JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

MAXIMA

&

MINIMA

(PRACTICE SHEET)

LEVEL-1

Question based on **Maxima & Minima of function**

Q.1 f(c) is a maximum value of f(x) if -
(A) $f'(c) = 0, f''(c) > 0$
(B) $f'(c) = 0, f''(c) < 0$
(C) $f'(c) \neq 0, f''(c) = 0$
(D) $f'(c) < 0, f''(c) > 0$

Q.2 f(c) is a minimum value of f(x) if -
(A) $f'(c) = 0, f''(c) > 0$
(B) $f'(c) = 0, f''(c) < 0$
(C) $f'(c) \neq 0, f''(c) = 0$
(D) $f'(c) < 0, f''(c) > 0$

Q.3 f(c) is a maximum value of f(x) when at x = c -
(A) $f'(x)$ changes sign from +ve to –ve
(B) $f'(x)$ changes sign from –ve to +ve
(C) $f'(x)$ does not change sign
(D) $f'(x)$ is zero

Q.4 f(c) is a minimum value of f(x) when at x = c-
(A) $f'(x)$ changes sign +ve to –ve
(B) $f'(x)$ changes sign from –ve to +ve
(C) $f'(x)$ does not change sign
(D) $f'(x)$ is zero

Q.5 The correct statement is -
(A) f(c) is an extreme value of f(x) if $f'(c) = 0$
(B) If f(c) is an extreme value of f(x) then $f'(c) = 0$
(C) If $f'(c) = 0$ then f(c) is an extreme value of f(x)
(D) All the above statements are incorrect

Q.6 If for a function f(x), $f'(a) = 0 = f''(a) = ... = f^{n-1}(a)$ but $f^n(a) \neq 0$ then at x = a, f(x) is minimum if -
(A) n is even and $f^n(a) > 0$
(B) n is odd and $f^n(a) > 0$
(C) n is even and $f^n(a) < 0$
(D) n is odd and $f^n(a) < 0$

Q.7 The point of maxima of sec x is -
(A) x = 0 (B) $x = \pi/2$
(C) $x = \pi$ (D) $x = 3\pi/2$

Q.8 $x^3 - 3x + 4$ is minimum at -
(A) x = 1 (B) x = –1
(C) x = 0 (D) No where

Q.9 The maximum value of $2x^3 - 9x^2 + 100$ is -
(A) 0 (B) 100
(C) 3 (D) 30

Q.10 If $f(x) = x^3 - kx + 7$ is maximum at x = –1, then the value of k is -
(A) 3 (B) 6 (C) –3 (D) –6

Q.11 Which of the following function has no extreme point-
(A) 2^x (B) [x]
(C) $\log_{10}x$ (D) All these functions

Q.12 If for a function f(x), $f'(a) = 0 = f''(a) = ... = f^{n-1}(a)$ but $f^n(a) \neq 0$ then at x = a, f(x) is maximum if -
(A) n is even and $f^n(a) > 0$
(B) n is odd and $f^n(a) > 0$
(C) n is even and $f^n(a) < 0$
(D) n is odd and $f^n(a) < 0$

Q.13 The maximum value of
$5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is -
(A) 5 (B) 10 (C) 11 (D) –1

Q.14 The function $f(x) = \sum_{K=1}^{5}(x - K)^2$ assumes minimum value for x given by
(A) 5 (B) 3 (C) 5/2 (D) 2

Q.15 If $f(x) = x^3 - 3x^2 + 3x + 7$, then -
(A) f(x) has a maximum at x = 1
(B) f(x) has a minimum at x = 1
(C) f(x) has a point of inflexion at x = 1
(D) None of these

Q.16 In [0, 2] the point of maxima of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ is –
(A) x = 0 (B) x = 1
(C) x = 1/2 (D) Does not exist

Q.17 If $f'(c)$ changes sign from negative to positive as x passes through c, then -

(A) f(c) is neither a maximum nor a minimum value of f(x)

(B) f(c) is a maximum value of f(x)

(C) f(c) is a minimum value of f(x)

(D) f(c) is either a maximum or a minimum value of f(x)

Q.18 If $f'(c)$ changes sign from positive to negative as x passes through c, then,

(A) f(c) is neither a maximum nor a minimum value of f(x)

(B) f(c) is a maximum value of f(x)

(C) f(c) is a minimum value of f(x)

(D) f(c) is either a maximum or minimum value of f(x)

Q.19 If $f'(c) < 0$ and $f''(c) > 0$, then at x = c, f(x) is -

(A) maximum

(B) minimum

(C) neither maximum nor minimum

(D) either maximum or minimum

Q.20 If for a function f(x), $f'(b)=0, f''(b) = 0$, $f'''(b) > 0$, then x = b is -

(A) a maximum point (B) a minimum point

(C) an extreme point (D) not an extreme point

Q.21 The maximum height of the curve $y = 6 \cos x - 8 \sin x$ above x axis is-

(A) 5 (B) 10

(C) 15 (D) None of these

Q.22 The minimum value of a sec x + b cosec x, $0 < a < b$, $0 < x < \pi/2$ is =

(A) a + b (B) $a^{2/3} + b^{2/3}$

(C) $(a^{2/3} + b^{2/3})^{3/2}$ (D) None of these

Q.23 The minimum value of $\frac{x}{\log x}$ $(x > 0)$ is -

(A) e (B) 1/e

(C) 0 (D) Does not exist

Q.24 For what value of x, $x^2 \log (1/x)$ is maximum-

(A) $e^{-1/2}$ (B) $e^{1/2}$ (C) e (D) e^{-1}

Q.25 For what value of k, the function:

$f(x)=kx^2+\frac{2k^2-81}{2}x-12$, is maximum at $x = 9/4$

(A) 9/2 (B) –9

(C) –9/2 (D) 9

Q.26 The greatest value of the function $f(x) = \cos [xe^{[x]} + 2x^2 - x]$, $-1 < x < \infty$ is-

