TRIGONOMETRICAL RATIO

(KEY CONCEPTS + SOLVED EXAMPLES)

══TRIGONOMETRICAL RATIO══

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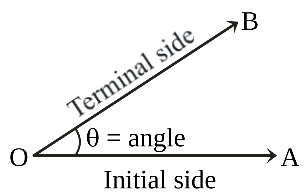
KEY CONCEPTS

1. Definition

Trigonometry is the branch of science in which we study about the angles and sides of a triangle.

1.1 Angle :

Consider a ray $\overrightarrow{OA}$. If this ray rotates about its end points O and takes the position OB, then the angle ∠AOB has been generated.

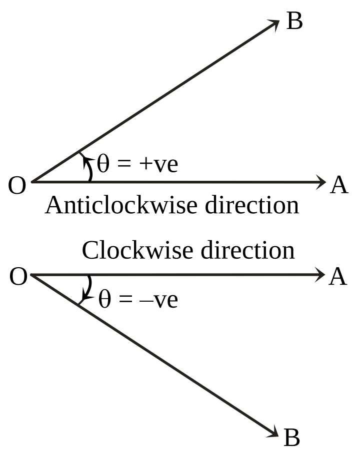


An angle is considered as the figure obtained by rotating a given ray about its end-point.

The initial position OA is called the initial side and the final position OB is called terminal side of the angle. The end point O about which the ray rotates is called the vertex of the angle.

1.2 Sense of an Angle :

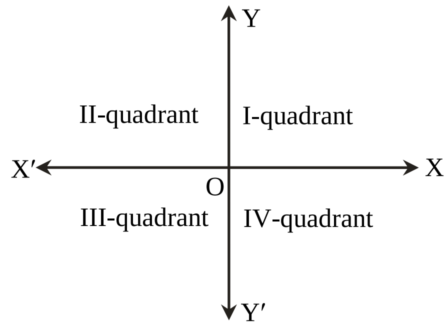
The sence of an angle is said to be positive or negative according as the initial side rotates in anticlockwise or clockwise direction to get to the terminal side.



1.3 Some Useful terms :

1.3.1 Quadrant :

Let XOX′ and YOY′ be two lines at right angles in the plane of the paper. These lines divide the plane of the paper into four equal parts which are known as quadrants.



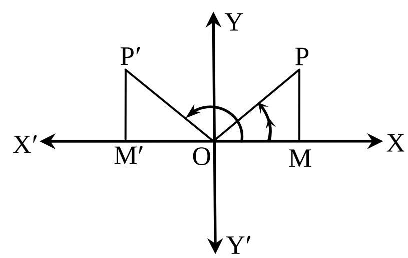
The lines XOX′ and YOY′ are known as x-axis and y-axis respectively. These two lines taken together are known as the coordinate axes. The regions XOY, YOX′, X′OY′ and Y′OX are known as the first, the second, the third and the fourth quadrant respectively.

1.3.2 Angle In Standard Position :

An angle is said to be in standard position if its vertex coincides with the origin O and the initial side coincides with OX i.e. the positive direction of x-axis.

1.3.3 Co–terminal Angles :

Two angles with different measures but having the same initial sides and the same terminal sides are known as co-terminal angles.



2. System of measurement of Angle

There are three system for measuring angles.

2.1 Sexagesimal or English system

2.2 Centesimal or French system

2.3 Circular system

2.3.1 Some Important Conversion :

π Radian $= 180^{\circ}$

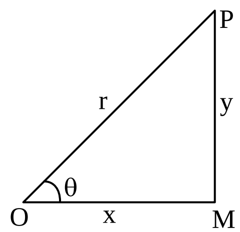
One radian $= \left(\frac{180}{\pi}\right)^{\circ}$

2.3.2 Relation between systems of measurement of angles :

$$\frac{D}{90} = \frac{G}{100} = \frac{2C}{\pi}$$

3. Trigonometrical ratios or Functions

In the right angled triangle OMP , we have base (OM) = x, perpendicular (PM) = y and hypotenuse (OP) = r, then we define the following trigonometric ratios which are known as trigonometric function.



