x is :

(A) increases (B) decreases
(C) remains constant (D) increases, then decreases

B-2. Value of x from the equation $x\sin\frac{\pi}{6}\cos^2\frac{\pi}{4} = \frac{\cot^2\frac{\pi}{6}\sec\frac{\pi}{3}\tan\frac{\pi}{4}}{\operatorname{cosec}^2\frac{\pi}{4}\operatorname{cosec}\frac{\pi}{6}}$ is :

(A) 4 (B) 6 (C) – 2 (D) 0

B-3. The area of a triangle is 12 sq. cm. Two sides are 6 cm and 12 cm. The included angle is θ, then sin θ =

(A) $\left(\frac{1}{2}\right)$ (B) $\left(\frac{1}{4}\right)$ (C) $\left(\frac{1}{6}\right)$ (D) $\left(\frac{1}{3}\right)$

B-4. If α + β = 90° and α = 2β, then cos² α + sin² β equals to :

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2

B-5. Given that sin A = $\frac{1}{2}$ and cos B = $\frac{1}{\sqrt{2}}$, where A and B are acute angles, then the value of A + B is :

(A) 30° (B) 45° (C) 75° (D) 15°

Section (C) : Complementary angles

C-1. If $\alpha + \beta = 90°$ and $\sin \alpha = \frac{1}{3}$, then $\sin \beta$ is :

(A) $\frac{\sqrt{2}}{3}$ (B) $\frac{2\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

C-2. The value of tan 5º. tan 10º. tan 15º. tan 20º ... tan 85º, is :

(A) 1 (B) 2 (C) 3 (D) None of these

C-3. $\sin(60° + \theta) - \cos(30° - \theta)$ is equal to :

(A) $2\cos\theta$ (B) $2\sin\theta$ (C) 0 (D) 1

C-4. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to :

(A) $\cos\beta$ (B) $\cos 2\beta$ (C) $\sin\alpha$ (D) $\sin 2\alpha$

C-5 If sec A = cosec B = $\frac{12}{7}$, then A + B is equal to :

(A) zero (B) 90º (C) < 90º (D) > 90º

Section (D) : Trigonometric identities

D-1. Given $3\sin\beta + 5\cos\beta = 5$, then the value of $(3\cos\beta - 5\sin\beta)^2$ is equal to :

(A) 9 (B) $\frac{9}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

D-2. The value of [(secA + tanA) (1 – sinA)] is equal to :

(A) $\tan^2 A$ (B) $\sin^2 A$ (C) cosA (D) sinA

D-3. If $x = 2\sin^2\theta$, $y = 2\cos^2\theta + 1$, then the value of x + y is :

(A) 2 (B) 3 (C) $\frac{1}{2}$ (D) 1

D-4. Value of $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$ is :

(A) 1 (B) – 1 (C) 2 (D) – 4

D-5. The value of $5\tan^2\theta - 5\sec^2\theta$ is :

(A) 1 (B) – 5 (C) 0 (D) 5

Exercise-2

OBJECTIVE QUESTIONS

1. If $\cot\theta = \frac{1}{\sqrt{3}}$, then the value of $\frac{1-\cos^2\theta}{2-\sin^2\theta}$ is :

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) None

2. Let $\tan A = \frac{\sqrt{3}}{4-\sqrt{3}}$ and $\tan B = \frac{\sqrt{3}}{4+\sqrt{3}}$ then $\frac{\tan A - \tan B}{1 + \tan A \tan B}$ is equal to :

(A) 0.385 (B) 0.375 (C) 0.575 (D) 0.465

3. Triangle PQR is right angled at Q and has side lengths PQ = 14 and QR = 48. If M is the mid-point of PR. If $\angle MQP = \theta$ then $\cos\theta$ is equal to :