(A) –1 (B) 1

(C) 0 (D) None of these

Q.27 For $f(x) = \sqrt{3} \sin x + 3 \cos x$, the point $x = \pi/6$ is -

(A) a local maximum

(B) a local minimum

(C) None of these

(D) a point of inflexion

Q.28 Which of the following functions has maximum or minimum value -

(A) sinh x (B) cosh x

(C) tanh x (D) None of these

Q.29 The maximum value of $5 \sin \theta + 3 \sin (\theta + \pi/3) + 3$ is -

(A) 11 (B) 12 (C) 10 (D) 9

Q.30 The maximum value of $(x-2)(x-3)^2$ is-

(A) 2/27 (B) 1/27

(C) 4/27 (D) 5/27

Q.31 A maximum point of cosecx is-

(A) x = 0 (B) $x = \pi/2$

(C) $x = \pi$ (D) $x = 3\pi/2$

Q.32 The function $f(x) = a \sin x + \frac{1}{3} \sin 3x$ has a maximum at $x = \pi/3$, then a equals-

(A) –2 (B) 2 (C) –1 (D) 1

Q.33 If $f(x) = x^3 + ax^2 + bx + c$ is minimum at x = 3 and maximum at x = –1, then-

(A) a = –3, b = –9, c = 0

(B) a = 3, b = 9, c = 0

(C) a = –3, b = –9, $c \in R$

(D) None of these

Q.34 If $a > 1$, $x > 1$, then minimum value of $\log_a x + \log_x a$ is -
(A) 2 (B) –2 (C) 2a (D) None

Q.35 If x be real, then the minimum value of $f(x) = 3^{x+1} + 3^{-(x+1)}$ is -
(A) 2 (B) 6 (C) 2/3 (D) 7/9

Q.36 If $\alpha < \beta$, $\alpha,\beta \in (0, \pi/2)$ then correct statement is -
(A) $\alpha - \sin\alpha > \beta - \sin\beta$
(B) $\alpha - \sin\alpha < \beta - \sin\beta$
(C) $\sin\alpha - \alpha < -\sin\beta + \beta$
(D) None of these

Q.37 Function $f(x) = e^x + e^{-x}$ has -
(A) one minimum point
(B) one maximum point
(C) many extreme points
(D) no extreme point

Q.38 Which of the following functions has infinite extreme points -
(A) tan x (B) cot x (C) sec x (D) cosh x

Q.39 The maximum value of the function $(x-2)^6 (x-3)^5$ is -
(A) 0 (B) 1
(C) –1 (D) does not exist

Q.40 If $f'(x) = (x-a)^{2n}(x-b)^{2p+1}$; $n, p \in N$, then-
(A) x = a is a minimum point
(B) x = a is a maximum point
(C) x = a is neither maximum nor minimum
(D) None of these

Q.41 At $x = 5\pi/6$, function $2\sin 3x + 3\cos 3x$ is-
(A) maximum (B) minimum
(C) zero (D) None of these

Q.42 The minimum value of $y = x(\log x)^2$ is -
(A) 0 (B) 1 (C) 2 (D) None

Question based on — Greatest a least value in an interval

Q.43 The local maximum value of $x(1-x)^2, 0 \le x \le 2$ is
(A) 2 (B) 4/27 (C) 5 (D) 0

Q.44 In the interval (–2, 2), the minimum value of $x^3 - 3x + 4$ is -
(A) 0 (B) 1 (C) 2 (D) 3

Q.45 The least value of $f(x) = x^3 - 12x^2 + 45x$ in [0, 7] is -
(A) 0 (B) 50 (C) 45 (D) 54

Q.46 The minimum value of $y = 7\cos\theta + 24\sin\theta$ $(0 \le \theta \le 2\pi)$ is -
(A) 25 (B) –25 (C) 50 (D) None

Q.47 If $0 \le x \le \pi$, then maximum value of $y = (1 + \sin x)\cos x$ is -
(A) $3\sqrt{3}$ (B) $3\sqrt{3}/2$
(C) $3\sqrt{3}/4$ (D) –1

Q.48 The highest point on the curve $y = xe^{-x}$ is-
(A) (1, 1/e) (B) (e, 1)
(C) (1/e, 1) (D) (1, e)

Q.49 The function $3x^4 - 2x^3 - 6x^2 + 6x + 1$ has a maximum in [0, 2] at -
(A) x = 1/2 (B) x = 1
(C) x = 0 (D) does not exist

Q.50 The function $f(x) = x^2 \log x$ in the interval [1, e] has -
(A) a point of maximum and minimum
(B) a point of maximum only
(C) a point of minimum only
(D) no point of maximum and minimum is [1, e]

Question based on — Maxima & Minima of function of Two variable

Q.51 If $xy = c^2$ then the minimum value of $ax + by$ $(a > 0, b > 0)$ is-
(A) $c\sqrt{ab}$ (B) $-c\sqrt{ab}$
(C) $2c\sqrt{ab}$ (D) $-2c\sqrt{ab}$

Q.52 The difference between two numbers is a. If their product is minimum, then number are-
(A) –a/2, a/2 (B) –a, 2a
(C) –a/3, 2a/3 (D) –a/3, 4a/3

Q.53 If the sum of the number and its square is minimum, then number is -
(A) 0 (B) 1/2
(C) –1/2 (D) None of these

Q.54 20 is divided into two parts so that product of cube of one quantity and square of the other quantity is maximum. The part are-

(A) 10, 10 (B) 16, 4
(C) 8, 12 (D) 12, 8

Q.55 Which of the following point lying on the line $x + 2y = 5$ is at minimum distance from the origin

(A) (1, 2) (B) (3, 1)
(C) (–1, 3) (D) (2, 3/2)

Q.56 The point on the curve $x^2 = 2y$ which is nearest to (0, 5) is -

(A) $(2\sqrt{2}, 0)$ (B) (0, 0)
(C) (2, 2) (D) None

Q.57 The maximum distance of the point (a, 0) from the curve $2x^2 + y^2 - 2x = 0$ is-

(A) $\sqrt{(1-2a+a^2)}$ (B) $\sqrt{(1+2a+2a^2)}$
(C) $\sqrt{(1+2a-a^2)}$ (D) $\sqrt{(1-2a+2a^2)}$

