QUADRATIC EQUATION & EXPRESSION

(KEY CONCEPTS + SOLVED EXAMPLES)

QUADRATIC EQUATION & EXPRESSION

1. *Polynomial*
2. *Quadratic Expression*
3. *Quadratic Equation*
4. *Solution of Quadratic Equation*
5. *Nature of Roots*
6. *Sum and Product of Roots*
7. *Formation of an equation with given roots*
8. *Roots Under Particular Cases*
9. *Condition for Common Roots*
10. *Nature of the Factors of the Quadratic Expression*
11. *Maximum and Minimum Value of Quadratic Expression*
12. *Sign of the Quadratic Expression*

KEY CONCEPTS

1. Polynomial

Algebraic expression containing many terms is called Polynomial.

e.g $4x^4 + 3x^3 - 7x^2 + 5x + 3$, $3x^3 + x^2 - 3x + 5$

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots a_{n-1}x^{n-1} + a_nx^n$

where x is a variable, $a_0, a_1, a_2 \ldots\ldots\ldots a_n \in C$.

1.1 Real Polynomial : Let $a_0, a_1, a_2 \ldots\ldots a_n$ be real numbers and x is a real variable.

Then $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$ is called real polynomial of real variable x with real coefficients.

eg. $-3x^3 - 4x^2 + 5x - 4$, $x^2 - 2x + 1$ etc. are real polynomials.

1.2 Complex Polynomial: If $a_0, a_1, a_2 \ldots a_n$ be complex numbers and x is a varying complex number, then $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots a_nx^n$ is called a complex polynomial of complex variable x with complex coefficients.

eg.- $3x^2 - (2+4i)x + (5i-4)$, $x^3 - 5ix^2 + (1+2i)x + 4$ etc. are complex polynomials.

1.3 Degree of Polynomial : Highest Power of variable x in a polynomial is called as a degree of polynomial.

e.g. $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots a_{n-1}x^{n-1} + a_nx^n$ is n degree polynomial.

$f(x) = 4x^3 + 3x^2 - 7x + 5$ is 3 degree polynomial

$f(x) = 3x - 4$ is single degree polynomial or Linear polynomial.

$f(x) = bx$ is odd Linear polynomial

2. Quadratic Expression

A Polynomial of degree two of the form $ax^2 + bx + c$ $(a \neq 0)$ is called a quadratic expression in x.

e.g $3x^2 + 7x + 5$, $x^2 - 7x + 3$

General form : – $f(x) = ax^2 + bx + c$

where $a, b, c \in C$ and $a \neq 0$

3. Quadratic Equation

A quadratic Polynomial f(x) when equated to zero is called Quadratic Equation.

e.g $3x^2 + 7x + 5 = 0$, $-9x^2 + 7x + 5 = 0$,

$x^2 + 2x = 0$, $2x^2 = 0$

General form :

$$ax^2 + bx + c = 0$$

Where, $a, b, c \in C$ and $a \neq 0$

3.1 Roots of a Quadratic Equation

The values of variable x which satisfy the quadratic equation is called as Roots (also called solutions or zeros) of a Quadratic Equation.

4. Solution of Quadratic Equation

4.1 Factorization Method :

Let $ax^2 + bx + c = a(x-\alpha)(x-\beta) = 0$

Then $x = \alpha$ and $x = \beta$ will satisfy the given equation.

Hence factorize the equation and equating each to zero gives roots of equation.

e.g. $3x^2 - 2x - 1 = 0 \quad \equiv (x-1)(3x+1) = 0$

$$x = 1, \frac{1}{3}$$

4.2 Hindu Method (Sri Dharacharya Method) :

By completing the perfect square as

$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Adding and substracting $\left(\frac{b}{2a}\right)^2$

$$\Rightarrow \left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = 0$$

Which gives, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hence the Quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has two roots, given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Note : Every quadratic equation has two and only two roots.

5. Nature of Roots

In Quadratic equation $ax^2 + bx + c = 0$, the term $b^2 - 4ac$ is called discriminant of the equation, which plays an important role in finding the nature of the roots. It is denoted by Δ or D.

(A) Suppose a, b, c $\in$ R and a $\neq$ 0 then

(i) If $D > 0 \Rightarrow$ Roots are Real and unequal

(ii) If $D = 0 \Rightarrow$ Roots are Real and equal and each equal to $-b/2a$

(iii) If $D < 0 \Rightarrow$ Roots are imaginary and unequal or complex conjugate.

(B) Suppose a, b, c $\in$ Q, a $\neq$ 0 then

(i) If $D > 0$ and D is perfect square

$\Rightarrow$ Roots are unequal and Rational

(ii) If $D > 0$ and D is not perfect square

$\Rightarrow$ Roots are irrational and unequal

5.1 Conjugate Roots :

The Irrational and complex roots of a quadratic equation are always occurs in pairs. Therefore $(a, b, c, \in Q)$

If One Root	then Other Root
$\alpha + i\beta$	$\alpha - i\beta$
$\alpha + \sqrt{\beta}$	$\alpha - \sqrt{\beta}$

6. Sum and Product of Roots

If α and β are the roots of quadratic equation $ax^2 + bx + c = 0$, then,

(i) Sum of Roots

$$S = \alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(ii) Product of Roots

$$P = \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

e.g. In equation

$$3x^2 + 4x - 5 = 0$$

Sum of roots $\quad S = -\frac{4}{3}$,

Product of roots $\quad P = -\frac{5}{3}$

6.1 Relation between Roots and Coefficients

If roots of quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are and then :

(i) $(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm\frac{\sqrt{b^2-4ac}}{a} = \frac{\pm\sqrt{D}}{a}$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2 - 2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha + \beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= -\frac{b\sqrt{b^2-4ac}}{a^2} = \frac{\pm\sqrt{D}}{a}$$

(iv) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -\frac{b(b^2 - 3ac)}{a^3}$

(v) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$

$= \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\left\{(\alpha+\beta)^2 - \alpha\beta\right\} \qquad = \frac{(b^2 - ac)\sqrt{b^2-4ac}}{a^3}$

