HEIGHT

&

DISTANCES

(KEY CONCEPTS + SOLVED EXAMPLES)

HEIGHT & DISTANCES

1. *Introduction*
2. *Definitions*
3. *Some useful results*
4. *Problems based on properties of a circle*

KEY CONCEPTS

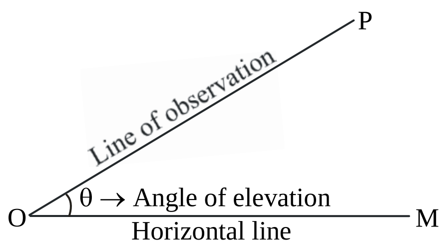
1. Introduction

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

2. Definitions

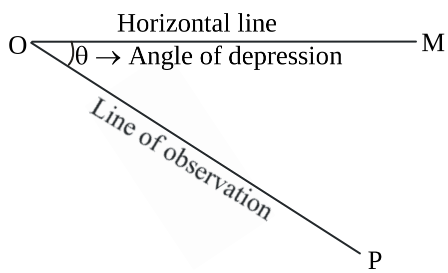
2.1 Angle of elevation :

Let O and P be two points where P is at a higher level than O. Let O be at the position of the observer and P be the position of the object. Draw a horizontal line OM through the point O. OP is called the line of observation or line of sight. Then ∠POM = θ is called the angle of elevation of P as observed from O.



2.2 Angle of depression :

In the above figure, if P be at a lower level than O, then ∠MOP = θ is called the angle of depression.



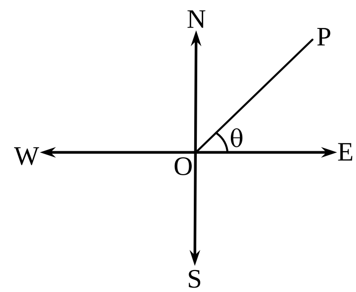
Note :

(1) The angle of elevation or depression is the angle between the line of observation and the horizontal line according as the object is at a higher or lower level than the observer.

(2) The angle of elevation or depression is always measured from horizontal line through the point of observation.

2.3 Bearing :

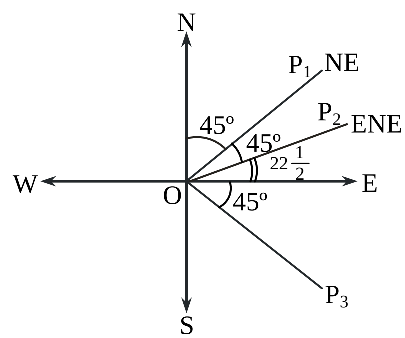
In the figure, if the observer and the object i.e., O and P be on the same level then bearing is defined. To measure the 'Bearing', the four standard directions East, West, North and South are taken as the cardinal directions. Angle between the line of observation i.e. OP and any one standard direction-east, west, north or south is measured. Thus, ∠POE = θ is called the bearing of point P with respect to O measured from east to north.



In other words the bearing of P as seen from O is the direction in which P is seen from O.

2.4 North-east :

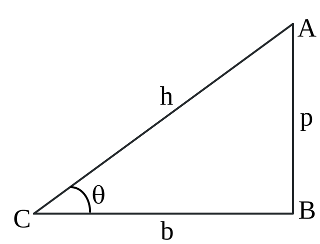
North-east means equally inclined to north and east, south-east means equally inclined to south and east. ENE means equally inclined to east and north-east.



3. Some useful results

3.1 In a triangle ABC,

$$\sin\theta = \frac{p}{h}, \quad \cos\theta = \frac{b}{h}$$

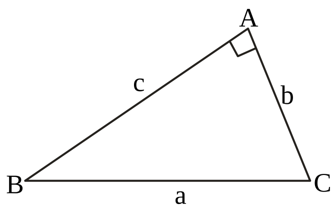


3.2 In any triangle ABC,

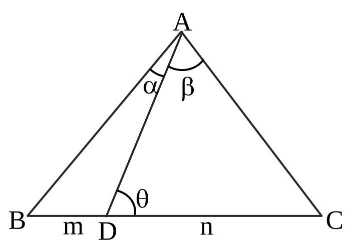
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{[By sine rule]}$$

or cosine formula

i.e., $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

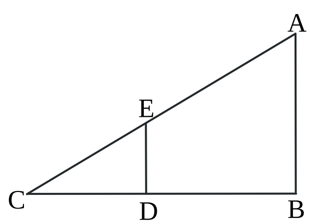


3.3 In any triangle ABC, if BD : DC = m : n and $\angle BAD = \alpha$, $\angle CAD = \beta$ and $\angle ADC = \theta$, then $(m + n)\cot\theta = m\cot\alpha - n\cot\beta$