Q.58 The sum of two non-zero number is 6. The minimum value of the sum of their reciprocals is-

(A) 3 (B) 6 (C) 2/3 (D) 6/5

Q.59 Divide 10 into two parts so that sum of double of one part and square of the other part is minimum, then the part are-

(A) 9, 1 (B) 5, 5
(C) 8, 2 (D) 4, 6

Q.60 The sum of two number is 12. If their product is maximum, then they are -

(A) 8, 4 (B) 9, 3
(C) 6, 6 (D) None of these

Q.61 If $xy = 4$ and $x < 0$ then maximum value of $x + 16y$ is -

(A) 8 (B) –8
(C) 16 (D) –16

Question based on — Geometrical result related to Maxima & Minima

Q.62 The area of a rectangle of maximum area inscribed in a circle of radius a is -

(A) πa^2 (B) a^2 (C) $2a^2$ (D) $2\pi a^2$

Q.63 The ratio between the height of a right circular cone of maximum volume inscribed in a sphere and the diameter of the sphere is -

(A) 2 : 3 (B) 3 : 4
(C) 1 : 3 (D) 1 : 4

Q.64 The point on the line y = x such that the sum of the squares of its distance from the point (a, 0), (–a, 0) and (0, b) is minimum will be -

(A) (a/6, a/6) (B) (a, a)
(C) (b, b) (D) (b/6, b/6)

Q.65 The minimum distance of the point (a, b, c) from x-axis is -

(A) $\sqrt{a^2+b^2}$ (B) $\sqrt{c^2+a^2}$
(C) $\sqrt{b^2+c^2}$ (D) $\sqrt{a^2+b^2+c^2}$

Q.66 An isosceles triangle with vertical angle 2θ is inscribed in a circle of radius a. The area of the triangle will be maximum when θ =

(A) π/6 (B) π/4 (C) π/3 (D) π/2

Q.67 The first and second order derivatives of a function f(x) exist at all points in (a, b) with $f'(c) = 0$, where $a < c < b$. Further more, if $f'(x) < 0$ at all points on the immediate left of c and $f'(x) > 0$ for all points on the immediate right of c, then at x = c, f(x) has a -

(A) local maximum (B) local minimum
(C) point of inflexion (D) none of these

Q.68 A wire of length p is cut into two parts. A circle and a square is formed with the help of these parts. The sum of the area of circle and square is minimum, if the ratio of sides of a square and diameter of circle is -

(A) 2 : 1 (B) 1 : 2
(C) 1 : 1 (D) None of these

LEVEL- 2

Q.1 The maximum value of $\sin^3 x + \cos^3 x$ is-

(A) 1 (B) 2

(C) 3/2 (D) None of these

Q.2 Let $f(x) = (x^2 - 4)^{n+1}(x^2 - x + 1)$, $n \in N$ and f(x) has a local extremum at x = 2 then -

(A) n = 2 (B) n = 6

(C) n = 3 (D) none

Q.3 Let $f(x) = (x - 1)^m (x - 2)^n$ $(m, n \in N)$, $x \in R$. Then at each point f(x) is either local maximum or local minimum if-

(A) m = 2, n = 3

(B) m = 2, n = 4

(C) m = 3, n = 5

(D) m = 3, n = 4

Q.4 If $\frac{x}{a} + \frac{y}{b} = 1$, then minimum value of $x^2 + y^2$ is-

(A) $\frac{2a^2b^2}{a^2+b^2}$ (B) $\frac{a^2b^2}{a^2+b^2}$

(C) $\frac{2ab}{a^2+b^2}$ (D) None of these

Q.5 Let $f(x) = \begin{cases} |x-1| + a, & x < 1 \\ 2x+3, & x \geq 1 \end{cases}$. If f(x) has a local minima at x = 1, then -

(A) $a \geq 5$ (B) $a > 5$

(C) $a > 0$ (D) none of these

Q.6 Which point of the parabola $y = x^2$ is nearest to the point (3, 0) -

(A) (–1, 1) (B) (1, 1)

(C) (2, 4) (D) (–2, 4)

Q.7 If value of the function $a^2 \sec^2\theta + b^2 \operatorname{cosec}^2\theta$ $(a > 0, b > 0)$ is minimum, then θ equals -

(A) $\tan^{-1}\sqrt{(a/b)}$ (B) $\tan^{-1}(a/b)$

(C) $\tan^{-1}(b/a)$ (D) $\tan^{-1}\sqrt{(b/a)}$

Q.8 For the curve $\frac{c^4}{r^2} = \frac{a^2}{\sin^2\theta} + \frac{b^2}{\cos^2\theta}$, the maximum value of r is -

(A) $\frac{c^2}{a+b}$ (B) $\frac{a+b}{c^2}$

(C) $\frac{c^2}{a-b}$ (D) $c^2(a + b)$

Q.9 If points of maxima and minima of a function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ $(a > 0)$ are respectively p and q, then for what value of a, the equation $p^2 = q$ is true -

(A) 0 (B) 0, 2 (C) 2 (D) None

Q.10 If $f(x) = \begin{cases} x^2 & x < 0 \\ 5 & x = 0 \\ 2\sin x & x > 0 \end{cases}$ then at x = 0, f(x) has

(A) a local maximum

(B) a local minimum

(C) an absolute minimum

(D) neither a maximum nor a minimum

Q.11 $f(x) = \begin{cases} \cos\frac{\pi x}{2}, & x > 0 \\ x + a, & x \leq 0 \end{cases}$ find the value of a if x = 0 is a point of maxima -

(A) $a \leq 1$ (B) $a \geq 1$

(C) $-1 \leq a \leq 1$ (D) none of these

Q.12 Find the value of a if $x^3 - 3x + a = 0$ has three real and distinct roots -

(A) a > 2 (B) a < 2

(C) –2 < a < 2 (D) none

Q.13 The maximum possible area that can be enclosed by a wire of length 20 cms by bending it into the form of a sector in square cms is -