(A) $\frac{7}{25}$ (B) $\frac{7}{50}$ (C) $\frac{7}{24}$ (D) $\frac{24}{25}$

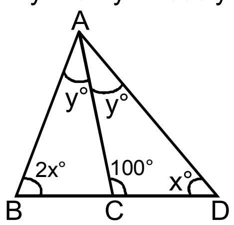
4. Given $8\tan\theta = 3\cos\theta$, then the value of the expression $E = \sin\theta + \cos\theta + \tan\theta + \sec\theta + \text{cosec}\theta + \cot\theta$ can be expressed as $\frac{a+b\sqrt{c}}{d}$ where b, c and d are primes then the value of (a + b + c + d) equal :

(A) 26 (B) 25 (C) 24 (D) 23

5. The value of expression $\frac{\sin 30^\circ + \tan 45^\circ - \sec 60^\circ}{\text{cosec } 30^\circ - \cot 45^\circ - \cos 60^\circ}$ =

(A) 0 (B) 1 (C) – 1 (D) $2 + \sqrt{3}$

6. In the diagram B, C and D lie on a straight line, with $\angle ACD = 100°$, $\angle ADB = x°$, $\angle ABD = 2x°$ and $\angle DAC = \angle BAC = y°$. The value of ($\sin y°. \tan y° + \sec y°$) equals :



(A) 7/2 (B) 3 (C) 5/2 (D) 5

7. Which one of the following quantities is not rational ?

(A) $\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}$ (B) $4\cos^3 30^\circ - 3\cos 30^\circ$

(C) $3\sin 30^\circ - 4\sin^3 30^\circ$ (D) $\frac{2\cot 30^\circ}{\cot^2 30^\circ - 1}$

8. A square DEAF is constructed inside a 30° – 60° – 90° triangle ABC with the hypotenuse BC = 4, D on side BC, E on side AC and F on side AB. The length of the side of the square is :

(A) $3 - \sqrt{3}$ (B) $3 - \sqrt{2}$ (C) 2 (D) 1.5

9. Let $S = \sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ$ and $P = \text{cosec}^2 45^\circ . \sec^2 30^\circ . \sin^3 90^\circ . \cos 60^\circ$, then the correct statement is :

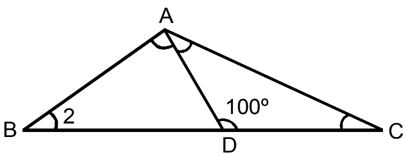
(A) S < P (B) S = P (C) S×P = 2 (D) S + P > 3

10. Suppose that θ is a number and $0 < \theta < 90^\circ$. Consider the following statements and select the correct choice,

I. $\sec(\theta^\circ) < 1$ **II.** $\sec(\theta^\circ) = 1$ **III.** $\sec(\theta^\circ) > 1$

(A) Only I is possible (B) All are possible (C) Only III is possible (D) Only I is impossible

11. In the given figure α and β are measured in degrees. Which one of the following statement is not correct ?



(A) $\beta > \alpha$ (B) $\sec\beta = 2$ (C) $\tan 3\alpha = \sqrt{3}$ (D) $\sin(\beta - \alpha) = \frac{1}{\sqrt{2}}$

12. A right triangle has angles which measure 30, 60 and 90 degrees. If the perimeter of this triangle is $15 + 5\sqrt{3}$ then the length of the hypotenuse of this triangle, is :

(A) 5 (B) 7.5 (C) 10 (D) 12.5

13. Let $T_1 = \dfrac{\sin 45^\circ - \sin 30^\circ + \cot 90^\circ}{\cos 45^\circ + \cos 60^\circ}$ and $T_2 = \dfrac{\sec 45^\circ - \tan 45^\circ}{\operatorname{cosec} 45^\circ + \cos 0^\circ + \cot 90^\circ}$, then :

(A) $T_1 + T_2 = 0$ (B) $T_1 - T_2 = 0$ (C) $T_1 = T_2$ (D) $T_2 = T_1$

14. Which one of the following when simplified is not equal to one ?

(A) $\tan 18^\circ \times \tan 36^\circ \times \tan 54^\circ \times \tan 72^\circ$ (B) $\sin^2 19^\circ + \sin^2 71^\circ$

(C) $\dfrac{2\sin 62^\circ}{\cos 28^\circ} - \dfrac{\sec 42^\circ}{\operatorname{cosec} 48^\circ}$ (D) None of these

