MATHEMATICS

Class-IX

Topic-10

CIRCLES

INDEX

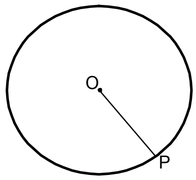
CH-10
CIRCLES

A. INTRODUCTION AND CHORD PROPERTIES OF CIRCLES

(a) Definition

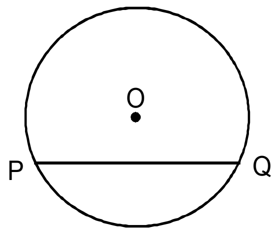
Circle : The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a **circle**.

The fixed point is called the **centre** of the circle and the fixed distance is called the **radius** of the circle.

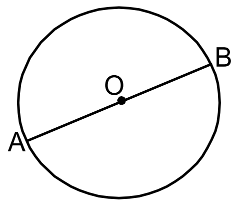


In figure, **O** is the **centre** and the length **OP** is the **radius** of the circle. So the line segment joining the centre and any point on the circle is called a radius of the circle.

Chord : If we take two points P and Q on a circle, then the line segment PQ is called a **chord** of the circle.

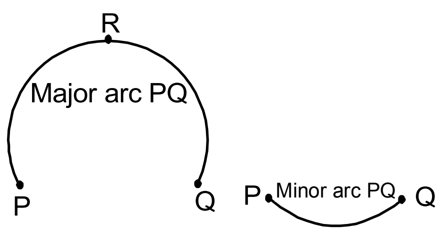


Diameter : The chord which passes through the centre of the circle, is called the **diameter** of the circle.



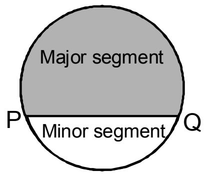
A diameter is the longest chord and all diameters of same circle have the same length, which is equal to two times the radius. In figure, AOB is a **diameter** of circle.

Arc : A piece of a circle between two points is called an **arc**. The longer one is called the **major arc PQ** and the shorter one is called the **minor arc PQ**. The minor arc PQ is also denoted by $\overset{\frown}{PQ}$ and the major arc PQ by $\overset{\frown}{QP}$. When P and Q are ends of a diameter, then both arcs are equal and each is called a **semi circle**.

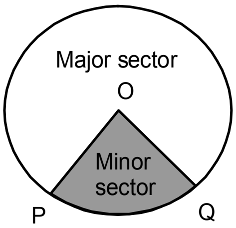


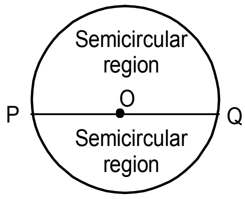
Circumference : The length of the complete circle is called its **circumference**.

Segment : The region between a chord and either of its arcs is called a **segment** of the circular region or simply a segment of the circle. There are two types of segments which are the **major segment** and the **minor segment** (as in **figure**).



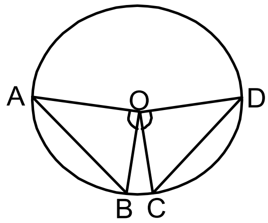
Sector : The region between an arc and the two radii, joining the centre to the end points of an arc is called a **sector**. Minor arc corresponds to the **minor sector** and the major arc corresponds to the **major sector**. When two arcs are equal, then both segments and both sectors become the same and each is known as a **semicircular region**.





(b) Important theorems related to chords

Theorem : Equal chords of a circle subtend equal angles at the centre.
Given : AB and CD are the two equal chords of a circle with centre O.

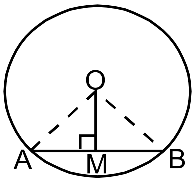


To Prove : $\angle AOB = \angle COD$.
Proof : In AOB and COD,

OA = OC [Radii of a circle]
OB = OD [Radii of a circle]
AB = CD [Given]
$\therefore$ $\Delta AOB \cong \Delta COD$ [By SSS congruency]
$\therefore$ $\angle AOB = \angle COD$. [By CPCT] **Hence Proved.**

Converse : If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Theorem : The perpendicular from the centre of a circle to a chord bisects the chord.



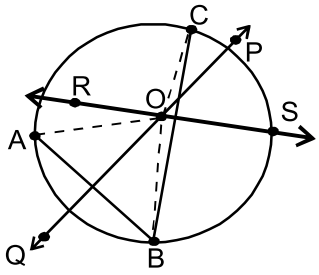
Given : A circle with centre O. AB is a chord of this circle. $OM \perp AB$.
To Prove : MA = MB.
Construction : Join OA and OB.
Proof : In right triangles OMA and OMB,

OA = OB [Radii of a circle]
OM = OM [Common]
$\angle OMA = \angle OMB$ [90° each]
$OMA \cong OMB$ [By RHS]
MA = MB [By CPCT] **Hence Proved.**

Converse : The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Theorem : There is one and only one circle passing through three given non-collinear points.

Proof : Take three points A, B and C, which are not in the same line, or in other words, they are not collinear [as in figure]. Draw perpendicular bisectors of AB and BC say, PQ and RS respectively. Let these perpendicular bisectors intersect at one point O.(Note that PQ and RS will intersect because they are not parallel) [as in figure].



$\therefore$ O lies on the perpendicular bisector PQ of AB.

$\therefore$ OA = OB

[Every point on the perpendicular bisector of a line segment is equidistant from its end points]

Similarly,

$\therefore$ O lies on the perpendicular bisector RS of BC.

$\therefore$ OB = OC

[Every point on the perpendicular bisector of a line segment is equidistant from its end points]

So, OA = OB = OC

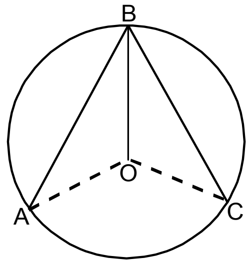
i.e., the points A, B and C are at equal distances from the point O.

So, if we draw a circle with centre O and radius OA it will also pass through B and C. This shows that there is a circle passing through the three points A, B and C. We know that two lines (perpendicular bisectors) can intersect at only one point, so we can draw only one circle with radius OA. In other words, there is a unique circle passing through A, B and C. **Hence Proved.**

Solved Examples

Example. 1

In figure, AB = CB and O is the centre of the circle. Prove that BO bisects $\angle$ABC.