(A) 25 (B) 10

(C) 15 (D) None of these

Q.14 The greatest value of the function $y = \dfrac{\sin 2x}{\sin(x + \pi/4)}$; $x \in (0, \pi/2)$ is -

(A) 1 (B) $-\sqrt{2}$

(C) $\sqrt{2}$ (D) $1/\sqrt{2}$

Q.15 The set of values of 'a' for which the function $f(x) = \dfrac{ax^3}{3} + (a + 2)x^2 + (a - 1)x + 2$ possesses a negative point of inflection -

(A) $(-\infty, -2) \cup (0,\infty)$ (B) $\{-4/5\}$

(C) $(-2, 0)$ (D) empty set

Q.16 If a, b, c, d are four positive real numbers such that abcd = 1 then minimum value of $(1 + a)(1 + b)(1+ c)(1 + d)$

(A) 8 (B) 12

(C) 16 (D) 20

Q.17 The difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $[-\pi/2, \pi/2]$ is

(A) $\dfrac{\sqrt{3}+\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}+\sqrt{2}}{2} + \dfrac{\pi}{3}$

(C) π (D) $\dfrac{\sqrt{3}+\sqrt{2}}{2} - \dfrac{\pi}{3}$

Q.18 A line is drawn through a fixed point (a, b), (a > 0, b > 0) to meet the positive direction of the coordinate axes in P, Q respectively. The minimum value of OP + OQ is -

(A) $\sqrt{a} + \sqrt{b}$ (B) $(\sqrt{a} + \sqrt{b})^2$

(C) $(\sqrt{a} + \sqrt{b})^3$ (D) None of these

Q.19 The equation of the line through (3, 4) which cuts the first quadrant a triangle of minimum area is-

(A) $4x + 3y - 24 = 0$ (B) $3x + 4y - 12 = 0$

(C) $2x + 4y - 12 = 0$ (D) $3x + 2y - 24 = 0$

Q.20 If $a < b < c < d$ & $x \in R$ then the least value of the function $f(x) = |x - a| + |x - b| + |x - c| + |x - d|$ is-

(A) $c - d + b - a$ (B) $c + d - b - a$

(C) $c + d - b + a$ (D) $c - d + b + a$

Q.21 Find the minimum and maximum value of $f(x, y) = 7x^2 + 4xy + 3y^2$ subjected to $x^2 + y^2 = 1$.

(A) $5, 5 - 2\sqrt{2}$ (B) $5 + 2\sqrt{2}, 5 - \sqrt{2}$

(C) $5 + 2\sqrt{2}, 5 - 2\sqrt{2}$ (D) None

Q.22 The least value of $2^{(x^2-3)^2+27}$ is -

(A) 2^{27} (B) 2

(C) 1 (D) None of these

Q.23 If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$ then -

(A) f(x) is increasing in [–1, 2]

(B) f(x) is continuous in [–1, 3]

(C) f(x) is maximum at x = 2

(D) All the above

Q.24 If the function $f(x) = x^3 + 3(a - 7)x^2 + 3(a^2 - 9)x - 1$ has a positive point of maximum, then -

(A) $a \in (3, \infty) \cup (-\infty, -3)$

(B) $a \in (-\infty, -3) \cup (3, 29/7)$

(C) $(-\infty, 7)$

(D) $(-\infty, 29/7)$

Q.25 Let the function f(x) be defined as below,

$$f(x) = \begin{cases} \sin^{-1}\lambda + x^2, & 0 < x < 1 \\ 2x, & x \ge 1 \end{cases}$$

f(x) can have a minimum at x = 1 then value of λ is -

(A) 1 (B) –1

(C) 0 (D) none of these

LEVEL- 3

Q.1 A differentiable function f(x) has a relative minimum at x = 0, then the function y = f(x) + ax + b has a relative minimum at x = 0 for -

(A) all a and all b (B) all b if a = 0

(C) all b > 0 (D) all a > 0

Q.2 If $a^2x^4 + b^2y^4 = c^6$, then the maximum value of xy is -

(A) $\frac{c^3}{2ab}$ (B) $\frac{c^3}{\sqrt{2ab}}$

(C) $\frac{c^3}{ab}$ (D) $\frac{c^3}{\sqrt{ab}}$

Q.3 The critical points of the function $f(x) = (x - 2)^{2/3}(2x + 1)$ are -

(A) 1 and 2 (B) 1 and –1/2

(C) –1 and 2 (D) 1

Q.4 If $f'(x) = (x - a)^{2n}(x-b)^{2m+1}$ where m, n ∈ N, then -

(A) x = b is point of minimum.

(B) x = b is a point of maximum.

(C) x = b is a point of inflexion.

(D) none of these

Q.5 The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is -

(A) 5 (B) – 5 (C) $\frac{1}{5}$ (D) none

Q.6 Let $f(x) = (x - p)^2 + (x - q)^2 + (x - r)^2$. Then f(x) has a minimum at x = λ where λ is equal to -

(A) $\frac{p+q+r}{3}$ (B) $3\sqrt{pqr}$

(C) $\frac{3}{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$ (D) none of these.

Q.7 If $xy = a^2$ and $S = b^2x + c^2y$ where a, b and c are constants then the minimum value of S is -

(A) abc (B) $bc\sqrt{a}$

(C) 2abc (D) none of these.

➢ Statement type Questions

All questions are Assertion & Reason type questions. Each of these questions contains two statements: Statement-I (Assertion) and Statement-II (Reason). Answer these questions from the following four option.

(A) If both Statement- I and Statement- II are true, and Statement - II is the correct explanation of Statement– I.

(B) If both Statement - I and Statement - II are true but Statement - II is not the correct explanation of Statement – I.

(C) If Statement - I is true but Statement - II is false.

(D) If Statement - I is false but Statement - II is true.

Q.8 **Statement I :** $e^\pi > \pi^e$.

Statement II : The function $x^{1/x}$ (x > 0) has a local maximum at x = e.

Q.9 **Statement I :** If 0 < a < b then absolute min^m value of |x – a| + |x – b| is b – a.

Statement II : The function |x – a| + |x – b| is differentiable at x = b.

