

# MATHEMATICS

## Class-X

### Topic-09

## AREA RELATED TO CIRCLE



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# CH-09

## AREA RELATED TO CIRCLE

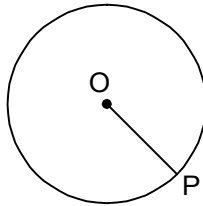
### A. PERIMETER AND AREA OF A CIRCLE

In our daily life we come across many objects which are circular in shape. For example cycle wheels, bangles, circular paths etc. That is why the problem of finding perimeters and areas related to circular figures is of great importance.

**Circle :** The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle. The diameter of a circle is twice its radius.

In figure, O is the centre and the length OP is the radius of the circle.



The length of the boundary of a circle is called its circumference or its perimeter. We know that the ratio of the circumference of a circle to its diameter is always a constant. This constant ratio is denoted by the greek letter  $\pi$ .

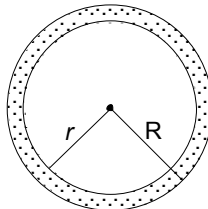
$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

$$\text{Circumference} = \pi \times 2r = 2\pi r \quad [ \because \text{Diameter} = 2r ]$$

The exact value of  $\pi$  is not known, because  $\pi$  is an irrational number. For all practical purposes, the value of  $\pi$  is approximately taken as  $\frac{22}{7}$  or 3.14

If  $r$  is the radius of a circle, then

- (i) Circumference =  $2\pi r$  or  $\pi d$ , where  $d = 2r$  is the diameter of the circle.
- (ii) Area =  $\pi r^2$ .
- (iii) Area of semi-circle =  $\frac{\pi r^2}{2}$ .
- (iv) Perimeter of the semi-circle =  $\pi r + 2r$ .
- (v) Area enclosed by two concentric circles  
 $= \pi R^2 - \pi r^2 = \pi (R^2 - r^2) = \pi(R + r)(R - r)$   
 Where  $R$  and  $r$  are radii of two concentric circles.



#### SOME IMPORTANT POINT

- (i) If two circles touch each other externally, then the distance between their centres is equal to sum of their radii.
- (ii) If two circles touch each other internally, then the distance between their centres is equal to difference of their radii.
- (iii) The distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

## Solved Examples

### Example. 1

Find the area of a circle whose circumference is 22 cm.

**Sol.** Let  $r$  be the radius of the circle.  
Then, Circumference = 22 cm

$$\Rightarrow 2\pi r = 22 \quad \Rightarrow \quad 2 \times \frac{22}{7} \times r = 22 \quad \Rightarrow \quad r = \frac{7}{2} \text{ cm}$$

$$\therefore \text{Area of the circle} = \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2 = 38.5 \text{ cm}^2.$$

### Example. 2

If the perimeter of a semi-circular protractor is 66 cm, find the diameter of the protractor  
(Take  $\pi = \frac{22}{7}$ ).

**Sol.** Let the radius of the protractor be  $r$  cm. Then, Perimeter = 66 cm

$$\Rightarrow \pi r + 2r = 66 \quad \Rightarrow \quad r \left[ \frac{22}{7} + 2 \right] = 66$$

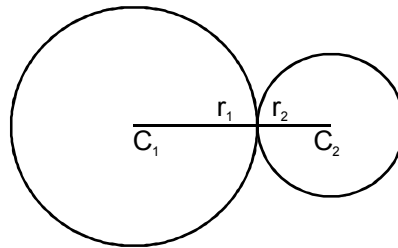
$$\Rightarrow r \left( \frac{36}{7} \right) = 66 \quad \Rightarrow \quad r = \frac{66 \times 7}{36} \quad \Rightarrow \quad r = \frac{77}{6} \text{ cm}$$

$$\therefore \text{Diameter} = 2r = \frac{77}{3} \text{ cm.}$$

### Example. 3

Two circles touch externally. The sum of their areas is  $130\pi$  sq. cm. and the distance between their centres is 14 cm. Find the radii of the circles.

**Sol.**



If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

Let the radii of the two circles be  $r_1$  cm and  $r_2$  cm respectively.

Let  $C_1$  and  $C_2$  be the centres of the given circles. Then,

$$\begin{aligned} C_1C_2 &= r_1 + r_2 && [\because C_1C_2 = 14 \text{ cm (given)}] \\ \Rightarrow 14 &= r_1 + r_2 && \Rightarrow r_1 + r_2 = 14 \quad \dots(i) \end{aligned}$$

It is given that the sum of the areas of two circles is equal to  $130\pi$  cm<sup>2</sup>.

$$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \quad \Rightarrow \quad r_1^2 + r_2^2 = 130 \quad \dots(ii)$$

Now,  $(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

$$\Rightarrow 14^2 = 130 + 2r_1r_2 \quad \text{[Using (i) and (ii)]}$$

$$\Rightarrow 196 - 130 = 2r_1r_2$$

$$\Rightarrow r_1r_2 = 33 \quad \dots(iii)$$

Now,  $(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$$\Rightarrow (r_1 - r_2)^2 = 130 - 2 \times 33 \quad \text{[Using (ii) and (iii)]}$$

$$\Rightarrow (r_1 - r_2)^2 = 64$$

$$\Rightarrow r_1 - r_2 = 8 \quad \dots(iv)$$

Solving (i) and (iv), we get  $r_1 = 11$  cm and  $r_2 = 3$  cm.

Hence, the radii of the two circles are 11 cm and 3 cm.

### Example. 4

A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

**Sol.** Let the outer and inner radii of the ring be  $R$  metres and  $r$  metres respectively.

Then,  $2\pi R = 396$  and  $2\pi r = 352$

$$2 \times \frac{22}{7} \times R = 396 \quad \text{and} \quad 2 \times \frac{22}{7} \times r = 352$$

$$R = 396 \times \frac{7}{22} \times \frac{1}{2} \quad \text{and} \quad r = 352 \times \frac{7}{22} \times \frac{1}{2}$$

$$R = 63 \text{ m and } r = 56 \text{ m}$$

Hence, width of the track =  $(R - r) \text{ m} = (63 - 56) \text{ m} = 7 \text{ m}$ .

### Example. 5

The diameter of a cycle wheel is 28 cm. How many revolution will it make in moving 13.2 km?

**Sol.** Distance travelled by the wheel in one revolution

$$= 2\pi r = 2 \times \frac{22}{7} \times \frac{28}{2} = 88 \text{ cm}$$

and the total distance covered by the wheel

$$= 13.2 \times 1000 \times 100 \text{ cm} = 1320000 \text{ cm}$$

$\therefore$  Number of revolutions made by the wheel

$$= \frac{1320000}{88} = 15000.$$

### Example. 6

A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of the side of the square.

**Sol.** We have,

Length of the wire = Circumference of the circle

$$\text{Length of the wire} = \left\{ 2 \times \frac{22}{7} \times 28 \right\} \text{ cm}$$

$$\text{Length of the wire} = 176 \text{ cm} \quad \dots(i)$$

Let the side of the square be  $x$  cm.

Then,

Perimeter of the square = Length of the wire

$$\Rightarrow 4x = 176$$

$$\Rightarrow x = 44 \text{ cm}$$

Hence, the length of the side of the square is 44 cm.

## Check Your Level

1. The difference between the circumference and the radius of a circle is 37 cm. Find the area of the circle.
2. A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.
3. A circular road runs round a circle. If the difference between the circumference of the outer circle and inner circle is 66 m, find the width of the road.
4. If the diameter of a circle is increased by 100%, find the percentage increase in its area.
5. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of the distances travelled by their tips in 2 days.

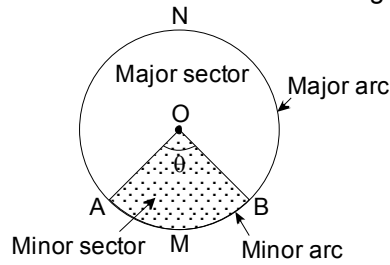
### Answers

1.  $154 \text{ cm}^2$       2.  $14 \text{ m}$       3.  $21/2 \text{ m}$       4.  $300\%$       5.  $1910.8 \text{ cm}$

## B. SECTOR AND SEGMENT OF A CIRCLE

### (a) Sector of a Circle and its Area

The region bounded by an arc of a circle and its two bounding radii is called a sector of the circle.

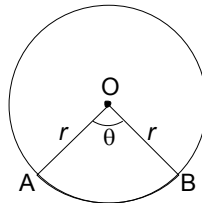


If the arc is a minor arc then the corresponding sector is called the minor sector and the remaining part bounded by the major arc is called the major sector.

In figure shaded region OAMB is the minor sector and the remaining portion OANB is the major sector.

#### (i) Length of Arc and Area of sector

Let  $r$  be the radius of the circle with centre  $O$  and  $AOB$  be a sector of the circle such that  $\angle AOB = \theta$ .  $0 < \theta < 180^\circ$ , then the arc  $AB$  is a minor arc of the circle.



Now, if  $\theta$  increases, the length of arc  $AB$  is also increases and if  $\theta$  becomes  $180^\circ$ , then arc  $AB$  becomes the circumference of a semi-circle.

When an arc subtends an angle  $180^\circ$  at the centre, then the length of the arc of the semi-circle =  $\pi r$ .