Sol. **Given :** In figure, AB = CB and O is the centre of the circle.

To Prove : BO bisects $\angle$ABC.

Construction : Join OA and OC.

Proof : In OAB and OCB,

OA = OC [Radii of the same circle]

AB = CB [Given]

OB = OB [Common]

Δ OAB $\cong \Delta$OCB [By SSS congruency]

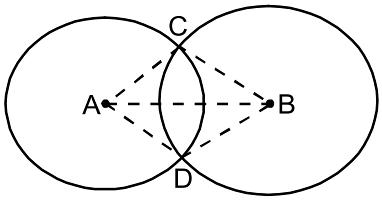
$\angle$ABO = $\angle$CBO [By CPCT]

BO bisects $\angle$ABC. **Hence Proved.**

Example. 2

Two circles with centres A and B intersect at C and D. Prove that $\angle ACB = \angle ADB$.

Sol. **Given :** Two circles with centres A and B intersect at C and D.

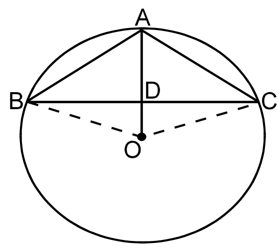


To Prove : $\angle ACB = \angle ADB$.
Construction : Join AC, AD, BC, BD and AB.
Proof : In ACB and ADB,

AC = AD [Radii of the same circle]
BC = BD [Radii of the same circle]
AB = AB [Common]
$\Delta ACB \cong \Delta ADB$ [By SSS congruency]
$\angle ACB = \angle ADB$. [By CPCT] **Hence Proved.**

Example. 3

In figure, $\overset{\frown}{AB} \cong \overset{\frown}{AC}$ and O is the centre of the circle. Prove that OA is the perpendicular bisector of BC.



Sol. **Given :** In figure, $\overset{\frown}{AB} \cong \overset{\frown}{AC}$ and O is the centre of the circle.
To Prove : OA is the perpendicular bisector of BC.
Construction : Join OB and OC.
Proof :

$\because$ $\overset{\frown}{AB} \cong \overset{\frown}{AC}$ [Given]

$\therefore$ chord AB = chord AC.

[If two arcs of a circle are congruent, then their corresponding chords are equal]

$\therefore$ $\angle AOB = \angle AOC$ ...(i)

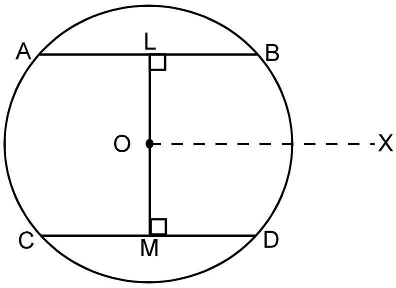
[$\because$ Equal chords of a circle subtend equal angles at the centre]

In Δ OBD and Δ OCD,
$\angle DOB = \angle DOC$ [From (i)]
OB = OC [Radii of the same circle]
OD = OD [Common]
$\therefore$ $\Delta OBD \cong \Delta OCD$ [By SAS congruency]
$\therefore$ $\angle ODB = \angle ODC$ ...(ii) [By CPCT]
And, BD = CD ...(iii) [By CPCT]
But $\angle BDC = 180^\circ$
$\therefore$ $\angle ODB + \angle ODC = 180^\circ$
$\angle ODB + \angle ODB = 180^\circ$ [From equation(ii)]
$2\angle ODB = 180^\circ$
$\angle ODB = 90^\circ$
$\therefore$ $\angle ODB = \angle ODC = 90^\circ$ ...(iv) [From (ii)]
So, by (iii) and (iv), OA is the perpendicular bisector of BC. **Hence Proved.**

Example. 4

Prove that the line joining the mid-points of the two parallel chords of a circle passes through the centre of the circle.

Sol. Let AB and CD be two parallel chords of a circle whose centre is O.
Let L and M be the mid-points of the chords AB and CD respectively. Join OL and OM.
Draw OX || AB or CD.



∵ L is the mid-point of the chord AB and O is the centre of the circle

∴ $\angle OLB = 90^\circ$ [The perpendicular drawn from the centre of a circle to a chord bisects the chord]

But, OX || AB

∴ $\angle LOX = 90^\circ$ ...(i)

[∵Sum of the consecutive interior angles on the same side of a transversal is 180º]

∵ M is the mid-point of the chord CD and O is the centre of the circle

∴ $\angle OMD = 90^\circ$

[∵ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

But OX || CD ...(ii)

[⇳ Sum of the consecutive interior angles on the same side of a transversal is 180º]

∴ $\angle MOX = 90^\circ$

From above equations, we get

$\angle LOX + \angle MOX = 90^\circ + 90^\circ = 180^\circ$

$\Rightarrow \quad \angle LOM = 180^\circ$

LM is a straight line passing through the centre of the circle. **Hence Proved.**

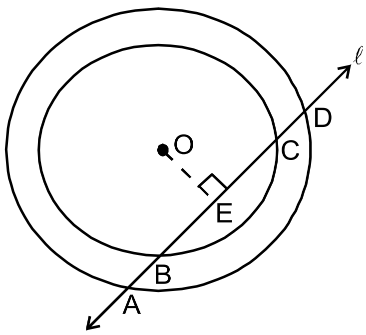
Example. 5

ℓ is a line which intersects two concentric circles (i.e., circles with the same centre) with common centre O at A, B, C and D (as in figure). Prove that AB = CD.

Sol. **Given :** ℓ is a line which intersects two concentric circles (i.e., circles with the same centre) with common centre O at A, B, C and D.

To Prove : AB = CD.

Construction : Draw OE ⊥ ℓ.



Proof : The perpendicular drawn from the centre of a circle to a chord bisects the chord.