Q.10 **Statement I :** The min^m value of the expression $x^2 + 2bx + c$ is $c - b^2$.

Statement II : The first order derivative of the expression at x = –b is zero

➢ Passage Based Questions

Passage:

Let $f(x) = 1 + a^2x - x^3$ where 'a' is real number, point of minima of f(x) must lie between [A, B] where A and B is the minimum and maximum value of $\frac{x^2+3x+1}{x^2+x+1}$ for all x ∈ R.

On the basis of above information, answer the following questions -

Q.11 Find the value of A

(A) – 1 (B) $-\frac{1}{2}$

(C) – 2 (D) None of these

Q.12 Find the value of B

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$

(C) $\frac{5}{4}$ (D) None of these

Q.13 If $a > 0$, then find the point of local minima

(A) $\frac{-a}{\sqrt{3}}$ (B) $\frac{a}{\sqrt{3}}$

(C) $\frac{a}{\sqrt{2}}$ (D) None of these

Q.14 If $a < 0$, then a must lie between -

(A) $[-\sqrt{2}, 0]$ (B) $[-\sqrt{3}, 0]$

(C) $[-\sqrt{3}, 0)$ (D) None of these

Q.15 For what value of a, the above information does not satisfy -

(A) $-\sqrt{2}$ (B) -1

(C) $-\sqrt{5}$ (D) None of these

➢ Column Matching Questions

Match the entry in Column 1 with the entry in Column 2.

Q.16 **For the function in column-I**

Column-I	**Column-II**
(A) $\cos x - 1 + \frac{x^2}{2!} - \frac{x^3}{3!}$	(P) minimum value is -4
(B) $\cos x - 1 + \frac{x^2}{2!}$	(Q) there is no extremum at $x = 0$
(C) $x^4 e^{-x^2}$	(R) the minimum is $f(0) = 0$
(D) $\sin 3x - 3\sin x$	(S) the functions reaches maximum at $x = \sqrt{2}$

Q.17 Let the function defined in column-I have domain $(0, \pi/2)$ the -

Column 1	**Column II**
(A) $x^2 + 2\cos x + 2$ on $(0, \pi/2)$ has	(P) local maximum at $\cos^{-1}(2/3)$
(B) $9x - 4\tan x$ on $(0, \pi/2)$ has	(Q) maximum at $x = 1/2$
(C) $(1/2 - x)\cos x + \sin x - \frac{x^2 - x}{4}$	(R) no local extremum
(D) $\left(\frac{1}{2} - x\right)\cos \pi(x+3) + (1/\pi)\sin \pi(x+3)$ on $(0, 4)$	(S) minimum at $x = 1$

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 If the function $f(x) = 2x^3 - 9ax^2 + 12a^2 x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals **[AIEEE 2003]**

(A) 1/2 (B) 3
(C) 1 (D) 2

Q.2 The real number x when added to its inverse gives the minimum value of the sum at x equal to- **[AIEEE 2003]**

(A) – 2 (B) 2
(C) 1 (D) – 1

Q.3 The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at – **[AIEEE 2006]**

(A) $x = -2$ (B) $x = 0$
(C) $x = 1$ (D) $x = 2$

Q.4 A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is – **[AIEEE 2006]**

(A) $\sqrt{\frac{x^3}{8}}$ (B) $\frac{1}{2}x^2$
(C) πx^2 (D) $\frac{3}{2}x^2$

Q.5 If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is- **[AIEEE 2007]**

(A) 2 (B) $\frac{1}{2}$
(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Q.6 Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds? **[AIEEE 2008]**

(A) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(B) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(C) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(D) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

Q.7 Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1,1]$- **[AIEEE 2009]**

(A) P(–1) is the minimum and P(1) is the maximum of P

(B) P(–1) is not minimum but P(1) is the maximum of P

(C) P(–1) is the minimum but P(1) is not the maximum of P

(D) Neither P(–1) is the minimum nor P(1) is the maximum of P

Q.8 The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is - **[AIEEE 2009, 11]**

(A) $\frac{3\sqrt{2}}{8}$ (B) $\frac{2\sqrt{3}}{8}$ (C) $\frac{3\sqrt{2}}{5}$ (D) $\frac{\sqrt{3}}{4}$

Q.9 Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} k - 2x, & if\ x \le -1 \\ 2x + 3, & if\ x > -1 \end{cases}$$

If f has a local minimum at $x = -1$, then a possible value of k is **[AIEEE 2010]**

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) –1

Q.10 For $x \in \left(0, \frac{5\pi}{2}\right)$, define

$$f(x) = \int_0^x \sqrt{t}\sin t\,dt$$

Then f has : **[AIEEE 2011]**

(A) local maximum at π and 2π

(B) local minimum at π and 2π

(C) local minimum at π and local maximum at 2π

(D) local maximum at π and local minimum at 2π

Q.11 Let a, b $\in$ R be such that the function f given by f (x) = ln |x| + bx^2 + ax, x $\neq$ 0 has extreme values at x = –1 and x = 2. **[AIEEE 2012]**

Statement 1 : f has local maximum at x = – 1 and at x = 2.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(A) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(B) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(C) Statement 1 is true, Statement 2 is false.

(D) Statement 1 is false, Statement 2 is true.