15. Evaluate :

$\sec^2 10^\circ - \cot^2 80^\circ + \dfrac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\cos\theta \sin(90^\circ - \theta) + \sin\theta \cos(90^\circ - \theta)}$

(A) 0 (B)1 (C)2 (D) 3

16. $\dfrac{1+\sec\theta}{\sec\theta} =$

(A) $\dfrac{\sin^2\theta}{1-\cos\theta}$ (B) $\dfrac{\sin^2\theta}{1-\sin\theta}$ (C) $\dfrac{\cos^2\theta}{1-\sin\theta}$ (D) $1+\sin\theta$

17. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then :

(A) $\tan\theta = \dfrac{1}{2}$ (B) $\tan\theta = 2$ (C) $\tan^5\theta + \cot^5\theta = 32$ (D) $\tan^7\theta + \cot^7\theta = 2$

18. The value of $\cos^4\theta + \sin^4\theta + 2\cos^2\theta \sin^2\theta$ when $\theta = 60^\circ$ is :

(A) 1 (B) 2 (C) $\dfrac{1}{\sqrt{2}}$ (D) $2\sqrt{2}$

19. The value of the expression $(\cos\theta - 1)(1 + \cos\theta)\ (1 + \cot^2\theta)$ is :

(A) 0 (B) 1 (C) $\sin^2\theta$ (D) – 1

20. If $\dfrac{\sin x}{1+\sec x} + \dfrac{\sin x}{\sec x - 1} = 2$, where $0^\circ < x < 90^\circ$ then cosec x has the value equal to :

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) $\sqrt{3}$

21. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is equal to :

(A) $\sqrt{2} - 1$ (B) $\sqrt{2}$ (C) $\sqrt{2}\sin\theta$ (D) $\sqrt{2} + \sin\theta$

22. If $x \sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$ then :

(A) $x^3 + y^3 = 1$ (B) $x^2 - y^2 = 1$ (C) $x^2 + y^2 = 1$ (D) $x^3 - y^3 = 1$

23. Let $x = r\cos\alpha \cdot \cos\beta$; $y = r\cos\alpha \cdot \sin\beta$ and $z = r\sin\alpha$ then $(x^2 + y^2 + z^2)$ is :

(A) independent of both α and β. (B) independent of α but dependent on β.

(C) independent of β but dependent on α. (D) dependent on both α and β.

24. Given $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then the value of $x^2 + 4y^2$ is equal to :

(A) 1 (B) 2 (C) 3 (D) 4

25. Which one of the following identities (wherever defined) is not correct ?

(A) $\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$ (B) $\frac{\cot x}{1+\tan x} = \frac{\cot x - 1}{2 - \sec^2 x}$

(C) $\operatorname{cosec}^2 x + \sec^2 x = \operatorname{cosec}^2 x \cdot \sec^2 x$ (D) $[(1 + \cot x - \operatorname{cosec} x)(1 + \tan x + \sec x)] = 1$

26. The expression, $2(1 + \cos x) - \sin^2 x$ is the same as :

(A) $(1 - \cos x)^2$ (B) $1 - \cos 2x$ (C) $(1 + \cos x)^2$ (D) $1 + \cos^2 x$

27. If $\sin x + \cos x = \frac{1}{2}$ then $\sin^4 x + \cos^4 x$ as a rational number equals :

(A) $\frac{3}{4}$ (B) $\frac{15}{32}$ (C) $\frac{19}{32}$ (D) $\frac{23}{32}$

28. The expression $1 + \frac{\tan^2\theta}{1+\sec\theta}$ when simplified, reduces to :

(A) $\sin\theta$ (B) $\sec\theta$ (C) $\operatorname{cosec}\theta$ (D) $\cot\theta$

