MATHEMATICS

Class-IX

Topic-8

QUADRILATERALS

INDEX

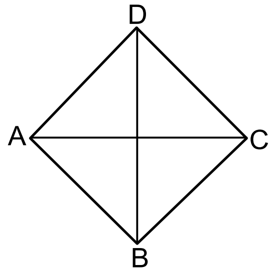
CH-08

QUADRILATERAL

(A) QUADRILATERALS

(a) Quadrilaterals

A **quadrilateral** is a four sided closed figure.

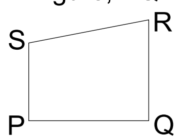


Let **A**, **B**, **C** and **D** be four points in a plane such that :

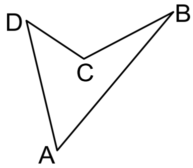
(i) No three of them are collinear.

(ii) The line segments AB, BC, CD and DA do not intersect except at their end points, then **figure** obtained by joining A, B, C & D is called a **quadrilateral**.

(i) Convex Quadrilaterals : A quadrilateral in which the measure of each interior angle is less than 180° is called a **convex quadrilateral**. In figure, PQRS is convex quadrilateral.

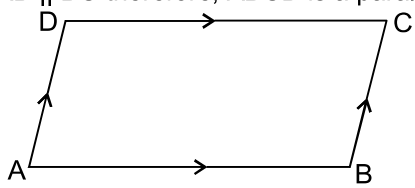


(ii) Concave Quadrilaterals : A quadrilateral in which the measure of one of the interior angles is more than 180° is called a **concave quadrilateral**. In figure, ABCD is concave quadrilateral**.**



(b) Special Quadrilaterals :

(i) Parallelogram : A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel. In figure, AB || DC, AD || BC therefore, ABCD is a parallelogram.



Properties :

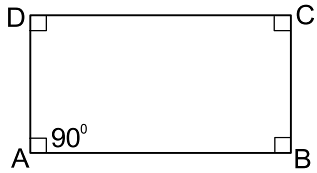
(a) A diagonal of a parallelogram divides it into two congruent triangles.

(b) In a parallelogram, opposite sides are equal.

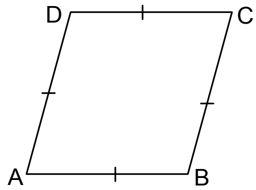
(c) The opposite angles of a parallelogram are equal.

(d) The diagonals of a parallelogram bisect each other.

(ii) Rectangle : A **rectangle** is a parallelogram, in which each of its angle is a right angle. If ABCD is a rectangle then ∠A = ∠B = ∠C = ∠D = 90°, AB = CD, BC = AD and diagonals AC = BD.

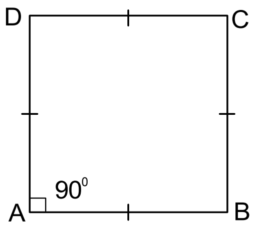


(iii) Rhombus : A **rhombus** is a parallelogram in which all its sides are equal in length. If ABCD is a rhombus then, AB = BC = CD = DA.



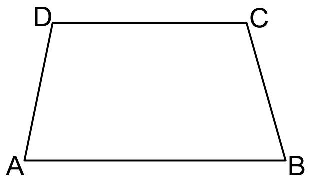
The diagonals of a rhombus are perpendicular to each other.

(iv) Square : A **square** is a parallelogram having all sides equal and each angle equal to right angle. If ABCD is a square then AB = BC = CD = DA, diagonal AC = BD and ∠A = ∠B = ∠C = ∠D = 90°.

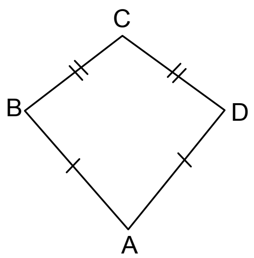


The diagonals of a square are perpendicular to each other.

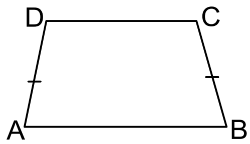
(v) Trapezium : A **trapezium** is a quadrilateral with one pair of opposite sides parallel. In figure, ABCD is a trapezium with AB || DC.



(vi) Kite : A **kite** is a quadrilateral in which two pairs of adjacent sides are equal. If ABCD is a kite then AB = AD and BC = CD.



(vii) Isosceles trapezium : A trapezium is said to be an **isosceles trapezium**, if its non-parallel sides are equal. Thus a quadrilateral ABCD is an isosceles trapezium, if AB || DC and AD = BC.

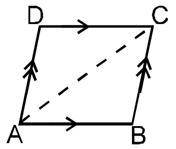


In isosceles trapezium ∠**A = **∠**B and **∠**C = **∠**D.**

NOTE : **(i)** Square, rectangle and rhombus are all parallelograms.
(ii) Kite and trapezium are not parallelograms.
(iii) A square is a rectangle.
(iv) A square is a rhombus.
(v) A parallelogram is a trapezium.

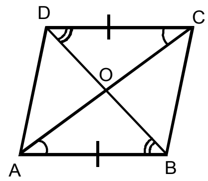
(c) Important theorems related to Quadrilaterals :

(i) Theorem : A diagonal of a parallelogram divides the parallelogram into two congruent triangles.
Given : A parallelogram ABCD.



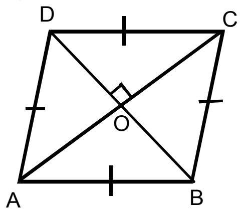
To Prove : A diagonal divides the parallelogram into two congruent triangles.
i.e., if diagonal AC is drawn then $\Delta ABC \cong \Delta CDA$.
and if diagonal BD is drawn then $\Delta ABD \cong \Delta CDB$
Construction : Join A and C.
Proof : Since, ABCD is a parallelogram.
$\therefore$ AB || DC and AD || BC.
In ΔABC and ΔCDA
$\angle BAC = \angle DCA$ [Alternate angles]
$\angle BCA = \angle DAC$ [Alternate angles]
And, AC = AC [Common side]
$\Delta ABC \cong \Delta CDA$ [By ASA congruency]
Similarly, we can prove that
$\Delta ABD \cong \Delta CDB$ **Hence Proved.**

(ii) Theorem : The diagonals of a parallelogram bisect each other.
Given : A parallelogram ABCD. Its diagonals AC and BD intersect each other at point O.
To Prove : Diagonals AC and BD bisect each other



i.e., OA = OC and OB = OD.
Proof : In ΔAOB and ΔCOD
$\because$ AB || DC and BD is a transversal line.
$\therefore$ $\angle ABO = \angle CDO$ [Alternate angles]
$\because$ AB || DC and AC is a transversal line.
$\therefore$ $\angle BAO = \angle DCO$ [Alternate angles]
And, AB = DC
$\Delta AOB \cong \Delta COD$ [By ASA congruency]
OA = OC and OB = OD [By CPCT] **Hence Proved.**

(iii) Theorem : The diagonals of a rhombus are perpendicular to each other.
Given : A rhombus ABCD whose diagonals AC and BD intersect at O.