SECTION-B

Q.1 If A > 0, B > 0 and A + B = $\pi/3$, then the maximum value of tan A tan B is **[IIT- 1993]**

(A) 1/3 (B) 2/3 (C) 1/2 (D) None

Q.2 On the interval [0, 1], the function $x^{25}(1-x)^{75}$ takes its maximum value at the point- **[IIT- 1995]**

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Q.3 The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is **[IIT- 1998]**

(A) 0 (B) 1 (C) 2 (D) infinite

Q.4 The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5 dt$ has a local minimum at x = **[IIT- 1999]**

(A) 0, 4 (B) 1, 3 (C) 0, 2 (D) 2, 4

Q.5 Let $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$, then at x = 0, f has - **[IIT Scr. 2000]**

(A) a local maximum (B) no local maximum

(C) a local minimum (D) no extremum

Q.6 Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and m (b) is minimum value of f(x). As b varies, the range of m (b) is- **[IIT Scr. 2001]**

(A) [0, 1] (B) (0, 1/2] (C) [1/2, 1] (D) (0, 1]

Q.7 The value of ‘θ’; θ $\in$ [0, π] for which the sum of intercepts on coordinate axes cut by tangent at point $(3\sqrt{3}\cos\theta, \sin\theta)$ to ellipse $\frac{x^2}{27} + y^2 = 1$ is minimum is : **[IIT Scr. 2003]**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

Q.8 $f(x) = x^2 - 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ if the minimum value of f(x) is always greater than maximum value of g(x) then. **[IIT Scr. 2003]**

(A) $|c| > \sqrt{2}|b|$ (B) $c > \sqrt{2b}$

(C) $c < -\sqrt{2b}$ (D) $|c| < \sqrt{2}|b|$

Q.9 If $f(x) = \sqrt{x^2 + x} + \frac{\tan^2\alpha}{\sqrt{x^2 + x}}$, α $\in$ (0, π/2), x > 0 then value of f(x) is greater than or equal to **[IIT Scr. 2003]**

(A) 2 (B) 2 tan α (C) $\frac{5}{2}$ (D) sec α

Q.10 Let f be a function defined on R (the set of all real numbers) such that

$f'(x) = 2010\ (x-2009)\ (x-2010)^2\ (x-2011)^3\ (x-2012)^4$, for all $x \in R$.

If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ell n\{g(x)\}$, for all $x \in R$, then the number of points in R at which g has a local maximum is **[IIT- 2010]**

(A) 0 (B) 1 (C) 2 (D) 5

Q.11 Let f , g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then **[IIT- 2010]**

(A) $a = b$ and $c \neq b$ (B) $a = c$ and $a \neq b$

(C) $a \neq b$ and $c \neq b$ (D) $a = b = c$

Q.12 Let $f : IR \to IR$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is **[IIT- 2012]**

Q.13 Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p′ (0) is **[IIT- 2012]**

Q.14 If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all $x \in (0, \infty)$, then **[IIT 2012]**

(A) f has a local maximum at x = 2

(B) f is decreasing on (2, 3)

(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D) f has a local minimum at x = 3

Q.15 A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are - **[JEE - Advance 2013]**

(A) 24 (B) 32 (C) 45 (D) 60

Q.16 The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at x = **[JEE - Advance 2013]**

(A) – 2 (B) $\frac{-2}{3}$ (C) 2 (D) $\frac{2}{3}$

Passage for Question 17 and 18

Let $f : [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, f(0) = f(1) = 0 and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0, 1]$.

[JEE - Advance 2013]

Q.17 If the function $e^{-x} f(x)$ assumes its minimum in the interval [0,1] at $x = \frac{1}{4}$, which of the following is true ?

(A) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$

(B) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$

(C) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$

(D) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

Q.18 Which of the following is true for 0 < x < 1 ?

(A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$

(C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

Q.19 A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[JEE - Advance 2013]**

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	A	B	B	A	C	A	B	A	D	C	B	B	C	C	C	B	C	D
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	B	C	A	A	B	B	A	B	C	C	D	B	C	A	A	B	A	C	A	C
Q.No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	D	A	B	C	A	B	C	A	A	D	C	A	C	D	A	D	D	C	A	C
Q.No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**												
Ans.	D	C	A	D	C	A	B	C												

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	B	B	A	B	D	A	C	A	A	C	A	A	A
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**					
Ans.	C	C	B	A	B	C	A	D	B	D					

LEVEL- 3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	B	A	A	A	A	C	A	C	A	A	B	A	C	C

16. A → Q ; B → R ; C → S ; D → P

17. (A) → R, (B) → P , (C) → Q, (D) → Q, S

LEVEL- 4

SECTION-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11
Ans.	D	C	D	B	D	D	B	A	D	D	A

SECTION-B

1.[A] If $A > 0$; $B > 0$

$A + B = \pi/3$

Product of tanA & tanB will be maximum when

$A = B = \pi/6$

$\therefore (\tan A \ \tan B)_{max} = \tan \pi/6 \times \tan \pi/6$

$$= \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$$

2.[B] $f(x) = x^{25}(1 - x)^{75}$

$f'(x) = x^{25}\, 75(1 - x)^{74}(-1) + (1 - x)^{75}.\, 25x^{24}$

$= 25x^{24}(1 - x)^{74}(-3x + 1 - x)$

$= 25x^{24}(x - 1)^{74}(-4x + 1)$

$= -25x^{24}(x - 1)^{74}(4x - 1)$

+   +   –   –
0   1/4   1

$\therefore \quad x = \frac{1}{4}$ is point of maxima

3.[B] $f(x) = \cos x + \cos\sqrt{2}x$

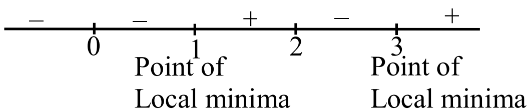
f(x) will be maximum when x = 0

$\therefore$ f(0) = 2

$\therefore$ only one value of x exist

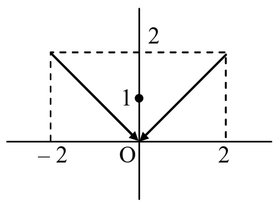
4.[B] $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5\,dt$

$f'(x) = x(e^x - 1)(x-1)(x-2)^3(x-3)^5$



x = 1, 3 point of local minima

5.[A]



$$(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$$

$$= \begin{cases} -x & ; \quad -2 \le x < 0 \\ 1 & ; \quad x = 0 \\ x & ; \quad 0 < x \le 2 \end{cases}$$

$\because$ f(0) > f(0 ± h)

$\therefore$ x = 0 is point of local maxima

6.[D] $f(x) = (1 + b^2)x^2 + 2bx + 1$

$$f(x)_{max} = \frac{4AC - B^2}{4A}$$

$$= \frac{4.(1+b^2).1 - 4b^2}{4(1+b^2)}$$

$$m(b) = \frac{1}{1+b^2} \quad \{\because F_{max} = m(b)\}$$

$\because \infty > b^2 \ge 0$

$\infty > 1 + b^2 \ge 1$

$$0 < \frac{1}{1+b^2} \le 1$$

$\therefore$ range of m(b) is (0, 1]