AE = ED ...(i)

And BE = EC ...(ii)

Subtracting (ii) from (i), we get

AE – BE = ED – EC

AB = CD. **Hence Proved.**

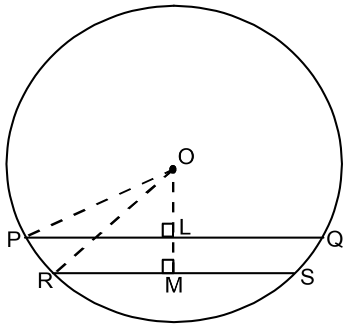
Example. 6

PQ and RS are two parallel chords of a circle whose centre is O and radius is 10 cm. If PQ = 16 cm and RS = 12 cm, find the distance between PQ and RS, if they lie

(i) on the same side of the centre O.

(ii) on the opposite sides of the centre O.

Sol. **(i)** Draw the perpendicular bisectors OL and OM of PQ and RS respectively.



$\because$ PQ || RS

$\therefore$ OL and OM are in the same line. $\Rightarrow$ O, L and M are collinear.

Join OP and OR.

In right triangle OLP,

$OP^2 = OL^2 + PL^2$ [By Pythagoras Theorem]

$\Rightarrow (10)^2 = OL^2 + \left(\frac{1}{2}\times PQ\right)^2$

[$\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow 100 = OL^2 + \left(\frac{1}{2}\times 16\right)^2$

$\Rightarrow 100 = OL^2 + (8)^2$

$\Rightarrow 100 = OL^2 + 64$

$\Rightarrow OL^2 = 100 - 64$

$\Rightarrow OL^2 = 36 = (6)^2$

$\Rightarrow$ OL = 6 cm

In right triangle OMR,

$OR^2 = OM^2 + RM^2$ [By Pythagoras Theorem]

$\Rightarrow OR^2 = OM^2 + \left(\frac{1}{2}\times RS\right)^2$

[$\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow (10)^2 = OM^2 + \left(\frac{1}{2}\times 12\right)^2$

$\Rightarrow (10)^2 = OM^2 + (6)^2$

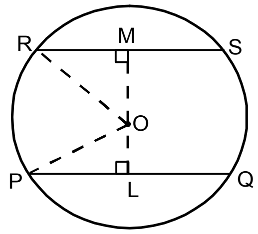
$\Rightarrow OM^2 = (10)^2 - (6)^2 = (10 - 6)(10 + 6) = (4)(16) = 64 = (8)^2$.

$\Rightarrow$ OM = 8 cm

$\therefore$ LM = OM – OL = 8 – 6 = 2 cm.

Hence, the distance between PQ and RS, if they lie on the same side of the centre O, is 2 cm.

(ii) Draw the perpendicular bisectors OL and OM of PQ and RS respectively.



$\because$ PQ || RS

$\therefore$ OL and OM are in the same line.

$\Rightarrow$ L, O and M are collinear.

Join OP and OR.

In right triangle OLP,

$OP^2 = OL^2 + PL^2$ [By Pythagoras Theorem]

$\Rightarrow$ $OP^2 = OL^2 + \left(\frac{1}{2}\times PQ\right)^2$

[$\because$The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$(10)^2 = OL^2 + \left(\frac{1}{2}\times 16\right)^2$

$100 = OL^2 + (8)^2$

$100 = OL^2 + 64$

$OL^2 = 100 - 64$

$OL^2 = 36 = (6)^2$

OL = 6 cm.

In right triangle OMR,

$OR^2 = OM^2 + RM^2$ [By Pythagoras Theorem]

$\Rightarrow$ $OR^2 = OM^2 + \left(\frac{1}{2}\times RS\right)^2$

[$\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\Rightarrow$ $(10)^2 = OM^2 + \left(\frac{1}{2}\times 12\right)^2$

$\Rightarrow$ $(10)^2 = OM^2 + (6)^2$

$\Rightarrow$ $OM^2 = (10)^2 - (6)^2$

$= (10 - 6)(10 + 6)$

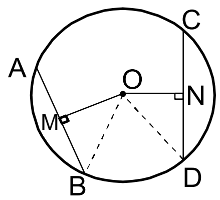
$= (4)(16) = 64 = (8)^2$.

$\Rightarrow$ OM = 8 cm

$\therefore$ LM = OL + OM = 6 + 8 = 14 cm

Hence, the distance between PQ and RS, if they lie on the opposite side of the centre O, is 14 cm.

Theorem : Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).



Given : A circle have two equal chords AB & CD. i.e. AB = CD and OM $\perp$ AB, ON $\perp$ CD.

To Prove : OM = ON
Construction : Join OB & OD
Proof : AB = CD (Given)
[The perpendicular drawn from the centre of a circle to a chord bisects the chord]

$\therefore \quad \frac{1}{2}AB = \frac{1}{2}CD \quad \Rightarrow \quad BM = DN$

In Δ OMB & Δ OND

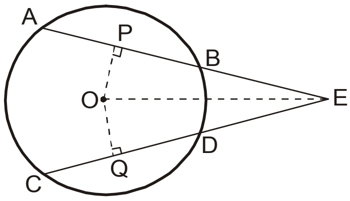
$\angle OMB = \angle OND = 90^\circ$ [Given]
OB = OD [Radii of same circle]
BM = DN [Proved above]
$\therefore \quad \Delta OMB \cong \Delta OND$ [By R.H.S. congruency]
$\therefore \quad$ OM = ON [By CPCT] **Hence Proved.**

Convers : Chords equidistant from the centre of a circle are equal in length.

Example. 7

AB and CD are equal chords of a circle whose centre is O. When produced, these chords meet at E. Prove that EB = ED.

Sol. **Given :** AB and CD are equal chords of a circle whose centre is O. When produced, these chords meet at E.
To Prove : EB = ED.



Construction : From O draw OP $\perp$ AB and OQ $\perp$ CD. Join OE.

