MATHEMATICS

Class-IX

Topic-9

AREA OF PARALLELOGRAMS AND TRIANGLES

CLASSROOM

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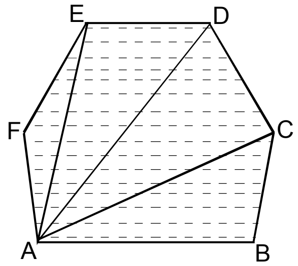
CH-09

AREA OF PARALLELOGRAMS AND TRIANGLES

(A) AREA OF PARALLELOGRAMS AND TRIANGLES

(a) Polygon region

Polygon region can be expressed as the union of a finite number of triangular regions in a plane such that if two of these intersect, their intersection is either a point or a line segment. It is the shaded portion including its sides as shown in the **figure**.



Area Axioms :

Every polygonal region R has an area, measured in square units and denoted by ar(R).

(i) Congruent area axiom : If R_1 and R_2 be two regions such that $R_1 \cong R_2$ then $ar(R_1) = ar(R_2)$.

(ii) Area addition axiom : If R_1 and R_2 are two polygonal regions, whose intersection is a finite number of points & line segments such that $R = R_1 \cup R_2$, then $ar(R) = ar(R_1) + ar(R_2)$.

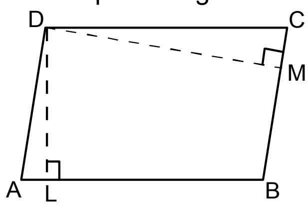
(iii) Rectangular area axiom : If AB = a metre and AD = b metre then,
ar (Rectangular region ABCD) = ab sq.m.

(b) Area of a parallelogram

Base and Altitude of a Parallelogram :

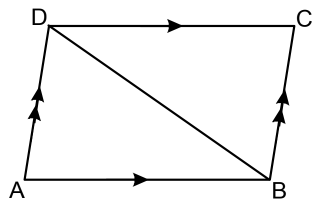
(i) Base : Any side of a parallelogram can be called its base.

(ii) Altitude : The length of the line segment which is perpendicular to the base from the opposite side is called the altitude or height of the parallelogram corresponding to the given base.



(i) DL is the altitude of **||gm ABCD**, corresponding to the base **AB**.
(ii) DM is the altitude of **||gm ABCD**, corresponding to the base **BC**.

Theorem : A diagonal of a parallelogram divides it into two triangles of equal area.
Given : A parallelogram ABCD whose one of the diagonals is BD.
To prove : ar (ΔABD) = ar (ΔCDB).
Proof



In ΔABD and ΔCDB

AB = DC [Opposite sides of a $||^{gm}$]

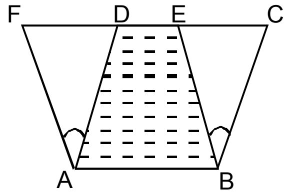
AD = BC [Opposite sides of a $||^{gm}$]

BD = BD [Common side]

ΔABD ≅ ΔCDB [By SSS congruency]

ar (ΔABD) = ar (ΔCDB) **Hence Proved.**

Theorem : Parallelograms on the same base and between the same parallels lines are equal in area.



Given : Two $||^{gms}$ ABCD and ABEF on the same base AB and between the same parallels AB and FC.

To prove : ar($||^{gm}$ ABCD) = ar($||^{gm}$ ABEF)

Proof : In ΔADF and ΔBCE, we have

AD = BC [Opposite sides of a $||^{gm}$]

AF = BE [Opposite sides of a $||^{gm}$]

∠DAF = ∠CBE [AD || BC and AF || BE]

[Angle between AD and AF = Angle between BC and BE]

∴ ΔADF ≅ ΔBCE [By SAS congruency]

∴ ar(ΔADF) = ar(ΔBCE) ...(i)

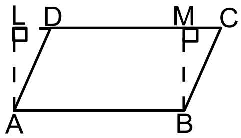
∴ ar($||^{gm}$ ABCD) = ar(ABED) + ar(ΔBCE)

= ar(ABED) + ar(ΔADF) [Using (i)]

= ar($||^{gm}$ ABEF).

Hence, ar($||^{gm}$ ABCD) = ar($||^{gm}$ ABEF). **Hence Proved.**

Theorem : The area of parallelogram is the product of its base and the corresponding altitude.



Given : A $||^{gm}$ ABCD in which AB is the base and AL is the corresponding height.

To prove : Area ($||^{gm}$ ABCD) = AB × AL.

Construction : Draw BM ⊥DC, so that rectangle ABML is formed.

Proof : $||^{gm}$ ABCD and rectangle ABML are on the same base AB and between the same parallel lines AB and LC.

ar($||^{gm}$ ABCD) = ar(rectangle ABML) = AB × AL.

area of a $||^{gm}$ = base × height. **Hence Proved.**

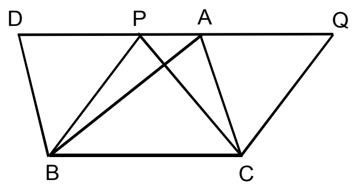
(c) Area of a Triangle

Theorem : Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Given : Two triangles ABC and PBC on the same base BC and between the same parallel lines BC and AP.

To prove : ar(ΔABC) = ar(ΔPBC)

Construction : Through B, draw BD || CA intersecting AP produced in D and through C, draw CQ || BP, intersecting PA produced in Q.



Proof : BD || CA [By construction]
And, BC || DA [Given]
∴ Quadrilateral BCAD is a parallelogram.
Similarly, Quadrilateral BCQP is a parallelogram.
Now, parallelogram BCQP and BCAD are on the same base BC, and between the same parallels.
∴ ar ($||^{gm}$ BCQP) = ar ($||^{gm}$ BCAD) ....(i)
Diagonals of a parallelogram divides it into two triangles of equal area.

∴ ar (ΔPBC) = $\frac{1}{2}$ ar ($||^{gm}$ BCQP) ...(ii)

And ar (ΔABC) = $\frac{1}{2}$ ar ($||^{gm}$ BCAD) ..(iii)

Now, ar ($||^{gm}$ BCQP) = ar ($||^{gm}$ BCAD) [From (i)]

⇒ $\frac{1}{2}$ ar ($||^{gm}$ BCAD) = $\frac{1}{2}$ ar ($||^{gm}$ BCQP)

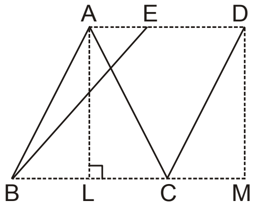
Hence, ar (ΔABC) = ar (ΔPBC) [Using (ii) and (iii)] **Hence Proved.**

Theorem : If a triangle and a parallelogram are on the same base and between the same parallels, the area of the triangle is equal to half of the parallelogram.
Given : A ΔABC and a paralellogram BCDE on the same ase BC and between the same parallels BC and AD.

