PARABOLA

(KEY CONCEPTS + SOLVED EXAMPLES)

══PARABOLA══

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KEY CONCEPTS

1. Definition

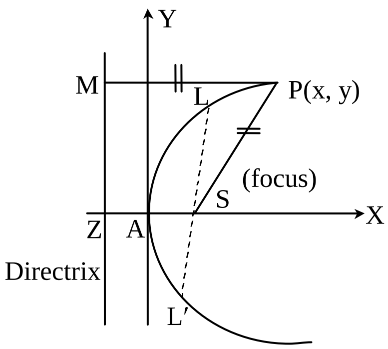
A parabola is the locus of a point which moves in such a way that its distance from a fixed point is equal to its perpendicular distance from a fixed straight line.

1.1 Focus :

The fixed point is called the focus of the Parabola.

1.2 Directrix :

The fixed line is called the directrix of the Parabola.



2. Terms related to Parabola

2.1 Eccentricity :

If P be a point on the parabola and PM and PS are the distances from the directrix and focus S respectively then the ratio PS/PM is called the eccentricity of the Parabola which is denoted by e.

Note: By the definition for the parabola e = 1.

If $e > 1 \Rightarrow$ Hyperbola, $e = 0 \Rightarrow$ circle, $e < 1 \Rightarrow$ ellipse

2.2 Axis :

A straight line passes through the focus and perpendicular to the directrix is called the axis of parabola.

2.3 Vertex :

The point of intersection of a parabola and its axis is called the vertex of the Parabola.

NOTE: The vertex is the middle point of the focus and the point of intersection of axis and directrix

2.4 Focal Length (Focal distance) :

The distance of any point P (x, y) on the parabola from the focus is called the focal length. i.e.

The focal distance of P = the perpendicular distance of the point P from the directrix.

2.5 Double ordinate :

The chord which is perpendicular to the axis of Parabola or parallel to Directrix is called double ordinate of the Parabola.

2.6 Focal chord :

Any chord of the parabola passing through the focus is called Focal chord.

2.7 Latus Rectum :

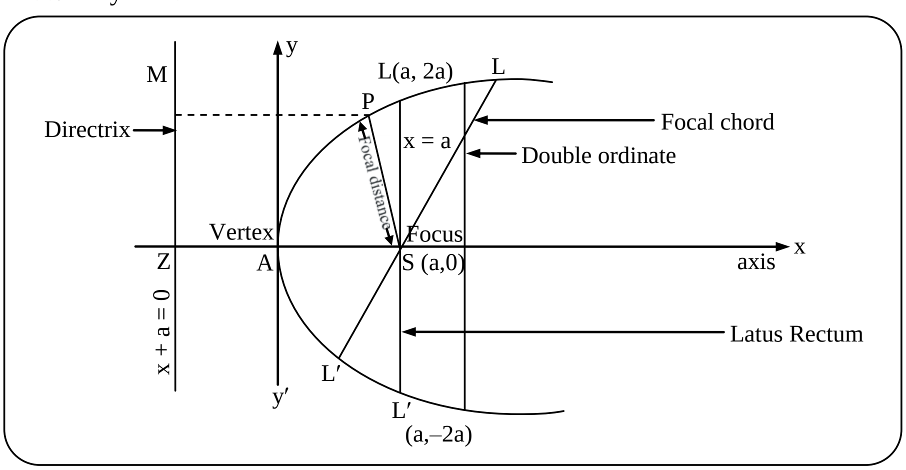
If a double ordinate passes through the focus of parabola then it is called as latus rectum.

2.7.1 Length of latus rectum :

The length of the latus rectum = 2 x perpendicular distance of focus from the directrix.