To prove : $\angle BOC = \angle DOC = \angle AOD = \angle AOB = 90^\circ$.

Proof : Parallelogram is a rhombus, if all of its sides are equal.

$AB = BC = CD = DA$ . ..(i)

Since the diagonals of a parallelogram bisect each other.

$\therefore$ $OB = OD$ and $OA = OC$ ... (ii)

In ΔBOC and ΔDOC

$BO = OD$ [From (ii)]

$BC = DC$ [From (i)]

$OC = OC$

So, by SSS criterion of congruence

$\Delta BOC \cong \Delta DOC$

$\angle BOC = \angle DOC$ [By CPCT]

But, $\angle BOC + \angle DOC = 180^\circ$ [Linear pair]

$\angle BOC = \angle DOC = 90^\circ$

Similarly, $\angle AOD = \angle AOB = 90^\circ$

Hence, $\angle BOC = \angle DOC = \angle AOD = \angle AOB = 90^\circ$.

Solved Examples

Example. 1

The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Let the angles be 3x, 5x, 9x and 13x.

$\therefore$ $3x + 5x + 9x + 13x = 360^\circ$

$\Rightarrow$ $30x = 360^\circ$ and $x = \dfrac{360^\circ}{30} = 12^\circ$

$\therefore$ 1^{st} angle $= 3x = 3 \times 12^\circ = 36^\circ$,

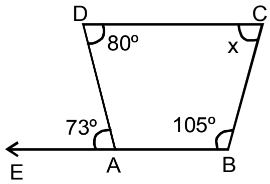
2^{nd} angle $= 5x = 5 \times 12^\circ = 60^\circ$,

3^{rd} angle $= 9x = 9 \times 12^\circ = 108^\circ$,

And, 4^{th} angle $= 13 \times 12^\circ = 156^\circ$.

Example. 2

Use the informations given in adjoining figure to calculate the value of x.



Sol. Since, EAB is a straight line.

$\therefore$ $\angle DAE + \angle DAB = 180^\circ$

$\Rightarrow$ $73^\circ + \angle DAB = 180^\circ$

$\Rightarrow$ $\angle DAB = 180^\circ - 73^\circ = 107^\circ$.

Since, the sum of the angles of quadrilateral ABCD is 360°.

$\therefore$ $107^\circ + 105^\circ + x + 80^\circ = 360^\circ$

$\Rightarrow$ $292^\circ + x = 360^\circ$

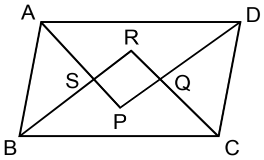
$\Rightarrow$ $x = 360^\circ - 292^\circ$

$\Rightarrow$ $x = 68^\circ$.

Example. 3

Prove that the angle bisectors of a parallelogram form a rectangle.

Sol. A parallelogram ABCD in which bisectors of angles A, B, C, D intersect at P, Q, R, S to form a quadrilateral PQRS.



Since, ABCD is a parallelogram, so AD || BC and transversal AB intersects them at A and B respectively.

$\angle A + \angle B = 180^o$

$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^o$

$\Rightarrow \quad \angle BAS + \angle ABS = 90^o$ ....(i) [AS and BS are bisectors of $\angle A$ and $\angle B$ respectively]

$\Rightarrow \quad \angle BAS + \angle ABS + \angle ASB = 180^o$

$\Rightarrow \quad 90^o + \angle ASB = 180^o$

$\Rightarrow \quad \angle ASB = 90^o$

$\Rightarrow \quad \angle RSP = 90^o$ [$\angle ASB$ and $\angle RSP$ are vertically opposite angles]

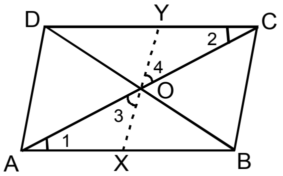
Similarly, $\angle SRQ = 90^o$, $\angle RQP = 90^o$ and $\angle SPQ = 90^o$.

Hence, PQRS is rectangle.

Example. 4

The diagonals of a parallelogram ABCD intersect at O. A line through O intersects AB at X and DC at Y. Prove that OX = OY.

Sol. Since AB || CD.



In ΔOAX and ΔOCY

$\angle 1 = \angle 2$ [Alternate angles]

OA = OC diagonals of parallelogram bisect each other

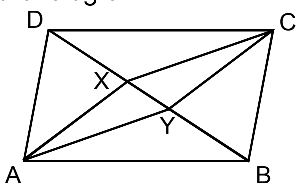
and $\angle 3 = \angle 4$ [Vertically opposite angles]

So, by ASA criterion of congruence,

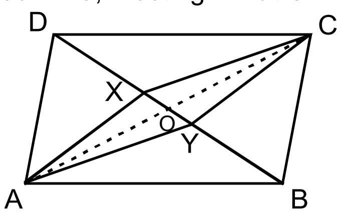
$\Delta OAX \cong \Delta OCY \Rightarrow$ OX = OY. [By CPCT]

Example. 5

In the adjoining figure, ABCD is a parallelogram and X, Y are the points on diagonal BD such that DX = BY. Prove that CXAY is a parallelogram.



Sol. Join AC, meeting BD at O.



Since the diagonals of a parallelogram bisect each other.

OA = OC and OD = OB

Now, OD = OB and DX = BY

OD – DX = OB – BY

OX = OY

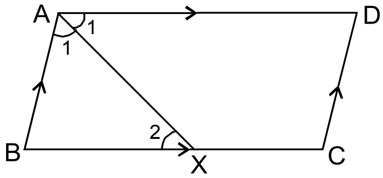
Now, OA = OC and OX = OY.

∴ CXAY is a quadrilateral whose diagonal bisect each other.