To prove : ar (ΔABC) = $\frac{1}{2}$ ar (parallelogram BCDE)

Construction : Draw AL ⊥ BC and DM ⊥ BC, meeting BC producted in M.
Proof : Since, E and D are colinear and BC || AD
∴ AL = DM ....(i) [∵ distance between parallel lines is always same]

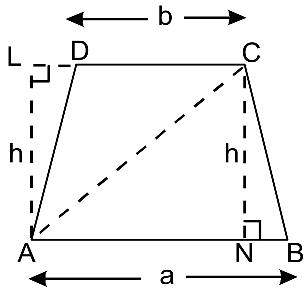
Now, ar (ΔABC) = $\frac{1}{2}$ (BC × AL)



⇒ ar (ΔABC) = $\frac{1}{2}$ (BC × DM) [∵ AL = DM (from (i)]

⇒ ar (ΔABC) = $\frac{1}{2}$ ar (parallelogam BCDE).

Theorem : The area of a trapezium is half the product of its height and the sum of the parallel sides.



Given : Trapezium ABCD in which AB || DC, AL ⊥ DC, CN ⊥ AB and AL = CN = h (say), AB = a, DC = b.

To prove : ar(trapezium ABCD) = $\frac{1}{2}$ h × (a + b).

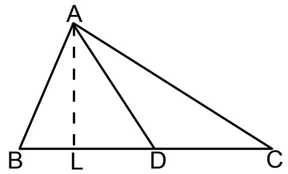
Construction : Join AC.

Proof : ∵ AC is a diagonal of quad. ABCD.

∴ ar(trapezium ABCD) = ar(ΔABC) + ar(ΔACD)

$= \frac{1}{2}h \times a + \frac{1}{2}h \times b = \frac{1}{2}h(a + b)$. **Hence Proved.**

Theorem : Median of a triangle divides it into two triangles of equal area.
Given : A ΔABC in which AD is the median.
To Prove : ar (ΔABD) = ar(ΔADC)
Construction : Draw AL⊥BC
Proof : Since, AD is the median of ΔABC. Therefore, D is the mid point of BC.



⇒ $ar(\Delta ABD) = \frac{1}{2}(BD \times AL)$ .. (i)

⇒ $ar(\Delta ADC) = \frac{1}{2}(CD \times AL)$

⇒ $ar(\Delta ADC) = \frac{1}{2}(BD \times AL)$ .. (ii) [∵ BD = CD, AD is the median of ΔABC]

From (i) & (ii)

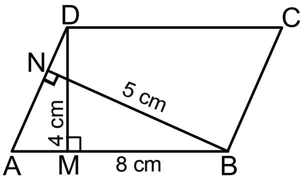
ar (ΔABD) = ar(ΔADC). **Hence Proved.**

Solved Examples

Example. 1

In a parallelogram ABCD, AB = 8 cm. The altitudes corresponding to sides AB and AD are respectively 4 cm and 5 cm. Find AD.

Sol. Area of a $||^{gm}$ = Base × corresponding altitude



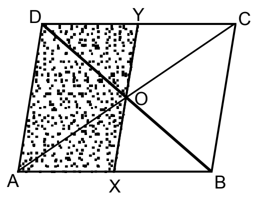
Area of parallelogram ABCD = AD × BN = AB × DM
AD × 5 = 8 × 4

$AD = \frac{8 \times 4}{5} = 6.4$ cm.

Example.2

The diagonals of a parallelogram ABCD intersect in O. A line through O meets AB in X and the opposite side CD in Y. Show that ar (quadrilateral AXYD) = $\frac{1}{2}$ ar(parallelogram ABCD).

Sol. AC is a diagonal of the parallelogram ABCD.



$ar(\Delta ACD) = \frac{1}{2}ar(|| \text{ gm } ABCD)$ ...(i)

Now, in Δs AOX and COY,

AO = CO [∵Diagonals of a parallelogram bisect each other]

∠AOX = ∠COY [Vertically opposite ∠s]

∠OAX = ∠OCY [Alternate interior ∠s]

[∵ AB || DC and transversal AC intersects them]

∴ ΔAOX ≅ ΔCOY [By ASA congruency]

ar(ΔAOX) = ar(ΔCOY) ...(ii)

Adding ar(quad. AOYD) to both sides of (ii), we get

ar(quad. AOYD) + ar(AOX) = ar (quad. AOYD) + ar (COY)

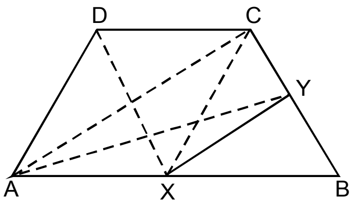
⇒ ar(quad. AXYD) = ar(ACD)

$= \frac{1}{2}$ ar(|| gm ABCD) **Hence Proved.**

Example. 3

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar(ΔADX) = ar(ΔACY).

Sol.



Join CX, DX and AY.

Clearly, triangles ADX and ACX are on the same base AX and between the parallels AB and DC.

∴ ar (ΔADX) = ar (ΔACX) ... (i)

Also, Δ ACX and Δ ACY are on the same base AC and between the parallels AC and XY.

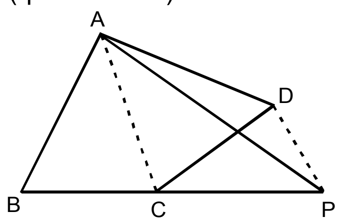
∴ ar (ΔACX) = ar (ΔACY) ...(ii)

From (i) and (ii), we get

ar (ΔADX) = ar (ΔACY).

Example. 4

ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P as shown in figure. Prove that ar (ΔABP) = ar(quad. ABCD).



Sol. Since Δs ACP and ACD are on the base AC and between the same parallels AC and DP.

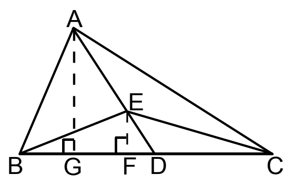
∴ ar(ΔACP) = ar(ΔACD)

⇒ ar(ΔACP) + ar(ΔABC) = ar(ΔACD) + ar(ΔABC)

⇒ ar(ΔABP) = ar(quad. ABCD).

Example. 5

In figure, E is any point on median AD of a ΔABC. Show that ar(ΔABE) = ar(ΔACE).



Sol. **Construction :** From A, draw AG⊥BC and from E draw EF⊥BC.