7.[A] $\frac{x^2}{27} + y^2 = 1$

equation of tangent to the curve at

$(3\sqrt{3}\cos\theta, \sin\theta)$

is $\frac{3\sqrt{3}\cos\theta.x}{27} + \frac{\sin\theta.y}{1} = 1$

$$\Rightarrow \frac{x}{3\sqrt{3}\sec\theta} + \frac{y}{\mathrm{cosec}\theta} = 1$$

sum of intercepts on coordinate axes is

Let $z = 3\sqrt{3}\sec\theta + \mathrm{cosec}\theta$

$\because$ we know $Z_{min} = ((3\sqrt{3})^{2/3} + (1)^{2/3})^{3/2} = 8$

& this minimum value of z occurs at $\theta = \frac{\pi}{6}$

8.[A] $f(x) = x^2 - 2bx + 2c^2$

$$f(x)_{min} = \frac{4.1.2c^2 - 4b^2}{4.1}$$

$$= 2c^2 - b^2$$

& $g(x) = -x^2 - 2cx + b^2$

$$g(x)_{max} = \frac{4.(-1).b^2 - 4c^2}{4.(-1)} = b^2 + c^2$$

According to question

$f(x)_{min} > g(x)_{max}$

$\Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow c^2 > 2b^2$

$\Rightarrow |c| > \sqrt{2}\,|b|$

9.[B] $f(x) = \sqrt{x^2 + x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}$; $\alpha \in \left(0, \frac{\pi}{2}\right)$; x > 0

AM ≥ GM

$$\therefore \frac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2} \ge \sqrt{\tan^2\alpha}$$

$$\Rightarrow f(x) \ge 2\tan\alpha \qquad \left(\because \alpha \in \left(0, \frac{\pi}{2}\right)\right)$$

$\Rightarrow f(x)_{min} = 2\tan\alpha$

10.[B] $f'(x) = 2010\,(x-2009)\,(x-2010)^2$

$(x-2011)^3\,(x-2012)^4$

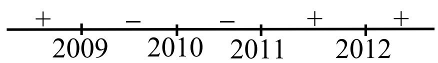
$f(x) = \ell n(g(x))$

$\Rightarrow g(x) = e^{f(x)}$

$\Rightarrow g'(x) = e^{f(x)}.\,f'(x)\ \{\because e^{f(x)} > 0\ \forall x \in R\}$

$\Rightarrow g'(x) = (+ve)\,f'(x)$

Sign convention for g'(x)



$\therefore$ x = 2009 is point of local maxima

$\therefore$ only one point

11.[D] $f(x) = e^{x^2} + e^{-x^2}$

$f'(x) = 2xe^{x^2} - 2xe^{-x^2}$

$$= 2x\left[\frac{e^{x^4}-1}{e^{x^2}}\right] \geq 0\ ;\ x \in [0, 1]$$

$\therefore$ f(x) is increasing function in [0, 1]

$a = f(1)_{max} = e + e = 2e$

$g(x) = xe^{x^2} + e^{-x^2}$

$g'(x) = xe^{x^2}\,2x + e^{x^2}.1 - 2xe^{-x^2}$

$$= \frac{2x^2e^{x^4} + e^{x^4} - 2x}{e^{x^2}} = \frac{(2x^2+1)e^{x^4} - 2x}{e^{x^2}} > 0$$

$\therefore$ g(x) is increasing function in [0, 1]

$\therefore b = g(1)_{max} = e + e = 2e$

$h(x) = x^2e^{x^2} + e^{-x^2}$

$h'(x) = x^2.2x\,e^{x^2} + e^{x^2}.2x - 2xe^{-x^2}$

$$= \frac{(2x^3+2x)e^{x^4} - 2x}{e^{x^2}} > 0$$

$\Rightarrow$ h(x) is increasing function in [0, 1]

$\therefore c = h(1)_{max} = e + e$

12.[5] $f(x) = |\,x\,| + |\,x-1|\,|\,x+1|$

$x \geq 1 f(x) = x^2 + x - 1 f'(x) = 2x + 1$

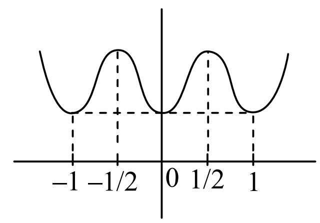
+ve

$0 \leq x < 1 f(x) = 1 - x^2 + x f'(x) = 1 - 2x x > \frac{1}{2}$ –ve

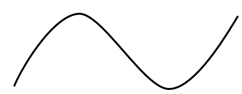
$-1 < x < 0 f(x) = 1 - x^2 - x f'(x)$

$= -2x - 1 x > -\frac{1}{2}$ –ve ; $x < -\frac{1}{2}$ +ve

$x \leq -1 f(x) = x^2 - x - 1 f'(x) = 2x - 1$ –ve



13.[9] p'(1) = 0, p'(3) = 0



$p'(x) = K(x-1)(x-3)$

$= K(x^2 - 4x + 3) p'(0) = 3K$

$p(x) = \frac{K}{3}x^3 - 2K\,x^2 + 3K\,x + \lambda$

$\frac{K}{3} - 2K + 3K + \lambda = 6, 9K - 18K + 9K + \lambda = 2$

$\frac{4}{3}K + \lambda = 6, \frac{4}{3}K = 4$

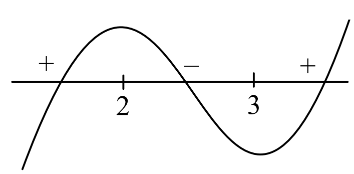
K = 3

p'(0) = 9

14.[A, B, C, D]

$$f(x) = \int_0^x e^{t^2}\,(t-2)\,(t-3)\,dt$$

$f'(x) = e^{x^2}\,(x-2)\,(x-3)$



$f'(x) < 0 \quad \forall x \in (2, 3)$

so f(x) is decreasing on (2, 3)

also at x = 2, $f'(x)$ changes its sign from +ve to –ve.