(vi) $\alpha^4 + \beta^4 = \left\{(\alpha+\beta)^2 - 2\alpha\beta\right\}^2 - 2\alpha^2\beta^2 \qquad = \left(\frac{b^2-2ac}{a^2}\right) - 2\frac{c^2}{a^2}$

(vii) $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2)$

$$= \frac{-b(b^2-2ac)\sqrt{b^2-4ac}}{a^4}$$

(viii) $\alpha^2 + \alpha\beta + \beta^2 = (\alpha + \beta)^2 - \alpha\beta$

(ix) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

(x) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$

(xi) $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2 = \frac{\alpha^4+\beta^4}{\alpha^4\beta^4} = \frac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

7. Formation of an Equation with given Roots

A quadratic equation whose roots are α and β is given by

$(x - \alpha)(x - \beta) = 0$

$\therefore\ x^2 - \alpha x - \beta x + \alpha\beta = 0$

$\therefore\ x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e x^2 – (sum of Roots)x + Product of Roots = 0

$\therefore\ x^2 - Sx + P = 0$

7.1 Equation in terms of the Roots of another Equation

If are roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are

(i) $-\alpha, -\beta \Rightarrow ax^2 - bx + c = 0$ (Replace x by – x)

(ii) $1/\alpha, 1/\beta \Rightarrow cx^2 + bx + a = 0$ (Replace x by 1/x)

(iii) α^n, β^n ; $n \in N$ $a(x^{1/n})^2 + b(x^{1/n}) + c = 0$ (Replace x by $x^{1/n}$)

(iv) $k\alpha, k\beta \Rightarrow ax^2 + kbx + k^2 c = 0$ (Replace x by x/k)

(v) $k + \alpha, k + \beta \Rightarrow a(x - k)^2 + b(x - k) + c = 0$ (Replace x by (x–k))

(vi) $\frac{\alpha}{k}, \frac{\beta}{k}$ $k^2 ax^2 + kbx + c = 0$ (Replace x by kx)

(vii) $\alpha^{1/n}, \beta^{1/n}$; $n \in N \Rightarrow a(x^n)^2 + b(x^n) + c = 0$ (Replace x by x^n)

7.2 Symmetric Expressions

The symmetric expressions of the roots , of an equation are those expressions in and , which do not change by interchanging and . To find the value of such an expression, we generally express that in terms of and .

Some examples of symmetric expressions are–

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^2 + \alpha\beta + \beta^2$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iv) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(v) $\alpha^2\beta + \beta^2\alpha$ (vi) $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2$

(vii) $\alpha^3 + \beta^3$ (viii) $\alpha^4 + \beta^4$

8. Roots Under Particular Cases

For the quadratic equation $ax^2 + bx + c = 0$

(i) If $b = 0 \Rightarrow$ roots are of equal magnitude but of opposite sign

(ii) If $c = 0 \Rightarrow$ one root is zero other is $-b/a$

(iii) If $b = c = 0 \Rightarrow$ both root are zero

(iv) If $a = c \Rightarrow$ roots are reciprocal to each other

(v) If $\left.\begin{matrix} a > 0 \ \ c < 0 \\ a < 0 \ c > 0 \end{matrix}\right\} \Rightarrow$ Roots are of opposite signs

(vi) If $\left.\begin{matrix} a > 0, b > 0, c > 0 \\ a < 0, b < 0, c < 0 \end{matrix}\right\} \Rightarrow$ Both roots are negative.

(vii) $\left.\begin{matrix} a > 0, b < 0, c > 0 \\ a < 0, b > 0, c < 0 \end{matrix}\right\} \Rightarrow$ Both roots are positive.

(viii) If sign of a = sign of b $\neq$ sign of c

$\Rightarrow$ Greater root in magnitude is negative.

(ix) If sign of b = sign of c $\neq$ sign of a $\Rightarrow$ Greater root in magnitude is positive.

(x) If a + b + c = 0 $\Rightarrow$ one root is 1 and second root is c/a.

(xi) If a = b = c = 0 then equation will become an identity and will be satisfy by every value of x.

9. Condition for Common Roots

9.1 Only One Root is Common : Let α be the common root of quadratic equations

$a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ then

$\therefore$ $a_1\alpha^2 + b_1\alpha + c_1 = 0$

$a_2\alpha^2 + b_2\alpha + c_2 = 0$

By Cramer's rule :

$$\frac{\alpha^2}{\begin{vmatrix} -c_1 & b_1 \\ -c_2 & b_2 \end{vmatrix}} = \frac{\alpha^2}{\begin{vmatrix} a_1 & -c_1 \\ a_2 & -c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

or

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\therefore\ \alpha = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1},\ \alpha^2 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1},\ \alpha \neq 0.$$

$\therefore$ The condition for only one Root common is $(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

9.2 Both roots are common : Then required conditions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Note : Two different quadratic equation with rational coefficient cannot have single common root which is complex or irrational, as imaginary and surd roots always occur in pair.

10. Nature of the Factors of the Quadratic Expression

The nature of factors of the quadratic expression $ax^2 + bx + c$ is the same as the nature of roots of the corresponding quadratic equation $ax^2 + bx + c = 0$ (a, b, c, $\in$R). Thus the factors of the expression are:

(i) Real and different, if $b^2 - 4ac > 0$.

(ii) Rational and different, if $b^2 - 4ac$ is a perfect square where (a, b, c, $\in$Q).

(iii) Real and equal , if $b^2 - 4ac = 0$.

(iv) Imaginary, if $b^2 - 4ac < 0$.

eg. The factors of $x^2 - x + 1$ are -

Sol. The factors of $x^2 - x + 1$ are imaginary because

$b^2 - 4ac = (-1)^2 - 4(1)(1)$

$= 1 - 4 = -3 < 0$

11. Maximum & Minimum Value of Quadratic Expression

In a Quadratic Expression $ax^2 + bx + c$

(i) If a > 0 Quadratic expression has least value at $x = -\frac{b}{2a}$. This least value is given by

$$\frac{4ac - b^2}{4a} = -\frac{D}{4a}$$

(ii) If a < 0, Quadratic expression has greatest value s at $x = -\frac{b}{2a}$. This greatest value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

12. Sign of the Quadratic Expression

Let $y = ax^2 + bx + c \quad (a \neq 0)$

$$y = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right] \qquad ...(1)$$

Where $D = b^2 - 4ac$ is the Discriminant of the quadratic equation $ax^2 + bx + c = 0$

Case 1.