29. If $\sin x + \sin^2 x = 1$, then the value of $\cos^8 x + 2\cos^6 x + \cos^4 x$ is :

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{3}{4}$ (D) 2

30. If $2\cos^2 x + 5\sin x = 4$ then which one of the following is correct ?

(A) $\operatorname{cosec}^2 x - \cot^2 x = 7$ (B) $\cot^2 x + \sec^2 x = 3$

(C) $\tan^2 x + \cot^2 x = 2$ (D) $\sec^2 x + \operatorname{cosec}^2 x = 16/3$

31. Let θ be an acute angle such that $\sec^2\theta + \tan^2\theta = 2$. The value of $(\operatorname{cosec}^2\theta + \cot^2\theta)$, is :

(A) 9 (B) 5 (C) 4 (D) 2

32. If $\operatorname{cosec}\theta - \sin\theta = \sqrt{5}$, where θ is an acute angle, then the value of $\sin\theta + \operatorname{cosec}\theta$ is :

(A) $\sqrt{3}$ (B) 1 (C) 3 (D) 9

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. If A + B = 90° and A = 2B then the value of cos 2B is - **[Raj. NTSE Stage-1 2005]**

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1

2. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, then the value of $x^2 - y^2$ is : **[Raj. NTSE Stage-1 2005]**

(A) a^2b^2 (B) $a^2 + b^2$ (C) $a^2 - b^2$ (D) 1

3. If $\frac{\cos\theta}{1+\sin\theta} - \frac{1-\sin\theta}{\cos\theta} = 2k$ then the value of k is : **[Raj. NTSE Stage-1 2005]**

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2

4. If $a\cos\theta + b\sin\theta = 3$ and $a\sin\theta - b\cos\theta = 4$, then the value of $a^2 + b^2$ is : **[Raj. NTSE Stage-1 2006]**

(A) 9 (B) 16 (C) 25 (D) None of these

5. The value of tan 15º. tan 25º. tan 45º. tan 65º. tan 75º is : **[Raj. NTSE Stage-1 2006]**
(A) 0 (B) 1 (C) – 1 (D) None of these

6. If tan θ + sec θ = 4, then the value of sin θ is : **[Raj. NTSE Stage-1 2007]**
(A) $\frac{15}{28}$ (B) $\frac{8}{15}$ (C) $\frac{15}{17}$ (D) $\frac{3}{5}$

7. The value of $\sin^2 1° + \sin^2 2° + \ldots\ldots\ldots\ldots + \sin^2 89°$ is ___________. **[Orissa NTSE stage-1 2012]**
(A) 1 (B) 44 (C) $44\frac{1}{2}$ (D) 45

8. In the right-angled ΔABC, cot A.cot B.cotC = **[Maharashtra NTSE stage 1 2013]**
(A) 1 (B) 0 (C) 2 (D) None of these

9. The valueof : tan 15° .tan 25° .tan 60°. tan 65° .tan 75° is **[UP NTSE_Stage-1 2013]**
(A) $\sqrt{3}$ (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

10. If secθ + tanθ = x, then the value of tanθ is : **[UP NTSE_Stage-1 2013]**
(A) $\frac{2x}{x^2+1}$ (B) $\frac{x^2+1}{2x}$ (C) $\frac{2x}{x^2-1}$ (D) $\frac{x^2-1}{2x}$

11. If $\sin^2\theta + \text{cosec}^2\theta = 6$, then sin θ + cosec θ =...........................**[Maharashtra NTSE stage 1 2013]**
(A) $3\sqrt{2}$ (B) $2\sqrt{2}$ (C) $4\sqrt{2}$ (D) $\sqrt{2}$

12. If sin(A + B) = $\frac{\sqrt{3}}{2}$, cos(A – B) = $\frac{\sqrt{3}}{2}$ and 0 < A + B $\leq$ 90°, if A > B then the value of A and B are : **[Raj. NTSE Stage-1 2013]**
(A) A = 45°, B = 15° (B) A = 60°, B = 30° (C) A = 0°, B = 30° (D) A = 30°, B = 0°

13. If $\cos A + \cos^2 A = 1$. then the value of $\sin^2 A + \sin^4 A$ is : **[Delhi NTSE Stage-1 2013]**
(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) 3