∴ CXAY is a parallelogram.

Example. 6

In the adjoining figure, ABCD is a parallelogram and the bisector of ∠A bisect BC at X. Prove that AD = 2AB.



Sol. ABCD is a parallelogram.

∴ AD || BC and AX cuts them.

∴ $\angle BXA = \angle DAX = \frac{1}{2}\angle A$ [Alternate interior angles]

∴ $\angle 2 = \frac{1}{2}\angle A$.

Also, $\angle 1 = \frac{1}{2}\angle A$

∴ $\angle 2 = \angle 1$ AB = BX

⇒ $AB = \frac{1}{2}BC$ $AB = \frac{1}{2}AD$

⇒ AD = 2 AB.

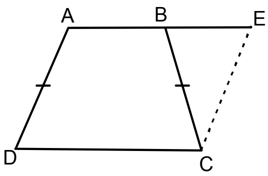
Example. 7

ABCD is a trapezium in which AB || CD and AD = BC. Show that :

(i) ∠A = ∠B **(ii)** ∠C = ∠D

(iii) ΔABC ≅ ΔBAD **(iv)** diagonal AC = diagonal BD.

Sol.



(i) Extend AB and draw a line through C parallel to DA intersecting AB produced at E.

Since, AD || CE and transversal AE cuts them at A and E respectively.

∴ ∠A + ∠E = 180º

⇒ 180º – ∠E = ∠A

Since, AB || CD and AD || CE

∴ AECD is a parallelogram.

⇒ AD = CE

⇒ BC = CE

Thus, in ΔBCE

BC = CE

⇒ ∠CBE = ∠CEB

⇒ 180º – ∠B = ∠E

⇒ 180º – ∠E = ∠B

⇒ ∠A = ∠B.

(ii) Consecutive interior angles on the same side of a transversal are supplementary.

∴ ∠A + ∠D = 180º and ∠E + ∠C = 180º

⇒ ∠A + ∠D = ∠E + ∠C

⇒ ∠B + ∠D = ∠E + ∠C

⇒ ∠D = ∠C [∠B = ∠E]

(iii) In ΔABC and ΔBAD

AB = BA

$\angle B = \angle A$

and BC = AD

So, by SAS congruence criterion

$\Delta ABC \cong \Delta BAD$

(iv) Since, $\Delta ABC \cong \Delta BAD$

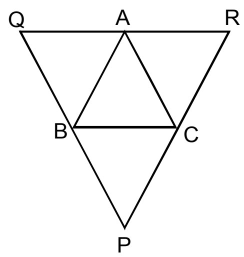
AC = BD

Hence, diagonal AC = diagonal BD.

Example. 8

In ΔABC, lines are drawn through A, B and C parallel respectively to the sides BC, CA and AB, forming ΔPQR. Show that $BC = \frac{1}{2}$ QR.

Sol.



AQ || CB and AC || QB

$\Rightarrow$ AQCB is parallelogram.

$\Rightarrow$ BC = AQ [$\because$Opposite sides of a ||gm are equal]

AR || CB and AB || RC

$\Rightarrow$ ARCB is parallelogram.

$\Rightarrow$ BC = AR [$\because$Opposite sides of a ||gm are equal]

So, AQ = AR

$\Rightarrow$ $AQ = AR = \frac{1}{2}QR$

$\Rightarrow$ $BC = \frac{1}{2}QR.$

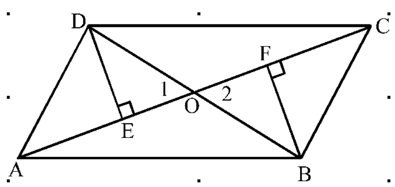
Check Your Level

1. Show that the diagonals of a rhombus are perpendicular to each other.

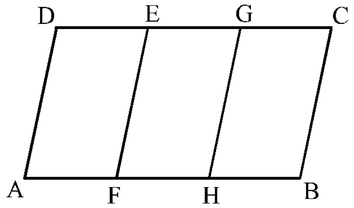
2. If ABCD is a rhombus, find $\angle AOD$, where O is the point of intersection of the diagonals.

3. If AD = (x + 2y) cm, BC = (2x + 3) cm, DC = (x + 7) cm and AB = (3y + 2) cm find AB and BC in parallelogram ABCD.

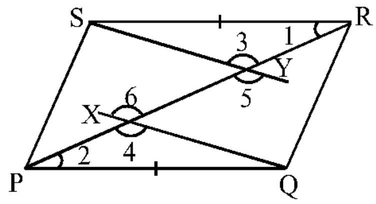
4. Given a parallelogram ABCD. DE perpendicular to AC and BF perpendicular to AC.
Prove that: DE = BF



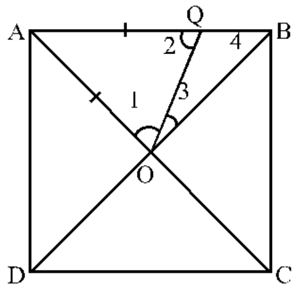
5. Given a parallelogram ABCD, EF || AD, GH || CB. Prove that EFHG is a parallelogram.



6. Given a parallelogram PQRS in which QX || SY. Prove that QX = SY



7. The diagonals of a square intersect at O. From AB a part AQ = AO is cut off. Prove that ∠AOQ = 3∠BOQ



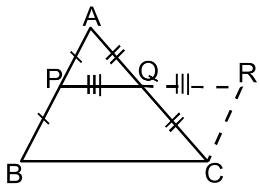
Answers

2. 90° **3.** AB = 8 cm, BC = 5 cm.

(B) MID POINT THEOREM AND ITS CONVERSE

(a) Mid point theorem

In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and is half of it.



Given : A triangle ABC in which P is the mid-point of side AB and Q is the mid-point of side AC.

To Prove : PQ is parallel to BC and is half of it i.e., PQ || BC and PQ = $\frac{1}{2}$BC.

Construction : Produce PQ upto point R such that PQ = QR. Join R and C.

Proof : In ΔAPQ and ΔCRQ

PQ = QR [By construction]

AQ = QC [Given]

And, ∠AQP = ∠CQR [Vertically opposite angles]

So, ΔAPQ ≅ ΔCRQ [By SAS]

∴ AP = CR [By CPCT]

And, ∠APQ = ∠CRQ [By CPCT]

But, ∠APQ and ∠CRQ are alternate angles and whenever the alternate angles are equal, the lines are parallel.