When an arc subtends angle  $\theta$  at the centre, then length of the arc =  $\frac{\pi r}{180^\circ} \times \theta = \frac{\pi r \theta}{180^\circ}$ .

If  $\ell$  be the length of the arc  $AB$ , then  $\ell = \frac{\pi r \theta}{180^\circ}$

Again, when an arc subtends angle  $180^\circ$  at the centre, the corresponding sector is a semi-circular region of area  $\frac{1}{2} \pi r^2$ .

$\therefore$  When an arc subtends an angle  $180^\circ$  at the centre, then the area of the corresponding sector is  $\frac{\pi r^2}{2}$ .

$\therefore$  When an arc subtends an  $\theta$  at the centre, then area of the sector =  $\frac{\pi r^2}{2} \times \frac{1}{180^\circ} \times \theta = \frac{\pi r^2 \theta}{360^\circ}$

If  $A$  be the area of the sector, then  $A = \frac{\pi r^2 \theta}{360^\circ}$  and length of an arc  $\ell = \frac{\pi r \theta}{180^\circ}$

$$\Rightarrow \frac{A}{\ell} = \frac{\pi r^2 \theta}{360^\circ} \times \frac{180^\circ}{\pi r \theta} = \frac{r}{2}. \text{ Hence, } A = \frac{\ell r}{2}$$

#### Some Important Points

(i) Angle described by minute hand in 60 minutes =  $360^\circ$ .

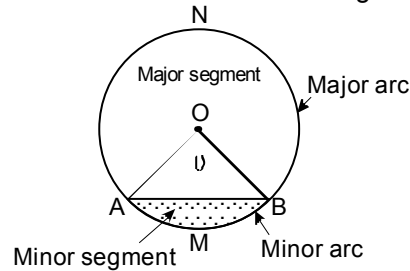
$$\therefore \text{Angle described by minute hand in one minute} = \left( \frac{360^\circ}{60} \right) = 6^\circ.$$

Thus, minute hand rotates through an angle of  $6^\circ$  in one minute.

- (ii) Angle described by hour-hand in -12 hours. =  $360^\circ$ .  
 $\therefore$  Angle described by hour hand in one hour =  $\left(\frac{360^\circ}{12}\right) = 30^\circ$ .  
 $\therefore$  Angle described by hour hand in one minute =  $\left(\frac{30^\circ}{60}\right) = \left(\frac{1}{2}\right)^\circ$ .  
 Thus, hour hand rotates through an angle of  $\left(\frac{1}{2}\right)^\circ$  in one minute.

**(b) Segment of a Circle and its Area**

The region enclosed by an arc and a chord is called the segment of the circle.

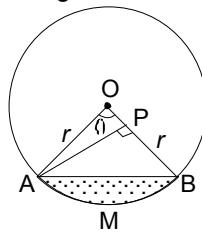


The segment containing the minor arc is called a minor segment and the remaining segment containing the major arc is called the major segment.

In the figure, the shaded region is the minor segment and the remaining part of the circle is major segment.

**(i) Area of a Segment of a Circle**

Let  $r$  be the radius of a circle with centre  $O$  and let  $AB$  be an arc subtending an angle  $\theta$  at the centre  $O$ . we shall find the area of the shaded segment  $AMB$ .



Let  $AP \perp OB$ .

Now, area of the segment  $AMB$

$$\begin{aligned}
 &= \text{Area of the sector } OAMB - \text{area of } \triangle OAB \\
 &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} \times OB \times AP \\
 &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r \times OA \sin \theta \quad [ \because \text{From } \triangle AOP, \sin \theta = \frac{AP}{OA} \Rightarrow AP = OA \sin \theta ] \\
 &= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta \quad [ \because OA = OB = r ]
 \end{aligned}$$

Hence, area of the segment =  $\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta$ .

## Solved Examples

**Example. 7**

A sector is cut from a circle of radius 21 cm. The angle of the sector is  $150^\circ$ . Find the length of its arc and area.

**Sol.** The length or arc  $\ell$  and area  $A$  of a sector of angle  $\theta$  in a circle of radius  $r$  are given by

$$\ell = \frac{\theta}{360^\circ} \times 2\pi r \quad \text{and} \quad A = \frac{\theta}{360^\circ} \times \pi r^2 \quad \text{respectively.}$$

Here,  $r = 21$  cm and  $\theta = 150$

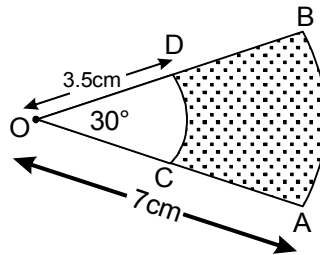
$$\therefore l = \left\{ \frac{150}{360} \times 2 \times \frac{22}{7} \times 21 \right\} \text{ cm} = 55 \text{ cm} \quad \text{and}$$

$$\begin{aligned} A &= \left\{ \frac{150}{360} \times \frac{22}{7} \times (21)^2 \right\} \text{ cm}^2 \\ &= \frac{1155}{2} \text{ cm}^2 = 577.5 \text{ cm}^2. \end{aligned}$$

**Example. 8**

In figure, there are shown sector of two concentric circles of radii 7 cm and 3.5 cm. Find the area of the shaded region. (Use  $\pi = \frac{22}{7}$ ).

**Sol.** Let  $A_1$  and  $A_2$  be the areas of sectors OAB and OCD respectively. Then,



$A_1$  = Area of a sector of angle  $30^\circ$  in a circle of radius 7 cm

$$\Rightarrow A_1 = \left\{ \frac{30}{360} \times \frac{22}{7} \times 7^2 \right\} \quad \left[ \text{Using : } A = \frac{\theta}{360} \times \pi r^2 \right]$$

$$\Rightarrow A_1 = \frac{77}{6} \text{ cm}^2$$

$A_2$  = Area of a sector of angle  $30^\circ$  in a circle of radius 3.5 cm.

$$\Rightarrow A_2 = \left\{ \frac{30}{360} \times \frac{22}{7} \times (3.5)^2 \right\} \quad \Rightarrow A_2 = \left\{ \frac{1}{12} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right\} = \frac{77}{24} \text{ cm}^2.$$

$$\begin{aligned} \therefore \text{Area of the shaded region} &= A_1 - A_2 = \left( \frac{77}{6} - \frac{77}{24} \right) \text{ cm}^2 \\ &= \frac{77}{24} \times (4 - 1) \text{ cm}^2 = \frac{77}{8} \text{ cm}^2 \\ &= 9.625 \text{ cm}^2. \end{aligned}$$

**Example. 9**

The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 A.M. and 9.35 A.M.

**Sol.** We have,

Angle described by the minute hand in one minute =  $6^\circ$ .

$\therefore$  Angle described by the minute hand in 35 minutes =  $(6 \times 35)^\circ = 210^\circ$

$\therefore$  Area swept by the minute hand in 35 minutes.

= Area of a sector of angle  $210^\circ$  in a circle of radius 10 cm

$$= \left\{ \frac{210}{360} \times \frac{22}{7} \times (10)^2 \right\} \text{ cm}^2 = 183.3 \text{ cm}^2.$$

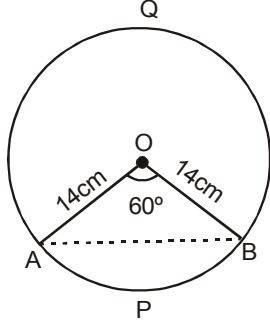
**Example. 10**

A chord of circle 14 cm makes an angle of  $60^\circ$  at the center of the circle. Find

- |                                                                       |                                                                             |
|-----------------------------------------------------------------------|-----------------------------------------------------------------------------|
| <p>(i) area of minor sector</p> <p>(iii) area of the major sector</p> | <p>(ii) area of the minor segment</p> <p>(iv) area of the major segment</p> |
|-----------------------------------------------------------------------|-----------------------------------------------------------------------------|



**Sol.** Given,  $r = 14$  cm,  $\theta = 60^\circ$



(i) Area of minor sector OAPB =  $\frac{\theta}{360^\circ} \pi r^2$   
 $= \frac{60^\circ}{360^\circ} \times 3.14 \times 14 \times 14 = 102.57 \text{ cm}^2$

(ii) Area of minor segment APB =  $\frac{\pi r^2 \theta}{360^\circ} - \frac{r^2}{2} \sin \theta$   
 $= 102.57 - \frac{14 \times 14}{2} \sin 60^\circ$   
 $= 102.57 - 98 \times \frac{\sqrt{3}}{2} = 17.80 \text{ cm}^2.$

(iii) Area of major sector = Area of circle – Area of minor sector OAPB =  $\pi (14)^2 - 102.57$   
 $= 615.44 - 102.57 = 512.87 \text{ cm}^2$

(iv) Area of major segment AQB  
 $= \text{Area of circle} - \text{Area of minor segment APB}$   
 $= 615.44 - 17.80 = 597.64 \text{ cm}^2.$

### Check Your Level

1. Find the area of a quadrant circle of circumference 22 cm.
2. The perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm. Find the area of the sector.
3. A pendulum swings through an angle of  $30^\circ$  and describes an arc 8.8 cm in length. Find the length of the pendulum.
4. The perimeter of a sector of a circle of radius 7 cm is 44 cm. Find area of the sector.
5. An arc of a circle is of length  $5\pi$  cm and the sector it bounds has an area of  $25\pi \text{ cm}^2$ . Find the radius of the circle.