$D > 0$: Suppose the roots of $ax^2 + bx + c = 0$ are α and β and $\alpha > \beta$ (say).

α, β are real and distinct.

Then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

Clearly $(x - \alpha)(x - \beta) > 0$ for $x < \beta$ and $x < \alpha$ since both factors are of the same sign and

$(x - \alpha)(x - \beta) < 0$ for $\alpha > x > \beta$

For $x = \beta$ or $x = \alpha$, $(x - \alpha)(x - \beta) = 0$

$\therefore$ If a > 0, then $ax^2 + bx + c > 0$ for all x outside the interval $[\beta, \alpha]$ and is negative for all x in (β, α).

If a < 0, then its viceversa.

Case 2.

D = 0 then from (1)

$$ax^2 + bx + c = a\left(x + \frac{b^2}{2a}\right)^2$$

$\therefore$ $\forall\, x \neq -\frac{b}{2a}$, the quadratic expression takes on values of the same sign as a;

If $x = -b/2a$ then $ax^2 + bx + c = 0$.

$\therefore$ If D = 0, then

(i) $ax^2 + bx + c > 0$ has a solution any $x \neq -\frac{b}{2a}$

if a > 0 and has no solution if a < 0;

(ii) $ax^2 + bx + c < 0$ has a solution any $x \neq -\frac{b}{2a}$

if a < 0 and has no solution if a > 0;

(iii) $ax^2 + bx + c \geq 0$ has any x as a solution

if a > 0 and the unique solution

$x = -\dfrac{b}{2a}$, if a < 0;

(iv) $ax^2 + bx + c \le 0$ has any x as a solution

if a < 0 and $x = -\dfrac{b}{2a}$, if a > 0;

Case 3.

D < 0 then from (1)

(i) if a > 0, then $ax^2 + bx + c > 0$ for all x;

(ii) if a < 0, then $ax^2 + bx + c < 0$ for all x.

eg. The sign of $x^2 + 2x + 3$ is positive for all $x \in R$, because here

$b^2 - 4ac = 4 - 12 = -8 < 0$ and a = 1 > 0.

eg. The sign of $3x^2 + 5x - 8$ is negative for all $x \in R$ because here

$b^2 - 4ac = 25 - 96 = -71 < 0$ and $a = -3 < 0$

12.1 Graph of Quadratic Expression :

Consider the expression $y = ax^2 + bx + c$, $a \neq 0$ and $a,b,c \in R$ then the graph between x, y is always a parabola. If a > 0 then the shape of the parabola in concave upward and if a < 0 then the shape of the parabola is concave downwards.

There is only 6 possible graph of a Quadratic expression as given below :

Case - I When a > 0

(i) If D > 0

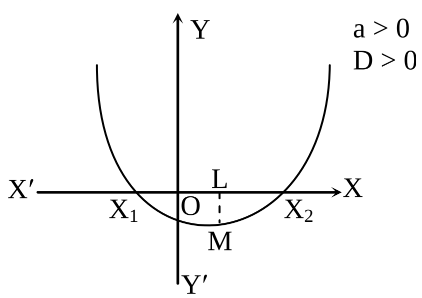
Roots are real and different (X_1 and X_2)

Minimum value $LM = \dfrac{4ac - b^2}{4a}$ at

$x = OL = -b/2a$

y is positive for all x out side interval $[x_1, x_2]$

and is negative for all x inside (x_1, x_2)

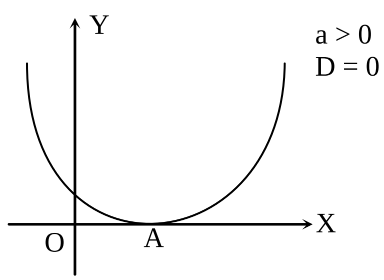


(ii) If D = 0

Roots are equal (OA)

Min. value = 0 at x = OA = –b/2a

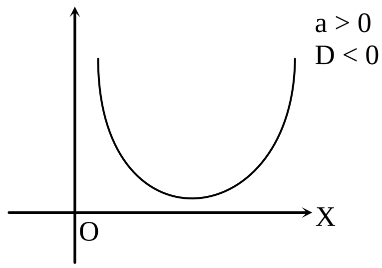
$y > 0$ for all $x \in \left\{R - \dfrac{-b}{2a}\right\}$



(iii) If D < 0

Roots are complex conjugate

y is positive for all $x \in R$.



Case- II When $a < 0$

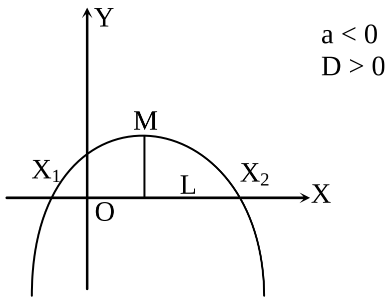
(i) If $D > 0$

Roots are real and different (x_1 and x_2)

Max. value = LM = $\dfrac{4ac - b^2}{4a}$ at x = OL = –b/2a

y is positive for all x inside (x_1, x_2) and

y is negative for all x outside [x_1, x_2]

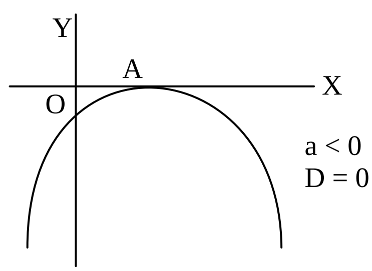


(ii) If D = 0

Roots are equal = OA

Max. value = 0 at x = OA = –b/2a

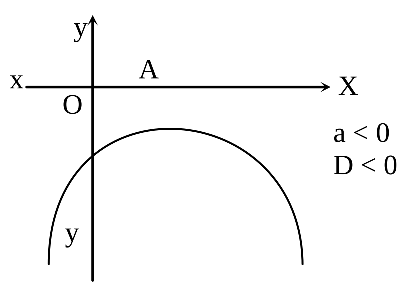
y is negative for all $x \in \left\{R - \dfrac{-b}{2a}\right\}$



(iii) If D < 0

Roots are complex conjugate

y is negative for all $x \in R$



13. Quadratic Expression in two Variables

The general form of a quadratic expression in two variable x & y is

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

The condition that this expression may be resolved into two linear rational factors is

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and $h^2 - ab > 0$

This expression is called discriminant of the above quadratic expression.