3. Standard form of Equation of Parabola

If we take vertex as the origin, axis as x-axis and distance between vertex and focus as 'a' then equation of the parabola in the simplest form will be. $y^2 = 4ax$



3.1 Parameters of the Parabola $y^2 = 4ax$

(i) Vertex A $\Rightarrow$ (0, 0)

(ii) Focus S $\Rightarrow$ (a, 0)

(iii) Directrix $\Rightarrow x + a = 0$

(iv) Axis $\Rightarrow y = 0$ or x– axis

(v) Equation of Latus Rectum $\Rightarrow x = a$

(vi) Length of L.R. $\Rightarrow$ 4a

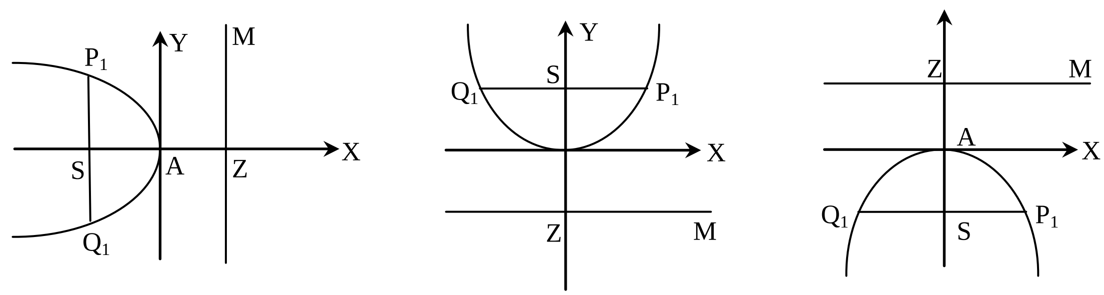
(vii) Ends of L.R. $\Rightarrow$ (a, 2a), (a, – 2a)

(viii) The focal distance $\Rightarrow$ sum of abscissa of the point and distance between vertex and L.R.

(ix) If length of any double ordinate of parabola $y^2 = 4ax$ is 2ℓ then coordinates of end points of this Double ordinate are

$$\left(\frac{\ell^2}{4a}, \ell\right) \text{ and } \left(\frac{\ell^2}{4a}, -\ell\right).$$

3.2 Other standard Parabola :



$x^2 = 4ay$ $\quad$ $y^2 = -4ax$ $\quad$ $x^2 = -4ay$

Equation of Parabola	Vertex	Axis	Focus	Directrix	Equation of Latus rectum	Length of Latus rectum
$y^2 = 4ax$	(0, 0)	$y = 0$	(a, 0)	$x = -a$	$x = a$	4a
$y^2 = -4ax$	(0, 0)	$y = 0$	(–a, 0)	$x = a$	$x = -a$	4a
$x^2 = 4ay$	(0, 0)	$x = 0$	(0, a)	$y = a$	$y = a$	4a
$x^2 = -4ay$	(0, 0)	$x = 0$	(0, –a)	$y = a$	$y = -a$	4a

4. Reduction to Standard Equation

If the equation of a parabola is not in standard form and if it contains second degree term either in y or in x (but not in both) then it can be reduced into standard form. For this we change the given equation into the following forms.

$(y - k)^2 = 4a(x - h)$ or $(x - p)^2 = 4b(y - q)$

And then we compare from the following table for the results related to parabola.

Equation of Parabola	Vertex	Axis	Focus	Directrix	Equation of L.R.	Length of L.R.
$(y - k)^2 = 4a(x - h)$	(h, k)	$y = k$	(h + a, k)	$x + a - h = 0$	$x = a + h$	4a
$(x - p)^2 = 4b(y - q)$	(p, q)	$x = p$	(p,b + q)	$y + b - q = 0$	$y = b + q$	4b

5. General Equation of a Parabola

If (h, k) be the locus of a parabola and the equation of directrix is $ax + by + c = 0$, then its equation is given by

$$\sqrt{(x-h)^2+(y-k)^2} = \frac{ax+by+c}{\sqrt{a^2+b^2}}$$

which gives $(bx-ay)^2 + 2gx + 2fy + d = 0$ where g, f, d are the constants.

Note:

(i) It is a second degree equation in x and y and the terms of second degree forms a perfect square and it contains at least one linear term.

(ii) The general equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a parabola, if

(a) $h^2 = ab$

(b) $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$

6. Equation of Parabola when its vertex and focus are given

6.1 If both lie on either of the coordinate axis :

In this case first find distance 'a' between these points and taking vertex as the origin suppose the equation as $y^2 = 4ax$ or $x^2 = 4ay$. Then shift the origin to the vertex.

6.2 When both do not lie on any coordinate axes :

In this case first find the coordinates of Z and equation of the directrix, then write the equation of the parabola by the definition.

7. Parametric equation of Parabola

The parametric equation of Parabola $y^2 = 4ax$ are $x = at^2$, $y = 2at$

Hence any point on this parabola is $(at^2, 2at)$ which is called as 't' point.