**Answers**

1.  $77/8 \text{ cm}^2$       2.  $44.8 \text{ cm}^2$       3.  $16.8 \text{ cm}$       4.  $105 \text{ cm}^2$   
 5.  $10 \text{ cm}$

## C. APPLICATIONS OF AREA RELATED TO CIRCLES

In our daily life we come across various plane figures, which are combinations of two or more plane figures. For example, window designs, flower beds, circular paths etc. In this section, we shall discuss problems of combinations of plane figures.

**IMPORTANT FORMULA**

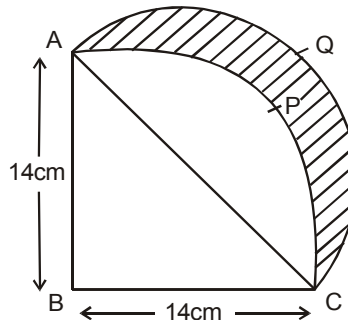
- (i) Heron's formula : Area of a triangle =  $\sqrt{s(s-a)(s-b)(s-c)}$   
 Where  $s =$  Semi-perimeter and  $a, b, c$  are the sides of the triangle.
- (ii) Area of a right angled triangle =  $\frac{1}{2} \times \text{base} \times \text{altitude}$

- (iii) Area of an equilateral triangle =  $\frac{\sqrt{3}}{4} a^2$ .
- (iv) Area of a rectangle = Length  $\times$  breadth
- (v) Perimeter of a rectangle of sides 'a' and 'b' =  $2(a + b)$ .
- (vi) Area of a square of side 'a' =  $a^2$ .
- (vii) Length of diagonal of a square of a side 'a' =  $\sqrt{2}a$ .
- (viii) Perimeter of a square of side 'a' =  $4a$ .
- (ix) Area of a parallelogram = Base  $\times$  Height
- (x) Area of a rhombus =  $\frac{1}{2} d_1 d_2$ .  
Where  $d_1$  and  $d_2$  are the lengths of its diagonals.
- (xi) Area of a trapezium =  $\frac{1}{2} (a + b)h$ .  
Where a and b are lengths of two parallel sides and h is the distance between them.

### Solved Examples

**Example. 11**

In the given figure ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semicircle is drawn. Find the area of the shaded portion.



**Sol.** In right angled triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 14^2 + 14^2$$

$$AC = \sqrt{2 \times 14^2} = 14\sqrt{2} \text{ cm}$$

Now required Area

$$= \text{Area APCQA}$$

$$= \text{Area ACQA} - \text{Area ACPA}$$

$$= \text{Area ACQA} - (\text{Area ABCPA} - \text{Area of } \triangle ABC)$$

$$= \frac{1}{2} \times \pi \times \left(\frac{14\sqrt{2}}{2}\right)^2 - \left[\frac{1}{4} \times \pi (14)^2 - \frac{1}{2} \times 14 \times 14\right]$$

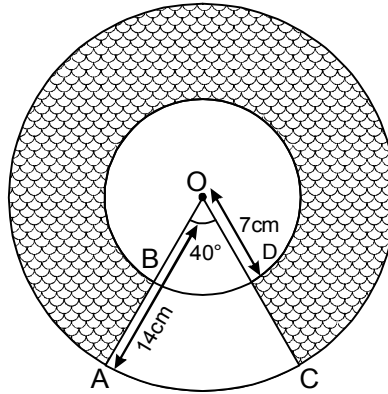
$$= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 + 7 \times 14$$

$$= 154 - 154 + 98 = 98 \text{ cm}^2.$$

**Example. 12**

Find the area of the shaded region in figure, where radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and  $\angle AOC = 40^\circ$ .

**Sol.** We have,



$$\begin{aligned} \text{Area of ring} &= \pi (R^2 - r^2) \\ &= \pi \times (14^2 - 7^2) \\ &= 462 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of the region ABDC} &= \text{Area of sector AOC} - \text{Area of sector BOD} \\ &= \left( \frac{40}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{40}{360} \times \frac{22}{7} \times 7 \times 7 \right) \text{ cm}^2 \\ &= \left( \frac{1}{9} \times 22 \times 14 \times 2 - \frac{1}{9} \times 22 \times 7 \times 1 \right) \text{ cm}^2 \\ &= \frac{22}{9} \times (28 - 7) \text{ cm}^2 = \frac{154}{3} \text{ cm}^2 \end{aligned}$$

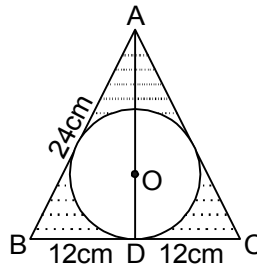
Hence, Required shaded area

$$= \left( 462 - \frac{154}{3} \right) \text{ cm}^2 = \frac{1232}{3} \text{ cm}^2 = 410.67 \text{ cm}^2$$

**Example. 13**

In an equilateral triangle of side 24 cm, a circle is inscribed touching its sides. Find the area of the remaining portion of the triangle [Take  $\sqrt{3} = 1.732$ ].

**Sol.** Let ABC be an equilateral triangle of side 24 cm, and let AD be perpendicular from A on BC. Since the triangle is equilateral, so AD bisects BC.



$\therefore$   $BD = CD = 12 \text{ cm}$   
The centre of the inscribed circle will coincide with the centroid of  $\triangle ABC$ .

$$\therefore OD = \frac{AD}{3}$$

In  $\triangle ABD$ , we have  $AB^2 = AD^2 + BD^2$  [Using Phythagoras Theorem]

$$\Rightarrow 24^2 = AD^2 + 12^2$$

$$\Rightarrow AD = \sqrt{24^2 - 12^2} = \sqrt{(24 - 12)(24 + 12)} = \sqrt{36 \times 12} = 12\sqrt{3} \text{ cm.}$$

$$\therefore OD = \frac{1}{3} AD = \left( \frac{1}{3} \times 12\sqrt{3} \right) \text{ cm} = 4\sqrt{3} \text{ cm}$$

$$\text{Area of the incircle} = \pi (OD)^2 = \left\{ \frac{22}{7} \times (4\sqrt{3})^2 \right\} \text{ cm}^2 = \left\{ \frac{22}{7} \times 48 \right\} \text{ cm}^2 = 150.85 \text{ cm}^2$$

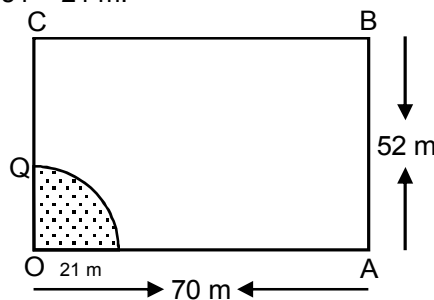
$$\text{Area of the triangle ABC} = \frac{\sqrt{3}}{4} (\text{Side})^2 = \frac{\sqrt{3}}{4} (24)^2 = 249.4 \text{ cm}^2$$

$$\therefore \text{Area of the remaining portion of the triangle} \\ = (249.4 - 150.85) \text{ cm}^2 = 98.55 \text{ cm}^2.$$

**Example. 14**

A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze ?

**Sol.** Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius  $r = 21$  m.



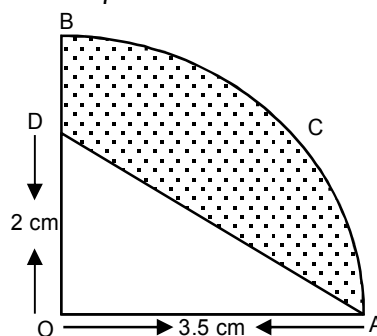
$$\text{Required area} = \frac{1}{4} \pi r^2$$

$$\text{Required area} = \left\{ \frac{1}{4} \times \frac{22}{7} \times (21)^2 \right\} \text{ cm}^2$$

$$\text{Required area} = \frac{693}{2} \text{ cm}^2 = 346.5 \text{ cm}^2.$$

**Example. 15**

In figure , AOBCA represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion (Take  $\pi = \frac{22}{7}$ ).



**Sol.** We have,

$$\text{Area of quadrant AOBCA} = \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = 9.625 \text{ cm}^2$$

$$\text{Area of } \triangle AOD = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} (3.5)(2) = 3.5 \text{ cm}^2$$

$$\text{Hence, Area of the shaded portion} = \text{Area of quadrant} - \text{Area of } \triangle AOD = (9.625 - 3.5) \text{ cm}^2 = 6.125 \text{ cm}^2$$

## Check Your Level

1. Find the area of largest possible square inscribed in a circle of unit radius?
2. Four circular cardboard pieces each of radius 7 cm are placed in such a way that each piece touches two other pieces. Find the area of the space enclosed by the four pieces.
3. Four horses are tethered at four corners of a square plot of side 63 metres so that they just cannot reach one another. Find the area left un-grazed inside the plot.
4. A building with base in the form of an equilateral triangle of side 14 m is built in a huge grass field. In one corner of the building a cow is tethered with a rope of length 21 m. Find the area grazed by the cow.
5. A square grass field has side 14 metres. Two goats are tethered with a rope of 14 metres long each at opposite corners. Find the common area grazed by both the goats.