14. In right triangle ABC. BC = 7 cm, AC – AB = 1 cm and $\angle$B = 90º. The value of cos A + cos B + cos C is : **[Delhi NTSE Stage-1 2013]**
(A) $\frac{1}{7}$ (B) $\frac{32}{24}$ (C) $\frac{31}{25}$ (D) $\frac{25}{31}$

15. $\frac{1}{\sin^2\theta} - \cot^2\theta$ is equal to : **[MP NTSE_Stage-1 2013]**
(A) 1 (B) – 1 (C) 2 (D) – 2

16. If sinθ + cosθ = 1, then sinθcosθ is equal to : **[MP NTSE_Stage-1 2013]**
(A) 0 (B) $\frac{1}{\sqrt{3}-1}$ (C) 1 (D) $\frac{1+\sqrt{2}}{1+\sqrt{3}}$

17. If sin θ – cos θ = $\sqrt{2}$ sin (90° – θ), then tan θ = **[Raj. NTSE Stage-1 2014]**
(A) $\sqrt{2} - 1$ (B) $\sqrt{2}$ (C) $1 - \sqrt{2}$ (D) $\sqrt{2} + 1$

18. What is the value of $\sin^2 0° + \sin^2 1° + \sin^2 2° + ... + \sin^2 90°$? **[Maharashtra NTSE stage 1 2014]**

(A) 0 (B) 45 (C) $\frac{89}{2}$ (D) $\frac{91}{2}$

19. If $a \cos \theta - b \sin \theta = c$, then $a \sin \theta + b \cos \theta =$ **[Raj. NTSE Stage-1 2014]**

(A) $\pm\sqrt{a^2+b^2+c^2}$ (B) $\pm\sqrt{a^2+b^2-c^2}$ (C) $\pm\sqrt{c^2-a^2-b^2}$ (D) None of these

20. $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \text{cosec } \theta)$ is equal to **[Raj. NTSE Stage-1 2014]**

(A) 0 (B) 2 (C) 1 (D) – 1

21. In sec 2A = Cosec (A – 42°) where 2A is acute Angle then value of A is **[UP NTSE_Stage-1 2014]**

(A) 44° (B) 22° (C) 21° (D) 66°

22. If $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$ then θ is **[UP NTSE_Stage-1 2014]**

(A) 30° (B) 45° (C) 60° (D) 90°

23. If $\cot\theta' + \text{cosec}\theta = 2$, then the value of $\frac{1+\cos\theta}{1-\cos\theta}$ is **[UP NTSE_Stage-1 2014]**

(A) 2 (B) 4 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

24. The value of $\sin^2\theta + \frac{1}{(1+\tan^2\theta)}$ is - **[UP NTSE_Stage-1 2014]**

(A) $\sin^2\theta$ (B) $\cos^2\theta$ (C) $\sec^2\theta$ (D) 1

25. If $\sec \theta + \tan \theta = P$ then the value of $\frac{P^2-1}{P^2+1}$ is **[UP NTSE_Stage-1 2014]**

(A) Cosec θ (B) Sin θ (C) $\frac{\tan\theta}{\sec\theta}$ (D) 1

26. If $\tan\theta = \frac{a}{b}$ then the value of $\frac{b\sin\theta - a\cos\theta}{b\sin\theta + a\cos\theta}$ **[UP NTSE_Stage-1 2014]**

(A) 1 (B) $\frac{a^2-b^2}{a^2+b^2}$ (C) $\frac{b^2-a^2}{b^2+a^2}$ (D) 0

27. Which of the following is not true **[MP NTSE_Stage-1 2014]**

(A) $\sin^2 25° + \sin^2 65° = 1$ (B) $\sin(90 - \theta)\cos(90 - \theta) = \frac{\tan\theta}{1+\tan^2\theta}$

(C) $\cos 60° = 1 - 2\cos^2(90° - 30°)$ (D) $\cos^2\theta - \sin^2\theta = \frac{\tan\theta}{1-\tan^2\theta}$