⇒ AP || CR

⇒ AB || CR

⇒ BP || CR

AP = BP [Given, P is mid-point of AB]

⇒ CR = BP [As, AP = CR]

Now, BP = CR and BP || CR

⇒ BCRP is a parallelogram.

[When any pair of opposite sides are equal and parallel, the quadrilateral is a parallelogram]

BCRP is a parallelogram and opposite sides of a parallelogram are equal and parallel.

∴ PR = BC and PR || BC

Since, PQ = QR

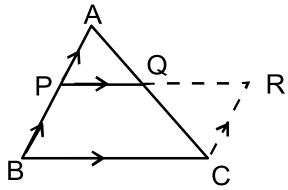
$PQ = \frac{1}{2}PR = \frac{1}{2}BC$ [As, PR = BC]

Also, PQ || BC [As, PR || BC]

∴ $PQ \parallel BC$ and $PQ = \frac{1}{2}BC$ **Hence Proved.**

(b) Converse of mid point theorem

The line drawn through the mid-point of one side of a triangle parallel to the another side; bisects the third side.



Given : A triangle ABC in which P is the mid-point of side AB and PQ is parallel to BC.

To prove : PQ bisects the third side AC i.e., AQ = QC.

Construction : Through C, draw CR parallel to BA, which meets PQ produced at point R.

Proof : Since, PQ || BC i.e., PR || BC [Given]

and CR || BA i.e., CR || BP. [By construction]

∴ Opposite sides of quadrilateral PBCR are parallel.

⇒ PBCR is a parallelogram

⇒ BP = CR

Also, BP = AP [As, P is mid-point of AB]

∴ CR = AP

∵ AB || CR and AC is transversal, ∠PAQ = ∠RCQ [Alternate angles]

∴ AB || CR and PR is transversal, ∠APQ = ∠CRQ [Alternate angles]

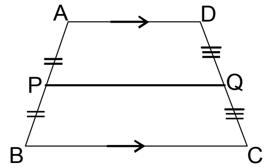
In APQ and CRQ

CR = AP, ∠PAQ = ∠RCQ and ∠APQ = ∠CRQ

⇒ ΔAPQ ≅ ΔCRQ [By ASA]

⇒ AQ = QC [By CPCT] **Hence Proved.**

NOTE : In quadrilateral ABCD, if side AD is parallel to side BC; ABCD is a trapezium.

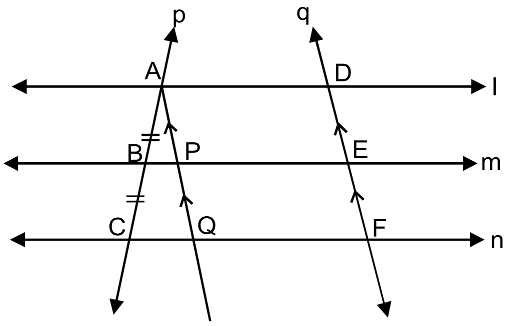


Now, P and Q are the mid-points of the non-parallel sides of the trapezium; then $PQ = \frac{1}{2}(AD + BC)$.

i.e. The length of the line segment joining the mid-points of the two non-parallel sides of a trapezium is always equal to half of the sum of the length of its two parallel sides.

(c) Intercept theorem

Theorem : If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.
Given : Three parallel lines l, m and n i.e., **l || m || n**. A transversal **p** meets these parallel lines at points A, B and C respectively such that AB = BC. Another transversal **q** also meets parallel lines **l, m** and **n** at points D, E and F respectively.



To Prove : DE = EF
Construction : Through point A, draw a line parallel to DEF; which meets BE at point P and CF at point Q.
Proof : In ΔACQ, B is mid-point of AC and BP is parallel to CQ and we know that the line through the mid-point of one side of the triangle and parallel to another side bisects the third side.

∴ AP = PQ ... (i)

When the opposite sides of a quadrilateral are parallel, it is a parallelogram and so its opposite sides are equal.

∴ AP || DE and AD || PE
⇒ APED is a parallelogram.
⇒ AP = DE ...(ii)
And PQ || EF and PE || QF
⇒ PQFE is a parallelogram
⇒ PQ = EF ...(iii)

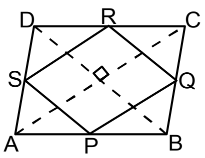
From above equations, we get
DE = EF **Hence Proved.**

Solved Examples

Example. 9

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Prove that the quadrilateral PQRS is a rectangle.

Sol.



In ΔABC, PQ || AC and PQ = $\frac{1}{2}$AC ... (i) [By mid-point theorem]

In ΔADC, SR || AC and SR = $\frac{1}{2}$AC ... (ii) [By mid-point theorem]

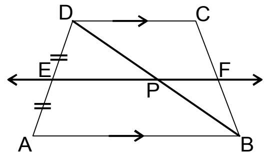
∴ PQ = SR and PQ || SR [From (i) and (ii)]
⇒ PQRS is a parallelogram.

Now, PQRS will be a rectangle if any angle of the parallelogram PQRS is 90º.

	PQ || AC	[By mid-point theorem]
	QR || BD	[By mid-point theorem]
But,	AC ⊥ BD	[Diagonals of a rhombus are perpendicular to each other]
∴	PQ ⊥ QR	[Angle between two lines = angle between their parallels]
⇒	PQRS is a rectangle.	**Hence Proved**

Example. 10

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (Figure). Prove that F is the mid-point of BC.



Sol. Given line EF is parallel to AB and AB || DC.

∴ EF || AB || DC.

According to the converse of the mid-point theorem, in ABD, E is the mid-point of AD and EP is parallel to AB. [As EF || AB]

∴ P is the mid-point of side BD.

[The line through the mid-point of a side of a triangle and parallel to the other side, bisects the third side]

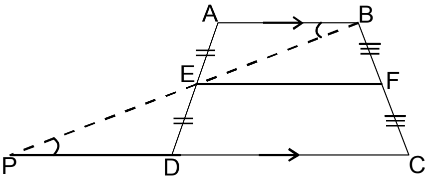
Now, in ΔBCD, P is mid-point of BD and, PF is parallel to DC. [As EF || DC]

∴ F is the mid-point of BC

[The line through the mid-point of a side of a triangle and parallel to the other side, bisects the third side] **Hence Proved.**

Example. 11

In the given figure, E and F are respectively, the mid-points of non-parallel sides of a trapezium ABCD.