Note:

(i) Parametric equation of the Parabola $x^2 = 4ay$ is $x = 2at$, $y = at^2$

(ii) Any point on Parabola $y^2 = 4ax$ may also be written as $(a/t^2, 2a/t)$

(iii) The ends of a double ordinate of a parabola can be taken as $(at^2, 2at)$ and $(at^2, -2at)$

(iv) Parametric equations of the parabola

$(y-h)^2 = 4a(x-k)^2$ is $x - k = at^2$ and $y - h = 2at$

8. Chord

8.1 Equation of chord joining any two points of a parabola

Let the points are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ then equation of chord is-

$$(y-2at_1) = \frac{2at_2-2at_1}{at_2^2-at_1^2}(x-at_1^2)$$

$$\Rightarrow y-2at_1 = \frac{2}{t_1+t_2}(x-at_1^2)$$

$$\Rightarrow (t_1+t_2)y = 2x + 2at_1t_2$$

Note :

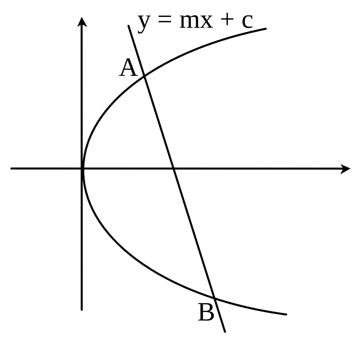
(i) If 't_1' and 't_2' are the Parameters of the ends of a focal chord of the Parabola $y^2 = 4ax$, then $t_1t_2 = -1$

(ii) If one end of focal chord of parabola is $(at^2, 2at)$, then other end will be $(a/t^2, -2a/t)$ and length of focal chord $= a(t + 1/t)^2$.

(iii) The length of the chord joining two points 't_1' and 't_2' on the parabola $y^2 = 4ax$ is

$$a(t_1-t_2)\sqrt{(t_1+t_2)^2+4}$$

8.2 Length of intercept $= \dfrac{4}{m^2}\sqrt{a(1+m^2)(a-mc)}$



9. Position of a Point and a Line with respect to a Parabola

9.1 Position of a point with respect to a parabola :

A point (x_1, y_1) lies inside, on or outside of the region of the parabola $y^2 = 4ax$ according as $y_1^2 - 4ax_1 <= \text{or} > 0$

9.2 Line and Parabola :

The line $y = mx + c$ will intersect a parabola $y^2 = 4ax$ in two real and different, coincident or imaginary point, according as $a - mc >, =, < 0$

10. Tangent to the Parabola

10.1 Condition of Tangency :

If the line $y = mx + c$ touches a parabola $y^2 = 4ax$ then $c = a/m$

Note:

(i) The line y = mx + c touches parabola $x^2 = 4ay$ if $c = -am^2$

(ii) The line $x \cos \alpha + y \sin \alpha = p$ touches the parabola $y^2 = 4ax$ if $a \sin^2 \alpha + p \cos \alpha = 0$

(iii) If the equation of parabola is not in standard form, then for condition of tangency, first eliminate one variable quantity (x or y) between equations of straight line and parabola and then apply the condition $B^2 = 4AC$ for the quadratic equation so obtained.

10.2 Equation of Tangent

10.2.1 Point Form :

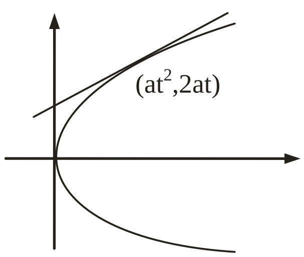
The equation of tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1) is $yy_1 = 2a(x + x_1)$ or T = 0

10.2.2 Parametric Form :

The equation of the tangent to the parabola at t i.e. $(at^2, 2at)$ is $ty = x + at^2$

10.2.3 Slope Form :

The equation of the tangent of the parabola $y^2 = 4ax$ is $y = mx + \frac{a}{m}$



Note :

(i) $y = mx + a/m$ is a tangent to the parabola $y^2 = 4ax$ for all value of m and its point of contact is $(a/m^2, 2a/m)$.