$$\sin\theta = \frac{P}{H} = \frac{y}{r}$$

$$\cos\theta = \frac{B}{H} = \frac{x}{r}$$

$$\tan\theta = \frac{P}{B} = \frac{y}{x}$$

$$\cot\theta = \frac{B}{P} = \frac{x}{y}$$

$$\sec\theta = \frac{H}{B} = \frac{r}{x}$$

$$\text{cosec}\theta = \frac{H}{P} = \frac{r}{y}$$

3.1 Fundamental Trigonometrical Identities :

(a) $\sin\theta = \frac{1}{\text{cosec}\theta}$

(b) $\cos\theta = \frac{1}{\sec\theta}$

(c) $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

(d) $1 + \tan^2\theta = \sec^2\theta$

or, $\sec^2\theta - \tan^2\theta = 1$

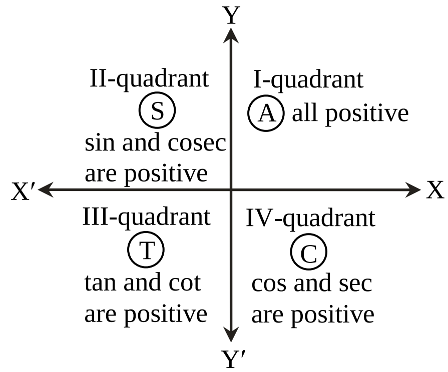
$(\sec\theta - \tan\theta) = \frac{1}{(\sec\theta + \tan\theta)}$

(e) $\sin^2\theta + \cos^2\theta = 1$

(f) $1 + \cot^2\theta = \text{cosec}^2\theta$

$(\text{cosec}\theta - \cot\theta) = \frac{1}{\text{cosec}\theta + \cot\theta}$

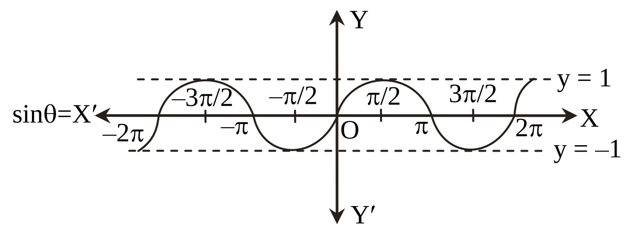
To be Remember :

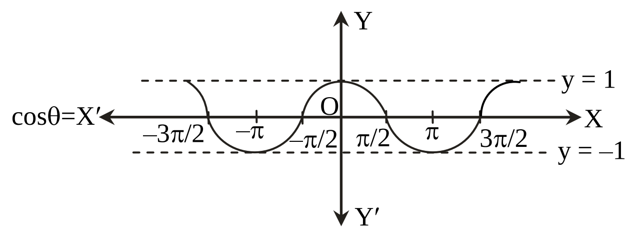


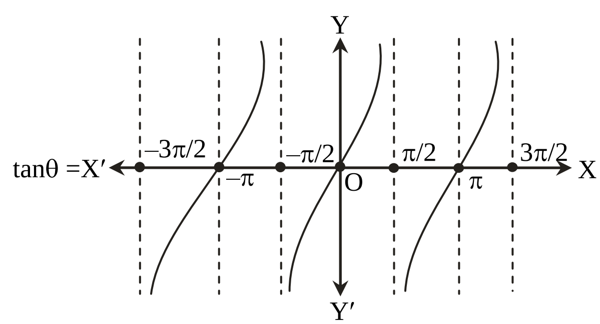
A crude aid to memorise the signs of trigonometrical ratio in different quadrant.