14. Some Important Points

(i) Every equation of n^{th} degree ($n \geq 1$) has exactly n roots and if the equation has more than n roots, it is an identity.

(ii) If α is a root of the equation f (x) = 0 then the polynomial f (x) is exactly divisible by $(x-\alpha)$ or $(x - \alpha)$ is a factor of f (x)

(iii) If quadratic equations $a_1 x^2 + b_1 x + c_1 = 0$ and $a_2 x^2 + b_2 x + c_2 = 0$ are in the same ratio $\left(\text{i.e.} \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}\right)$ then

$$\frac{b_1^2}{b_2^2} = \frac{a_1 c_1}{a_2 c_2}$$

(iv) If one root is k times the other root of quadratic equation $a_1 x^2 + b_1 x + c_1 = 0$ then

$$\frac{(k+1)^2}{k} = \frac{b^2}{ac}$$

(v) Quadratic equations containing modulas sign are solved considering both positive and negative values of the quantity containing modulus sign. Finally the roots of the given equation will be those values among the values of the variable so obtained which satisfy the given equation.

SOLVED EXAMPLES

Ex.1 The number of real roots of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ is-

(A) 2 (B) 1

(C) infinite (D) None

Sol. Let $e^{\sin x} = y$ then given equation reduces to

$$y - \frac{1}{y} - 4 = 0$$

$\Rightarrow$ $y^2 - 4y - 1 = 0$

$\Rightarrow$ $y = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$

$= 4.23, - 0.23$

But $y = e^{\sin x}$ is never negative. So

$y = e^{\sin x} = 4.23$

$\Rightarrow$ $\sin x = \log 4.23 > 1$

which is not possible. Hence the equation has no real root. **Ans.[D]**

Ex.2 If r and s are positive, then roots of the equation $x^2 - rx - s = 0$ are -

(A) imaginary

(B) real and both positive

(C) real and of opposite signs

(D) real and both negative

Sol. Here Discriminant

$= r^2 + 4s > 0$ $(\because r, s > 0)$

$\Rightarrow$ roots are real

Again $a = 1 > 0$ and $c = - s < 0$

$\Rightarrow$ roots are of opposite signs.

Ans.[C]

Ex.3 Both roots of the equation $(x - b)(x - c) + (x - c)(x - a) + (x - a)(x - b) = 0$ are -

(A) positive (B) negative

(C) real (D) imaginary

Sol. The given equation can be written in the following form :

$3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

Here discriminant

$= 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4[(a^2 + b^2 + c^2) - (ab + bc + ca)] > 0$

$[\because a^2 + b^2 + c^2 > ab + bc + ca]$

$\therefore$ Both roots are real. **Ans.[C]**

Ex.4 If the roots of the quadratic equation $x^2 - 4x - \log_3 a = 0$ are real, then the least value of a is-

(A) 81 (B) 1/81

(C) 1/64 (D) None of these

Sol. Since the roots of the given equation are real.

$\therefore$ Disc. $> 0 \Rightarrow 16 + 4\log_3 a \ge 0$

$\Rightarrow \log_3 a \ge -4 \Rightarrow a \ge 3^{-4} \Rightarrow a \ge 1/81$

Hence, the least value of a is 1/81.

Ans.[B]

Ex.5 If $a < b < c < d$, then roots of $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are-

(A) real and equal (B) real and unequal

(C) imaginary (D) rational

Sol. Here

$3x^2 - (a + c + 2b + 2d)x + (ac + 2bd) = 0$

$\therefore$ Discriminant

$= (a + c + 2b + 2d)^2 - 12(ac + 2bd)$

$= [(a + 2d) - (c + 2b)]^2 + 4(a + 2d)(c + 2b) - 12(ac + 2bd)$

$= [(a + 2d) - (c + 2b)]^2 + 8(c - b)(d - a) > 0.$

Ans.[B]

Ex.6 If roots of the equation $ax^2 + 2bx + c = 0$ are real and different, then roots of the equation $(a^2 + 2b^2 - ac)x^2 + 2b(a + c)x + (2b^2 + c^2 - ac) = 0$ are-

(A) real and equal (B) real and unequal

(C) imaginary (D) None of these

Sol. The second equation can be written as

$(a + c)(ax^2 + 2bx + c) = 2(ac - b^2)(x^2 + 1)$

$\Rightarrow 2(ac - b^2)(x^2 + 1) = 0$ ...(1)

Since roots of $ax^2 + 2bx + c = 0$ are real and different, therefore

$D = 4b^2 - 4ac > 0 \Rightarrow b^2 > ac$

Thus from (1), we get

$x^2 + 1 = 0$

or $x = \pm i$

So roots of the second equation are imaginary.

Ans.[C]

Ex.7 For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of roots is zero, then the sum of roots is-

(A) 0 (B) $\frac{2ab}{b+c}$ (C) $\frac{2bc}{b+c}$ (D) $\frac{-2bc}{b+c}$

Sol. $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$

$\frac{b-a}{x^2+(b+a)x+ab} = \frac{1}{x+c}$

or $x^2 + (a + b)x + ab = (b - a)x + (b - a)c$

or $x^2 + 2ax + ab + ca - bc = 0$

Since product of the roots = 0

$ab + ca - bc = 0 \Rightarrow a = \frac{bc}{b+c}$

Thus sum of roots $= -2a = \frac{-2bc}{b+c}$

Ans.[D]

Ex.8 If p and q are roots of the equation $x^2 - 2x + A = 0$ and r and s be roots of the equation $x^2 - 18x + B = 0$ if $p < q < r < s$ be in A.P., then A and B are respectively-

(A) – 3, 77 (B) 3, 77

(C) 3,– 77 (D) None of these

Sol. Here p, q are roots of $x^2 - 2x + A = 0$

$\therefore\ p + q = 2$ ...(1)

Also r, s are roots of $x^2 - 18x + B = 0$

$\therefore\ r + s = 18$ ...(2)

Now since p,q,r,s in A.P. say with common difference d.