### Answers

1. 2                      2. 42 cm<sup>2</sup>                      3. 850.5 m<sup>2</sup>                      4. 1257.66 m<sup>2</sup>  
5. 112 m<sup>2</sup>
-

## Exercise Board Level

**TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS :**

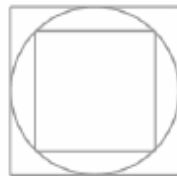
**[01 MARK EACH]**

1. If the circumference of a circle and the perimeter of a square are equal, then show that area of the circle is greater than area of the square.
2. If the perimeter of a circle is equal to that of a square, then find the ratio of their areas
3. Find the area of the circle that can be inscribed in a square of side 6 cm
4. Find the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm
5. Find the area of a sector of circle of radius 21 cm and central angle  $120^\circ$ .
6. The areas of two sectors of two different circles with equal corresponding arc lengths are equal. Is this statement true? If no, give reasons in support of your answer.

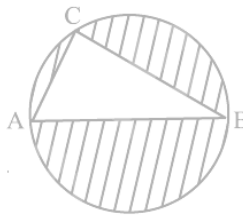
**TYPE (II) : SHORT ANSWER TYPE QUESTIONS :**

**[02 MARKS EACH]**

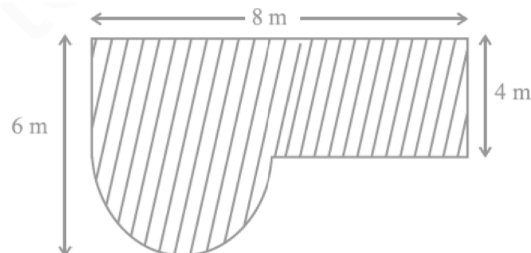
7. In Figure , a square is inscribed in a circle of diameter  $d$  and another square is circumscribing the circle. Find the ratio of area of the outer square to the area of the inner square.



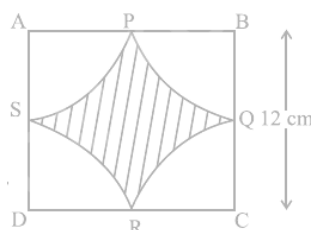
8. In figure,  $AB$  is a diameter of the circle,  $AC = 6$  cm and  $BC = 8$  cm. Find the area of the shaded region (Use  $\pi = 3.14$ ).



9. Find the area of the shaded field shown in figure.



10. Find the area of the minor segment of a circle of radius 14 cm, when the angle of the corresponding sector is  $60^\circ$ .
11. Find the area of the shaded region in figure, where arcs drawn with centres  $A, B, C$  and  $D$  intersect in pairs at mid-points  $P, Q, R$  and  $S$  of the sides  $AB, BC, CD$  and  $DA$ , respectively of a square  $ABCD$  (Use  $\pi = 3.14$ ).

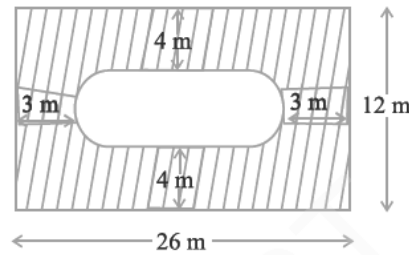


12. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road.
13. A circular pond is 17.5 m is of diameter. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of Rs 25 per  $m^2$ .
14. Find the difference of the areas of a sector of angle  $120^\circ$  and its corresponding major sector of a circle of radius 21 cm.

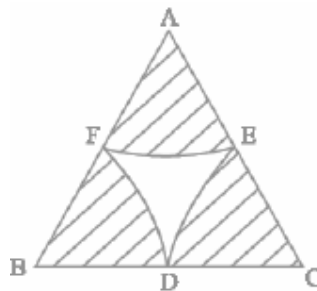
**TYPE (III) : LONG ANSWER TYPE QUESTIONS:**

**[03 MARK EACH]**

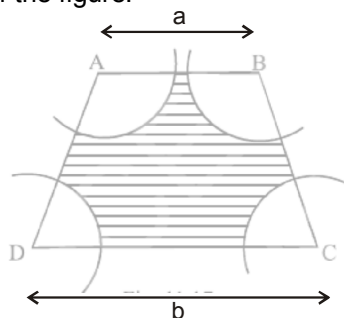
15. Find the area of the shaded region in figure.



16. In figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm. to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. (Use  $\pi = 3.14$ ).

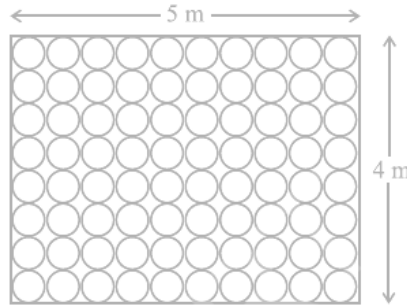


17. The diameters of front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.
18. In figure, ABCD is a trapezium with  $AB \parallel DC$ ,  $AB = 18$  cm,  $DC = 32$  cm and distance between AB and DC = 14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the shaded region of the figure.

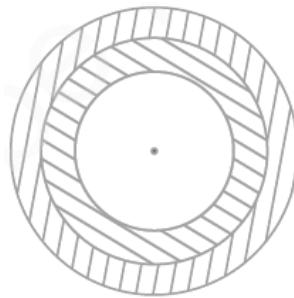


19. Three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these circles.
20. On a square cardboard sheet of area  $784 \text{ cm}^2$ , four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.

21. Floor of a room is of dimensions 5 m × 4 m and it is covered with circular tiles of diameters 50 cm each as shown in figure. Find the area of floor that remains uncovered with tiles. (Use  $\pi = 3.14$ )



22. An archery target has three regions formed by three concentric circles as shown in figure. If the diameters of the concentric circles are in the ratio 1: 2:3, then find the ratio of the areas of three regions.



23. Area of a sector of central angle  $200^\circ$  of a circle is  $770 \text{ cm}^2$ . Find the length of the corresponding arc of this sector.

**TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS**

**[04 MARK EACH]**

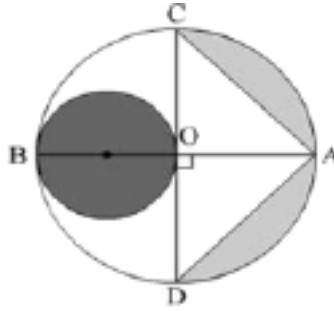
24. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.
25. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is  $1256 \text{ cm}^2$ . (Use  $\pi = 3.14$ ).

**Previous Year Problems**

1. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions  $14 \text{ cm} \times 7 \text{ cm}$ . Find the area of the remaining card board.  
[use  $\pi = \frac{22}{7}$ ]. **[2 MARKS/CBSE 10TH BOARD: 2012]**
2. If the difference between the circumference and the radius of a circle is 37cm, then using  $\pi = \frac{22}{7}$ , the circumference (in cm) of the circle is : **[1 MARK /CBSE 10TH BOARD: 2012]**  
(A) 154                      (B) 44                      (C) 14                      (D) 7



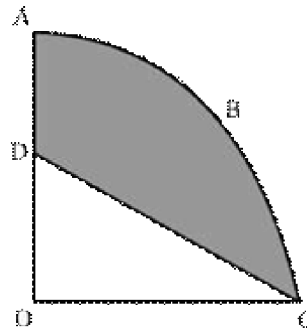
3. In Figure, AB and CD are two diameters of a circle with centre O, which are perpendicular to each other. OB is the diameter of the smaller circle. If OA = 7cm, find the area of the shaded region. [using  $\pi = \frac{22}{7}$ ]. **[3 MARKS /CBSE 10TH BOARD: 2012]**



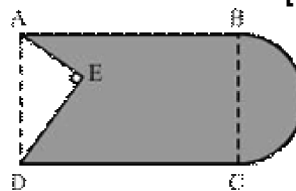
4. In a circle of radius 21 cm, an arc subtends an angle of  $60^\circ$  at the centre. Find : (i) the length of the arc (ii) area of the sector formed by the arc [ use  $\pi = \frac{22}{7}$  ].

**[3 MARKS/ CBSE 10TH BOARD: 2013]**

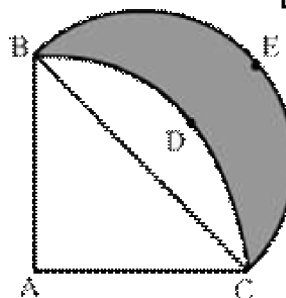
5. In Figure, OABC is a quadrant of a circle of radius 7 cm. If OD = 4 cm, find the area of the shaded region. [Use  $\pi = \frac{22}{7}$ ]. **[2 MARKS/ CBSE 10TH BOARD: 2013]**



6. In Figure, from a rectangular region ABCD with AB = 20 cm, a right triangle AED with AE = 9 cm and DE = 12 cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region. [Use  $\pi = 3.14$ ]. **[3 MARKS/ CBSE 10TH BOARD: 2013]**



7. In Figure, ABCD is a quadrant of a circle of radius 28 cm and a semi circle BEC is drawn with BC as diameter. Find the area of the shaded region. [Use  $\pi = \frac{22}{7}$ ]. **[3 MARKS/ CBSE 10TH BOARD: 2014]**

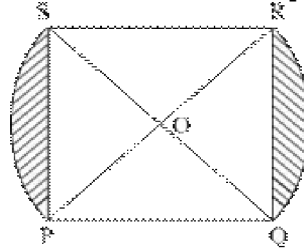


8. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is  $60^\circ$ . Also find the area of the corresponding major segment. [Use  $\pi = \frac{22}{7}$ ]

**[3 MARKS/ CBSE 10TH BOARD: 2014]**

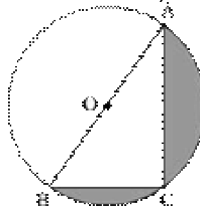
9. In Figure PQRS is a square lawn with side  $PQ = 42$  metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).