Hence x = 2 is point of maxima

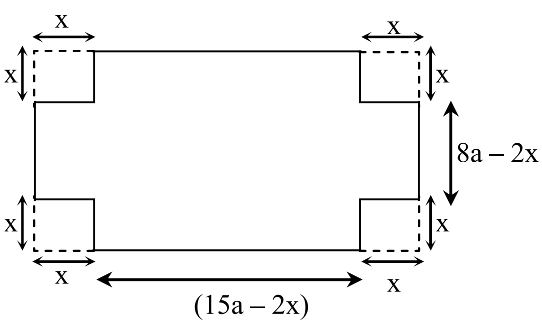
At x = 3, $f'(x)$ changes its sign from –ve to +ve.

Hence x = 3 is point of minima.

Also $f'(2) = f'(3) = 0$

So from Rolle's Theorem there exist a point c such that $f''(c) = 0$

15.[A,C]



$V = (15a - 2x)(8a - 2x)x$

$V = 4x^3 - 46ax^2 + 120a^2x$

$$\frac{dV}{dx} = 12x^2 - 92ax + 120a^2$$

$= 4(3x^2 - 23ax + 30a^2)$

at x = 5, $\frac{dV}{dx} = 0$

$30a^2 - 115a + 75 = 0$

$\Rightarrow 6a^2 - 23a + 15 = 0$

$\Rightarrow (a - 3)(6a - 5) = 0$

$\Rightarrow$ So, a = 3 or $a = \frac{5}{6}$

Now $\frac{d^2V}{dx^2} = 24x - 92a$

For a = 3, $\frac{d^2V}{dx^2} < 0$

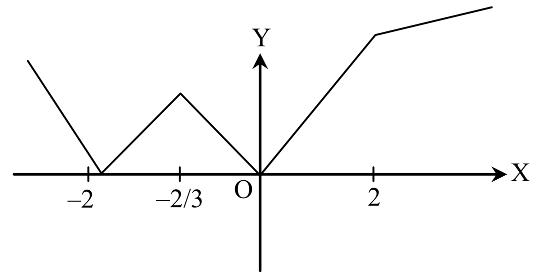
So, V is maximum for a = 3.

Hence lengths are 24 and 45.

16.[A,B]

$f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$

$$= \begin{cases} -2x - 4, & x < -2 \\ 2x + 4, & -2 \le x < -\frac{2}{3} \\ -4x, & -\frac{2}{3} \le x < 0 \\ 4x, & 0 \le x < 2 \\ 2x + 4, & x \ge 2 \end{cases}$$



Clearly point of minima x = – 2, 0

Point of maxima $x = \frac{-2}{3}$

17.[C] Let $\phi(x) = e^{-x}f(x)$; $x \in [0,1]$

$\phi'(x) = e^{-x}(f'(x) - f(x))$

as $\phi_{min}(x) = \phi(1/4)$ so $\phi'(1/4) = 0$

and $\phi''(x) = e^{-x}(f''(x) - 2f'(x) + f(x)) > 0$; for $x \in [0,1]$ (given)

so $\phi'(x)$ increases for $x \in [0,1]$ and $\phi'(1/4) = 0$

so $\phi'(x) < 0 \Rightarrow f'(x) < f(x)$ for $x \in (0, 1/4)$

and $\phi'(x) > 0 \Rightarrow f'(x) > f(x)$ for $x \in (1/4, 1)$

18.[D]

From previous questions

$$\left.\begin{matrix} \phi_{min}(x) = \phi(1/4), \phi(0) = f(0) = 0 \\ \& \phi(1) = e^{-1}f(1) = 0 \end{matrix}\right\} \text{given}$$

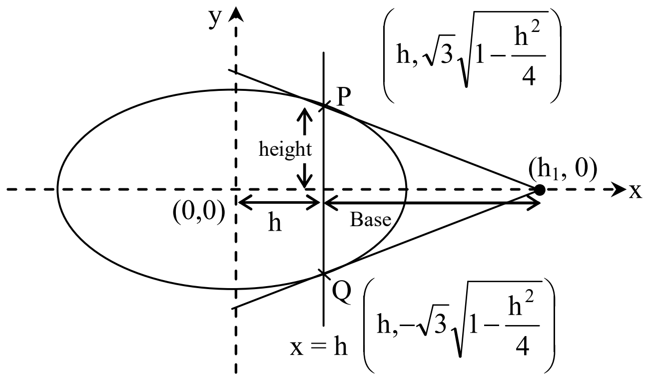
hence $\phi_{max} = \phi(0) = \phi(1) = 0$

so, $\phi(x) < 0$; for $x \in (0,1)$

$e^{-x}f(x) < 0$; for $x \in (0,1)$

$f(x) < 0$; for $x \in (0,1)$

19.[9]



Line PQ is chord of contact

$\Rightarrow \frac{xh_1}{4} + 0 = 1….(1)$

$x = h…..(2)$

Compare (1) & (2)

$h_1 = \frac{4}{h}$

So area $= \left(\frac{4}{h} - h\right) \times \sqrt{3}\sqrt{1-\frac{h^2}{4}}$

$= \frac{\sqrt{3}}{2} \frac{(4-h^2)^{3/2}}{h}$ regular decreasing

$\left(\text{Area}_{\max}\right)_{h=1/2} = \frac{\sqrt{3}}{2} \frac{\left(4-\frac{1}{h}\right)^{3/2}}{1/2}$,

$(\text{Area})_{\min \atop h=1} = \frac{\sqrt{3}}{2}(3)^{3/2}$

$= \frac{\sqrt{3}}{2}(\sqrt{15})^{3/2}$

So, $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = \frac{8}{\sqrt{5}} \times \frac{\sqrt{3}}{8} \times (\sqrt{15})^{3/2} - 8$

$\times \frac{\sqrt{3}}{2}(3)^{3/2}$

$= 5 \times 9 - 4 \times 9$

$= 45 - 36$

$= 9$