(ii) $y = mx - am^2$ is a tangent to the parabola $x^2 = 4ay$ for all value of m and its point of contact is $(2am, am^2)$

(iii) Point of intersection of tangents at points t_1 and t_2 of parabola is $[at_1t_2, a(t_1 + t_2)]$

(iv) Two perpendicular tangents of a parabola meet on its directrix. So the director circle of a parabola is its directrix or tangents drawn from any point on the directrix are always perpendicular.

(v) The tangents drawn at the end points of a focal chord of a parabola are perpendicular and they meet at the directrix.

11. Geometrical properties of the Parabola

(i) The semi latus rectum of a parabola is the H.M. between the segments of any focal chord of a parabola i.e. if PQR is a focal chord, then $2a = \frac{2PQ.QR}{PQ + QR}$

(ii) The tangents at the extremities of any focal chord of a parabola intersect at right angles and their point of intersection lies on directrix i.e. the locus of the point of intersection of perpendicular tangents is directrix.

(iii) If the tangent and normal at any point P of parabola meet the axes in T and G respectively, then

(a) ST = SG = SP

(b) $\angle$ PSK is a right angle, where K is the point where the tangent at P meets the directrix.

(c) The tangent at P is equally inclined to the axis and the focal distance.

(iv) The locus of the point of intersection of the tangent at P and perpendicular from the focus on this tangent is the tangent at the vertex of the parabola.

(v) If a circle intersect a parabola in four points, then the sum of their ordinates is zero.

(vi) The area of triangle formed inside the parabola $y^2 = 4ax$ is

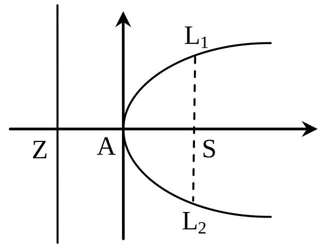
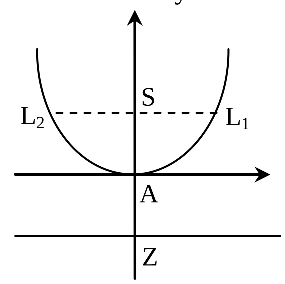
$\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ where y_1, y_2, y_3 are ordinate of vertices of the triangle.

(vii) The abscissa of point of intersection R of tangents at $P(x_1, y_1)$ and $Q(x_2, y_2)$ on the parabola is G.M. of abscissa of P and Q and ordinate of R is A.M. of ordinate of P and Q thus R

$$\left(\sqrt{x_1x_2}, \frac{x_1 + x_2}{2}\right)$$

(viii) The area of triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points

Review Chart for Standard Parabolas

	$y^2 = 4ax$	$x^2 = 4ay$
Diagram		
Vertex (A)	(0, 0)	(0, 0)
Focus (S)	(a, 0)	(0,a)
Axis	$y = 0$	$x = 0$
Directrix	$x + a = 0$	$y + a = 0$
Equation of LR	$x – a = 0$	$y – a = 0$
Length of LR	4a	4a
Extremities of LR(L_1, L_2)	(a, 2a); (a, –2a)	(2a, a) ; (–2a, a)
Focal distance of (x, y)	$x + a$	$y + a$
Parametric equations	$x = at^2$, $y = 2at$	$x = 2at$, $y = at^2$
Parametric points	$(at^2, 2at)$	$(2at, at^2)$
Condition of tangency (for $y = mx + c$)	$c = a/m$	$c = – am^2$
Tangent at (x_1, y_1)	$yy_1 = 2a (x + x_1)$	$xx_1 = 2a (y + y_1)$
Tangent in slope form	$y = mx + a/m$	$y = mx – am^2$
point of contact of above	$(a/m^2, 2a/m)$	$(2am, am^2)$
Tangent at ' t' point	$ty = x + at^2$	$tx = y + at^2$
Slope of tangent at ' t '	1/t	t
Normal at (x_1, y_1)	$y – y_1 = – \frac{y_1}{2a} (x – x_1)$	$y – y_1 = – \frac{2a}{x_1} (x – x_1)$
Normal in slope form	$y = mx – 2am – am^3$	$y = mx + 2a + a/m^2$
Foot of above normal	$(am^2, – 2am)$	$(– 2a/m, a/m^2)$
Normal at ' t ' point	$y + tx = 2at + at^3$	$ty + x = 2at + at^3$
Slope of normal at 't'	– t	– 1/t
Condition of normal (for $y = mx + c$)	$c = – 2am – am^3$	$c = 2a + a/m^2$
Director circle	$x + a = 0$	$y + a = 0$
Diameter w.r.t. $(y = mx + \alpha)$	$y = 2a/m$	$x = 2am$