Prove that :

(i) EF || AB **(ii)** EF = $\frac{1}{2}$ (AB + DC).

Sol. Join BE and produce it to intersect CD produced at point P. In ΔAEB and ΔDEP, AB || PC and BP is transversal.

	∠ABE = ∠DPE	[Alternate interior angles]
	∠AEB = ∠DEP	[Vertically opposite angles]
And	AE = DE	[E is mid-point of AD]
So,	ΔAEB ≅ ΔDEP	[By AAS congruency]
	BE = PE	[By CPCT]
And	AB = DP	[By CPCT]

Since, the line joining the mid-points of any two sides of a triangle is parallel and half of the third side, Therefore, in ΔBPC,

E is mid-point of BP [As, BE = PE]

and F is mid-point of BC [Given]

⇒ EF || PC and EF = $\frac{1}{2}$PC

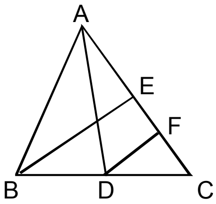
⇒ EF || DC and EF = $\frac{1}{2}$ (PD + DC)

⇒ EF || AB and EF = $\frac{1}{2}$ (AB + DC) [As, DC || AB and PD = AB] **Hence Proved.**

Example. 12

AD and BE are medians of ΔABC and BE || DF. Prove that CF = $\frac{1}{4}$AC.

Sol.



In ΔBEC, DF is a line through the mid - point D of BC and parallel to BE intersecting CE at F. Therefore, F is the midpoint of CE. Because the line drawn through the mid point of one side of a triangle and parallel to another sides bisects the third side.

Now, F is the mid point of CE

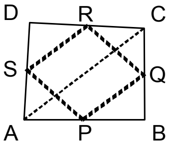
$\Rightarrow$ CF = $\frac{1}{2}$CE $\Rightarrow$ CF = $\frac{1}{2}$ ($\frac{1}{2}$AC) $\Rightarrow$ CF = $\frac{1}{4}$ AC.

Example.13

Prove that the figure formed by joining the mid - points of the pairs of consecutive sides of a quadrilateral is a parallelogram.

Sol. ABCD is a quadrilateral in which P, Q, R and S are the midpoints of the sides AB, BC, CD and DA respectively.

Join A and C.



In ΔABC, P and Q are the midpoints of sides AB and AC respectively.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC [By midpoint theorem]

In ΔABC, P and Q are the midpoints of sides AB and AC respectively.

$\therefore$ RS || AC and RS = $\frac{1}{2}$ AC [By midpoint theorem]

$\therefore$ PQ = RS and PQ || RS.

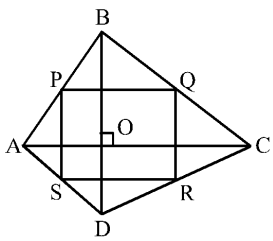
Thus in quadrilateral PQRS one pair of opposite sides are equal and parallel.

Hence, PQRS is a parallelogram.

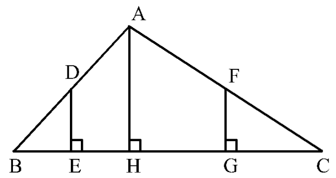
Check Your Level

1. Prove that the median to the hypotenuse of a right angled triangle is half the length of the hypotenuse.

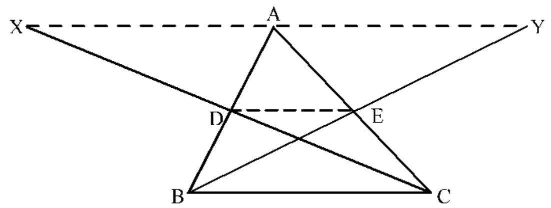
2. The diagonals of a quadrilateral ABCD are perpendicular to each other. Show that the quadrilateral, formed by joining the middle points of its sides is a rectangle.



3. Perpendiculars dropped from the mid points of two sides of a triangle to the third side are equal.



4. In △ABC, the medians CD and BE are produced to X and Y such that CD = DX and BE = EY. Prove that the points X, A, Y are collinear.



5. Show that the three line segments which join the middle points of the sides of a triangle, divide it into four triangles which are congruent to each other.

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]

1. Diagonals of a parallelogram ABCD intersect at O. If $\angle$BOC = 90º and $\angle$BDC = 50º, then find $\angle$OAB.

2. Three angles of a quadrilateral are 75º, 90º and 75º. Find the fourth angle

3. A diagonal of a rectangle is inclined to one side of the rectangle at 25º. Find the acute angle between the diagonals.

4. ABCD is a rhombus such that $\angle$ACB = 40º. Then find $\angle$ADB.

5. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then identify the type of quadrilateral ABCD ?

6. If APB and CQD are two parallel lines, then find the type of quadrilateral formed by the bisectors of the angles APQ, BPQ, CQP and PQD.

7. Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

8. One angle of a quadrilateral is of 108º and the remaining three angles are equal. Find each of the three equal angles.

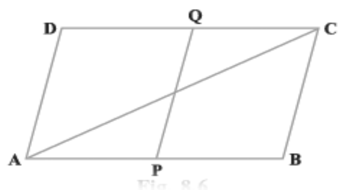
9. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram.

10. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

11. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

12. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that ΔDEF is also an equilateral triangle.

13. Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ see figure. Show that AC and PQ bisect each other.



14. A diagonal of a parallelogram bisects one of its angle. Prove that it will bisect its opposite angle also.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[03 MARK EACH]**

15. Show that the quadrilateral formed by joining the mid-points the sides of a rhombus, taken in order, form a rectangle.

16. In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of $\angle$ A meets DC in E. AE and BC produced meet at F. Find the length of CF.

17. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.

18. ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that $\angle$ A = $\angle$ B and $\angle$ C = $\angle$ D.

19. E is the mid-point of a median AD of Δ ABC and BE is produced to meet AC at F. Show that $AF = \frac{1}{3}$ AC.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[04 MARK EACH]**

20. PQ and RS are two equal and parallel line-segments. Any point M not lying on PQ or RS is joined to Q and S and lines through P parallel to QM and through R parallel to SM meet at N. Prove that line segments MN and PQ are equal and parallel to each other.

21. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which
AC = BD and AC $\perp$ BD. Prove that PQRS is a square.

22. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.

23. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA =AR and CQ = QR.

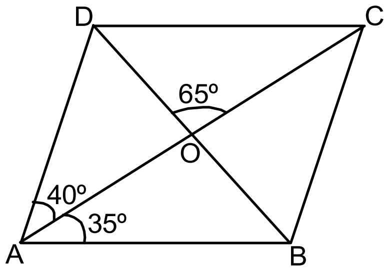
Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Quadrilaterals

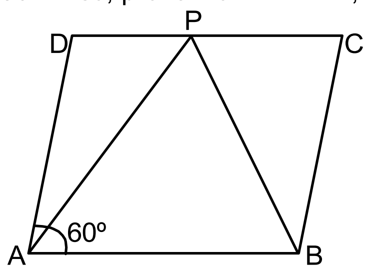
A-1. In the following figure, ABCD is a parallelogram $\angle DAO = 40^o$, $\angle BAO = 35^o$ and $\angle COD = 65^o$. Find :



(i) $\angle$ABO **(ii)** $\angle$ODC **(iii)** $\angle$ACB **(iv)** $\angle$CBD

A-2. ABCD is a parallelogram. P is a point on AD such that AP = $\frac{1}{3}$AD. Q is a point on BC such that CQ = $\frac{1}{3}$BC. Prove that AQCP is a parallelogram.

A-3. In the following figure, ABCD is a parallelogram in which ∠A = 60º. If the bisectors of ∠A and ∠B meet at P, prove that ∠APB = 90º. Also, prove that AD = DP, PC = BC and DC = 2AD.



A-4. In a parallelogram ABCD, the bisector of ∠A also bisects BC at P. Prove that AD = 2AB.

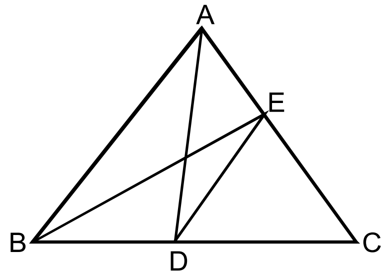
A-5. ABCD is a parallelogram and X and Y are points on the diagonal BD such that DX = BY. Prove that

(i) AXCY is a parallelogram **(ii)** AX = CY, AY = CX

(iii) ΔAYB ≅ ΔCXD **(iv)** ΔAXD ≅ ΔCYB

Section (B) : Mid-point theorem and its converse

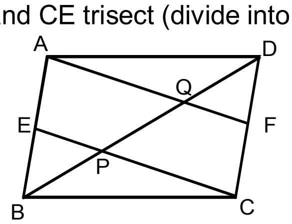
B-1. In the following figure, AD is a median and DE || AB. Prove that BE is a median.



B-2. ABCD is a trapezium in which side AB is parallel to side DC and E is the mid-point of side AD. If F is a point on side BC such that segment EF is parallel to side DC. Prove that EF = $\frac{1}{2}$ (AB + DC).

B-3. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half of the difference of these sides.

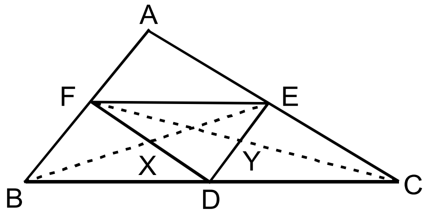
B-4. In figure, ABCD is a parallelogram. E and F are the mid-points of the sides AB and CD respectively. Prove that the line segments AF and CE trisect (divide into three equal parts) the diagonal BD.



B-5. P is the mid-point of side AB of a parallelogram ABCD. A line through B parallel to PD meet DC at Q and AD produced at R. Prove that :

(i) AR = 2BC **(ii)** BR = 2BQ

B-6. In the adjoining figure, D, E, F are the midpoints of the sides BC, CA and AB of ΔABC. If BE and DF intersect at X while CF and DE intersect at Y, prove that $XY = \frac{1}{4}BC$.



OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Quadrilaterals

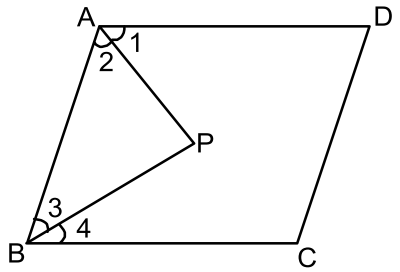
A-1. In a parallelogram ABCD, $\angle D = 105^\circ$, then the $\angle A$ and $\angle B$ will be :
(A) 105º, 75º (B) 75º, 105º (C) 105º, 105º (D) 75º, 75º

A-2. When the diagonals of a parallelogram are perpendicular to each other then it is called :
(A) square (B) rectangle (C) rhombus (D) parallelogram

A-3. ABCD is a rhombus with $\angle ABC = 56^\circ$, then the $\angle ACD$ will be :
(A) 56º (B) 62º (C) 124º (D) 34º

A-4. In an Isosceles trapezium ABCD if $\angle A = 45^\circ$ then $\angle C$ will be :
(A) 90º (B) 135º (C) 120º (D) none of these

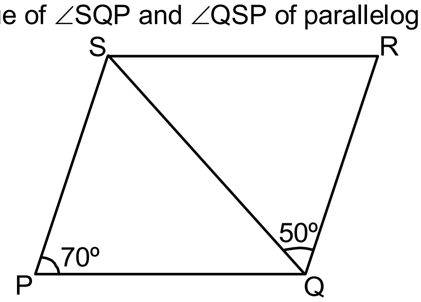
A-5. In the adjoining figure, AP and BP are angle bisectors of $\angle A$ and $\angle B$ which meets at P on the parallelogram ABCD. Then $2\angle APB$ =



(A) $\angle C + \angle D$ (B) $\angle A + \angle C$ (C) $\angle B + \angle D$ (D) $2\angle C$

A-6. In a quadrilateral ABCD, AO & DO are angle bisectors of $\angle A$ and $\angle D$ and given that $\angle C = 105^\circ$, $\angle B = 70^\circ$, then the $\angle AOD$ is :
(A) 67.5º (B) 77.5º (C) 87.5º (D) 99.75º

A-7. From the figure, find the value of $\angle SQP$ and $\angle QSP$ of parallelogram PQRS.