**[4 MARKS/ CBSE 10TH BOARD: 2014]**



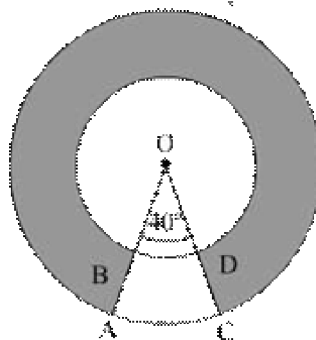
10. In figure, O is the centre of a circle such that diameter  $AB = 13$  cm and  $AC = 12$  cm. BC is joined. Find the area of the shaded region. (Take  $\pi = 3.14$ )

**[3 MARKS/ CBSE 10TH BOARD: 2014]**



11. In figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm, where  $\angle AOC = 40^\circ$ . (Use  $\pi = \frac{22}{7}$ ).

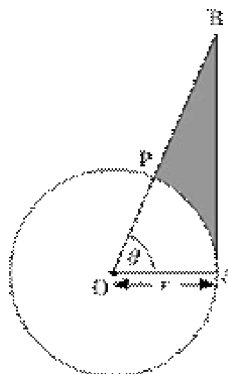
**[3 MARKS/ CBSE 10TH BOARD: 2014]**



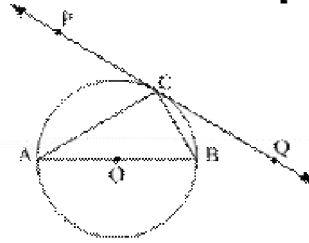
12. In Figure, is shown a sector OAP of a circle with centre O, containing  $\angle \theta$ . AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is

$$r \left[ \tan \theta + \sec \theta + \frac{\pi \theta}{180} - 1 \right].$$

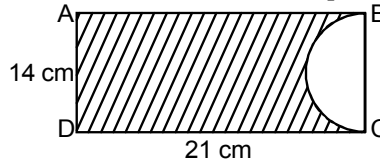
**[4 MARKS/ CBSE 10TH BOARD: 2014]**



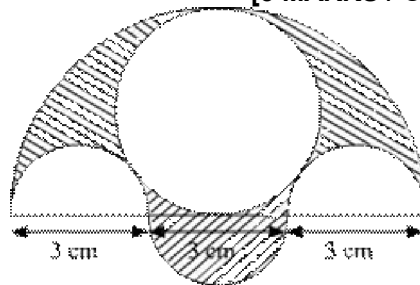
13. In figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and  $\angle CAB = 30^\circ$ , find  $\angle PCA$ .  
[1 MARK / CBSE 10TH BOARD: 2015]



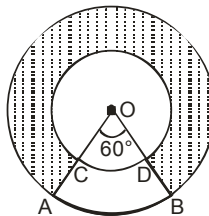
14. In the given figure, ABCD is rectangle of dimensions 21 cm  $\times$  14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.  
[4 MARKS / CBSE 10TH BOARD: 2015]



15. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.  
[3 MARKS / CBSE 10TH BOARD: 2015]



16. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If  $\angle AOB = 60^\circ$ , find the area of the shaded region. [Use  $\pi = \frac{22}{7}$  ]  
[3 MARKS / CBSE 10TH BOARD: 2017]



## Exercise-1

### SUBJECTIVE QUESTIONS

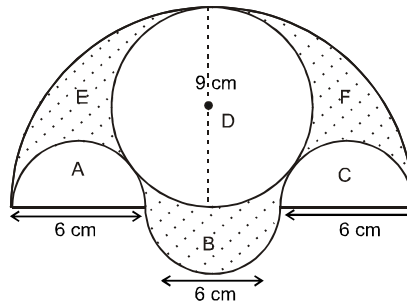
#### Subjective Easy, only learning value problems

#### Section (A) : Perimeter and Area of a Circle

- A-1. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.
- A-2. Diameter of a tyre is 1.26 m. Find the distance covered by it in 500 revolutions.
- A-3. Two circles touch internally. The sum of their areas is  $116 \pi \text{ cm}^2$  and distance between their centres is 6 cm. Find the radii of the circles.

**A-4.** The inner circumference of a circular track is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of Rs 2 per metre. [Use  $\pi = \frac{22}{7}$ ]

**A-5.** In **figure** there are three semicircles, A, B and C having diameter 6 cm each, and another semicircle E having a circle D with diameter 9 cm are shown.



Calculate :

- (i) the area of the shaded region
- (ii) the cost of painting the shaded region at the rate of 25 paise per  $\text{cm}^2$ , to the nearest rupee.

**Section (B) : Sector and Segment of a Circle**

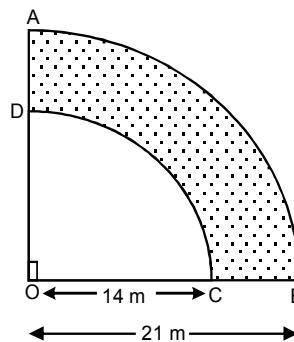
**B-1.** A chord of a circle of radius 12 cm subtends an angle of  $60^\circ$  at the centre. Find the area of the corresponding shaded segment of the circle. (Use  $\pi = 3.14$  and  $\sqrt{3} = 1.73$ ).

**B-2.** The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. [Use  $\pi = \frac{22}{7}$ ]

**B-3.** In a circle with centre O and radius 5 cm, AB is a chord of length  $5\sqrt{3}$  cm. Find the area of sector AOB.

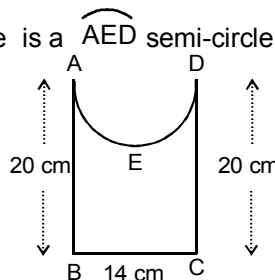
**B-4.** A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of the major and minor segments (Take  $\pi = 3.14$ )

**B-5.** ABCD is a flower bed. If OA = 21 cm and OC = 14 m, find the area of the bed. [Use  $\pi = \frac{22}{7}$ ]

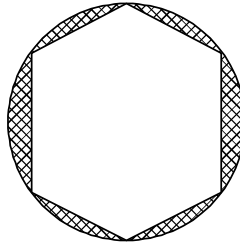


**Section (C) : Applications of area related to Circles**

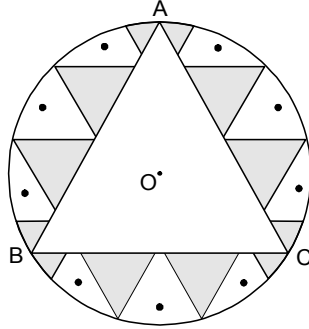
**C-1.** Find the perimeter of figure, where  $\widehat{AED}$  is a semi-circle and ABCD is a rectangle.



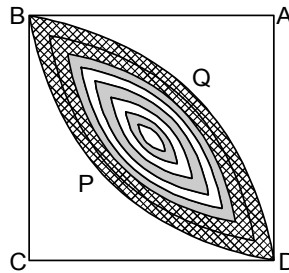
- C-2.** A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 3.50 per  $\text{cm}^2$ . (Use  $\sqrt{3} = 1.7$ ).



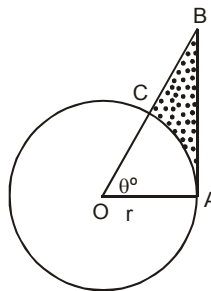
- C-3.** In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. O is the centre of circle. Find the area of the design (shaded region).



- C-4.** Calculate the area of the designed region in figure common between the two quadrants of circles of radius 8 cm each.



- C-5.** Figure, shows a sector of a circle, centre O, containing an angle  $\theta^\circ$ . Prove that :



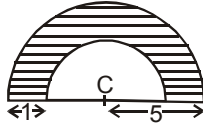
- (i) Perimeter of the shaded region is  $r \left( \tan \theta + \sec \theta + \frac{\pi \theta}{180} - 1 \right)$
- (ii) Area of the shaded region is  $\frac{r^2}{2} \left( \tan \theta - \frac{\pi \theta}{180} \right)$

## OBJECTIVE QUESTIONS

### Single Choice Objective, straight concept/formula oriented

#### Section (A) : Perimeter and Area of a Circle

**A-1.** The area of the shaded portion in the given figure is :



- (A)  $7.5 \pi$  sq.units      (B)  $6.5 \pi$  sq.units      (C)  $5.5 \pi$  sq.units      (D)  $4.5 \pi$  sq.units

**A-2.** The radius of a circle is increased by 1 cm, then the ratio of the new circumference to the new diameter is :

- (A)  $\pi + 2$                       (B)  $\pi + 1$                       (C)  $\pi$                               (D)  $\pi - \frac{1}{2}$

**A-3.** If the sum of the circumferences of two circles with radii  $R_1$  and  $R_2$  is equal to the circumference of a circle of radius  $R$ , then :

- (A)  $R_1 + R_2 = R$               (B)  $R_1 + R_2 > R$               (C)  $R_1 + R_2 < R$   
 (D) Nothing definite can be said about the relation among  $R_1$ ,  $R_2$  and  $R$