28. If sin x + cosec x = 2, then $(\sin^{10}x + \text{cosec}^{10}x)$ is equal to : **[Delhi NTSE Stage-1 2014]**

(A) 3 (B) 0 (C) 1 (D) 2

29. If Cosec 39° = x, the value of $\frac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$, is : **[Delhi NTSE Stage-1 2014]**

(A) $\sqrt{x^2-1}$ (B) $\sqrt{1-x^2}$ (C) $x^2 - 1$ (D) $1 - x^2$

30. If $\frac{\tan\theta+\cot\theta}{\tan\theta-\cot\theta}$ = 2, $(0 \le \theta \le 90°)$, then the value of θ is : **[Delhi NTSE Stage-1 2014]**
(A) 60° (B) 30° (C) 90° (D) 45°

31. If $\tan\theta + 4\cot\theta = 4$, the value of $\tan^3\theta + \cot^3\theta$ is : **[West Bengal NTSE Stage-1 2014]**
(A) $8\frac{1}{8}$ (B) 16 (C) $7\frac{9}{8}$ (D) $27\frac{1}{27}$

32. ϕ is an acute angle such that $\tan\phi = 2/3$ then evaluate $\left(\frac{1+\tan\phi}{\sin\phi+\cos\phi}\right).\left(\frac{1-\cot\phi}{\sec\phi+\operatorname{cosec}\phi}\right)$ **[NTSE Stage-2 2014]**
(A) $-\frac{1}{5}$ (B) $-\frac{4}{\sqrt{13}}$ (C) $\frac{1}{5}$ (D) $\frac{4}{\sqrt{13}}$

33. If $\sin x + \sin^2 x = 1$ the $\cos^8 x + 2\cos^6 x + \cos^4 x$ = ... **[Bihar NTSE Stage-1 2014]**
(A) 0 (B) –1 (C) 2 (D) 1

34. $1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$ = ... **[Bihar NTSE Stage-1 2014]**
(A) 0 (B) 1 (C) siny (D) cosy

35. If $\sin A = \frac{1}{2}$ $(0° < A < 90°)$ then $4\cos^3 A - 3\cos A$ = **[Jharkhand NTSE stage 1 2014]**
(A) 0 (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

36. If $\sin\theta + \operatorname{cosec}\theta = 2$ then $\sin^{100}\theta + \operatorname{cosec}^{100}\theta$ = ... **[Jharkhand NTSE stage 1 2014]**
(A) 1 (B) 2 (C) 4 (D) none of these

37. $\frac{\tan x}{\sec x - 1} - \frac{\sin x}{1+\cos x}$ = ... **[Bihar NTSE Stage-1 2014]**
(A) 2 tanx (B) 2 sinx (C) 6 cosx (D) 2 cotx

38. The value of $\cos^2 26° + \cos 64° \sin 26° + \frac{\tan 36°}{\cot 54°}$ **[Chattisgarh NTSE Stage-1 2014]**
(A) 2 (B) 1 (C) 0 (D) none of these

39. If $\operatorname{cosec} x - \cot x = \frac{1}{3}$, where $x \ne 0$, then the value of $\cos^2 x - \sin^2 x$ is **[NTSE Stage-2 /2015]**
(A) $\frac{16}{25}$ (B) $\frac{9}{25}$ (C) $\frac{8}{25}$ (D) $\frac{7}{25}$

40. If $\sin\theta + \sin^2\theta = 1$, then the value of $\cos^2\theta + \cos^4\theta$ is **[Raj. NTSE Stage-1 2015]**
(A) 3 (B) 2 (C) 1 (D) 0

41. tan 43° tan 45° tan 47° is equal to **[Raj. NTSE Stage-1 2015]**
(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) 2.