Proof : ar(ΔABD) = $\frac{BD \times AG}{2}$ and ar(ΔADC) = $\frac{DC \times AG}{2}$

But, BD = DC [∵D is the mid-point of BC, AD being the median]

∴ ar(ΔABD) = ar(ΔADC) ... (i)

Again, ar(ΔEBD) = $\frac{BD \times EF}{2}$ and ar(ΔEDC) = $\frac{DC \times EF}{2}$

But, BD = DC

ar(ΔEBD) = ar(ΔEDC) .. (ii)

Subtracting (ii) from (i), we get

ar(ΔABD) – ar(ΔEBD) = ar(ΔADC) – ar(ΔEDC)

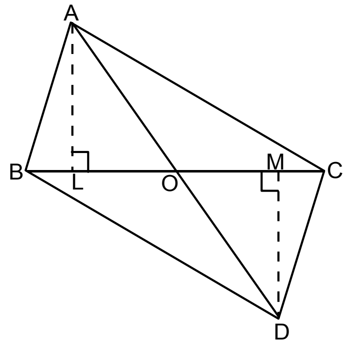
⇒ ar(ΔABE) = ar(ΔACE). **Hence Proved.**

Example. 6

Triangles ABC and DBC are on the same base BC; with A, D on opposite sides of the line BC, such that ar(ΔABC) = ar(ΔDBC). Show that BC bisects AD.

Sol. **Construction :** Draw AL⊥BC and DM⊥BC

Proof :



ar(ΔABC) = ar(ΔDBC) [Given]

⇒ $\frac{BC \times AL}{2} = \frac{BC \times DM}{2}$

⇒ AL = DM ...(i)

Now in Δs OAL and OMD

AL = DM [From (i)]

⇒ ∠ALO = ∠DMO [Each = 90º]

⇒ ∠AOL = ∠MOD [Vertically opposite ∠s]

∴ ΔOLA ⊥ ΔOMD [By AAS congruency]

∴ OA = OD [By CPCT]

i.e., BC bisects AD. **Hence Proved.**

Example. 7

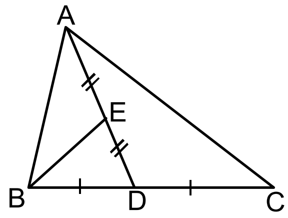
ABC is a triangle in which D is the mid-point of BC and E is the mid-point of AD. Prove that the area of ΔBED = $\frac{1}{4}$ area of ΔABC.

Sol. **Given :** A ΔABC in which D is the mid-point of BC and E is the mid-point of AD.

To prove : ar(ΔBED) = $\frac{1}{4}$ar(ΔABC).

Proof : ∵ AD is a median of ΔABC.

∴ ar (ΔABD) = ar(ΔADC) = $\frac{1}{2}$ar (ΔABC) ... (i)



[∵ Median of a triangle divides it into two triangles of equal area]

Again,

∵ BE is a median of ΔABD.

∴ $ar(\Delta BEA) = ar(\Delta BED) = \frac{1}{2} ar(\Delta ABD)$

[∵ Median of a triangle divides it into two triangles of equal area]

And $ar(\Delta BED) = \frac{1}{2} ar(\Delta ABD) = \frac{1}{2} \times \frac{1}{2} ar(\Delta ABC)$ [From (i)]

$ar(\Delta BED) = \frac{1}{4} ar(\Delta ABC)$. **Hence Proved.**

Example. 8

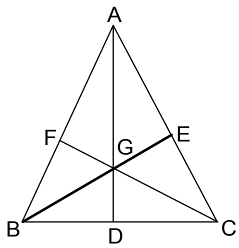
If the medians of a ΔABC intersect at G, show that

$$ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC).$$

Sol. **Given :** A ΔABC and its medians AD, BE and CF intersect at G.

To prove :

$ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$.



Proof : A median of a triangle divides it into two triangles of equal area.

In ΔABC, AD is the median.

∴ $ar(\Delta ABD) = ar(\Delta ACD)$ ...(i)

In ΔGBC, GD is the median.

∴ $ar(\Delta GBD) = ar(\Delta GCD)$ ...(ii)

Subtract equation (ii) from (i), we get

$ar(\Delta ABD) - ar(\Delta GBD) = ar(\Delta ACD) - ar(\Delta GCD)$

$ar(\Delta AGB) = ar(\Delta AGC)$ ..(iii)

Similarly, $ar(AGB) = ar(BGC)$ ..(iv)

From (iii) & (iv)

$ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC)$

But, $ar(\Delta ABC) = ar(\Delta AGB) + ar(\Delta AGC) + ar(\Delta BGC) = 3\, ar(\Delta AGB)$

$ar(\Delta AGB) = \frac{1}{3} ar(\Delta ABC)$.

Hence, $ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$. **Hence proved.**

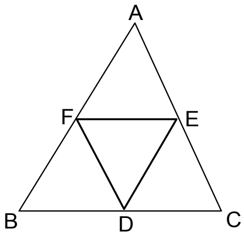
Example. 9

D, E and F are respectively the mid points of the sides BC, CA and AB of a ABC. Show that :

(i) BDEF is a parallelogram **(ii)** ar(||^{gm}BDEF) = $\frac{1}{2}$ar(ABC)

(iii) ar(ΔDEF) = $\frac{1}{4}$ar(ABC)

Sol.



(i) In ΔABC,

∵ F is the mid-point of side AB and E is the mid point of side AC.

∴ EF || BD

[∵ Line joining the mid-points of any two sides of a Δ is parallel to the third side.]

Similarly, ED || FB.

Hence, BDEF is a parallelogram. **Hence Proved.**

(ii) Similarly, we can prove that AFDE and FDCE are parallelograms.

∵ FD is a diagonal of parallelogram BDEF.

∴ ar(ΔFBD) = ar(ΔDEF) ...(i)

Similarly, ar(ΔFAE) = ar(ΔDEF) ...(ii)

And , ar(ΔDCE) = ar(ΔDEF) ...(iii)

From above equations, we have

ar(ΔFBD) = ar(ΔFAE) = ar(ΔDCE) = ar(ΔDEF)

and ar (ΔFBD) + ar (ΔDCE) + ar (ΔDEF) + ar (ΔFAE) = ar (ΔABC)

⇒ 2 [ar(ΔFBD)+ ar(ΔDEF)] = ar(ΔABC) [By using (ii) and (iii)]

⇒ 2 [ar. (II^{gm} BDEF)] = ar (ΔABC) ⇒ ar (||^{gm} BDEF) = $\frac{1}{2}$ ar (ΔABC).

(iii) Since, ΔABC is divided into four non-overlapping triangles FBD, FAE, DCE and DEF.

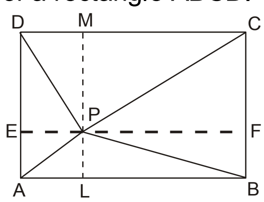
∴ ar(ΔABC) = ar(ΔFBD) + ar(ΔFAE) + ar(ΔDCE) + ar(ΔDEF)

⇒ ar (ΔABC) = 4 ar (ΔABC) [Using (i), (ii) and (iii)]

⇒ ar(ΔDEF) = $\frac{1}{4}$ ar(ΔABC). **Hence Proved.**

Example.10

In figure, P is a point in the interior of a rectangle ABCD. Show that



(i) ar(ΔAPD) + ar(ΔPBC) = $\frac{1}{2}$ar(rectangle ABCD)

(ii) ar(ΔAPD) + ar(ΔPBC) = ar(ΔAPB) + ar(ΔPCD)

Sol. **Construction :** Draw EPF || AB || CD and LPM || AD || BC.