$\therefore\ q = p + d,\ r = p + 2d,\ s = p + 3d$

Form (1) and (2)

$\left.\begin{matrix} 2p+d=2 \\ 2p+5d=18 \end{matrix}\right\} \Rightarrow 4d = 16 \Rightarrow d = 4$

$\therefore 2p + 4 = 2 \Rightarrow p = -1$

Hence $p = -1,\ q = -1 + 4 = 3$

$r = -1 + 8 = 7,\ s = -1 + 12 = 11$

$A = pq = -3,\ B = rs = 77$

Ans.[A]

Ex.9 If α, β are roots of the equation $ax^2 + 3x + 2 = 0$ $(a < 0)$, then $\alpha^2/\beta + \beta^2/\alpha$ is greater than-

(A) 0 (B) 1

(C) 2 (D) None of these

Sol. Since $a < 0$, therefore discriminant $D = 9 - 8a > 0$. So, α and β are real.

We have $\alpha + \beta = \frac{-3}{a}$ and $\alpha\beta = \frac{2}{a}$

$\therefore\ \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{(\alpha+\beta)^3}{\alpha\beta} - 3(\alpha + \beta)$

$= -\frac{27}{2a^2} + \frac{9}{a} < 0$ $[\because a < 0]$ **Ans.[D]**

Ex.10 If α, β are the roots of $x^2 - p(x+1) - c = 0$ then $\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+c} + \frac{\beta^2+2\beta+1}{\beta^2+2\beta+c}$ is equal to-

(A) 0 (B) 1

(C) 2 (D) None of these

Sol. Here the equation is $x^2 - p(x + 1) - c = 0$

$\therefore\ \alpha + \beta = p,\ \alpha\beta = -(p + c)$

$\Rightarrow\ (\alpha + 1)(\beta + 1) = 1 - c$

Now given expression

$= \frac{(\alpha+1)^2}{(\alpha+1)^2-(1-c)} + \frac{(\beta+1)^2}{(\beta+1)^2-(1-c)}$,

Putting value of $1 - c = (\alpha + 1)(\beta + 1)$

$= \frac{\alpha+1}{\alpha-\beta} + \frac{\beta+1}{\beta-\alpha}$

$= \frac{\alpha+1-\beta-1}{\alpha-\beta} = 1$ **Ans.[B]**

Note :– Some times an intermediate step calculation is necessary before the main problem is attempted. As above $(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$

$= -p - c + p + 1 = 1 - c$

Without which the main result would be difficult to find.

Ex. 11 If α, β be the roots $x^2 + px - q = 0$ and γ, δ be the roots of $x^2 + px + r = 0$ then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} =$

(A) 1 (B) q (C) r (D) q + r

Sol. Here $\left.\begin{array}{l}\alpha+\beta=-p\\ \gamma+\delta=-p\end{array}\right\} \Rightarrow \alpha+\beta=\gamma+\delta$ (note)

Now $(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta$

$= \alpha^2 - \alpha(\alpha + \beta) + r$

$= -\alpha\beta + r$

$= -(-q) + r = q + r$

By symmetry of the results

$(\beta - \gamma)(\beta - \delta) = q + r$

Hence the ratio is 1 **Ans.[A]**

Note :– If we ignore the equality of $\alpha + \beta$ and $\gamma + \delta$, the problem may be seen to be difficult, or at least the calculations are increased unnecessarily.

Ex.12 If α, β be roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$ then

$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$

(A) $p^2 + q^2$ (B) $p^2 - q^2$

(C) $q^2 - p^2$ (D) none of these

Sol. Here $\left.\begin{array}{l}\alpha+\beta=-p;\ \alpha\beta=1\\ \gamma+\delta=-q;\gamma\delta=1\end{array}\right\} \Rightarrow \alpha\beta=\gamma\delta$

Now $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$= \{\alpha\beta - \gamma(\alpha + \beta) + \gamma^2\}\{\alpha\beta + \delta(\alpha + \beta) + \delta^2\}$

$= \{1 + \gamma p + \gamma^2\}\{1 - p\delta + \delta^2\}$

$= [(\gamma^2+1) + \gamma p][(\delta^2 + 1) - p\delta)]$

$= (-q\gamma + \gamma p)(-q\delta - p\delta)$

$= \gamma\delta(q^2 - p^2) = 1(q^2 - p^2)$

Ans.[C]

Note:– Remember that the root always satisfy the equation and hence this fact may be used to find some values which may be occurring in the calculations.

Ex.13 If α and β are roots of the equation $x^2 + px + q = 0$ and α^4 and β^4. are roots of $x^2 - rx + s = 0$, then the roots of $x^2 - 4qx + 2q^2 - r = 0$ are–

(A) both real

(B) both positive

(C) both negative

(D) one negative and one positive

Sol. The discriminant of the equation

$x^2 - 4qx + 2q^2 - r = 0$ is

$D = 16q^2 - 4(2q^2 - r) = 8q^2 + 4r$ ...(1)

But α, β are roots of the equation

$x^2 + px + q = 0$

$\Rightarrow \alpha + \beta = -p$ and $\alpha\beta = q$

and α^4, β^4 are roots of the equation

$x^2 - rx + s = 0$

$\Rightarrow \alpha^4 + \beta^4 = r$ and $\alpha^4\beta^4 = s$

$\therefore D = 8\alpha^2\beta^2 + (\alpha^4 + \beta^4)$

$= 4(\alpha^2 + \beta^2)^2 \geq 0$

Thus both roots are real.