**A-4.** It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park could be :

- (A) 10 m                          (B) 15 cm                          (C) 20 m                          (D) 24 cm

**A-5.** The circumference of a circle exceeds its diameter by 15 cm then, the circumference of the circle is :

- (A) 22.00 cm                      (B) 22.5 cm                      (C) 21.01 cm                      (D) None

**A-6.** If the circumference of a circle increases from  $4\pi$  to  $8\pi$ , then its area is :

- (A) halved                          (B) doubled                          (C) tripled                          (D) quadrupled

**A-7.** The ratio of the outer and inner circumferences of a circular path is 23: 22. If the path is 5 m wide, the radius of the inner circle is :

- (A) 55 m                          (B) 110 m                          (C) 220 m                          (D) 230 m

**A-8.** If a wire is bent into the shape of a square, then the area of the square is  $81 \text{ cm}^2$ . When the same wire is bent into a semi-circular shape, then the area of the semi circle will be :

- (A)  $22 \text{ cm}^2$                       (B)  $44 \text{ cm}^2$                       (C)  $77 \text{ cm}^2$                       (D)  $154 \text{ cm}^2$

**A-9.** A circular park has a path of uniform width around it. The difference between the outer and inner circumferences of the circular park is 132 m. Its width is :

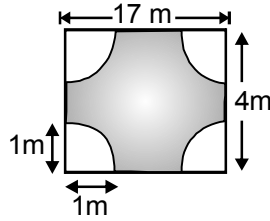
- (A) 20 m                          (B) 21 m                          (C) 22 m                          (D) 24 m

**A-10.** A circular ground whose diameter is 140 meters is to be fenced by wire three times around its circumference. Find the length of wire needed. [ Use  $\pi = \frac{22}{7}$  ]

- (A) 440 m                          (B) 1320 m                          (C) 660 m                          (D) None of these

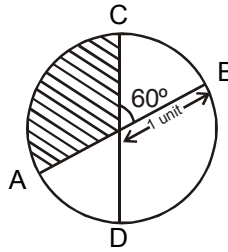
**Section (B) : Sector and Segment of a Circle**

**B-1.** The perimeter of the following shaded portion of the figure is :



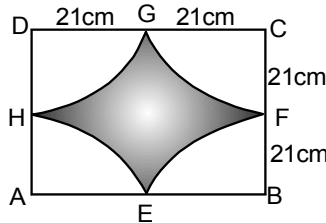
- (A) 40 m                      (B) 40.07 m                      (C) 40.28 m                      (D) 35 m

**B-2.** Find the area of the shaded region in the given figure where AB and CD are diameters :



- (A)  $\frac{\pi}{3}$  sq. units                      (B)  $\frac{\pi}{2}$  sq. units                      (C)  $\frac{\pi}{4}$  sq. units                      (D)  $\pi^2$  sq. units

**B-3.** In figure, ABCD is square of side 42 cm. HEA, EFB, FGC and GHD are four quadrants of circles, then shaded area is :

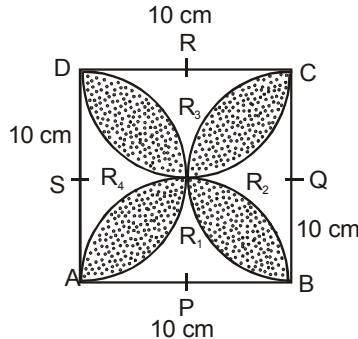


- (A) 374 cm<sup>2</sup>                      (B) 372 cm<sup>2</sup>                      (C) 376 cm<sup>2</sup>                      (D) 378 cm<sup>2</sup>

**B-4.** A lawn is the form of a square of side 30 m. A cow is tied with a rope of 10 m to a pole standing at one of its corner. The maximum area of the lawn grazed by this cow is :

- (A) 300 m<sup>2</sup>                      (B) 150 m<sup>2</sup>                      (C) 78.5 m<sup>2</sup>                      (D) 450 m<sup>2</sup>

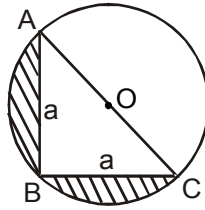
**B-5.** Find the area of the shaded region in figure. where ABCD is a square of side 10 cm. (Use  $\pi = 3.14$ )



- (A) 57 cm<sup>2</sup>                      (B) 55 cm<sup>2</sup>                      (C) 60 cm<sup>2</sup>                      (D) 63 cm<sup>2</sup>

**Section (C) : Applications of area related to Circles**

**C-1.** If AC passes through the centre of the circle, then the area of the shaded region in the given figure is



- (A)  $\frac{a^2}{2}(3 - \pi)$       (B)  $a^2\left(\frac{\pi}{2} - 1\right)$       (C)  $2a^2(\pi - 1)$       (D)  $\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$

**C-2.** If a rectangle of sides 5 cm and 15 cm is divided into three squares of equal area, then the sides of the squares will be :

- (A) 4 cm      (B) 6 cm      (C) 7 cm      (D) None

**C-3.** The area of the square that can be inscribed in a circle of radius 12 cm is :

- (A) 72 cm<sup>2</sup>      (B) 144 cm<sup>2</sup>      (C) 288 cm<sup>2</sup>      (D) 576 cm<sup>2</sup>

**C-4.** From a square metal sheet of side 28 cm, a circular sheet of largest possible radius is cut off the area of the remaining sheet is :

- (A) 784 cm<sup>2</sup>      (B) 78.4 cm<sup>2</sup>      (C) 168 cm<sup>2</sup>      (D) 84 cm<sup>2</sup>

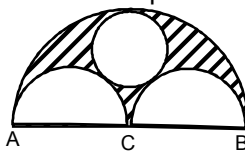
**C-5.** A square with side length 1 is inscribed in a semicircle such that one side of the square is on the diameter of the semicircle. The perimeter of the semicircle is :

- (A)  $\pi\sqrt{5}$       (B)  $\frac{\pi\sqrt{5}}{2}$       (C)  $\sqrt{5}\left(\frac{\pi}{2} + 1\right)$       (D)  $\frac{\sqrt{5}}{2}(\pi + 1)$

**Exercise-2**

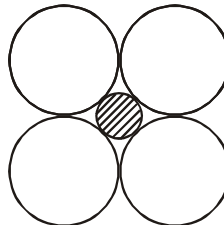
**OBJECTIVE QUESTIONS**

**1.** In the given figure, the diameter of the biggest semi-circle is 56 cm and the radius of the smallest circle is 7 cms. then find the area of the shaded portion



- (A) 462 cm<sup>2</sup>      (B) 231 cm<sup>2</sup>      (C) 924 cm<sup>2</sup>      (D) None of these

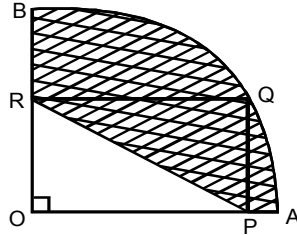
**2.** In the figure given, find the radius of the inner circle, if outer circles are of radii 1 m.



- (A)  $(\sqrt{2} + 1)$  m      (B)  $\sqrt{2}$  m      (C)  $(\sqrt{2} - 1)$  m      (D) None of these



3. In this figure, AOB is a quarter circle of radius 10 and PQRO is a rectangle of perimeter 26. Find the perimeter of the shaded region.



- (A)  $17 - 5\pi$                       (B)  $17 + 5\pi$                       (C)  $13 + 5\pi$                       (D)  $13 - 5\pi$

4.  $\triangle ABC$  is an isosceles right triangle with area P. The radius of the circle that passes through the point A, B and C is

- (A)  $\sqrt{P}$                       (B)  $\sqrt{\frac{P}{2}}$                       (C)  $\frac{\sqrt{P}}{2}$                       (D)  $\sqrt{2P}$

5. A circle passes through the three vertices of an isosceles triangle that has sides of length 3 and a base of length 2. The area of the circle is

- (A)  $\frac{9\pi}{4}$                       (B)  $\frac{81\pi}{32}$                       (C)  $\frac{27\pi}{16}$                       (D)  $\frac{5\pi}{2}$

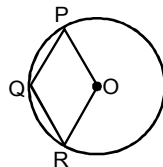
6. Two circles, each with radius  $\sqrt{6}$ , intersect at the two points A and B. For each of the circles diameters from point A are drawn and the opposite ends C and D connected to point B. If the area of figure ACBD is  $2\sqrt{11}$ , then the length of AB may have :

- (A) two rational values                      (B) only one rational value  
(C) one irrational and one rational value                      (D) two irrational values

7. Two circles I and II are external tangent. A tangent to the circle I passes through the centre of the circle II. The distance from the point of tangency to the centre of the circle II is three times the radius of the circle II. The ratio of the circumference of the circle I to the circumference of the circle II.