Proof : $\because$ AB = CD [Given]

$\therefore$ OP = OQ [$\because$ Equal chords of a circle are equidistant from the centre]

Now in ΔOPE and ΔOQE,

$\angle OPE = \angle OQE$ [Each 90º]
OE = OE [Common]
OP = OQ [Proved above]
$\therefore \quad \Delta OPE \cong \Delta OQE$ [By RHS congruency]
$\therefore$ PE = QE [By CPCT]

$\Rightarrow \quad PE - \frac{1}{2}AB = QE - \frac{1}{2}CD$ [$\because$ AB = CD (Given)]

$\Rightarrow$ PE – PB = QE – QD
$\Rightarrow$ EB = ED. **Hence Proved.**

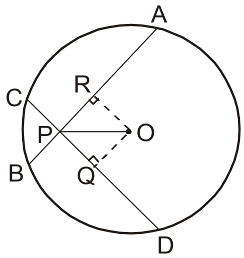
Example. 8

AB and CD are the chords of a circle whose centre is O. They intersect each other at P. If PO be the bisector of $\angle$APD, prove that AB = CD.

OR

In the given figure, O is the centre of the circle and PO bisects the angle APD. Prove that AB = CD.

Sol. **Given :** AB and CD are the chords of a circle whose centre is O. They intersect each other at P. PO is the bisector of $\angle$APD.
To Prove : AB = CD.
Construction : Draw OR $\perp$ AB and OQ $\perp$ CD.



Proof : In ΔOPR and ΔOPQ,

	$\angle OPR = \angle OPQ$	[Given]
	$OP = OP$	[Common]
And	$\angle ORP = \angle OQP$	[Each = 90°]
$\therefore$	$\Delta OPR \cong \Delta OPQ$	[By AAS congruency]
$\therefore$	$OR = OQ$	[By CPCT]
$\therefore$	$AB = CD$	[Chords of a circle which are equidistant from the centre are equal].

Check Your Level

1. Find the number of least non-collinear points required to draw a circle passing through them.

2. Given an arc of a circle, show how to complete the circles ?

3. If two circles intersect each other, prove that the line joining their centres bisects the common chord at right angle.

4. Find the length of a chord which is at a distance of 16 cm from the centre of a circle whose radius is 20 cm.

5. Two circles of radii 10 cm and 17 cm intersect at two points and the distance between their centres is 21 cm. Find the length of the common chord.

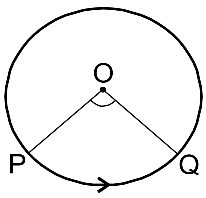
Answers

1. 3 **4.** 24cm **5.** 16cm

B. RESULTS ON ANGLES SUBTENDED BY ARCS AND CYCLIC QUADRILATERAL

(a) Result on angles subtended by arcs

In figure, the angle subtended by the minor arc PQ at O is $\angle POQ$ and the angle subtended by the major arc PQ at O is reflex angle $\angle POQ$.

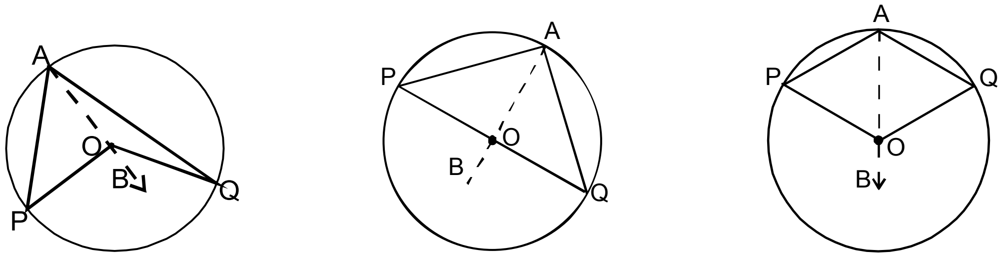


Theorem : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Given : An arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.

To Prove : $\angle POQ = 2\angle PAQ$.

Construction : Join AO and extend it to a point B.



(A) (B) (C)

Proof : There arises three cases:-

(A) arc PQ is minor (B) arc PQ is a semi-circle (C) arc PQ is major.

In all the cases, $\angle BOQ = \angle OAQ + \angle AQO$ ...(i)

[An exterior angle of a triangle is equal to the sum of the two interior opposite angles]

In $\triangle OAQ$,

$OA = OQ$ [Radii of a circle]

$\therefore$ $\angle OQA = \angle OAQ$ ...(ii)

[Angles opposite equal sides of a triangle are equal]

From (i) and (ii)

$\angle BOQ = 2\angle OAQ$ ...(iii)

Similarly,

$\angle BOP = 2\angle OAP$ ...(iv)

Adding (iii) and (iv), we get

$\angle BOP + \angle BOQ = 2(\angle OAP + \angle OAQ)$

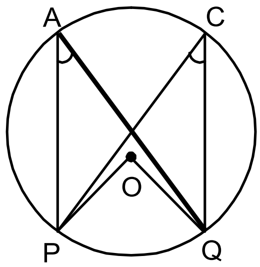
$\angle POQ = 2\angle PAQ.$ ...(v)

NOTE : For the case (C), where PQ is the major arc, (v) is replaced by reflex angles.

Thus, reflex $\angle POQ = 2\angle PAQ$.

Theorem : Angles in the same segment of a circle are equal.

Proof : Let P and Q be any two points on a circle to form a chord PQ, A and C any other points on the remaining part of the circle and O be the centre of the circle. Then,



$\angle POQ = 2\angle PAQ$ ...(i)

And $\angle POQ = 2\angle PCQ$ ...(ii)

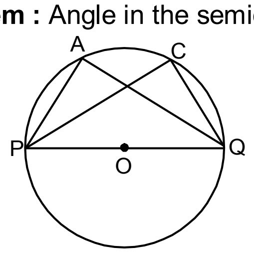
[Angle subtended at the centre is double than the angle subtended by it on the remaining part of the circle]

From equation (i) & (ii)

$2\angle PAQ = 2\angle PCQ$

$\angle PAQ = \angle PCQ.$ **Hence Proved.**

Theorem : Angle in the semicircle is a right angle.



Proof : $\angle PAQ$ is an angle in the segment, which is a semicircle.