(A) 60º, 50º (B) 60º, 45º (C) 70º, 35º (D) 35º, 70º

A-8. Two opposite angles of a parallelogram are $(3x - 2)^\circ$ and $(50 - x)^\circ$ then the value of x will be:-
(A) 17° (B) 16° (C) 15° (D) 13°

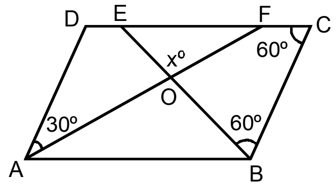
A-9. Which of the following properties are not TRUE for parallelogram ?
(A) Its diagonals are perpendicular to each other
(B) The diagonals divide the figure into four congruent triangles
(C) Its diagonals are equal
(D) All of the above

A-10. If the diagonals of a parallelogram are equal then it is a :
(A) rectangle (B) trapezium (C) rhombus (D) square

A-11. The diagonals of a rectangle ABCD meet at O. If $\angle BOC = 44°$, then $\angle OAD$ is :
(A) 68° (B) 44° (C) 54° (D) None of these

A-12. If ABCD is a trapezium in which AB || CD and AD = BC, then $\angle A$ is equal to :
(A) 180° (B) 90° (C) $\angle B$ (D) None of these

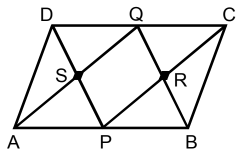
A-13. In the adjoining figure ABCD is a parallelogram, then the measure of x is :



(A) 45° (B) 60° (C) 90° (D) 135°

A-14. ABCD is a parallelogram and AP and CQ are the perpendiculars from A and C on its diagonal BD, respectively. Then AP is equal to :
(A) DP (B) CQ (C) PQ (D) AB

A-15. In fig. ABCD is a parallelogram. P and Q are mid points of the sides AB and CD, respectively. Then PRQS is :



(A) parallelogram (B) trapezium (C) rectangle (D) none of these

Section (B) : Mid-point theorem and its converse.

B-1. In a triangle, P, Q and R are the mid-points of the sides BC, CA and AB respectively. If AC = 16 cm, BC = 20 cm and AB = 24 cm, then the perimeter of the quadrilateral ARPQ will be :
(A) 60 cm (B) 30 cm (C) 40 cm (D) none of these

B-2. LMNO is a trapezium with LM || NO. If P and Q are the mid-points of LO and MN respectively and LM = 5 cm and ON = 10 cm, then PQ =
(A) 2.5 cm (B) 5 cm (C) 7.5 cm (D) 15 cm

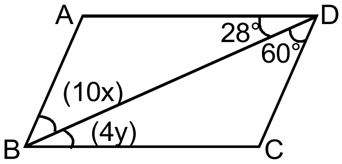
B-3. In a right angle triangle ABC is right angled at B. Given that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, then the area of ΔADE =
(A) 67.5 cm^2 (B) 13.5 cm^2 (C) 27 cm^2 (D) data insufficient

B-4. In ΔABC, AD is the median through A and E is the mid-point of AD. BE produced meets AC in F. Then
(A) $AF = \frac{1}{4}AC$ (B) $AF = \frac{1}{3}AC$ (C) $AF = \frac{1}{2}AC$ (D) None of these

Exercise-2

OBJECTIVE QUESTIONS

1. In given figure, ABCD is a parallelogram. Compute the values of x and y.



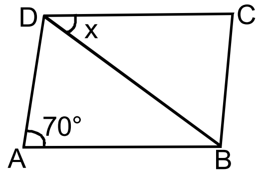
(A) x = 6° and y = 7° (B) x = 3° and y = 2°
(C) x = 5° and y = 6° (D) None of these

2. If an angle of an a parallelogram is two-third the adjacent angles, Then find both angles of a parallelogram.
(A) 60°, 120° (B) 36°, 144° (C) 72°, 108° (D) None of these

3. A quadrilateral having only one pair of opposite sides parallel is called a
(A) Square (B) Rhombus (C) Trapezium (D) Parallelogram

4. The angles of a quadrilateral are x° , (x – 10)° , (x + 30°) and (2 x)° , the smallest angle is equal to
(A) 68° (B) 52° (C) 58° (D) 47°

5. In the given figure, ABCD is a rhombus. If ∠ A = 70° , then ∠ CDB is equal to



(A) 65° (B) 55° (C) 75° (D) 80°

6. Which is not correct about rectangle EFGH ?
(A) ∠ E = ∠ F = ∠ G = ∠ H = 90° (B) EG = FH
(C) EF = GH and HE = FG (D) EG and FH are angle bisectors.

7. PQRS is a square, PR and SQ intersect at O. The measure of ∠ POQ is
(A) 45° (B) 90° (C) 180° (D) None of these

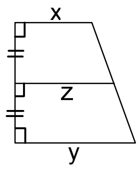
8. The measure all the angles of a parallelogram, If an angle is 24° less than twice the smallest angle, is
(A) 37° , 143° , 37° , 143° (B) 108° , 72° , 108° , 72°
(C) 68° , 112° , 68° , 112° (D) None of these

9. To construct rhombus uniquely it is necessary to know atleast ____of its parts.
(A) 1 (B) 2 (C) 3 (D) 5

10. ABCD is a trapezium in which AB || CD. If ∠ADC = 2∠ABC, AD = a cm and CD = b cm, then the length (in cm) of AB is :
(A) $\frac{a}{2} + 2b$ (B) a + b (C) $\frac{2}{3}a + b$ (D) $a + \frac{2}{3}b$

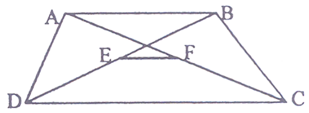
11. ABCD is a quadrilateral whose diagonals intersect each othe at the point O such that OA = OB = OD. If ∠OAB = 30º, then the measure of ∠ODA is :
(A) 30º (B) 45º (C) 60º (D) 90º

12. A quadrilateral ABCD has four angles x°, 2x°, $\frac{5x^o}{2}$ and $\frac{7x^o}{2}$ respectively. What is the difference between the value of biggest and the smallest angles.
(A) 40° (B) 100° (C) 80° (D) 20°

13. 