- (A) 2                      (B) 3                      (C) 4                      (D) 16

8. In the given figure, OPQR is a rhombus, three of whose vertices lie on a circle with centre O. If the area of rhombus is  $32\sqrt{3}$  cm<sup>2</sup>. The radius of circle is :

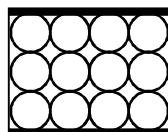


- (A) 8                      (B) 9                      (C) 10                      (D) 11

9. A wire in the shape of an equilateral triangle encloses an area of S sq. cm. If the same wire is bent in form of a circle. The area of the circle will be :

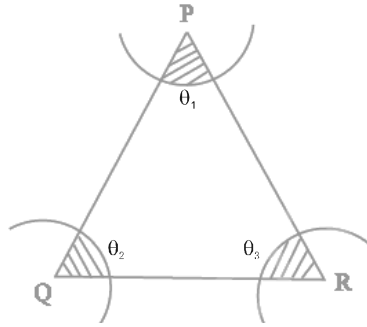
- (A)  $\frac{\pi S^2}{9}$                       (B)  $\frac{3S^2}{\pi}$                       (C)  $\frac{3S}{\pi}$                       (D)  $\frac{3\sqrt{3}S}{\pi}$

10. Each of the congruent circles shown is external tangent to other circles and/or to the side(s) of the rectangle as shown. If each circle has circumference  $16\pi$ , then the length of a diagonal of the rectangle, is

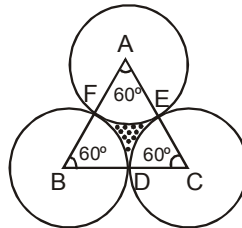


- (A) 80                      (B) 40                      (C) 20                      (D) 15

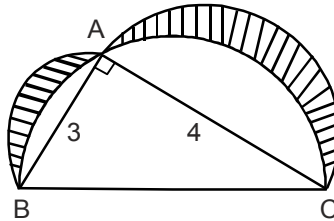
11. In figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



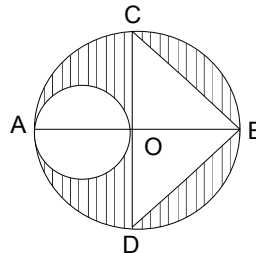
- (A) 204 cm<sup>2</sup>                      (B) 308 cm<sup>2</sup>                      (C) 320 cm<sup>2</sup>                      (D) None of these
12. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.
- (A)  $\frac{60}{\pi}$  cm                      (B)  $\frac{30}{\pi}$  cm                      (C)  $\frac{40}{\pi}$  cm                      (D)  $\frac{20}{\pi}$  cm
13. The area of an equilateral triangle is  $49\sqrt{3}$  cm<sup>2</sup>. Taking each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle as shown in figure. Find the area of the triangle not included in the circle. [Take  $\sqrt{3} = 1.73$ ]



- (A) 7.77 cm<sup>2</sup>                      (B) 7.24 cm<sup>2</sup>                      (C) 7 cm<sup>2</sup>                      (D) None of these
14. In figure ABC is a right-angled triangle right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



- (A) 4 cm<sup>2</sup>                      (B) 3 cm<sup>2</sup>                      (C) 12 cm<sup>2</sup>                      (D) 6 cm<sup>2</sup>.
15. In figure, AB and CD are two perpendicular diameters of a circle with centre O. If OA = 7 cm, find the area of the shaded region. [Use  $\pi = \frac{22}{7}$ ]



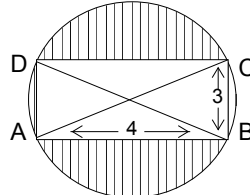
- (A) the area of the shaded region is 12.5 cm<sup>2</sup>                      (B) the area of the shaded region is 66.5 cm<sup>2</sup>  
 (C) the area of the shaded region is 10.5 cm<sup>2</sup>                      (D) None of these

**Exercise-3**

**NTSE PROBLEMS (PREVIOUS YEARS)**

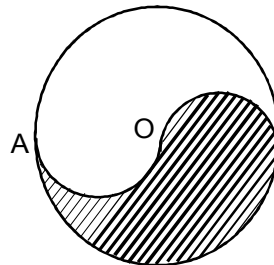
1. The circumference of a circle and perimeter of a square are equal. The ratio of their areas is-  
**[Raj. NTSE Stage-1 2005]**  
 (A)  $\pi : 4$                       (B)  $2 : \pi$                       (C)  $\pi : 2$                       (D)  $4 : \pi$

2. In the given figure,  $AB = 4$  cm,  $BC = 3$ cm, the area of shaded portion is : **[Raj. NTSE Stage-1 2005]**



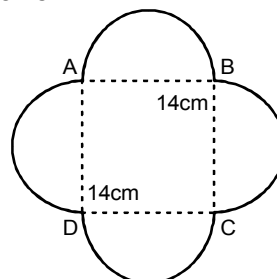
- (A)  $6.25 \text{ cm}^2$                       (B)  $7.64 \text{ cm}^2$                       (C)  $12 \text{ cm}^2$                       (D)  $19.64 \text{ cm}^2$

3. In the following figure, if O is the centre of the circle and radius  $OA = 14$  cm, then the area of the shaded portion is : **[Raj. NTSE Stage-1 2006]**



- (A)  $7\pi \text{ cm}^2$                       (B)  $49\pi \text{ cm}^2$                       (C)  $98\pi \text{ cm}^2$                       (D)  $196\pi \text{ cm}^2$

4. In the following figure the perimeter is **[Raj. NTSE Stage-1 2007]**

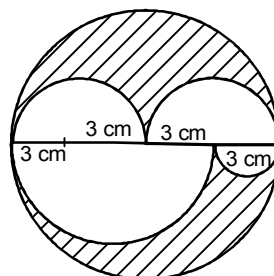


- (A) 56 cm                      (B) 88 cm                      (C) 196 cm                      (D) None

5. A thin wire is bent into the form of a circle of radius 7 cm. If a square is made out of this wire, the side of the square would be : **(NTSE Stage -I/Bihar/2009)**

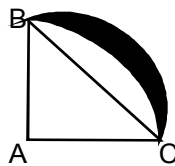
- (A) 7 cm                      (B) 14 cm                      (C) 11 cm                      (D) 22 cm

6. As shown in the figure diameter of outer circle is 12 cm. Ratio of shaded area of upper semi circle to that of the shaded area in the lower semicircle. **[Rajasthan NTSE Stage -I 2009]**



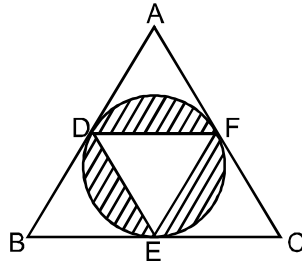
- (A) 1 : 1                      (B) 11 : 3                      (C) 4 : 3                      (D) 8 : 27

7. The area of a circle inscribed in an equilateral triangle is  $48\pi$  square units. The perimeter of the triangle is \_\_\_\_\_ units. **[UP NTSE Stage-1 2012]**  
 (A) 24 (B) 32 (C) 36 (D) 72
8. The perimeter of square and circumference of Circle are equal, the area of square is  $121 \text{ m}^2$ , then the area of circle is : **[Raj. NTSE Stage-1 2013]**  
 (A)  $7\pi \text{ m}^2$  (B)  $14\pi \text{ m}^2$  (C)  $21\pi \text{ m}^2$  (D)  $49\pi \text{ m}^2$
9. A circle is inscribed in an equilateral triangle of side 'a' cm. The area (in  $\text{cm}^2$ ) of a square inscribed in the circle is : **[Harayana NTSE Stage-1 2013]**  
 (A)  $a^2/6$  (B)  $a^2/3$  (C)  $3a^2/4$  (D)  $a^2/12$
10. Four circular cardboard pieces, each of radius 7 cm. are placed in such a way that each piece touches the two other pieces. The area of the space enclosed by the four pieces is: **[Delhi NTSE Stage-1 2013]**  
 (A)  $21 \text{ cm}^2$  (B)  $42 \text{ cm}^2$  (C)  $84 \text{ cm}^2$  (D)  $168 \text{ cm}^2$
11. In figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. The area of the shaded region is **[Raj. NTSE Stage-1 2014]**



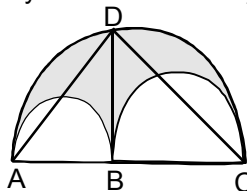
- (A)  $98 \text{ cm}^2$  (B)  $154 \text{ cm}^2$  (C)  $56 \text{ cm}^2$  (D) None of these

12. In the given figure, ABC is an equilateral triangle whose side is  $2\sqrt{3}$  cm. A circle is drawn which passes through the midpoints D, E and F of its sides. The area of the shaded region is **[Raj. NTSE Stage-1 2014]**



- (A)  $\frac{1}{4}(4\pi - 3\sqrt{3}) \text{ cm}^2$  (B)  $\frac{1}{4}(2\pi - \sqrt{3}) \text{ cm}^2$  (C)  $\frac{1}{4}(\pi - 3\sqrt{3}) \text{ cm}^2$  (D)  $\frac{1}{4}(3\pi - \sqrt{3}) \text{ cm}^2$

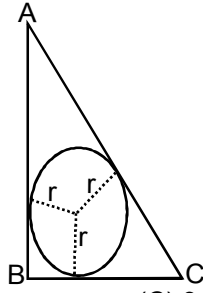
13. In the given figure,  $AB = 4 \text{ cm}$  and  $BD = 4\sqrt{3} \text{ cm}$ . Then the relation between  $\Delta$  [area of triangle ADC] and S (shaded area bounded by three semicircles) is : **[Harayana NTSE Stage-1 2014]**