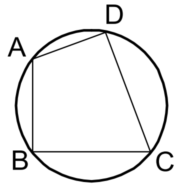
$\therefore \quad \angle PAQ = \frac{1}{2} \angle POQ = \frac{1}{2} \times 180^\circ = 90^\circ$ [POQ is straight line angle or $\angle POQ = 180^\circ$]

If we take any other point C on the semicircle, then again we get

$\angle PCQ = \frac{1}{2} \angle POQ = \frac{1}{2} \times 180^\circ = 90^\circ$. **Hence Proved.**

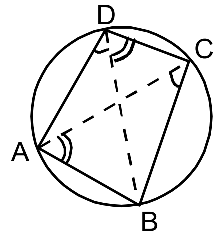
(b) Cyclic quadrilaterals

A quadrilateral ABCD is called **cyclic** if all the four vertices of it lie on a circle.



Theorem : The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Given : A cyclic quadrilateral ABCD.



To Prove : $\angle A + \angle C = \angle B + \angle D = 180^\circ$.

Construction : Join AC and BD.

Proof : $\angle ACB = \angle ADB$ ...(i)

And $\angle BAC = \angle BDC$ ...(ii) [Angles of same segment of a circle are equal]

Adding equation (i) & (ii)

$\therefore \quad \angle ACB + \angle BAC = \angle ADB + \angle BDC$

$\angle ACB + \angle BAC = \angle ADC$.

Adding $\angle ABC$ to both sides, we get

$\angle ACB + \angle BAC + \angle ABC = \angle ADC + \angle ABC$.

$\therefore \quad \angle ADC + \angle ABC = 180^\circ$

i.e., $\angle D + \angle B = 180^\circ$

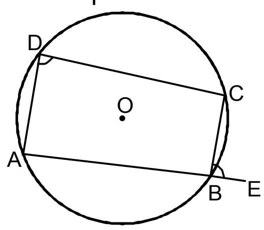
$\therefore \quad \angle A + \angle C = 360^\circ - (\angle B + \angle D) = 180^\circ$ [$\because \angle A + \angle B + \angle C + \angle D = 360^\circ$]. **Hence Proved.**

Solved Examples

Example. 9

If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle.

Sol. Let ABCD be a cyclic quadrilateral inscribed in a circle with centre O. The side AB of quadrilateral ABCD is produced to E. Then, we have to prove that $\angle CBE = \angle ADC$.



Since the sum of opposite pairs of angles of a cyclic quadrilateral is 180°

$\therefore \quad \angle ABC + \angle ADC = 180^\circ$

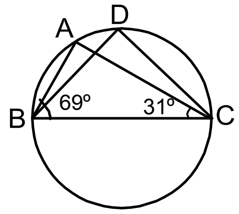
But $\angle ABC + \angle CBE = 180^\circ$ [$\because \angle ABC$ and $\angle CBE$ form a linear pair]

$\therefore \quad \angle ABC + \angle ADC = \angle ABC + \angle CBE$

$\Rightarrow \quad \angle ADC = \angle CBE$ or $\angle CBE = \angle ADC$. **Hence proved**

Example. 10

In figure, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.



Sol. In ABC,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$\Rightarrow$ $\angle BAC + 69^\circ + 31^\circ = 180^\circ$

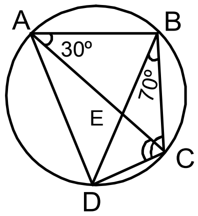
$\Rightarrow$ $\angle BAC + 100^\circ = 180^\circ$

$\Rightarrow$ $\angle BAC = 180^\circ - 100^\circ = 80^\circ$

Now, $\angle BDC = \angle BAC = 80^\circ$. [Angles in the same segment of a circle are equal]

Example. 11

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC = 30^\circ$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.



Sol. $\angle CDB = \angle BAC = 30^\circ$ ...(i) [Angles in the same segment of a circle are equal]

$\angle DBC = 70^\circ$ ...(ii)

In ΔBCD,

$\angle BCD + \angle DBC + \angle CDB = 180^\circ$

$\Rightarrow$ $\angle BCD + 70^\circ + 30^\circ = 180^\circ$ [Using (i) and (ii)]

$\Rightarrow$ $\angle BCD + 100^\circ = 180^\circ$

$\Rightarrow$ $\angle BCD = 180^\circ - 100^\circ$

$\Rightarrow$ $\angle BCD = 80^\circ$ ...(iii)

In ΔABC, AB = BC

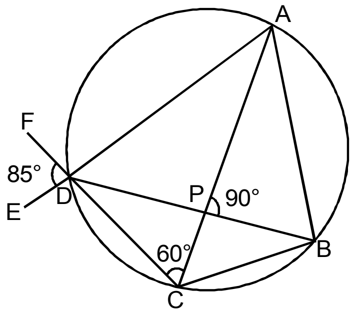
$\therefore$ $\angle BCA = \angle BAC = 30^\circ$ ...(iv) [Angles opposite to equal sides of a triangle are equal]

$\Rightarrow$ $\angle BCA + \angle ECD = 80^\circ$ $\Rightarrow$ $30^\circ + \angle ECD = 80^\circ$

$\Rightarrow$ $\angle ECD = 80^\circ - 30^\circ$ $\Rightarrow$ $\angle ECD = 50^\circ$.

Example. 12

In figure, find the measures of $\angle ABD$, $\angle CDP$, $\angle PDA$, $\angle CAB$ and $\angle CBD$.



Sol. Clearly, $\angle ABD$ and $\angle ACD$ are in the same segment determined by chord AD.

$\angle ABD = \angle ACD$

$\angle ABD = 60^\circ$ [Given $\angle ACD = 60^\circ$]

Now, $\angle CPD = \angle BPA$ [Vertically opp. angles]

$\angle CPD = 90^\circ$ [Given $\angle BPA = 90^\circ$]

In ΔCPD , we have

$\angle CDP + \angle CPD + \angle PCD = 180°$

$\angle CDP + 60° + 90° = 180°$

$\angle CDP + 150° = 180°$

$\angle CDP = 180° - 150° = 30°$

Since $\angle EDF$ and $\angle ADC$ are vertically opposite angles.

$\angle ADC = \angle EDF$

$\angle ADC = 85°$

$\angle PDA + \angle PDC = 85°$

$\angle PDA + 30° = 85°$

$\angle PDA = 85° - 30° = 55°$

Since, $\angle CDB = \angle CDP$ and $\angle CAB$ are the angles in the same segment determined by chord CB.