42. If $2\tan x = 1$, then value of $\frac{\cos x + 2\sin x}{\cos x - \sin x}$ is **[Delhi NTSE stage 1 2015]**
(A) 1 (B) 0 (C) 4 (D) 2

43. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^{23}\theta + \cot^{23}\theta$ = **[Jharkhand NTSE stage 1 2015]**
(A) 23 (B) 4 (C) 1 (D) 2

44. The value of $\frac{1}{1+\cot^2\alpha} + \frac{1}{1+\tan^2\alpha}$
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 2

45. If $\cos 43^\circ = \frac{x}{\sqrt{x^2+y^2}}$, then the value of $\tan 47^\circ$ **[Jharkhand NTSE stage 1 2015]**
(A) x/y (B) y/x (C) $\frac{x}{\sqrt{x^2+y^2}}$ (D) $\frac{y}{\sqrt{x^2+y^2}}$

46. If sin7x = cos11x, then the value of tan9x + cot9x : **[Jharkhand NTSE stage 1 2015]**
(A) 4 (B) 2 (C) 1 (D) 3

47. If $\cos(\alpha+\beta) = 0$, then $\sin(\alpha-\beta)$ = ? **[Jharkhand NTSE stage 1 2015]**
(A) $\cos2\beta$ (B) $\cos\beta$ (C) $\sin\alpha$ (D) $\sin2\alpha$

48. If $\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$ and $0^\circ < \theta < 90^\circ$, write the value of θ. **[Orissa NTSE Stage-1 2015]**
(A) 30° (B) 60° (C) 75° (D) 88°

49. $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ is equal to.................... **[MP NTSE_Stage-1 2015]**
(A) $\text{cosec}\,\theta - \cot\theta$ (B) $\tan\theta - \sec\theta$ (C) $\sec\theta - \tan\theta$ (D) $\cot\theta - \text{cosec}\,\theta$

50. The value of sin 12° cos 78° + cos 12° sin 78° is **[MP NTSE_Stage-1 2015]**
(A) 0 (B) 1 (C) –1 (D) None of these

51. Value of $\tan20^\circ \tan40^\circ \tan50^\circ \tan70^\circ$ **[Raj. NTSE Stage-1 2016]**
(A) 0 (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) 1

52. $\frac{\cos\theta}{1-\tan\theta} - \frac{\sin\theta}{\cot\theta - 1}$ is equal to **[Rajasthan NTSE Stage-1 2016]**
(A) $\sin\theta + \cos\theta$ (B) $\text{Sin}\,\theta - \cos\theta$ (C) $2\sin\theta$ (D) $\frac{1}{\cos\theta - \sin\theta}$

53. If ABCD is a cyclic quadrilateral, the value of $\tan\frac{A}{2}\tan\frac{C}{2} + \tan\frac{B}{2}\tan\frac{D}{2}$ is
[West Bengal NTSE Stage-1 2016]
(A) 0 (B) 1 (C) –1 (D) 2

54. If A + B = 90° then $\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}$ is equal to **[Delhi NTSE stage 1 2016]**
(A) $\cot^2A$ (B) $\cot^2B$ (C) $-\tan^2A$ (D) $-\cot^2A$

55. If $2^{\sin x + \cos y} = 1$, and $16^{\sin^2 x + \cos^2 y} = 4$, then values of sin x and cos y respectively are
[Delhi NTSE stage 1 2016]
(A) $-\frac{1}{2}, \frac{1}{2}$ (B) $\frac{1}{2}, \frac{-1}{3}$ (C) 1 , – 1 (D) $\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

56. If in a right angled triangle ABC, tan B = $\sqrt{3}$, then the value of Sin B and Cos B is ? **[MP NTSE_Stage-1 2016]**

(A) 0 , 1 (B) $\frac{1}{2}, \frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

57. $\cos^4 x - \sin^4 x =$ **[MP NTSE_Stage-1 2016]**

(A) $2 \sin^2 x - 1$ (B) $1 - 2 \cos^2 x$ (C) $\sin^2 x - \cos^2 x$ (D) None of these

58. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} =$ **[MP NTSE_Stage-1 2016]**

(A) $\frac{2}{\sin\theta}$ (B) $\frac{2}{\cos\theta}$ (C) $\frac{2}{\tan\theta}$ (D) $\frac{2}{\cot\theta}$

59. If sin (A + B) = 1 and cos (A - B) = $\frac{\sqrt{3}}{2}$, then the values of A and B are : **[MP NTSE_Stage-1 2016]**