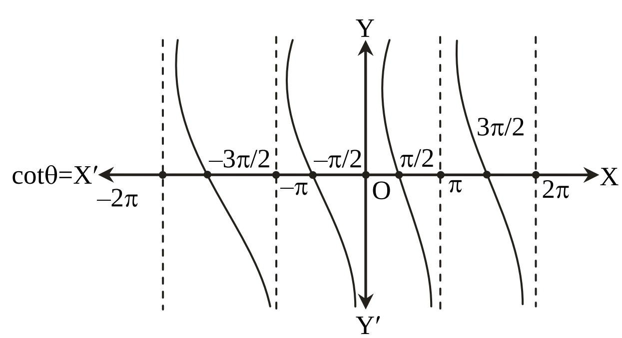
" All Students To Career Point "

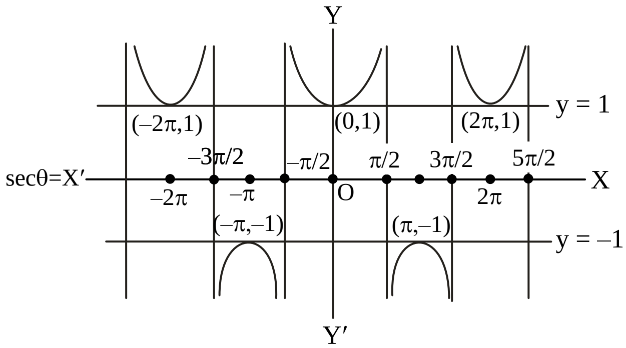
4. Graph of different trigonometrical ratios

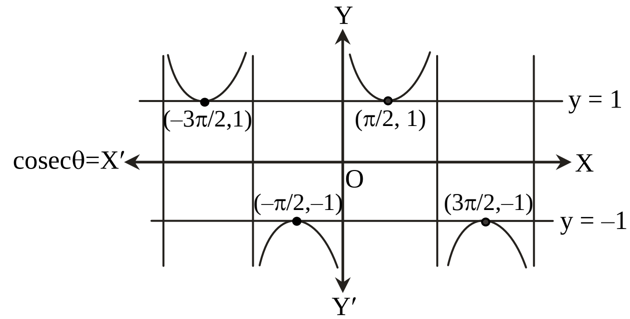












5. Trigonometrical ratio of allied angles

Two angles are said to be allied when their sum or difference is either zero or a multiple of 90º.

similarly,

Allied angles	(– θ)	(90° – θ)	(90° + θ)	(180° – θ)	(180° + θ)	(270° – θ)	(270° + θ)	(360° – θ)
Trigo. ratio		or $\left(\frac{\pi}{2}-\theta\right)$	or $\left(\frac{\pi}{2}+\theta\right)$	or (π – θ)	or (π + θ)	or $\left(\frac{3\pi}{2}-\theta\right)$	or $\left(\frac{3\pi}{2}+\theta\right)$	or (2π – θ)
sin	–sin θ	cos θ	cos θ	sin θ	– sin θ	– cos θ	– cos θ	– sin θ
cos	cos θ	sin θ	– sin θ	– cos θ	– cos θ	– sin θ	sin θ	cos θ
tan	– tan θ	cot θ	– cot θ	– tan θ	tan θ	cot θ	– cot θ	– tan θ

cot (90° + θ) = – tan θ

sec(90° + θ) = – cosec θ

cosec (90° + θ) = sec θ

5.1 Periodic Function :

All the trigonometric function are periodic function. They repeat their value after a certain period

$\sin(2n\pi + \theta) = \sin\theta$

$\cos(2n\pi + \theta) = \cos\theta$

$\tan(n\pi + \theta) = \tan\theta$

6. Sum & Difference formulae

(a) sin (A + B) = sin A cos B + cos A sin B

(b) sin(A – B) = sinA cosB – cosA sinB

(c) cos (A + B) = cos A cos B – sin A sin B

(d) cos (A – B) = cos A cos B + sin A sin B

(e) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(f) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(g) $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(h) $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Some More Results :

*(a) $\sin(A + B).\sin(A - B) = \sin^2 A - \sin^2 B$

$= \cos^2 B - \cos^2 A$

*(b) $\cos(A + B).\cos(A - B) = \cos^2 A - \sin^2 B$

$= \cos^2 B - \sin^2 A$

(c) sin (A + B + C) = sin A cos B cos C

+ cosA sin B sin C + cos A cos B sin C

– sin A sin B sin C

(d) cos (A + B + C) = cos A cos B cos C

– cos A sin B sin C – sin A cos B sin C

– sin A sinB cosC

(e) tan (A + B + C)