SOLVED EXAMPLES

Ex.1 The vertex of the parabola $y^2 + 6x - 2y + 13 = 0$ is

(A) $(1, -1)$ (B) $(-2, 1)$

(C) $(3/2, 1)$ (D) $(-7/2, 1)$

Sol. We have $y^2 + 6x - 2y + 13 = 0$

$\Rightarrow y^2 - 2y = -6x - 13$

$\Rightarrow (y - 1)^2 = -6(x + 2)$

Clearly, the vertex of this parabola is $(-2, 1)$

Ans. [B]

Ex.2 If vertex of parabola is $(2, 0)$ and directrix is y-axis, then its focus is -

(A) $(2, 0)$ (B) $(-2, 0)$

(C) $(-4, 0)$ (D) $(4, 0)$

Sol. Since the axis of the parabola is the line which passes through vertex and perpendicular to the directrix, therefore x-axis is the axis of the parabola.

Obviously $Z \equiv (0, 0)$.

Let focus of the parabola is $S(a, 0)$. Since vertex $(2,0)$ is mid point of ZS, therefore

$$\frac{a+0}{2} = 2 \Rightarrow a = 4.$$

$\therefore$ Focus is $(4, 0)$

Ans. [D]

Ex.3 If the focus of a parabola is $(1, 0)$ and its directrix is $x + y = 5$, then its vertex is-

(A) $(0, 1)$ (B) $(0, -1)$

(C) $(2, 1)$ (D) $(3, 2)$

Sol. Since axis is a line perpendicular to directrix, so it will be $x - y = k$. It also passes from focus, therefore $k = 1$.

So equation of axis is $x - y = 1$.

Solving it with $x + y = 5$, we get

$Z \equiv (3, 2)$.

If vertex is (a, b), then $a = 2$, $b = 1$.

Hence vertex is $(2, 1)$.

Ans. [C]

Ex.4 The directrix and axis of the parabola $4y^2 - 6x - 4y = 5$ are respectively.

(A) $8x + 11 = 0;\ y - 1 = 0$

(B) $8x - 11 = 0,\ 2y - 1 = 0$

(C) $8x + 11 = 0;\ 2y - 1 = 0$

(D) None of these

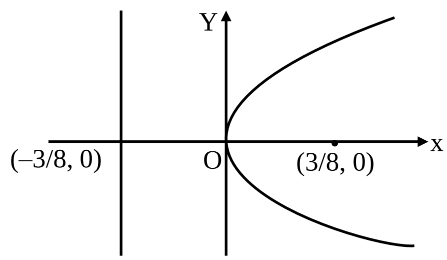
Sol. Here $4y^2 - 4y = 6x + 5$

$$\Rightarrow 4\left(y - \frac{1}{2}\right)^2 = 6(x + 1)$$

Put $y - \frac{1}{2} = Y$, $x + 1 = X$

The equation in standard form $Y^2 = \frac{3}{2}X$

$4a = \frac{3}{2} \Rightarrow a = \frac{3}{8}$



Directrix, $X + a = 0$

$\Rightarrow x + 1 + \frac{3}{8} = 0 \qquad \Rightarrow 8x + 11 = 0$

Axis is $Y = 0 \Rightarrow y - \frac{1}{2} = 0 \Rightarrow 2y - 1 = 0$

Ans. [C]

Ex.5 The angle subtended by double ordinate of length 8a at the vertex of the parabola $y^2 = 4ax$ is -

(A) 45º (B) 90º (C) 60º (D) 30º

Sol. Let (x_1, y_1) be any point on the parabola $y^2 = 4ax$, then length of double ordinate

$2y_1 = 8a \Rightarrow y_1 = 4a$

$y_1^2 = 4ax_1 \Rightarrow x_1 = 4a$

$\therefore$ vertices of double ordinate are

$P(4a, 4a);\ Q(4a, -4a)$

If A is the vertex$(0, 0)$, then

Slope of $AP = 1 = m_1$

Slope of $AQ = -1 = m_2$

$\therefore m_1 m_2 = -1 \Rightarrow \angle PAQ = 90º$ **Ans. [B]**

Ex.6 The length of latus rectum of a parabola, whose focus is $(2, 3)$ and directrix is the line $x - 4y + 3 = 0$ is -