Proof :

(i) EPF || AB and DA cuts them.

∴ $\angle DEP = \angle EAB = 90^\circ$ [Corresponding angles]

∴ $PE \perp AD$.

Similarly, $PF \perp BC$; $PL \perp AB$ and $PM \perp DC$.

$\therefore ar(\Delta APD) + ar(\Delta BPC) = \left(\frac{1}{2}\times AD\times PE\right)+\left(\frac{1}{2}\times BC\times PF\right)=\frac{1}{2}AD\times(PE+PF)$ [$\because BC = AD$]

$= \frac{1}{2} \times AD \times EF = \frac{1}{2} \times AD \times AB$ [$\because EF = AB$]

$= \frac{1}{2} \times$ ar(rectangle ABCD).

(ii) $ar(\Delta APB) + ar(\Delta PCD) = \left(\frac{1}{2}\times AB\times PL\right)+\left(\frac{1}{2}\times DC\times PM\right)=\frac{1}{2}\times AB\times(PL+PM)$ [$\because DC = AB$]

$= \frac{1}{2} \times AB \times LM = \frac{1}{2} \times AB \times AD$ [$\because LM = AD$]

$= \frac{1}{2} \times$ ar(rect. ABCD).

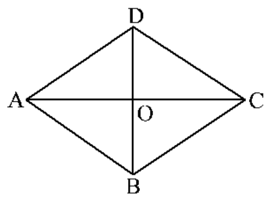
∴ $ar(\Delta APD) + ar(PBC) = ar(\Delta APB) + ar(PCD)$ **Hence Proved.**

Check Your Level

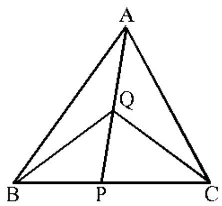
1. Find the area of the parallelogram whose base is 8.5 cm and height 4 cm.

2. Find the base of a parallelogram whose area is 85 sq. cm and the altitude is 17 cm.

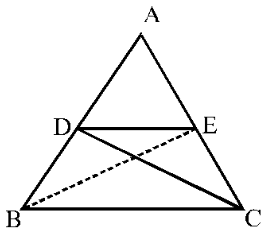
3. Prove that the area of a rhombus is equal to half the product of its diagonals.



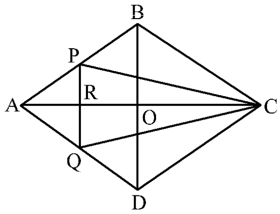
4. In Δ ABC, P is any point on the base BC. Q is the mid point of AP. Show that area of the $\Delta QBC = \frac{1}{2}$ area of ΔABC.



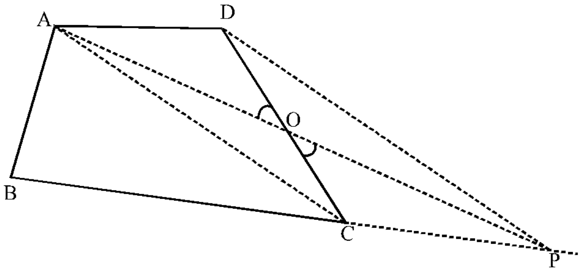
5. ABC is a triangle and a straight line DE, drawn parallel to BC cuts the sides AB and AC at D and E respectively. Prove that area of Δ ABE = area of Δ ACD.



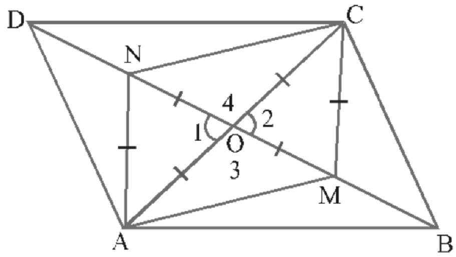
6. In the quadrilateral ABCD, diagonal BD bisects AC at right angles. If P and Q are the middle points of AB and AD respectively, prove that Δ PQC = $\frac{3}{8}$ quadrilateral ABCD.



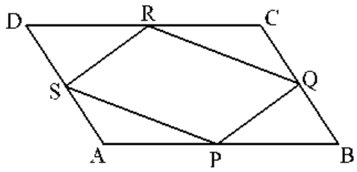
7. ABCD is a quadrilateral. A line drawn through D parallel to AC meets BC produced at P. Prove that
(i) Area of Δ BAP = Area of quadrilateral ABCD
(ii) Area of Δ AOD = Area of Δ COP



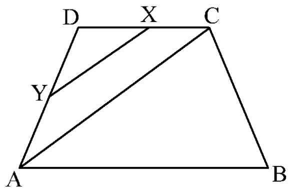
8. ABCD is a parallelogram whose diagonals AC and BD meet at O. M and N are the mid points of OB and OD respectively. Prove that AMCN is a parallelogram whose area is half that of ABCD.



9. In the figure ABCD is a parallelogram. P, Q, R and S are the mid points of AB, BC, CD and DA respectively. Prove that the area of the parallelogram PQRS in equal to the half the area of the parallelogram ABCD.



10. In the figure ABCD is a trapezium and YX || AC. Show that area of triangle BCX is equal to area of triangle ACY.



Answers

1. 34 cm^2 **2.** 5 cm

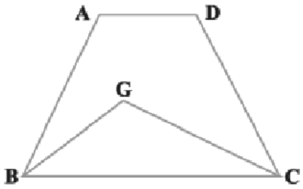
Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]

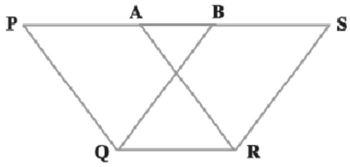
1. Find the area of the figure formed by joining the mid-points of the adjacent sides of a rhombus with diagonals 12 cm and 16 cm.

2. In which of the following figures (Figure), you find two polygons on the same base and between the same parallels?

(i) 

(ii) 

(iii) 

(iv) 

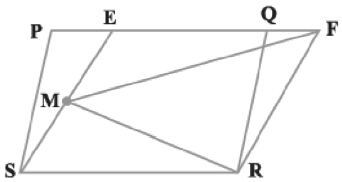
3. Name the figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm and also find its area.

4. The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to x (ar ΔABC). Find x.

5. If a triangle and a parallelogram are on the same base and between same parallels, then find the ratio of the area of the triangle to the area of parallelogram.