$\angle CAB = \angle CDP$

$\angle CAB = 30°$

In ΔACD, we have

$\angle CAD + \angle ACD + \angle ADC = 180°$

$\angle CAD + 60° + 85° = 180°$

$\angle CAD = 180° - (60° + 85°) = 35°$

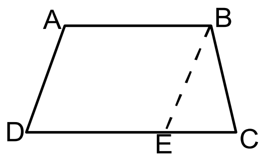
Clearly, $\angle CAD$ and $\angle CBD$ are angles in the same segment determined by chord CD.

$\angle CBD = \angle CAD = 35°$.

Example. 13

If the non parallel side of a trapezium are equal, prove that it is cyclic.

Sol. **Given :** ABCD is a trapezium whose two non-parallel sides AD and BC are equal.



To Prove : Trapezium ABCD is a cyclic.

Construction : Draw BE || AD.

Proof : AB || DE [Given]

and AD || BE [By construction]

Quadrilateral ABED is a parallelogram.

$\angle BAD = \angle BED$ ...(i) [Opp. angles of a ||gm]

And AD = BE ...(ii) [Opp. sides of a ||gm]

But AD = BC ...(iii) [Given]

From (ii) and (iii),

BE = BC

$\angle BCE = \angle BEC$ ...(iv) [Angles opposite to equal sides]

$\angle BEC + \angle BED = 180°$ [Linear Pair Axiom]

$\angle BCE + \angle BAD = 180°$ [From (iv) and (i)]

Trapezium ABCD is cyclic.

[If a pair of opposite angles of a quadrilateral is 180º, then the quadrilateral is cyclic]

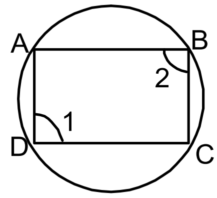
Hence Proved.

Example. 14

Prove that a cyclic parallelogram is a rectangle.

Sol. **Given :** ABCD is a cyclic parallelogram.

To Prove : ABCD is a rectangle.



Proof : ABCD is a cyclic quadrilateral

$\angle 1 + \angle 2 = 180^\circ$ ...(i)

[Opposite angles of a cyclic quadrilateral are supplementary]

ABCD is a parallelogram

$\angle 1 = \angle 2$ ...(ii) [Opp. angles of a || gm]

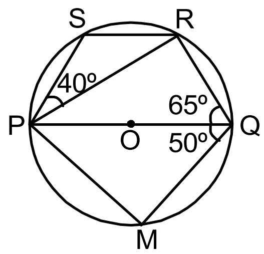
From (i) and (ii),

$\angle 1 = \angle 2 = 90^\circ$

Parallelogram ABCD is a rectangle. **Hence Proved.**

Example. 15

In figure, PQ is a diameter of a circle with centre O. If $\angle PQR = 65^\circ$, $\angle SPR = 40^\circ$, $\angle PQM = 50^\circ$, find $\angle QPR$, $\angle PRS$ and $\angle QPM$.



Sol. **(i)** **$\angle QPR$**

$\because$ PQ is a diameter

$\therefore$ $\angle PRQ = 90^\circ$ [Angle in a semi-circle is 90º] In PQR,

$\angle QPR + \angle PRQ + \angle PQR = 180^\circ$

$\Rightarrow$ $\angle QPR + 90^\circ + 65^\circ = 180^\circ$

$\Rightarrow$ $\angle QPR + 155^\circ = 180^\circ$

$\Rightarrow$ $\angle QPR = 180^\circ - 155^\circ$

$\Rightarrow$ $\angle QPR = 25^\circ$.

(ii) **$\angle PRS$**

$\because$ PQRS is a cyclic quadrilateral.

$\angle PSR + \angle PQR = 180^\circ$[Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow$ $\angle PSR + 65^\circ = 180^\circ$

$\Rightarrow$ $\angle PSR = 180^\circ - 65^\circ$

$\Rightarrow$ $\angle PSR = 115^\circ$

In ΔPSR,

$\because$ $\angle PSR + \angle SPR + \angle PRS = 180^\circ$

$\therefore$ $115^\circ + 40^\circ + \angle PRS = 180^\circ$

$155^\circ + \angle PRS = 180^\circ$

$\Rightarrow$ $\angle PRS = 180^\circ - 155^\circ$

$\Rightarrow$ $\angle PRS = 25^\circ$.

(iii) **$\angle QPM$**

$\because$ PQ is a diameter

$\therefore$ $\angle PMQ = 90^\circ$ [Angle in a semi-circle is 90º]

In PMQ,

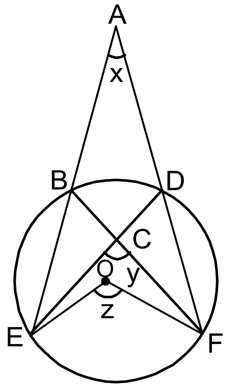
$\angle PMQ + \angle PQM + \angle QPM = 180^\circ$

$\Rightarrow$ $90^\circ + 50^\circ + \angle QPM = 180^\circ$ $140^\circ + \angle QPM = 180^\circ$

$\Rightarrow$ $\angle QPM = 180^\circ - 140^\circ$ $\angle QPM = 40^\circ$.

Example. 16

In figure, O is the centre of the circle. Prove that $\angle x + \angle y = \angle z$.



Sol. $\angle EBF = \frac{1}{2}\angle EOF = \frac{1}{2}\angle z$

[Angle subtended by an arc of a circle at the centre is twice the angle subtended by it at any point of the remaining part of the circle]

$\angle ABF = 180^\circ - \frac{1}{2}\angle z$ ...(i) [Linear Pair Axiom]

$\angle EDF = \frac{1}{2}\angle EOF = \frac{1}{2}\angle z$

[Angle subtended by any arc of a circle at the centre is twice
the angle subtended by it at any point of the remaining part of the circle]

$\angle ADE = 180^\circ - \frac{1}{2}\angle z$ ...(ii) [Linear Pair Axiom]

$\angle BCD = \angle ECF = \angle y$ [Vertically Opp. Angles]

$\angle BAD = \angle x$

In quadrilateral ABCD

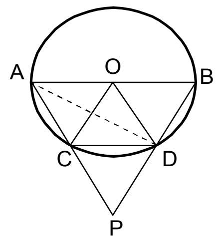
$\angle ABC + \angle BCD + \angle CDA + \angle BAD = 2 \times 180^\circ$

$180^\circ - \frac{1}{2}\angle z + \angle y + 180^\circ - \frac{1}{2}\angle z + \angle x = 2 \times 180^\circ$ $\angle x + \angle y = \angle z$. **Hence Proved.**

Example. 17

AB is a diameter of the circle with centre O and chord CD is equal to radius OC. AC and BD produced meet at P. Prove that $\angle CPD = 60^\circ$.