$= \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

(Note : * Important)

7. Formulae for product into sum or difference conversion

Formulae :

(a) $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

(b) $2 \cos A \sin B = \sin (A + B) - \sin (A - B)$

(c) $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

(d) $2 \sin A \sin B = \cos (A - B) - \cos (A + B)$

8. Formulae for sum or difference into product conversion

(a) $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2}\right) . \cos\left(\frac{C-D}{2}\right)$

similarly other formula are,

(b) $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2}\right) . \sin\left(\frac{C-D}{2}\right)$

(c) $\cos C + \cos D = 2 \cos \left(\frac{C+D}{2}\right) . \cos\left(\frac{C-D}{2}\right)$

☞ (d) $\cos C - \cos D = 2 \sin \left(\frac{C+D}{2}\right) . \sin\left(\frac{C-D}{2}\right)$

9. Trigonometrical ratios of multiple angles

(i) $\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2\tan\theta}{1+\tan^2\theta}$

(ii) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$

$= 1 - 2 \sin^2 \theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(iii) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(iv) $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

(v) $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

(vi) $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

(vii) $\sin \theta/2 = \sqrt{\frac{1-\cos\theta}{2}}$

(viii) $\cos \theta/2 = \sqrt{\frac{1+\cos\theta}{2}}$

(ix) $\tan \theta/2 = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

10. Conditional trigonometrical identities

We have certain trigonometric identities like,

$\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$ etc.

Such identities are identities in the sense that they hold for all value of the angles which satisfy the given condition among them and they are called conditional identities.

If A, B, C denote the angle of a triangle ABC, then the relation $A + B + C = \pi$ enables us to establish many important identities involving trigonometric ratios of these angles.

(I) If $A + B + C = \pi$, then $A + B = \pi - C$, $B + C = \pi - A$ and $C + A = \pi - B$

(II) If $A + B + C = \pi$, then $\sin(A + B)$

$= \sin(\pi - C) = \sin C$

similarly, $\sin (B + C) = \sin(\pi - A) = \sin A$

and $\sin (C + A) = \sin (\pi - B) = \sin B$

(III) If $A + B + C = \pi$, then $\cos(A + B) = \cos(\pi - C)$

$= - \cos C$

similarly, $\cos (B + C) = \cos(\pi - A) = - \cos A$

and $\cos (C + A) = \cos (\pi - B) = - \cos B$

(IV) If $A + B + C = \pi$, then $\tan(A + B) = \tan(\pi - C)$

$= - \tan C$

similarly, $\tan (B + C) = \tan(\pi - A) = - \tan A$

and, $\tan (C + A) = \tan (\pi - B) = - \tan B$

(V) If $A + B + C = \pi$, then $\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

and $\frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2}$ and $\frac{C+A}{2} = \frac{\pi}{2} - \frac{B}{2}$

$\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\left(\frac{C}{2}\right)$

$\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = \sin\left(\frac{C}{2}\right)$

$\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\left(\frac{C}{2}\right)$

All problems on conditional identities are broadly divided into the following four types:

(I) Identities involving sines and cosines of the multiple or sub-multiples of the angles involved.

(II) Identities involving squares of sines and cosines of the multiple or sub-multiples of the angles involved.

(III) Identities involving tangents and cotangents of the multiples or sub-multiples of the angles involved.

(IV) Identities involving cubes and higher powers of sines and cosines and some mixed identities.

10.1 TYPE I : Identities involving sines and cosines of the multiple or sub-multiple of the angles involved.

Working Method :

Step – 1 :

Use C & D formulae.

Step – 2 :

Use the given relation ($A + B + C = \pi$) in the expression obtained in step -1 such that a factor can be taken common after using multiple angles formulae in the remaining term.

Step – 3 :

Take the common factor outside.