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

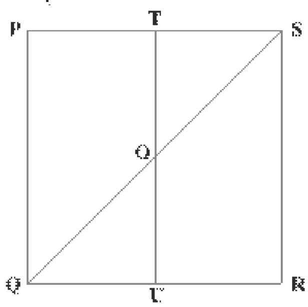
6. If in Figure , PQRS and EFRS are two parallelograms, then prove that ar (MFR) = $\frac{1}{2}$ ar (PQRS).



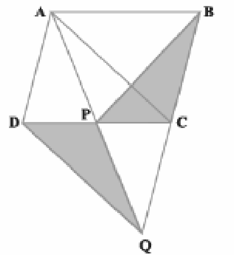
7. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on QR. If PS = 5 cm, then prove that ar (PAS) = 30 cm^2.

8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then prove that ar (BDE) = $\frac{1}{4}$ ar (ABC).

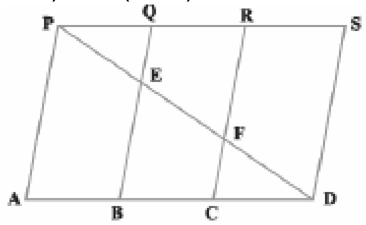
9. PQRS is a square. T and U are respectively, the mid-points of PS and QR (Figure). Find the area of ΔOTS, if PQ = 8 cm, where O is the point of intersection of TU and QS.



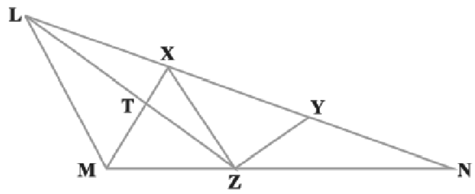
10. ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ (Figure). If AQ intersects DC at P, show that ar (BPC) = ar (DPQ)



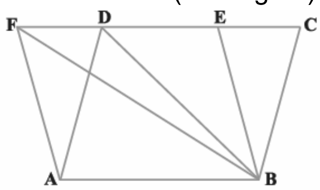
11. In Figure , PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC. Prove that ar (PQE) = ar (CFD).



12. X and Y are points on the side LN of the triangle LMN such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (See Figure). Prove that ar (LZY) = ar (MZYX)

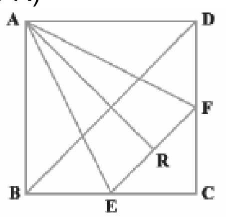


13. The area of the parallelogram ABCD is 90 cm^2 (see Figure). Find



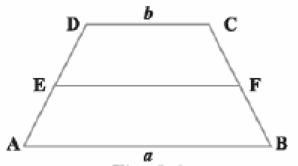
(i) ar (ABEF) **(ii)** ar (ABD) **(iii)** ar (BEF)

14. ABCD is a square. E and F are respectively the mid- points of BC and CD. If R is the mid-point of EF (Figure), prove that ar (AER) = ar (AFR)

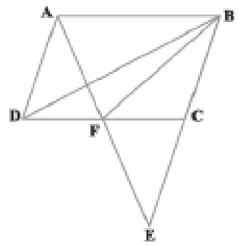


TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[04 MARK EACH]**

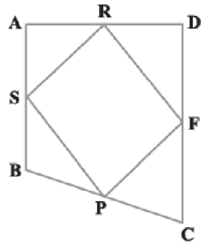
15. ABCD is a trapezium with parallel sides AB = a cm and DC = b cm (Figure). E and F are the mid-points of the non-parallel sides. Then find ratio of ar (ABFE) and ar (EFCD).



16. ABCD is a parallelogram in which BC is produced to E such that CE = BC (Figure). AE intersects CD at F. If ar (DFB) = 3 cm^2, find the area of the parallelogram ABCD.

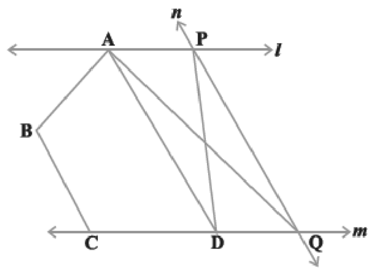


17. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral (Figure).

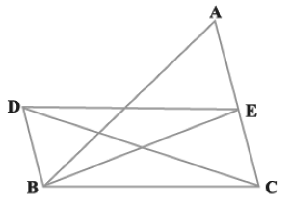


[Hint: Join BD and draw perpendicular from A on BD.]

18. In Figure l, m, n, are straight lines such that l || m and n intersects l at P and m at Q. ABCD is a quadrilateral such that its vertex A is on l. The vertices C and D are on m and AD || n. Show that ar (ABCQ) = ar (ABCDP)



19. In Figure, BD || CA, E is mid-point of CA and BD = $\frac{1}{2}$ CA. Prove that ar (ABC) = 2ar(DBC).

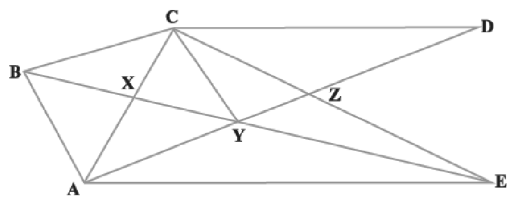


20. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that ar (ADF) = ar (ABFC)

21. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

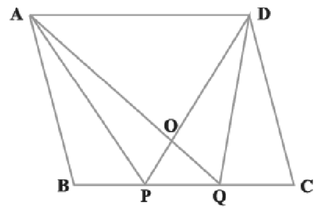
22. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of ΔGBC = area of the quadrilateral AFGE.

23. In Figure, CD || AE and CY || BA. Prove that ar (CBX) = ar (AXY)



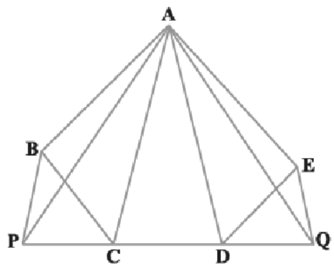
TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS [05 MARK EACH]

24. In Figure, ABCD is a parallelogram. Points P and Q on BC trisects BC in three equal parts. Prove that ar (APQ) = ar (DPQ) = $\frac{1}{6}$ ar (ABCD)



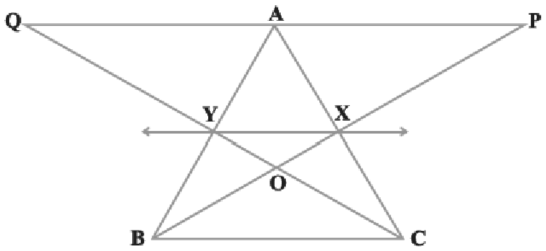
25. ABCD is a trapezium in which AB || DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that ar (DCYX) = $\frac{7}{9}$ ar (XYBA).