Sol. **Given :** AB is a diameter of the circle with centre O and chord CD is equal to radius OC. AC and BD produced meet at P.



To Prove : $\angle CPD = 60^\circ$.

Construction : Join AD.

Proof : In OCD,

OC = OD ...(i)

[Radii of the same circle]

OC = CD ...(ii) [Given]

From (i) and (ii),

OC = OD = CD

$\therefore$ ΔOCD is an equilateral triangle.

$\therefore$ $\angle COD = 60^\circ$

$\therefore$ $\angle CAD = \frac{1}{2}\angle COD = \frac{1}{2}\angle(60^\circ) = 30^\circ$

[Angle subtended by any arc of a circle at the centre is twice the angle subtended by it at any point of the remaining part of the circle]

$\Rightarrow$ $\angle PAD = 30^\circ$ . ..(iii)

And , $\angle ADB = 90^\circ$ ...(iv) [Angle in a semi-circle]

$\Rightarrow$ $\angle ADB + \angle ADP = 180^\circ$ [Linear Pair Axiom]

$\Rightarrow$ $90^\circ + \angle ADP = 180^\circ$ [From (iv)]

$\Rightarrow$ $\angle ADP = 90^\circ$ ...(v)

In ΔADP,

$\angle APD + \angle PAD + \angle ADP = 180^\circ$

$\Rightarrow$ $\angle APD + 30^\circ + 90^\circ = 180^\circ$ [From (iii) and (v)]

$\Rightarrow$ $\angle APD + 120^\circ = 180^\circ$

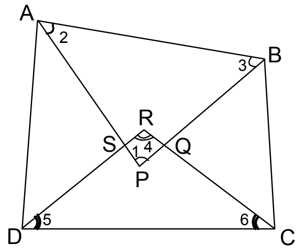
$\Rightarrow$ $\angle APD = 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow$ $\angle CPD = 60^\circ$. **Hence Proved.**

Example. 18

Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

Sol. **Given :** ABCD is a cyclic quadrilateral. Its angle bisectors form a quadrilateral PQRS.



To Prove : PQRS is a cyclic quadrilateral.

Proof : In ΔAPB

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$ ...(i)

In ΔDRC

$\angle 4 + \angle 5 + \angle 6 = 180^\circ$ ...(ii)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$ ...(iii)

[Adding (i) and (ii)]

But $\angle 2 + \angle 3 + \angle 6 + \angle 5 = \frac{1}{2}[\angle A + \angle B + \angle C + \angle D] = \frac{1}{2}(360^\circ) = 180^\circ$

$\therefore$ $\angle 1 + \angle 4 = 360^\circ - (\angle 2 + \angle 3 + \angle 6 + \angle 5) = 360^\circ - 180^\circ = 180^\circ$.

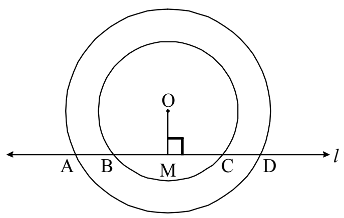
$\therefore$ PQRS is a cyclic quadrilateral.

[$\because$ If the sum of any pair of opposite angles of a quadrilateral is 180º, then the quadrilateral is a cyclic] **Hence Proved.**

Example. 19

Two concentric circles with centre O have A, B, C, D as the points of intersection with the line as shown in the figure. If AD = 12 cm and BC = 8 cm, find the length of AB, CD, AC and BD.

Sol. Since $OM \perp BC$, a chord of the circle,



$\therefore$ It bisects BC.

$\therefore$ $BM = CM = \frac{1}{2}(BC) = \frac{1}{2}(8) = 4$ cm

Since, $OM \perp AD$, a chord of the circle,

$\therefore$ It bisects AD.

$\therefore$ $AM = MD = \frac{1}{2}AD = \frac{1}{2}(12) = 6$ cm

Now, $AB = AM - BM = 6 - 4 = 2$ cm

$CD = MD - MC = 6 - 4 = 2$ cm

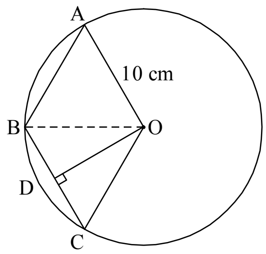
$AC = AM + MC = 6 + 4 = 10$ cm

$BD = BM + MD = 4 + 6 = 10$ cm

Example. 20

OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O. If the radius of the circle is 10 cm. Find the area of the rhombus.

Sol. Since, OABC is a rhombus



$\therefore$ $OA = AB = BC = OC = 10$ cm

Now, $OD \perp BC \Rightarrow CD = \frac{1}{2}BC = \frac{1}{2}(10) = 5$ cm

$\therefore$ By pythagoras theorem,

$OC^2 = OD^2 + DC^2$

$\Rightarrow$ $OD^2 = OC^2 - DC^2 = (10)^2 - (5)^2 = 100 - 25 = 75$

$\Rightarrow$ $OD = \sqrt{75} = 5\sqrt{3}$

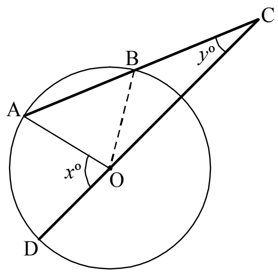
$\therefore$ Area $(\Delta OBC) = \frac{1}{2}BC \times OD = \frac{1}{2}(10) \times 5\sqrt{3} = 25\sqrt{3}$ sq. cm.