Step – 4 :

Again use the given relation ($A+ B + C =\pi$) within the bracket in such a manner so that we can apply C & D formulae.

Step – 5

Find the result according to the given options.

10.2 TYPE II :Identities involving squares of sines and cosines of multiple or sub-multiples of the angles involved.

Working Method :

Step – 1 :

Arrange the terms of the identity such that either $\sin^2 A – \sin^2 B = \sin(A + B). \sin(A – B)$

or $\cos^2 A – \sin^2 B = \cos(A + B).\cos(A – B)$ can be used.

Step – 2 :

Take the common factor outside.

Step – 3 :

Use the given relation ($A + B + C = \pi$) within the bracket in such a manner so that we can apply C & D formulae.

Step – 4 :

Find the result according to the given options.

10.3 Type III :Identities for tan and cot of the angles

Working Method :

Step – 1 :

Express the sum of the two angles in terms of third angle by using the given relation ($A + B + C = \pi$).

Step – 2 :

Taking tangent or cotangent of the angles of both the sides.

Step – 3 :

Use sum and difference formulae in the left hand side.

Step – 4 :

Use cross multiplication in the expression obtained in the step 3

Step – 5 :

Arrange the terms as per the result required.

11. The greatest and least value of the expression [a sinθ + b cosθ]

The greatest and least values of $a \sin\theta + b \cos\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

12. Miscellaneous points

(A) Some useful Identities :

(a) $\tan (A + B + C) = \dfrac{\sum \tan A - \tan A \tan B \tan C}{1 - \sum \tan A . \tan B}$

(b) $\tan\theta = \cot \theta – 2 \cot 2\theta$

(c) $\tan 3\theta = \tan\theta.\tan(60^\circ – \theta).\tan (60^\circ + \theta)$

(d) $\tan(A + B) – \tan A – \tan B$

$= \tan A.\tan B.\tan(A + B)$

(e) $\sin \theta \sin (60^\circ – \theta) \sin (60^\circ + \theta) = \dfrac{1}{4} \sin 3\theta$

(f) $\cos \theta \cos (60^\circ – \theta) \cos (60^\circ + \theta) = \dfrac{1}{4} \cos 3\theta$

(B) Some useful series :

(a) $\sin \alpha + \sin (\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots\ldots + $ to n terms

$$= \frac{\sin\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin\left(\frac{\beta}{2}\right)} ; \beta \neq 2n\pi$$

(b) $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta)$

+..... + to n terms

$$= \frac{\cos\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]\left[\sin\left(\frac{n\beta}{2}\right)\right]}{\sin\left(\frac{\beta}{2}\right)} ; \beta \neq 2n\pi$$

(C) Sine, cosine and tangent of some angle less than 90º.

	15º	18º	22½º	36º
sin	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$
cos	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

(D) Domain and Range of Trigonometrical Function

Trig. Function	Domain	Range
$\sin\theta$	R	$[-1, 1]$
$\cos\theta$	R	$[-1, 1]$
$\tan\theta$	$R - \{2n+1)\pi/2, n \in z\}$	(∞, ∞) or R
cosec θ	$R - \{n\pi, n \in z\}$	$(-\infty, -1] \cup [1, \infty)$
$\sec\theta$	$R - (\{2n+1)\pi/2, n \in z\}$	$(-\infty, -1] \cup [1, \infty)$
$\cot\theta$	$R - \{n\pi, n \in z\}$	$(-\infty, \infty) = R$

(E) An Increasing Product series :

$$p = \cos\alpha . \cos 2\alpha . \cos 2^2\alpha .... \cos(2^{n-1}\alpha) = \begin{cases} \frac{\sin 2^n\alpha}{2^n \sin\alpha}, \text{if } \alpha \neq n\pi \\ 1, \text{ if } \alpha = 2k\pi \\ -1, \text{if } \alpha = (2k+1)\pi \end{cases}$$

(F) Conversion 1 radian = 180º/π = 57º 17′ 45″

and $1^\circ = \frac{\pi}{180}$ = 0.01475 radians (approximately)

(G) Basic right angled triangle are (pythogerian Triplets)

3, 4, 5 ; 5, 12, 13; 7, 24, 25; 8, 15, 17;9,40, 41;11, 60, 61; 12, 35, 37; 20, 21, 29 etc.