(A) 45° , 45° (B) 30° , 45° (C) 60° , 30° (D) 0° , 90°

60. $\frac{\sin^4\theta - \cos^4\theta}{1-\sin^2\theta}$ = how much? **[Maharashtra NTSE stage 1 2017]**

(A) $1-\cot^2\theta$ (B) $1-\tan^2\theta$ (C) $\tan^2\theta - 1$ (D) $\cot^2\theta - 1$

61. If in a right angles triangle ABC, $\cos A = \frac{9}{41}$, then the value of cot A and cosec A will be : **[MP NTSE_Stage-1 2017]**

(A) $\frac{40}{9}, \frac{40}{41}$ (B) $\frac{9}{40}, \frac{41}{40}$ (C) $\frac{9}{41}, \frac{41}{9}$ (D) $\frac{9}{40}, \frac{40}{41}$

62. In a Triangle ABC, if ∠B=90°, AB=5, BC=12, then sin C= ..... **[MP NTSE_Stage-1 2017]**

(A) $\frac{12}{13}$ (B) $\frac{5}{13}$ (C) $\frac{5}{12}$ (D) $\frac{13}{5}$

63. If $\tan\theta = \frac{1}{\sqrt{3}}$, then the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ is : **[MP NTSE_Stage-1 2017]**

(A) $\sqrt{3}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

64. $\cos\theta \sqrt{\sec^2\theta - 1}$ is equal to **[UP NTSE_Stage-1 2017]**

(A) Sinθ (B) Cotθ (C) Secθ (D) 1

Answer Key

Exercise Board Level

TYPE (I)

1.	1	**2.**	$\sqrt{3}$	**3.**	0	**4.**	$\frac{\sqrt{b^2-a^2}}{b}$	**5.**	0
6.	90°	**7.**	2	**8.**	$\frac{1}{2}$	**9.**	$\frac{-1}{2}$		

TYPE (II)

16. 90°

PREVIOUS YEAR PROBLEMS

1.	(B)	**2.**	(B)	**3.**	(C)	**4.**	– 2	**5.**	1 **OR** (B)
8.	0	**9.**	(D)	**10.**	(B)	**11.**	(B)		
12.	A = 30° , B = 15°			**17.**	10	**19.**	(A)	**20.**	(D)
21.	(C)	**22.**	(A)	**23.**	(B)	**26.**	(D)	**27.**	(D)
28.	(C)	**29.**	(A)	**30.**	49/64	**31.**	$\angle$ B = 60° , BC = 25 , AC = $\frac{50\sqrt{3}}{2}$		
37.	(B)	**38.**	(D)	**39.**	(C)				

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-5. $\frac{225}{64}$

Section (B)

B-1. 0 **B-2.** $\sqrt{3}$ **B-3.** 60° **B-4** $\frac{113}{24}$ **B-5** 30°

Section (C)

C-1. 1 **C-2.** 24° **C-4** 0

Section (D)

D-7. 1 **D-11.** **(i)** 2 **(ii)** – 1

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A)

A-1. (B) **A-2.** (B) **A-3.** (D) **A-4.** (B)

Section (B)

B-1. (B) **B-2.** (B) **B-3.** (D) **B-4.** (A) **B-5.** (C)

Section (C)

C-1. (B) **C-2.** (A) **C-3.** (C) **C-4.** (B) **C-5** (B)

Section (D)

D-1. (A) **D-2.** (C) **D-3.** (B) **D-4.** (C) **D-5.** (B)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	A	A	C	A	D	A	C	C	D	C	B	D	C	A	D	A	D	C
Ques.	21	22	23	24	25	26	27	28	29	30	31	32								
Ans.	C	C	A	D	D	C	D	B	A	D	B	C								

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	A	C	B	C	C	B	A	D	B	A	A	C	A	A	D	D	B	B
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	C	B	D	B	D	D	D	C	A	A	D	D	D	B	B	D	A	D	C
Ques.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	C	D	A	A	B	A	B	C	B	D	A	D	B	A	D	D	B	C	C
Ques.	61	62	63	64																
Ans.	B	B	D	A																