In the figure, z =
(A) $\frac{2xy}{x+y}$ (B) $\sqrt{x}\ \ y$ (C) $\sqrt{x^2+y^2}$ (D) $\frac{x+y}{2}$

14. In the trapezium shown, AB II DC, and E and F are the midpoints of the two diagonals. If DC = 60 and EF = 5 then the length of AB is equal to :



(A) 40 (B) 45 (C) 50 (D) 55

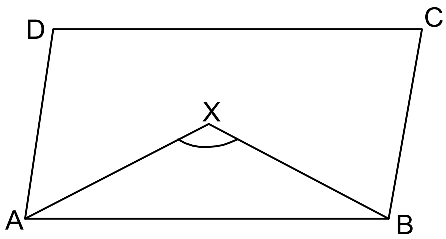
15. Suppose the triangle ABC has an obtuse angle at C and let D be the midpoint of side AC. Suppose E is on BC such that the segment DE is parallel to AB. Consider the following three statements.
(i) E is the midpoint of BC
(ii) The length of DE is half the length of AB
(iii) DE bisects the altitude from C to AB
(A) only (i) is true (B) only (i) and (ii) are true
(C) only (i) and (iii) are true (D) all three are true.

16. The line joining the mid points of the diagonals of a trapezium has length 3. If the longer base is 97, then the shorter base is :
(A) 94 (B) 92 (C) 91 (D) 90

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. In the given diagram, ABCD is a parallelogram. Bisectors of ∠A and ∠B meet at the point X. Then the value of ∠A X B is - **[Raj. NTSE Stage-1 2005]**

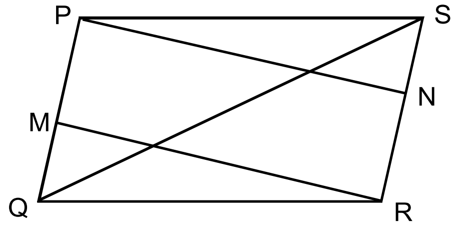


(A) 150° (B) 120° (C) 90° (D) 60°

2. The short diagonal of a rhombus is equal to its side, then the value of angle opposite to its longer diagonal is **[Raj. NTSE Stage-1 2005]**
(A) 30° (B) 60° (C) 90° (D) 120°

3. The bisectors of angles of a parallelogram makes a figure which is **[Raj. NTSE Stage-1 2013]**
(A) Rectangle (B) Circle (C) Pentagon (D) Octagon

4. PQRS is a parallelogram and M, N are the mid-points of PQ and RS respectively. Which of the following is not true ? **[M.P. NTSE Stage-1 2013]**

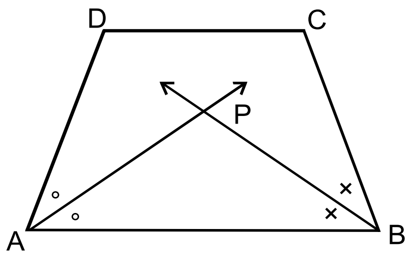


(A) RM trisects QS (B) PN trisects QS
(C) $\Delta PSN \cong \Delta RQM$ (D) MS is not parallel to QN

5. An obtuse angle of a rhombus is greater than twice the acute angle by 30°. Find the measure of each angle. **[Maharashtra NTSE Stage- 1_2014]**
(A) 50° (B) 130° (C) 80° (D) 60°

6. ABCD is a rhombus and P, Q, R, S are respectively mid points of sides AB, BC, CD, DA. Then $\angle RSP$ is : **[MP NTSE Stage-I_2014]**
(A) 120° (B) 90° (C) 60° (D) 30°

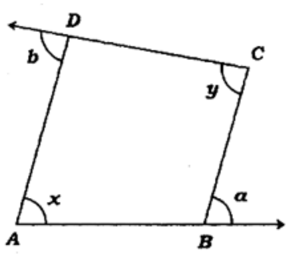
7. If the angle bisectors of $\angle DAB$ and $\angle CBA$ of any quadrilateral ABCD intersects at the Point P, then find $2m\angle APB$: **[Maharashtra NTSE Stage- 1_2014]**



(A) $\angle C + \angle D$ (B) $\angle C + \angle B$ (C) $\angle A + \angle B$ (D) $\angle A + \angle D$

8. The side of a rhombus is 10 cm. The smaller diagonal is $\frac{1}{3}$ of the greater diagonal. Find the length of the greater diagonal : **[MP NTSE Stage-I_2015]**
(A) $6\sqrt{10}$ cm (B) $10\sqrt{10}$ cm (C) $6\sqrt{10}$ cm (D) $5\sqrt{10}$ cm

9. Sides AB and CD of a quadrilateral. ABCD are extended as in figure. Then a + b is equal to **[Raj. NTSE Stage-1 2016]**



(A) x+2y (B) x-y (C) x+y (D) 2x+y.

10. Angle at A in trapezium ABCD if AB = 18 cm, BC = 10 cm, CD = 12 cm, DA = 8cm, AB || CD, will be : **[West Bengal NTSE Stage-1 2016]**

(A) 80° (B) 45° (C) 90° (D) None of these

11. The line segment joining the mid-points of the adjacent sides of a quadrilateral form : **[MP NTSE Stage-I_2016]**

(A) parallelogram (B) Square (C) Rhombus (D) Rectangle

12. In a rhombus of side 10 cm, one of the diagonal is 12 cm long, the length of second diagonal will be **[MP NTSE Stage-I_2016]**

(A) 4 cm (B) 8 cm (C) 12 cm (D) 16 cm

Answer Key

BOARD LEVEL EXERCISE

TYPE (I) :

1. 40° **2.** 120° **3.** 50° **4.** 50°

5. Trapezium **6.** Rectangle **7.** AC = 6 cm, BD = 4 cm

TYPE (II) :

8. 84° **9.** 60°, 120°, 60°, 120° **10.** $\angle A = \angle C = 60°$, $\angle B = \angle D = 120°$

TYPE (III)

16. 4 cm

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-1. **(i)** 80º **(ii)** 80º **(iii)** 40º **(iv)** 25º

OBJECTIVE QUESTIONS

Section (A)

A-1. (B) **A-2.** (C) **A-3.** (B) **A-4.** (B) **A-5.** (A)

A-6. (C) **A-7.** (A) **A-8.** (D) **A-9.** (D) **A-10.** (A)

A-11. (A) **A-12.** (C) **A-13.** (C) **A-14.** (B) **A-15.** (A)

Section (B)

B-1. (C) **B-2.** (C) **B-3.** (B) **B-4.** (B)

Exercise-2

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	A	C	C	C	B	D	B	C	B	B	C	B	D	C	D	C

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	C	D	A	D	B	B	A	A	C	C	A	D