(A) $\frac{7}{\sqrt{17}}$ (B) $\frac{14}{\sqrt{21}}$ (C) $\frac{7}{\sqrt{21}}$ (D) $\frac{14}{\sqrt{17}}$

Sol. The length of latus rectum

= 2 × perp. from focus to the directrix

$= 2 \times \frac{2-4(3)+3}{\sqrt{(1)^2+(4)^2}} = \frac{-14}{\sqrt{17}}$

The numerical length = $\frac{14}{\sqrt{17}}$

Ans. [D]

Note:- The negative sign of the latus rectum may only be ignored if its length is asked. For other calculations it should be used.

Ex.7 The coordinates of an endpoint of the latus rectum of the parabola $(y-1)^2 = 4(x+1)$ are

(A) (0,–3) (B) (0,–1)

(C) (0,1) (D) (1, 3)

Sol. Shifting the origin at (–1, 1) we have

$$\left.\begin{array}{l} x = X-1 \\ y = Y+1 \end{array}\right\} \quad ...(i)$$

Using (i), the given parabola becomes.

$Y^2 = 4X$

The coordinates of the endpoints of latus rectum are

(X = 1, Y = 2) and (X =1, Y= –2)

Using (i), the coordinates of the end point of the latus rectum are (0,3) and (0, –1)

Ans. [B]

Ex.8 The length of the chord of parabola $x^2 = 4ay$ passing through the vertex and having slope tan α is -

(A) 4a cosec α cot α (B) 4a tan α sec α

(C) 4a cos α cot α (D) 4a sin α tan α

Sol. Let A be the vertex and AP be a chord of $x^2 = 4ay$ such that slope of AP is tan α. Let the coordinates of P be$(2at, at^2)$ Then,

Slope of AP = $\frac{at^2}{2at} = \frac{t}{2}$

$\Rightarrow \tan\alpha \ \frac{t}{2} = \Rightarrow t = 2\tan\alpha$

Now, AP

$= \sqrt{(2at-0)^2+(at^2-0)^2} = at\sqrt{4+t^2}$

$= 2a\tan\alpha \sqrt{4+4\tan^2 a} = 4a\tan\alpha\sec\alpha$

Ans. [B]

Ex.9 The point on $y^2 = 4ax$ nearest to the focus has its abscissa equal to -

(A) –a (B) a (C) a/2 (D) 0

Sol. Let $P(at^2, 2at)$ be a point on the parabola $y^2 = 4ax$ and S be the focus of the parabola. Then, $SP = a + at^2$

[∵ focal distance = x + a]

Clearly, SP is least for t = 0.

Hence, the abscissa of P is $at^2 = a \times 0 = 0$

Ans. [D]

Ex.10 The common tangent of the parabola $y^2 = 8ax$ and the circle $x^2 + y^2 = 2a^2$ is -

(A) y = x + a (B) y = x – a

(C) y = x –2a (D) y = x + 2a

Sol. Any tangent to parabola is

$y = mx + \frac{2a}{m}$

Solving with the circle

$x^2 + (mx + \frac{2a}{m})^2 = 2a^2$

$B^2 - 4AC = 0$ gives $m = \pm 1$

Otherwise

Perp. from (0, 0) = radius $a\sqrt{2}$

$\therefore \frac{\left(\frac{2a}{m}\right)}{\sqrt{1+m^2}} = a\sqrt{2} \Rightarrow m = \pm 1$

Tangent $y = \pm x \pm 2a$

∴ y = x + 2a is correct option.