Ans.[A]

Ex.14 If one root of the equation $4x^2 + 2x - 1 = 0$ is α, then other root is-

(A) 2α (B) $4\alpha^3 - 3\alpha$

(C) $4\alpha^3 + 3\alpha$ (D) None of these

Sol. Let α and β are roots of the given equation, then

$\alpha + \beta = -\frac{1}{2} \Rightarrow \beta = -\frac{1}{2} - \alpha$

Now $4\alpha^2 + 2\alpha - 1 = 0$

$\Rightarrow 4\alpha^2 = 1 - 2\alpha$ ...(1)

Now $4\alpha^3 = \alpha - 2\alpha^2$

$= \alpha - \frac{1}{2}(1 - 2\alpha)$ [from (1)]

$\therefore 4\alpha^3 - 3\alpha = -2\alpha - \frac{1}{2}(1 - 2\alpha)$

$= -\frac{1}{2} - \alpha = \beta$ **Ans.[B]**

Ex.15 If α, β are roots of $Ax^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$, then p is equal to-

(A) $(B^2 - 2AC)/A^2$ (B) $(2AC - B^2)/A^2$

(C) $(B^2 - 4AC)/A^2$ (D) $(4AC - B^2)A^2$

Sol. $\alpha + \beta = -B/A$, $\alpha\beta = C/A$

$\alpha^2 + \beta^2 = -p, \alpha^2\beta^2 = q$

$\therefore (\alpha + \beta)^2 = B^2/A^2$

$\Rightarrow (\alpha^2 + \beta^2) + 2\alpha\beta = B^2/A^2$

$\Rightarrow -p + 2C/A = B^2/A^2$

$\Rightarrow p = \dfrac{2CA - B^2}{A^2}$ **Ans.[B]**

Ex.16 If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then $a\,S_{n+1} + c\,S_{n-1} =$

(A) bS_n (B) b^2S_n

(C) $2bS_n$ (D) $-bS_n$

Sol. Here α, β are roots

$\therefore$ $a\alpha^2 + b\alpha + c = 0$ ...(1)

$a\beta^2 + b\beta + c = 0$ ...(2)

Now let us consider (Keeping results (1), (2) in mind)

$a\,S_{n+1} + b\,S_n + c\,S_{n-1}$

$= a[\alpha^{n+1} + \beta^{n+1}] + b[\alpha^n + \beta^n] + c[\alpha^{n-1} + \beta^{n-1}]$

$= [a\alpha^{n+1} + b\alpha^n + c\alpha^{n-1}] + [a\beta^{n+1} + b\beta^n + c\beta^{n-1}]$

$= \alpha^{n-1}[a\alpha^2 + b\alpha + c] + \beta^{n-1}[a\beta^2 + b\beta + c]$

$= 0 + 0 = 0$

Hence $aS_{n+1} + cS_{n-1} = -bS_n$. **Ans.[D]**

Ex.17 The quadratic equation whose one root is $\dfrac{1}{2+\sqrt{5}}$ will be-

(A) $x^2 + 4x - 1 = 0$ (B) $x^2 - 4x - 1 = 0$

(C) $x^2 + 4x + 1 = 0$ (D) None of these

Sol. Given root $= \dfrac{1}{2+\sqrt{5}} = \sqrt{5} - 2$

So the other root $= -\sqrt{5} - 2$. Then sum of the roots $= -4$, product of the roots $= -1$

Hence the equation is $x^2 + 4x - 1 = 0$

Ans.[A]

Ex.18 If the roots of equation $x^2 + bx + ac = 0$ are α, β and roots of the equation $x^2 + ax + bc = 0$ are α, γ then the value of α, β, γ respectively-

(A) a, b, c (B) b, c, a

(C) c, a, b, (D) None of these

Sol. From the given two equation

$\alpha + \beta = -b$ ...(1)

$\alpha\beta = ac$ ...(2)

$\alpha + \gamma = -a$ ...(3)

$\alpha\gamma = bc$ ...(4)

(1) – (3) $\Rightarrow \beta - \gamma = a - b$ ...(5)

(2) /(4) $\Rightarrow \beta/\gamma = a/b$

$\beta = \dfrac{a\gamma}{b}$ ..(6)

putting the value of β in (5)

$\dfrac{a\gamma}{b} - \gamma = a - b \Rightarrow \gamma\,\dfrac{(a-b)}{b} = (a-b)$

$\therefore \gamma = b$

$\therefore \beta = a$ & $\alpha = c$. **Ans.[C]**

Ex.19 If every pair from among the equations $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and $x^2 + rx + pq = 0$ has a common root, then the sum of the three common roots is-

(A) $2(p + q + r)$ (B) $p + q + r$

(C) $-(p + q + r)$ (D) pqr

Sol. The given equations are

$x^2 + px + qr = 0,$ ...(i)

$x^2 + qx + rp = 0,$ ...(ii)

and $x^2 + rx + pq = 0$ ...(iii)

Let α, β be the roots of (i) β, γ be the roots of (ii) and γ, α be the roots of (iii). Since β is a common root of (i) and (ii).

$\therefore \beta^2 + p\beta + qr = 0$ and $\beta^2 + q\beta + rp = 0$

$\Rightarrow (p - q)\beta + r(q - p) = 0 \Rightarrow \beta = r$

Now, $\alpha\beta = qr \Rightarrow \alpha r = qr \Rightarrow \alpha = q$.

Since, β and γ are roots of (ii). Therefore,

$\beta\gamma = rp \Rightarrow r\gamma = rp \Rightarrow \gamma = p$

$\alpha + \beta + \gamma = q + r + p = p + q + r$.

Ans.[B]

Ex.20 If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ $(b \neq c)$ have a common root, then $a + 4b + 4c$ is equal to-

(A) –2 (B) –1

(C) 0 (D) 1

Sol. Let be the common root of the given equations. Then

$$a\alpha^2 + 2c\alpha + b = 0$$

and $a\alpha^2 + 2b\alpha + c = 0$

$\Rightarrow 2\,\alpha(c - b) + (b - c) = 0$

$\Rightarrow \alpha = \frac{1}{2}$ $[\because b \neq c]$

Putting $\alpha = 1/2$ in $a\alpha^2 + 2c\alpha + b = 0$,

we get $a + 4b + 4c = 0$. **Ans.[C]**

Ex.21 The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is-

(A) 0,2 (B) 0,–2

(C) 2,–2 (D) None of these

Sol. Let α be the root of $x^2 - x + m = 0$ and 2α be the root of $x^2 - 3x + 2m = 0$. Then,