26. In Figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar (ABCDE) = ar (APQ)

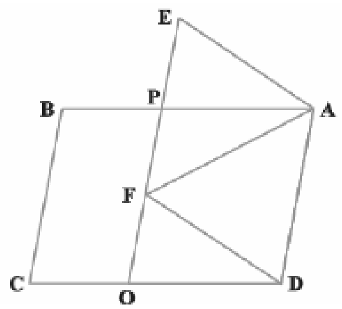


27. If the medians of a ΔABC intersect at G, show that ar (AGB) = ar (AGC) = ar (BGC) = $\frac{1}{3}$ ar (ABC).

28. In Figure , X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (ABP) = ar (ACQ).



29. In Figure, ABCD and AEFD are two parallelograms. Prove that ar (PEA) = ar (OFD)



[**Hint:** Join PD].

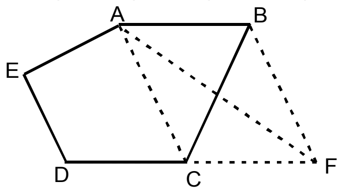
Exercise-1

SUBJECTIVE QUESTIONS

<u>Subjective Easy, only learning value problems</u>

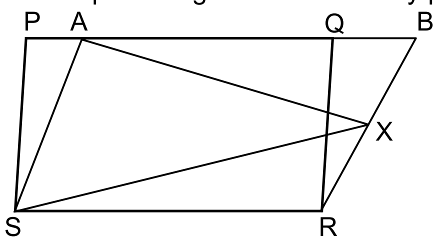
Section (A) : Area of parallelograms and triangles

A-1 ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that :
(i) ar (ΔACB) = ar (ΔACF) **(ii)** ar (AEDF) = ar (ABCDE)



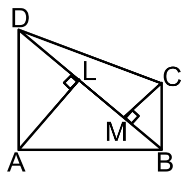
A-2. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Prove that : ar (ΔAPB) = ar (ΔBQC).

A-3. In the figure, PQRS and ABRS are parallelograms and X is any point on side BR. Prove that :

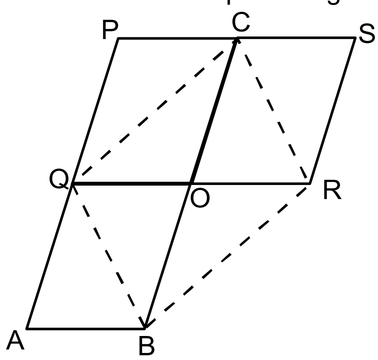


(i) ar(||^{gm} PQRS) = ar(||^{gm} ABRS) **(ii)** ar(AXS) = $\frac{1}{2}$ ar(||^{gm} PQRS)

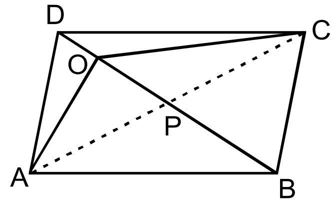
A-4. BD is one of the diagonals of a quadrilateral ABCD. If ALBD and CMBD, show that : ar(quadrilateral ABCD) = $\frac{1}{2}$ × BD × (AL + CM).



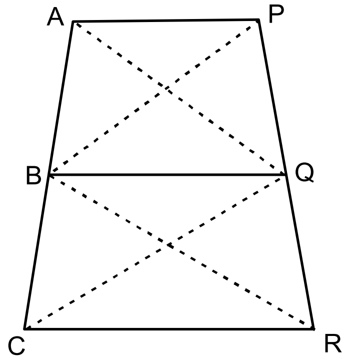
A-5. In the adjoining figure, PQRS and PABC are two parallelograms of equal area. Prove that QC || BR.



A-6. O is any point on the diagonal BD of the parallelogram ABCD. Prove that ar (ΔOAB) = ar(ΔOBC).

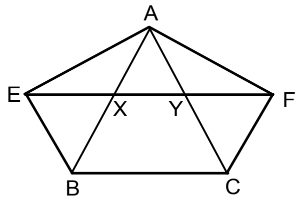


A-7. In figure, AP || BQ || CR. Prove that ar(ΔAQC) = ar(ΔPBR).

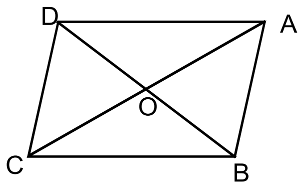


A-8. The base BC of ΔABC is divided at D such that BD = $\frac{1}{2}$DC. Prove that ar(ΔABD) = $\frac{1}{3}$ar(ΔABC).

A-9. In the given figure, XY is a line parallel to side BC of a ΔABC. BE || AC and CF || AB meet XY in E and F respectively. Show that ar(ΔABE) = ar(ΔACF).

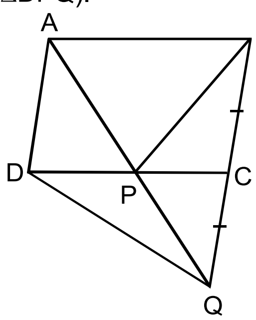


A-10. Diagonals AC and BD of a quadrilateral ABCD intersect at O, such that OB = OD. If AB = CD, then show that :

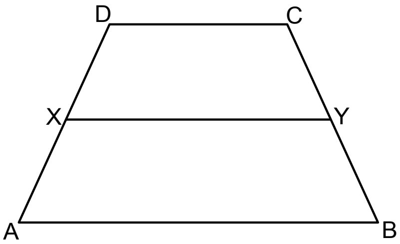


(i) ar (ΔDOC) = ar (ΔAOB) **(ii)** ar (ΔDCB) = ar (ΔACB)
(iii) DA || CB or ABCD is a parallelogram.

A-11. In figure, ABCD is a parallelogram and BC is produced to point Q such that AD = CQ. If AQ intersect DC at P, show that ar(ΔBPC) = ar(ΔDPQ).



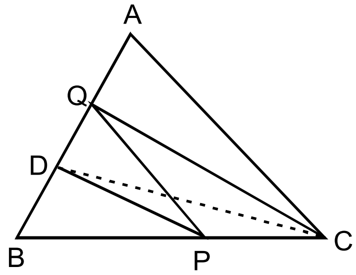
A-12. In figure, ABCD is a trapezium in which AB || DC and DC = 40 cm and AB = 60 cm. If X and Y are, respectively, the mid-points of AD and BC, prove that :



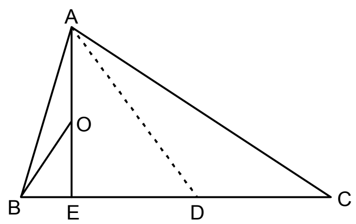
(i) XY = 50 cm **(ii)** DCYX is a trapezium

(iii) Area (trapezium DCYX) = $\frac{9}{11}$ Area (trapezium XYBA)

A-13. In ΔABC, D is the midpoint of AB. P is any point on BC. CQ || PD meets AB in Q. Show that ar(ΔBPQ) = $\frac{1}{2}$ ar(ΔABC) .