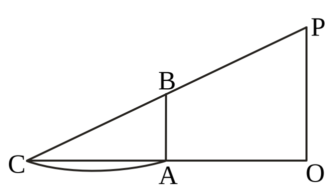


3.4 In a triangle ABC, if DE || AB.

then, $\dfrac{AB}{DE} = \dfrac{BC}{DC}$



3.5 To find the shadow of line object AB with respect to the light source P, we first join the upper points P and B. Let O be the projection of P on the plane on which object AB is situated, join OA. The section AC obtained by the intersection of the lines PB and OA, extended represents the shadow of AB with respect to light source P.

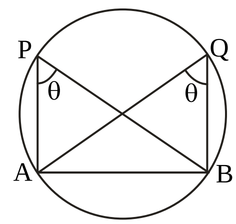


4. Problems based on properties of a circle

Whenever a line subtends equal angles at two points or the greatest angles at some points on a given line, such problems can be solved easily using the properties of a circle.

Mainly the following geometrical properties of a circle will be used :

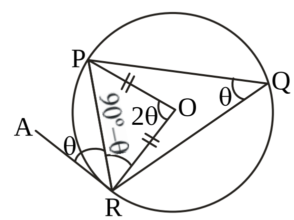
(a) Angles on the same segment of a circle are equal.



In other words, we can say that if the angles APB and AQB subtended on the segment AB are equal, a circle will pass through the points A, B, Q, and P, i.e. the point A, B, Q and P will be concyclic.

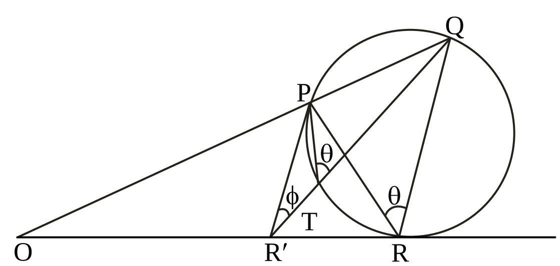
(b) If AR be the tangent to the circle through the points P, Q, R then

$\angle PRA = \angle PQR = \theta$.



[angle between any chord and the tangent to the circle is equal to the angle subtended by the chord at the circumference].

(c) Greatest Angle :



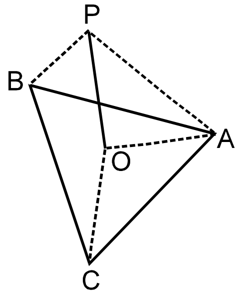
If PQ subtends greatest angle at R which lies on the line OR, then point R will be the point of contact of the tangent to the circle through P, Q, R.

SOLVED EXAMPLES

Ex.1 Each side of an equilateral triangle subtends an angle of 60º at the top of a tower h m high located at the centre of the triangle. If a is the length of each side of the triangle, then-

(A) $3a^2 = 2h^2$ (B) $2a^2 = 3h^2$

(C) $a^2 = 3h^2$ (D) $3a^2 = h^2$



Sol. Let O be the centre of the equilateral triangle ABC and OP the tower of height h. Then each of the triangles PAB, PBC and PCA equilateral. Thus, PA = PB = PC = a. Therefore, from right-angle triangle POA, we have $PA^2 = PO^2 + OA^2$

$$\Rightarrow a^2 = h^2 + \left(\frac{a}{2}\sec 30^0\right)^2$$

$$= h^2 + \frac{a^2}{4}\cdot\frac{4}{3} = h^2 + \frac{a^2}{3}$$

$$\frac{2}{3} \Rightarrow a^2 = h^2 \text{ or } 2a^2 = 3h^2$$ **Ans. (B)**

Ex.2 A tower subtends an angle of 30º at a point on the same level as its foot, and at a second point h m above the first, the depression of the foot of tower is 60º. The height of the tower is

(A) h m (B) 3h m

(C) $\sqrt{3}$ h m (D) $\frac{h}{3}$ m.