$\alpha^2 - \alpha + m = 0$ and $4\alpha^2 - 6\alpha + 2m = 0$

$\Rightarrow \frac{\alpha^2}{-4m} = \frac{\alpha}{-2m} = \frac{1}{2} \Rightarrow m^2 = -2m$

$\Rightarrow m = 0, m = -2$ **Ans.[B]**

Ex.22 If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then, the common factor is –

(A) $(x - 3)$ (B) $(x - 6)$

(C) $(x - 8)$ (D) None of these

Sol. Here Let $x - \alpha$ is the common factor

then $x = \alpha$ is root of the corresponding equation

$\therefore$ $\alpha^2 - 11\alpha + a = 0$

$\alpha^2 - 14\alpha + 2a = 0$

Subtracting $3\alpha - a = 0 \Rightarrow \alpha = a/3$

Hence $\frac{a^2}{9} - 11\frac{a}{3} + a = 0$, $a = 0$ or $a = 24$

since $a \neq 0$, $a = 24$

$\therefore$ The common factor of $\left\{\begin{array}{l} x^2 - 11x + 24 \\ x^2 - 14x + 48 \end{array}\right.$ is

clearly $x - 8$ **Ans.[C]**

Note :– Shorter method Eliminating a from both

$$\left.\begin{array}{l} 2x^2 - 22x + 2a \\ x^2 - 14x + 2a \\ - \quad + \quad - \end{array}\right\} x^2 - 8x = 0 \Rightarrow x(x - 8) = 0$$

$\therefore x \neq 0$, $\therefore (x - 8)$ Ans.

Ex.23 If x is real then the value of the expression $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between

(A) –3 and 3 (B) –4 and 5

(C) –4 and 4 (D) –5 and 4

Sol. Let $\frac{x^2 + 14x + 9}{x^2 + 2x + 3} = y$

$\Rightarrow x^2(1 - y) + 2x(7 - y) + 3(3 - y) = 0$

Hence $4(7 - y)^2 - 12(1 - y)(3 - y) \geq 0$

gives $-2y^2 - 2y + 40 \geq 0$

$\Rightarrow y^2 + y - 20 \leq 0$

$\Rightarrow (y + 5)(y - 4) \leq 0 \Rightarrow -5 \leq y \leq 4$

Ans.[D]

Note:– Theory of max. minima may be used to find the extreme values for example in the above $(x^2 + 2x + 3)(2x + 14) - (x^2 + 14x + 9)(2x + 2) = 0$. Which gives two value of x, corresponding find values of y.

Ex.24 $\frac{8x^2 + 16x - 51}{(2x - 3)(x + 4)} > 3$ if x is such that –

(A) $x < -4$ (B) $-3 < x < 3/2$

(C) $x > 5/2$ (D) All these true

Sol. Consider $\frac{8x^2 + 16x - 51}{(2x - 3)(x + 4)} - 3 > 0$

$\Rightarrow \frac{2x^2 + x - 15}{2x^2 + 5x - 12} > 0$

$\Rightarrow \frac{(2x - 5)(x + 3)}{(2x - 3)(x + 4)} > 0$

Hence both Nr and Dr are positive if

$$x < -4 \text{ or } x > \frac{5}{2}$$

and both negative if $-3 < x < \frac{3}{2}$

Hence all the statements are true, as such (D) is the correct option.

Note:– If $a < c < b < d$, then $\frac{(x - a)(x - b)}{(x - c)(x - d)} > 0$

if $x < a$ or $x > d$ or if $c < x < b$

Ex.25 The real values of a for which the quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs are given by-

(A) $a > 5$ (B) $0 < a < 4$

(C) $a > 0$ (D) $a > 7$

Sol. The roots of the given equation will be of opposite signs if they are real and their product is negative, i.e.

Discriminant ≥ 0 and product of roots < 0

$\Rightarrow (a^3 + 8a - 1)^2 - 8(a^2 - 4a) \geq 0$ and

$$\frac{a^2 - 4a}{2} < 0$$

$\Rightarrow a^2 - 4a < 0$

$[\because a^2 - 4a < 0 \Rightarrow (a^3 + 8a - 1)^2 - 8(a^2 - 4a) \geq 0]$

$\Rightarrow 0 < a < 4.$

Ans.[B]

Ex.26 The value of the expression $x^2 + 2bx + c$ will be positive if-

(A) $b^2 - 4c > 0$ (B) $b^2 - 4c < 0$

(C) $c^2 < b$ (D) $b^2 < c$

Sol. Expression $= (x + b)^2 - b^2 + c$

$= (x + b)^2 + (c - b^2)$

$\therefore$ expression will be positive if $c - b^2 > 0$

$\Rightarrow b^2 < c$ **Ans.[D]**

Aliter : Here a = 1 > 0. Hence exp. > 0 when $B^2 - 4AC < 0$ i.e. when $4b^2 - 4c < 0 \Rightarrow b^2 < c$.

Ex.27 If roots of the equation $x^2 + ax + 25 = 0$ are in the ratio of 2 : 3 then the value of a is -

(A) $\frac{\pm 5}{\sqrt{6}}$ (B) $\frac{\pm 25}{\sqrt{6}}$

(C) $\frac{\pm 5}{6}$ (D) None of these

Sol. Here k = 2/3

so from the condition $= \frac{(k+1)^2}{k} = \frac{b^2}{ac}$

$$\frac{(2/3+1)^2}{2/3} = \frac{a^2}{25}$$

$$\Rightarrow \frac{25}{9} \times \frac{3}{2} = \frac{a^2}{25} \Rightarrow \frac{25}{6} = \frac{a^2}{25}$$

$\therefore a^2 = \frac{25 \times 25}{6} \Rightarrow a = \frac{\pm 25}{\sqrt{6}}$ **Ans.[B]**

Ex.28 If the roots of the equations $x^2 + 3x + 2 = 0$ and $x^2 - x + \lambda = 0$ are in the same ratio then the value of λ is given by-

(A) 2/7 (B) 2/9

(C) 9/2 (D) 7/2

Sol. If roots are in same ratio then

$$\frac{3^2}{(-1)^2} = \frac{(1).(2)}{(1).(\lambda)} \Rightarrow 9 = \frac{(2)}{(\lambda)} \Rightarrow \lambda = \frac{2}{9}$$

Ans.[B]

Ex.29 Let α, β be the roots of $ax^2 + bx + c = 0$ & γ, δ be the roots of $px^2 + qx + r = 0$; and D_1, D_2 the respective Discriminants of these equations. If $\alpha, \beta, \gamma, \delta$ are in A.P., then $D_1 : D_2$

(A) $\frac{a^2}{p^2}$ (B) $\frac{a^2}{b^2}$ (C) $\frac{b^2}{q^2}$ (D) $\frac{c^2}{r^2}$

Sol. We have $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

and $\gamma + \delta = \frac{-q}{p}$, $\gamma\delta = \frac{r}{p}$

Now $\alpha, \beta, \gamma, \delta$ are in A. P.