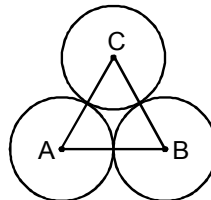
- (A)  $\Delta > S$  (B)  $\Delta < S$  (C)  $\Delta = S$  (D) None of the above

14. A chord of a circle of radius 7 cm. subtends an angle of  $90^\circ$  at its centre. The ratio of areas of smaller and larger segment is : **[Bihar NTSE Stage-1 2014]**  
 (A) 2 : 7 (B) 1 : 10 (C) 1 : 11 (D) None of these
15. If the radius of circle is  $\pi$ , then its area will be- **[Chattisgarh NTSE Stage-1 2014]**  
 (A)  $\pi$  (B)  $\pi^2$  (C)  $\pi^3$  (D)  $3\pi$

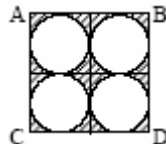
16. In below figure  $\triangle ABC$  is a right angle triangle in which  $\angle B = 90$  and  $BC = 6$  cm and  $AB = 8$  cm. Then radius of incircle will be : **[Chattisgarh NTSE Stage-1 2014]**



- (A) 5 cm                      (B) 4 cm                      (C) 3 cm                      (D) 2 cm
17. The area of triangle formed by joining the centres of three congruent externally touching circles, is  $49\sqrt{3}$  sq-cm, then find the radius of circle. **[Maharashtra NTSE Stage-1 2014]**
- (A)  $\sqrt{7}$  cm                      (B) 7 cm                      (C)  $7\sqrt{3}$  cm                      (D)  $4\sqrt{3}$  cm
18. The radii of two circles are 9 cm and 12 cm. The circumference of a circle whose area is equal to sum of the areas of the two circles is : **[Delhi NTSE Stage-1 2014]**
- (A) 15 cm                      (B)  $15\pi$  cm                      (C)  $30\pi$  cm                      (D) 225 cm
19. The area of a rectangle is same as that of a circle of radius  $\sqrt{\frac{35}{11}}$  cm. If the length of the rectangle exceeds its breadth by 3 cm., then the length of the rectangle is : **[Jharkhand NTSE Stage-1 2014]**
- (A) 2 cm                      (B) 3 cm                      (C) 4 cm                      (D) 5 cm
20. The sum of the areas of two circles which touch each other externally is  $153\pi$  sq. units. If the sum of their radii is 15 units, then the ratio of large radius to the smaller radius is equal to : **[Jharkhand NTSE Stage-1 2014]**
- (A) 4                      (B) 2                      (C) 3                      (D) none of these
21. In the given figure, three circles with centres A, B, C respectively touch each other externally. If  $AB = 5$  cm,  $BC = 7$ cm. and  $CA = 6$  cm., then the radius of the circle with centre A is **[Jharkhand NTSE Stage-1 2015]**



- (A) 1.5 cm.                      (B) 2 cm.                      (C) 2.5 cm.                      (D) 3 cm.
22. In the given figure ABCD is a square of side 14 cm. Find the area of the shaded region.



- [Jharkhand NTSE Stage-1 2015]**
- (A)  $56\text{ cm}^2$                       (B)  $48\text{ cm}^2$                       (C)  $42\text{ cm}^2$                       (D)  $44\text{ cm}^2$
23. Find the area of circle in sq. cms, in which the chord of length 18 cm is at the distance of half of the radius of circle from the centre of circle : **[MP NTSE Stage-1 2015]**
- (A)  $108\pi$                       (B)  $54\pi$                       (C)  $27\pi$                       (D)  $81\pi$
24.  $\frac{1}{9}$ th region of the circle with the radius  $3\sqrt{7}$  cm is shaded. Then find the area of unshaded region. **[MP NTSE Stage-1 2015]**
- (A) 154 sq. cm                      (B) 176 sq. cm                      (C) 44 sq. cm                      (D) 22 sq. cm

25. A circle with area  $A \text{ cm}^2$  is contained in the interior of a larger circle with area  $(A + B) \text{ cm}^2$  and the radius of the larger circle is 4 cm. If  $A, B, A + B$  are in arithmetic progression, then the diameter (in cm) of the smaller circle is : **(NTSE Stage-2 /2015)**
- (A)  $\frac{\sqrt{3}}{2}$                       (B)  $\frac{4\sqrt{3}}{3}$                       (C)  $\frac{8\sqrt{3}}{3}$                       (D)  $2\sqrt{3}$
26.  $\Delta ABC$  is a right angled triangle with  $\angle A = 90^\circ$ ,  $AB = b \text{ cm}$ ,  $AC = a \text{ cm}$ , and  $BC = c \text{ cm}$ . A circle is inscribed in this triangle. The radius of the circle, in cm, is : **[Haryana NTSE Stage-1 2016]**
- (A)  $a + b - c$                       (B)  $\frac{1}{2} (a + b - c)$                       (C)  $\frac{1}{2} (a - b + c)$                       (D)  $\sqrt{a^2 + b^2 + c^2}$
27. The wheel of a motor car makes 1000 revolutions in moving 440 m. The diameter of the wheel is **[Bihar NTSE Stage-1 2016]**
- (A) 0.44 m                      (B) 0.14 m                      (C) 0.24 m                      (D) 0.34 m
28. The area of the largest circle that can be drawn inside a square side 28 cm is **[Bihar NTSE Stage-1 2016]**
- (A) 17248                      (B) 784                      (C) 8624                      (D) 616
29. The radius of a wheel is 0.25m The number of revolution to travel a distance of 11 km will be- **[UP NTSE Stage-1 2017]**
- (A) 1000                      (B) 4000                      (C) 8000                      (D) 7000

## Answer Key

### Exercise Board Level

**TYPE (I)**

2. 14 : 11      3.  $9\pi \text{ cm}^2$       4. 50 cm      5.  $462 \text{ cm}^2$   
 6. No

**TYPE (II)**

7. 2 : 1      8.  $54.5 \text{ cm}^2$       9.  $(32 + 2\pi) \text{ m}^2$       10.  $17.8 \text{ cm}^2$   
 11.  $30.96 \text{ cm}^2$       12.  $15246 \text{ cm}^2$       13. Rs 3061.50      14.  $462 \text{ cm}^2$

**TYPE (III)**

15.  $248 - 4\pi$       16.  $39.25 \text{ cm}^2$       17. 560      18.  $196 \text{ cm}^2$   
 19.  $1.967 \text{ cm}^2$       20.  $168 \text{ cm}^2$       21.  $4.3 \text{ m}^2$       22. 1 : 3 : 5  
 23.  $73\frac{1}{3} \text{ cm}$

**TYPE (IV)**

24.  $24\sqrt{21} - 77$       25.  $800 \text{ cm}^2$

### Previous Year Problems

1.  $21 \text{ cm}^2$       2. (B)      3.  $66.5 \text{ cm}^2$   
 4. (i) 22 cm      (ii)  $231 \text{ cm}^2$   
 5.  $24.5 \text{ cm}^2$   
 $\text{cm}^2$       6.  $334.39 \text{ cm}^2$       7.  $392 \text{ cm}^2$       8.  $17.79 \text{ cm}^2, 598.21$   
 9.  $1386 \text{ m}^2$       10.  $36.39 \text{ cm}^2$       11.  $410.66 \text{ cm}^2$       13.  $60^\circ$   
 14.  $217 \text{ cm}^2, 78 \text{ cm}$       15.  $99/8 \text{ cm}^2$       16.  $3465 \text{ cm}^2$

### Exercise-1

#### SUBJECTIVE QUESTIONS

**Section (A)**

- A-1. 36 cm      A-2. 1980 m  
 A-3. radii of the given circles are 10 cm and 4 cm respectively.  
 A-4. Rs 528      A-5. (i)  $49.5 \text{ cm}^2$       (ii) Rs. 12.375 (Approximately)

**Section (B)**

- B-1.** 13.08 cm<sup>2</sup>      **B-2.** 10.26      **B-3.**  $\frac{25\pi}{3}$  cm<sup>2</sup>      **B-4.** 28.5 cm<sup>2</sup>, 285.5 cm<sup>2</sup>  
**B-5.** 192.5 m<sup>2</sup>

**Section (C)**

- C-1.** 76 cm.      **C-2.** Cost of making the design = Rs. 1626.8.  
**C-3.**  $\left(\frac{22528}{7} - 768\sqrt{3}\right)$  cm<sup>2</sup>.      **C-4.**  $\frac{256}{7}$  cm<sup>2</sup>.

**OBJECTIVE QUESTIONS**

**Section (A)**

- A-1.** (D)      **A-2.** (C)      **A-3.** (A)      **A-4.** (A)      **A-5.** (A)  
**A-6.** (D)      **A-7.** (B)      **A-8.** (C)      **A-9.** (B)      **A-10.** (B)

**Section (B)**

- B-1.** (C)      **B-2.** (A)      **B-3.** (D)      **B-4.** (C)      **B-5.** (A)

**Section (C)**

- C-1.** (D)      **C-2.** (D)      **C-3.** (C)      **C-4.** (C)      **C-5.** (C)

**Exercise-2**

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	B	A	B	D	C	A	D	A	B	A	A	D	B

**Exercise-3**

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	B	C	B	C	C	D	D	A	B	A	A	A	B	C	D	B	C	D	A
Ques.	21	22	23	24	25	26	27	28	29											
Ans.	B	C	A	B	C	B	B	D	D											