A-14. D is the midpoint of side BC of ΔABC and E is the midpoint of BD. If O is the midpoint of AE, prove that ar(ΔBOE) = $\frac{1}{8}$ ar(ΔABC).

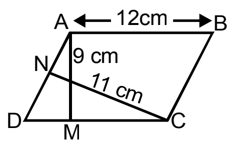


OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

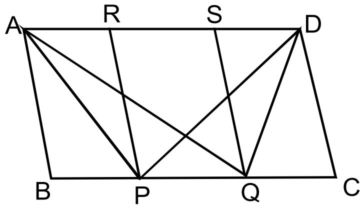
Section (A) : Area of parallelograms and triangles

A-1. In parallelogram ABCD, AB = 12 cm. The altitudes corresponding to the sides AB and AD are respectively 9 cm and 11 cm. Find AD.



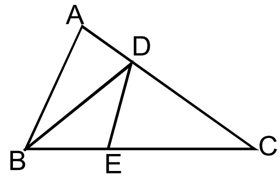
(A) $\frac{108}{11}$ cm (B) $\frac{108}{10}$ cm (C) $\frac{99}{10}$ cm (D) $\frac{108}{17}$ cm

A-2. ABCD is a parallelogram. Points P and Q, on BC trisects it in three equal parts. PR and QS are also drawn parallel to AB, then ar(APQ) = ... ar(ABCD).



(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

A-3. In the figure, D and E are the mid-point of the sides AC and BC respectively of ΔABC. If ar(ΔBED) = 12 cm^2, then ar (ΔAEC) =

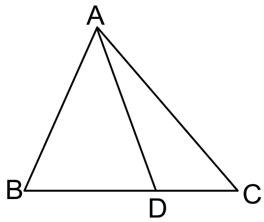


(A) 48 cm^2 (B) 24 cm^2 (C) 36 cm^2 (D) none of these

A-4. In ABC, AD is a median and P is a point on AD such that AP : PD = 1 : 2, then the area of ABP =

(A) $\frac{1}{2}$ × Area of ABC (B) $\frac{2}{3}$ × Area of ABC (C) $\frac{1}{3}$ × Area of ABC (D) $\frac{1}{6}$ × Area of ABC

A-5. In ΔABC, if AD divides BC in the ratio m : n then area of ΔABD : area of ΔABC is :

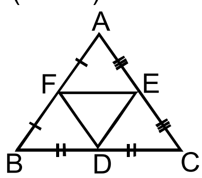


(A) m : n (B) (m + 1) : n (C) m : (n + m) (D) n : m

A-6 The area of figure formed by joining the mid points of the adjacent sides of a rhombus with diagonals 12 cm and 16 cm is

(A) 48 cm^2 (B) 64 cm^2 (C) 96 cm^2 (D) 192 cm^2

A-7. In figure, if ar(ΔABC) = 28 cm^2 then ar (AEDF) =



(A) 21 cm^2 (B) 18 cm^2 (C) 16 cm^2 (D) 14 cm^2

A-8. If the area of ΔABC is 120 cm^2 and the median AD is bisected at point P. then find $\frac{ar(\Delta ABP)}{ar(\Delta ACD)}$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$

Exercise-2

OBJECTIVE QUESTIONS

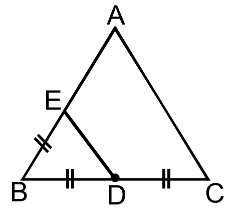
1. In quadrilateral ABCD, diagonals AC and BD intersect at point E. Then
(A) ar (AED) + ar (BCE) = ar (ABE) + ar (CDE) (B) ar (AED) – ar (BCE) = ar (ABE) – ar (CDE)
(C) ar (AED) ÷ ar (BCE) = ar (ABE) ÷ ar (CDE) (D) ar (AED) × ar (BCE) = ar (ABE) × ar (CDE)

2. AD is a median of ΔABC. If X is any point on AD, then find ratio of ar (ΔABX) to the ar(ΔACX).
(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) None of these

3. In ΔABC, P is mid-point of median AD. Then $\frac{ar(BPD)}{ar(ABC)}$ =
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

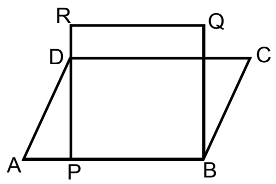
4. In ΔABC,D is a point on BC such that it divides BC in the ratio 3 : 5 i.e., BD : DC = 3 : 5 . Find ar (ADC) : ar (ABC).
(A) 3 : 5 (B) 5 : 8 (C) 3 : 8 (D) None of these

5. In the given figure, ABC and BDE are two equilateral triangles and D is the mid-point of BC. Then $\frac{ar(BDE)}{ar(ABC)}$ =



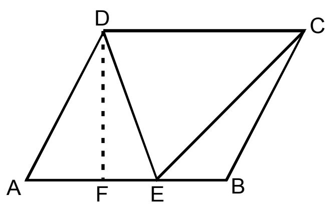
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

6. In the figure, ABCD is a parallelogram and PBQR is a rectangle.



If AP : PB = 1 : 2 = PD : DR, what is the ratio of the area of ABCD to the area of PBQR ?
(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 2 : 3

7. ABCD is a parallelogram. ΔDEC is drawn such that BE = $\frac{1}{3}$AE. Sum of the areas of ΔADE and ΔBEC is:



(A) $\frac{1}{3}$ area of parallelogram ABCD (B) $\frac{1}{2}$ area of parallelogram ABCD
(C) $\frac{2}{3}$ area of ΔDEC (D) $\frac{1}{2}$ area of ΔDEC

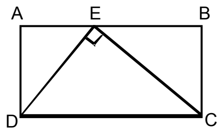
8. E is the midpoint of diagonal BD of a parallelogram ABCD. If the point E is joined to a point F on DA such that DF = $\frac{1}{3}$DA, then the ratio of the area of ΔDEF to the area of quadrilateral ABEF is :

(A) 1 : 3 (B) 1: 4 (C) 1 : 5 (D) 2 : 5

9. ABCD (in order) is a rectangle with AB = CD = $\frac{12}{5}$ and BC = DA = 5. Point P is taken on AD such that $\angle$BPC = 90º. The value of (BP + PC) is equal to :

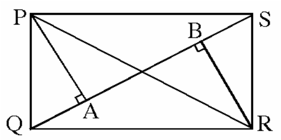
(A) 5 (B) 6 (C) 7 (D) 8

10. In the diagram, ABCD is a rectangle and point E lies on AB. Triangle DEC has $\angle$DEC = 90º, DE = 3 and EC = 4. The length of AD is :



(A) 2.4 (B) 2.8 (C) 1.8 (D) 3.2

11. In the figure PQRS is a rectangle, which one is true?



(A) area of Δ APS = area of Δ QRB (B) PA = RB
(C) area of Δ PQS = area of Δ QRS (D) all of these

12. ABCD is parallelogram, AE $\perp$ DC and CF $\perp$ AD. If AB = 16 cm, AE = 8 cm, CF = 10 cm, find AD.

(A) 16 cm (B) 12 cm (C) 12.8 cm (D) 10.2 cm

13. The perimeter of an isosceles triangle is 32 cm and its base is 12 cm. One of its equal sides forms the diagonal of a parallelogram. Find the area of parallelogram.