$\Rightarrow \beta - \alpha = \delta - \gamma; (\beta - \alpha)^2 = (\delta - \gamma)^2$

$\Rightarrow (\beta + \alpha)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$$\Rightarrow \frac{b^2}{a^2} - \frac{4c}{a} = \frac{q^2}{p^2} - \frac{4r}{p}$$

$$\Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$$

$$\Rightarrow \frac{D_1}{a^2} = \frac{D_2}{p^2} \Rightarrow \frac{D_1}{D_2} = \frac{a^2}{p^2}$$

Ans.[A]

Ex.30 If α_1, α_2 and β_1, β_2, are respectively roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$, then the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a none zero solution then-

(A) $p^2 br = a^2 qc$ (B) $b^2 pr = q^2 ac$

(C) $r^2 pb = c^2 ar$ (D) None of these

Sol. Obviously

$\alpha_1 + \alpha_2 = -b/a$, $\alpha_1\alpha_2 = c/a$

$\beta_1 + \beta_2 = -q/p$, $\beta_1\beta_2 = r/p$

Since the system of equations

$\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$

has a non zero solution

$\therefore \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0 \Rightarrow \alpha_1 \beta_2 - \alpha_2 \beta_1 = 0$

$$\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} \Rightarrow \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \mp \sqrt{\left\{\frac{\alpha_1 \alpha_2}{\beta_1 \beta_2}\right\}}$$

$$\therefore \frac{-b/a}{-q/p} = \sqrt{\left\{\frac{c/a}{r/p}\right\}} \Rightarrow \frac{b^2p^2}{q^2a^2} = \frac{cp}{ar}$$

$\Rightarrow b^2pr = q^2 ac$

Ans.[B]

Ex.31 The sum of all real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$, is-

(A) 0 (B) 8

(C) 4 (D) None of these

Sol. **Case I :** $x - 2 > 0, x > 2$

Putting $x-2 = y, y > 0$

$\therefore \quad y^2 + y - 2 = 0 \Rightarrow y = -2, 1$

$\Rightarrow \quad x = 0, 3$ But $0 \not> 2$,

Hence x = 3 is the real root.

Case II : $x - 2 < 0 \Rightarrow x < 2, y < 0$

$y^2 - y - 2 = 0 \Rightarrow y = 2, -1 \Rightarrow x = 4, x = 1$

since 4 |< 2, only x = 1 is the real root

Hence the sum of the real roots = 3+ 1 = 4

(C) is correct option. **Ans.[C]**

Note : The problem is asked in 'Fill up the blank form" in IIT–97(July)

Ex.32 If $0 \le x \le \pi$, then the solution of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is given by x equal to

(A) $\frac{\pi}{6}, \frac{\pi}{3}$ (B) $\frac{\pi}{3}, \frac{\pi}{2}$

(C) $\frac{\pi}{6}, \frac{\pi}{2}$ (D) None of these

Sol. Let $16^{\sin^2 x} = y$, then $16^{\cos^2 x} = 16^{1-\sin^2 x} = \frac{16}{y}$

Hence $y + \frac{16}{y} = 10 \Rightarrow y^2 - 10y + 16 = 0$

or y = 2, 8

Now $16^{\sin^2 x} = 2 \Rightarrow (2)^{4\sin^2 x} = (2)^1$

$\Rightarrow 4 \sin^2 x = 1 \quad \therefore \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$

Hence (A) is correct answer.

Ans.[A]

Note: If restriction $0 \le x \le \pi$ is not given as in the original problem then $\sin^2 x = \frac{1}{4}$

$\Rightarrow x = n\pi \pm \frac{\pi}{6}$.

Hence there will be infinite roots.

Since sin x is positive in $[0, \pi]$ negative values are not considered.

Ex.33 If p, q, r be in H. P. and p and r be different having same sign, then the root of the equation $px^2 + 2qx + r = 0$ will be-

(A) real (B) equal

(C) Imaginary (D) None of these

Sol. Here p, q, r in H. P.

$\Rightarrow \quad q = \frac{2pr}{p+r}$ ...(1)

Now $D = 4q^2 - 4pr$

$= -4\left[pr - \left(\frac{2pr}{p+r}\right)^2\right]$ using (1)

$= -(pr)\left[2\left(\frac{p-r}{p+r}\right)^2\right]$

Since $pr > 0, p \ne r$ given,

$D \ne 0$ and $D < 0$ Hence the roots are imaginary.

Ans.[C]

Note :– The students should develop a practice of arguing, using given conditions. The discriminant should be expressed in perfect square form as far as possible.

Ex.34 If $x = 2 + \sqrt{3}$ then the value of $x^3 - 7x^2 + 13x - 12$ is –

(A) 3 (B) 6 (C) –9

(D) 9

Sol. $x = 2 + \sqrt{3}$

$\Rightarrow x - 2 = \sqrt{3}$

$\Rightarrow x^2 + 4 - 4x = 3$

$\Rightarrow x^2 - 4x = -1$

$\Rightarrow x^2 - 4x + 1 = 0$

Now we can write the given equation as

$x^3 - 7x^2 + 13x - 12$

$= x(x^2 - 4x + 1) - 3x^2 + 12x - 12$

$= x(x^2 - 4x + 1) - 3(x^2 - 4x + 1) - 9$

Now putting the value of $x^2 - 4x + 1 = 0$

$= x(0) - 3(0) - 9 = -9$ **Ans.[C]**