Ans. [D]

Ex.11 The slope of tangents drawn from a point (4, 10) to the parabola $y^2 = 9x$ are-

(A) $\frac{1}{4}, \frac{3}{4}$ (B) $\frac{1}{4}, \frac{9}{4}$

(C) $\frac{1}{4}, \frac{1}{3}$ (D) None of these

Sol. The equation of a tangent of slope m to the parabola $y^2 = 9x$ is

$y = mx + \frac{9}{4m}$

If it passes through (4, 10), then

$10 = 4m + \frac{9}{4m} \Rightarrow 16m^2 - 40m + 9 = 0$

$\Rightarrow (4m - 1)(4m - 9) = 0 \Rightarrow m\ \frac{1}{4}, \frac{9}{4}$ **Ans. [B]**

Ex.12 Tangents are drawn from the point (–2, –1) to the parabola $y^2 = 4x$. If α is the angle between these tangents then tan α equals -

(A) 3 (B) 2 (C) 1/3 (D) 1/2

Sol. Any tangent to $y^2 = 4x$ is

$y = mx + 1/m$

If it is drawn from (–2, –1), then

$-1 = -2m + 1/m$

$\Rightarrow 2m^2 - m - 1 = 0$

If $m = m_1, m_2$ then $m_1 + m_2 = 1/2$,

$m_1m_2 = -1/2$

$\therefore$ tan a $\frac{m_1 - m_2}{1 + m_1m_2} = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1m_2}}{1 + m_1m_2}$

$\frac{\sqrt{1/4 + 2}}{1 - 1/2} = 3$

Ans. [A]

Ex.13 If the straight line x + y = 1 is a normal to the parabola $x^2 = ay$, then the value of a is -

(A) 4/3 (B) 1/2 (D) 3/4 (D) 1/4

Sol. We know that equation of normal to the parabola $x^2 = ay$ is

$y = mx + \frac{2a}{4} + \frac{a}{4m^2}$

Given that x + y = 1 or y = –x + 1 is normal to the parabola therefore

$m = -1$ and $\frac{a}{2} + \frac{a}{4m^2} = 1$

$\therefore \frac{a}{2} + \frac{a}{4} = 1 \Rightarrow \frac{3a}{4} = 1 \Rightarrow a = \frac{4}{3}$

Ans. [A]

Ex.14 If line y = 2x + k is normal to the parabola $y^2 = 4x$ at the point $(t^2, 2t)$, then -

(A) k = –12, t = –2 (B) k = 12, t = –2

(C) k = 12, t = 2 (D) None of these

Sol. Since normal to the parabola $y^2 = 4x$ at $(t^2, 2t)$ is $y + tx = 2t + t^3$.

Comparing it with y = 2x + k, we get

$t = -2,\ k = 2t + t^3 = -12$

Ans. [A]

Ex.15 Which of the following lines, is a normal to the parabola $y^2 = 16x$

(A) $y = x - 11\cos\theta - 3\cos 3\theta$

(B) $y = x - 11\cos\theta - \cos 3\theta$

(C) $y = (x - 11)\cos\theta + \cos 3\theta$

(D) $y = (x - 11)\cos\theta - \cos 3\theta$

Sol. Here a = 4

condition of normality $c = -2am - am^3$

(1) and (2) are not clearly the answer as

m = 1 for (3), (4) m = $\cos\theta$

$c = -2(4)\cos\theta - 4\cos^3\theta$

$= -8\cos\theta - (3\cos\theta + \cos 3\theta)$

$= -11\cos\theta - \cos 3\theta$

Hence (D) is correct

Ans. [D]

Ex.16 If the tangents at P and Q on a parabola (whose focus is S) meet in the point T, then SP, ST and SQ are in -

(A) H.P. (B) G.P.

(C) A.P. (D) None of these

Sol. Let P $(at_1^2, 2at_1)$ and Q $(at_2^2, 2at_2)$ be any two points on the parabola $y^2 = 4ax$, then point of intersection of tangents at P and Q will be

$T \equiv [at_1t_2, a(t_1 + t_2)]$

Now $SP = a(t_1^2 + 1)$

$SQ = a(t_2^2 + 1)$

$ST = a\sqrt{(t_1^2 + 1)(t_2^2 + 1)}$

$\therefore$ $ST^2 = SP.SQ$

$\therefore$ SP, ST and SQ are in G.P.

Ans. [B]

Ex.17 If the distance of 2 points P and Q from the focus of a parabola $y^2 = 4ax$ are 4and 9 respectively, then the distance of the point of intersection of tangents at P and Q from the focus is

(A) 8 (B) 6

(C) 5 (D) 13

Sol. If S is the focus of the parabola and T is the point of intersection of tangents at P and Q, then

$ST^2 = SP \times SQ \Rightarrow ST^2 = 4 \times 9 \Rightarrow ST = 6$

Ans.

[B]