(A) 48 cm^2 (B) 38 cm^2 (C) 96 cm^2 (D) None of these

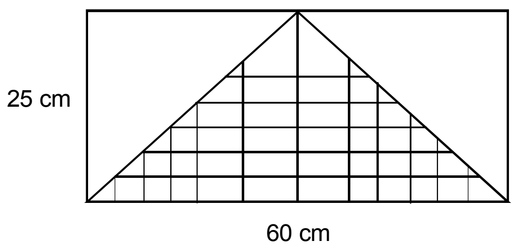
Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. The area of a rhombus is 36 cm^2 . If one diagonal is double of second, then the length of bigger diagonal is **[RAJASTHAN NTSE Stage-1 2005]**

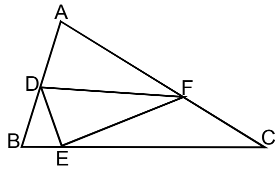
(A) 6 cm (B) 12 cm (C) 16 cm (D) 36 cm

2. In the following figure, the area of the shaded portion is **[RAJASTHAN NTSE Stage-1 2007]**



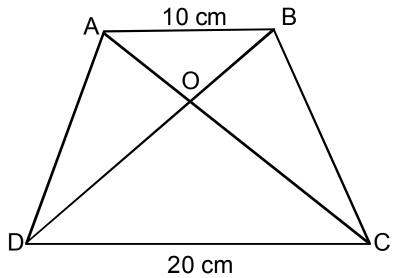
(A) 85 cm^2 (B) 420 cm^2 (C) 750 cm^2 (D) 1500 cm^2

3. In the figure AD = $\frac{1}{2}$DB, BE = EC and CF = $\frac{1}{3}$ AF. If the area of ΔABC = 120 cm^2, the area (in cm^2) of ΔDEF is : **[Harayana NTSE Stage-1 2013]**



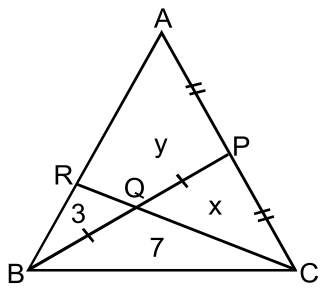
(A) 21 (B) 35 (C) 40 (D) 45

4. □ ABCD is a trapezium, AB || DC. Diagonals of trapezium intersect to each other at point O :
Ar (Δ AOB) = 3 sq.cm, Ar (Δ COD) = 12 sq. cm, Ar (□ABCD) = ...
[MAHARASHTRA NTSE Stage-1 2013]



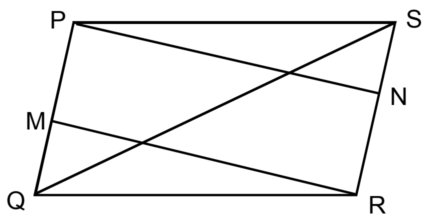
(A) 27 sq. cm (B) 45 sq. cm (C) 36 sq. cm (D) 18 sq. cm

5. In the figure given below, points P and Q are mid points on the sides AC and BP respectively. Area of each part is shown in the figure, then find the value of x + y. **[Maharashtra NTSE Stage-1 2013]**



(A) 11 (B) 4 (C) 7 (D) 18

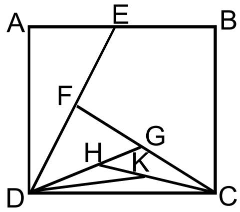
6. PQRS is a parallelogram and M, N are the mid-points of PQ and RS respectively. Which of the following is not true ? **[M.P. NTSE Stage-1 2013]**



(A) RM trisects QS (B) PN trisects QS
(C) ΔPSN $\cong$ ΔRQM (D) MS is not parallel to QN

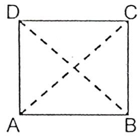
7. In ΔABC, E divides AB in the ratio 3 : 1 and F divides BC in the ratio 3 : 2, then the ratio of areas of ΔBEF and ΔABC is : **[Jharkhand NTSE Stage-1 2014]**
(A) 3 : 5 (B) 3 : 10 (C) 1 : 5 (D) 3 : 20

8. In the figure, the area of square ABCD is 4 cm^2 and E any point on AB. F, G, H and K are the mid point of DE, CF, DG, and CH respectively. The area of ΔKDC is - **[Delhi NTSE Stage-1 2016]**



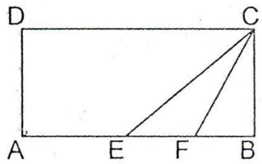
(A) $\frac{1}{4}$ cm^2 (B) $\frac{1}{8}$ cm^2 (C) $\frac{1}{16}$ cm^2 (D) $\frac{1}{32}$ cm^2

9. ABCD is a square of area of 4 square units which is divided into 4 non overlapping triangles as shown in figure, then sum of perimeters of the triangles so formed is **[Delhi NTSE Stage-1 2016]**



(A) $8(2 + \sqrt{2})$ (B) $8(1 + \sqrt{2})$ (C) $4(1 + \sqrt{2})$ (D) $4(2 + \sqrt{2})$

10. In the diagram ABCD is a rectangle with AE = EF = FB, the ratio of the areas of triangle CEF and that of rectangle ABCD is **[Delhi NTSE Stage-1 2016]**



(A) 1 : 6 (B) 1 : 8 (C) 1 : 9 (D) 1 : 10

Answer Key

Exercise Board Level

TYPE (I)

1. 48 cm² **2.** **(iv)** **3.** a rhombus of area 24 cm² **4.** $\frac{1}{2}$ar (ABC)

5. 1 : 2 **9.** 8 cm²

TYPE (II)

13. **(i)** 90 cm² **(ii)** 45 cm² **(iii)** 45 cm²

TYPE (III)

15. (3a + b) : (a + 3b) **16.** 12 cm²

Exercise-1

OBJECTIVE QUESTIONS

Section (A)

A-1. (A) **A-2.** (D) **A-3.** (B) **A-4.** (D) **A-5.** (C) **A-6** (A)

A-7. (D) **A-8.** (D)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	D	A	C	B	C	A	B	C	C	A	D	C	C

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	B	A	D	D	D	B	D	A