(H) Each interior angle of a regular polygon of n sides

$$= \frac{n-2}{n} \times 180 \text{ degrees}$$

SOLVED EXAMPLES

Ex.1 The value of the expression -

$$1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y} \text{ is equal}$$

to -

(A) 0 (B) 1

(C) sin y (D) cos y

Sol. $1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$

$$= \frac{1+\cos y - \sin^2 y}{1+\cos y} + \frac{1-\cos^2 y - \sin^2 y}{\sin y\,(1-\cos y)}$$

$$= \frac{\cos y + \cos^2 y}{1+\cos y} + 0 = \cos y$$ **Ans.[D]**

Ex.2 If cosec θ – sin θ = m and sec θ – cos θ = n then $(m^2n)^{2/3} + (n^2m)^{2/3}$ equals to -

(A) 0 (B) 1 (C) –1 (D) 2

Sol. cosec θ – sin θ = m

$$m = \frac{1}{\sin\theta} - \sin\theta = \frac{\cos^2\theta}{\sin\theta} \quad ...(i)$$

$$n = \frac{1}{\cos\theta} - \cos\theta = \frac{\sin^2\theta}{\cos\theta} \quad ...(ii)$$

$$m \times n = \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} = \sin\theta\cos\theta$$

from (i) and (ii)

from (i) $\cos^2\theta = m \cdot \sin\theta$

or $\cos^3\theta = m\sin\theta\cos\theta$

$= m\,.\,(mn) = m^2n$

Similarly $\sin^3\theta = n^2m$

since $\sin^2\theta + \cos^2\theta = 1$

$(n^2m)^{2/3} + (m^2n)^{2/3} = 1$ **Ans.[B]**

Ex.3 The value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is -

(A) $\frac{1}{2}$ (B) $\cos\frac{\pi}{8}$

(C) $\frac{1}{8}$ (D) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

Sol. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)$

$\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$

$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)$

$\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$

$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$= \frac{1}{4}\left(2-1-\cos\frac{\pi}{4}\right)\left(2-1-\cos\frac{3\pi}{4}\right)$

$= \frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$

$= \frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right) = \frac{1}{4}\left(1-\frac{1}{2}\right) = \frac{1}{8}$

Ans.[C]

Ex.4 The value of sin 20° sin 40° sin 60° sin 80° is -

(A) $\frac{3}{8}$ (B) $\frac{1}{8}$

(C) $\frac{3}{16}$ (D) None of these

Sol. sin 20° sin 40° sin 60° sin 80°

$= \frac{\sqrt{3}}{2}\sin 20°\sin(60° - 20°)\sin(60° + 20°)$

$= \frac{\sqrt{3}}{2}\sin 20°(\sin^2 60° - \sin^2 20°)$

$= \frac{\sqrt{3}}{2}\sin 20°\left(\frac{3}{4} - \sin^2 20°\right)$

$= \frac{\sqrt{3}}{8}(3\sin 20° - 4\sin^3 20°)$

$= \frac{\sqrt{3}}{8}\sin 60°$

$= \frac{\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2} = \frac{3}{16}$ **Ans.[C]**

Alternate : By direct formula

$\sin\theta\,.\sin(60^o - \theta).\sin(60^o + \theta) = \frac{1}{4}\sin 3\theta$

⇒ sin 60º [sin 20º sin (60º – 20º) sin (60º + 20º)]

$= \sin 60^o\left[\frac{1}{4}\sin 60^o\right] = \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{16}$

Ex.5 $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$ equals to -

(A) 1/2 (B) 1/4

(C) 3/2 (D) 3/4

Sol. $= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{\pi}{8}$

$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right)$

$= \frac{1}{2}\left[\left(2\cos^2\frac{\pi}{8}\right)^2 + \left(2\cos^2\frac{3\pi}{8}\right)^2\right]$