Sol. Let OP be the tower of height x., A the point on the same level as the foot O of the tower and B be the point h m above A (see Fig.). Then ∠AOB = 60º and ∠PAO = 30º . From right-angled triangle AOP, we have

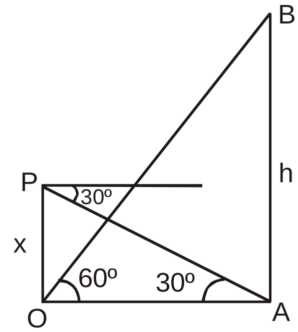
OA = x cot 30º

and from right-angled triangle OAB, we have

OA = h cot 60º

Therefore, from (1) and (2), we get

x cot 30º = h cot 60º



$$\sqrt{3}\,x = \frac{1}{\sqrt{3}}h,$$

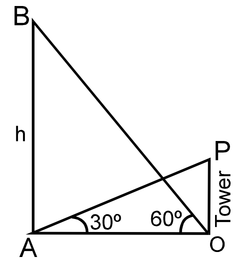
$$x = \frac{1}{3}h$$ **Ans.[D]**

Ex.3 A tower subtends an angle of 30º at a point on the same level as the foot of the tower . At a second point, h metre above first, the depression of the foot of the tower is 60º, the horizontal distance of the tower from the points is

(A) h cos 60º (B) (h/3) cot 30º

(C) (h/3) cot 60º (D) h cot 30º

Sol. Let the tower OP subtend an angle of 30º at A. A point on the same level as the foot of the tower. From a point B at the height h, vertically above A, from where the angle of depression of foot O of the tower is 60º i.e., ∠AOP = 60º.



From Δ OAP ,

OA = h cot 60º .

Also from Δ OAP ,

OA = OP cot 30º .

∴ OA = h cot 60º = OP cot 30º

⇒ OP = h/3.

∴ OA = (h/3) cot 30º,

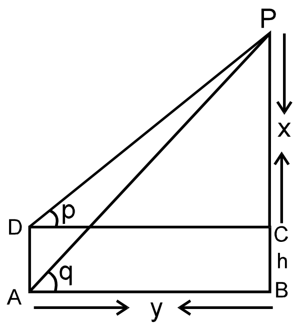
which is given in (B) **Ans. [B]**

Ex.4 The top of a hill observed from the top and bottom of a building of height h is at angles of elevation p and q respectively. The height of the hill is-

(A) $\frac{h\cot q}{\cot q - \cot p}$ (B) $\frac{h\cot q}{\cot p - \cot q}$

(C) $\frac{h\tan p}{\tan p - \tan q}$ (D) None of these

Sol. Let AD be the building of height h and BP be the hill. Then



$\tan q = \frac{h+x}{y}$ and $\tan p = \frac{x}{y}$

$\Rightarrow \tan q = \frac{h+x}{x\cot p}$

$\Rightarrow x \cot p = (h + x) \cot q$

$x = \frac{h\cot q}{\cot p - \cot q}$

$\Rightarrow h + x = h + \frac{h\cot q}{\cot p - \cot q}$

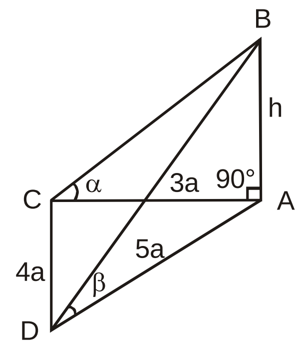
$= \frac{h\cot q}{\cot p - \cot q}$ **Ans.[B]**

Ex.5 A person standing at the foot of a tower walks a distance 3a away from the tower and observes that the angle of elevation of the top of the tower is α. He then walks a distance 4a perpendicular to the previous direction and observes the angle of elevation to be β. The height of the tower is

(A) 3a tan β (B) 5a tan β

(C) 4a tan β (D) 7a tan β

Sol.



In ΔABC

$\tan \alpha = \frac{h}{3a}$

$\therefore h = 3a \tan \alpha$

In ΔABD; $\tan \beta = \frac{h}{5a}$

So, $h = 5a \tan \beta$ **Ans.[B]**