$= \frac{1}{2}\left[\left(1+\cos\frac{\pi}{4}\right)^2 + \left(1+\cos\frac{3\pi}{4}\right)^2\right]$

$= \frac{1}{2}\left[\left(1+\frac{1}{\sqrt{2}}\right)^2 + \left(1-\frac{1}{\sqrt{2}}\right)^2\right]$

$= \frac{1}{2}[2+1] = \frac{3}{2}$ **Ans.[C]**

Ex.6 If $A + B + C = \frac{3\pi}{2}$, then

$\cos 2A + \cos 2B + \cos 2C =$

(A) $1 - 4\cos A \cos B \cos C$

(B) $4\sin A \sin B \sin C$

(C) $1 + 2\cos A \cos B \cos C$

(D) $1 - 4\sin A \sin B \sin C$

Sol. $\cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A + B)\cos(A - B) + \cos 2C$

$= 2\cos\left(\frac{3\pi}{2} - C\right)\cos(A - B) + \cos 2C$

$\because A + B + C = \frac{3\pi}{2}$

$= -2\sin C\cos(A - B) + 1 - 2\sin^2 C$

$= 1 - 2\sin C\,[\cos(A - B) + \sin C)$

$= 1 - 2\sin C\left[\cos(A-B) + \sin\left(\frac{3\pi}{2} - (A+B)\right)\right]$

$= 1 - 2\sin C\,[\cos(A - B) - \cos(A + B)]$

$= 1 - 4\sin A \sin B \sin C$ **Ans.[D]**

Ex.7 In any triangle ABC, $\sin A - \cos B = \cos C$, then angle B is -

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Sol. We have, $\sin A - \cos B = \cos C$

$\sin A = \cos B + \cos C$

$2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

$2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{\pi-A}{2}\right)\cos\left(\frac{B-C}{2}\right)$

$\because A + B + C = \pi$

$2\sin\frac{A}{2}\cos\frac{A}{2} = 2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$\cos\frac{A}{2} = \cos\frac{B-C}{2}$ or $A = B - C$

But $A + B + C = \pi$

Therefore $2B = \pi \Rightarrow B = \pi/2$ **Ans.[A]**

Ex.8 $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is equals to -

(A) 0 (B) 1 (C) –1 (D) 4

Sol. $\tan 9° + \tan 81° - (\tan 27° + \tan 63°)$

$(\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$= \left(\frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ}\right) - \left(\frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\cos 27^\circ}{\sin 27^\circ}\right)$

$= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\cos 27^\circ \sin 27^\circ}$

$= \frac{2}{\sin 18} - \frac{2}{\sin 54^\circ} = \frac{2}{\sin 18^\circ} - \frac{2}{\sin 36^\circ}$

$= \frac{2\times4}{\sqrt{5}-1} - \frac{2\times4}{\sqrt{5}+1} = 8\left[\frac{\sqrt{5}+1-\sqrt{5}+1}{(\sqrt{5}-1)(\sqrt{5}+1)}\right]$

$= \frac{16}{4} = 4$ **Ans.[D]**

Ex.9 $\cos^3 x.\sin 2x = \sum_{m=1}^{n} a_m \sin mx$ is an identity in x. Then -

(A) $a_3 = \frac{3}{8}$, $a_2 = 0$ (B) $n = 5$, $a_1 = \frac{1}{4}$

(C) $\sum a_m = \frac{3}{4}$ (D) All the above

Sol. $\cos^3 x.\sin 2x = \frac{\cos 3x + 3\cos x}{4}.\sin 2x$

$= \frac{1}{8}(\sin 5x - \sin x) + \frac{3}{8}(\sin 3x + \sin x)$

$= \frac{1}{4}\sin x + \frac{3}{8}\sin 3x + \frac{1}{8}\sin 5x.$

$\therefore n = 5, a_1 = \frac{1}{4}, a_2 = 0, a_3 = \frac{3}{8},$

$a_4 = 0, a_5 = \frac{1}{8}$ **Ans.[D]**