Area of Rhombus $= 2$ (Area of ΔOBC) $= 2(25\sqrt{3}) = 50\sqrt{3}$ sq. cm.

Example. 21

In the given figure, AB is the chord of a circle with centre O. AB is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If $\angle ACD = y^o$ and $\angle AOD = x^o$, prove that $x^o = 3y^o$.

Sol. Since BC = OB [Given]



$\therefore$ $\angle OCB = \angle BOC = y^o$ [Angles opposite to equal sides are equal]

$\angle OBA = \angle BOC + \angle OCB = y^o + y^o = 2y^o$.

[Exterior angle of a Δ is equal to the sum of the opposite interior angles]

Also OA = OB [Radii of the same circle]

$\angle OAB = \angle OBA = 2y^o$ [Angles opposite to equal sides of a triangle are equal]

$\angle AOD = \angle OAC + \angle OCA = 2y^o + y^o = 3y^o$

[Exterior angle of a Δ is equal to the sum of the opposite interior angles]

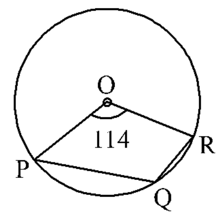
Hence, $x^o = 3y^o$. **Hence Proved.**

Check Your Level

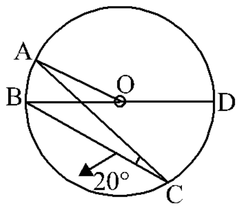
1. Two angles in the same segment of a circle are $(2x + 10)^\circ$ and $(x + 45^\circ)$. Then x is equal to

2. One angle of a cyclic quadrilateral is twice its opposite angle. Then find the smaller of the two angles

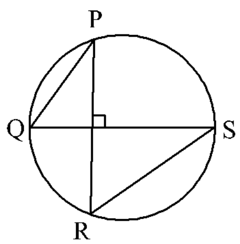
3. In the figure, O is the centre of the circle, find $\angle PQR$



4. In the figure, O is the centre and $\angle ACB = 20^\circ$,find $\angle AOD$ -



5. In this figure, find sum of $\angle Q$ and $\angle S$.



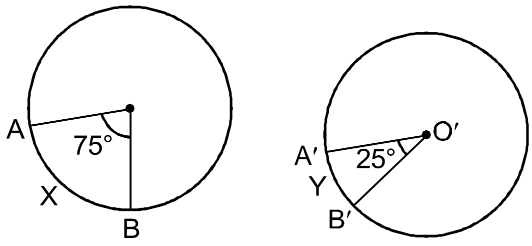
Answers

1. 35º **2.** 60º **3.** 123º **4.** 140° **5.** 90º

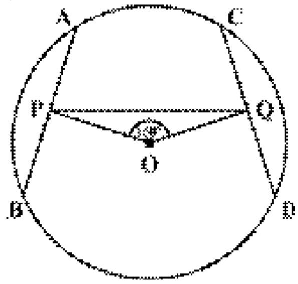
Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : **[01 MARK EACH]**

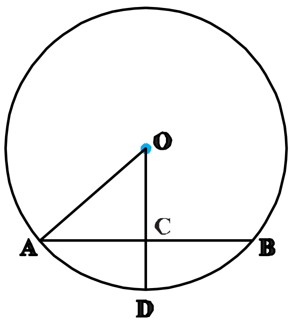
1. In Figure, two congruent circles have centres O and O'. Arc AXB subtends an angle of 75º at the centre O and arc A' Y B' subtends an angle of 25º at the centre O'. Then the ratio of arcs A X B and A'Y B' is :



2. In Figure, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD, respectively. If ∠ POQ = 150º, then find ∠ APQ ?

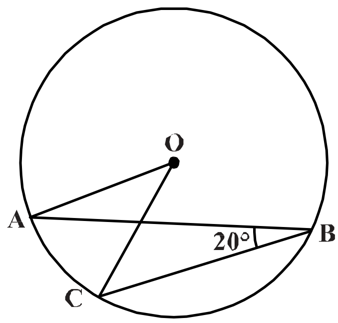


3. In Figure, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then find CD.

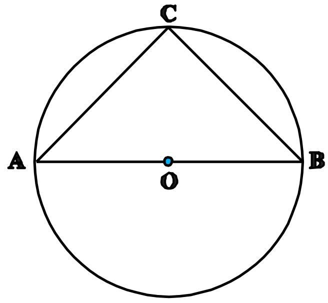


4. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then find the radius of the circle passing through the points A, B and C.

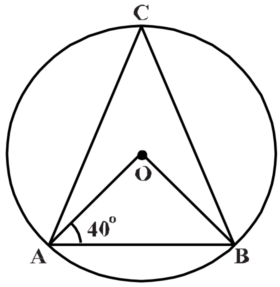
5. In Figure, if ∠ ABC = 20º, then find ∠ AOC.



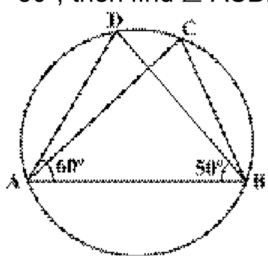
6. In Figure, if AOB is a diameter of the circle and AC = BC, then find ∠ CAB.



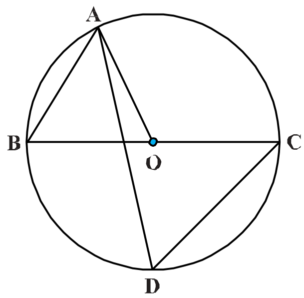
7. In Figure, if ∠ OAB = 40º, then find ∠ ACB.



8. In Figure, if ∠ DAB = 60º, ∠ ABD = 50º, then find ∠ ACB.

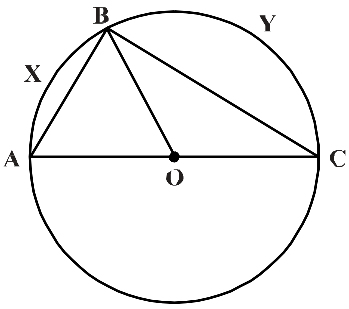


9. In Figure, BC is a diameter of the circle and ∠ BAO = 60º. Then find ∠ ADC.

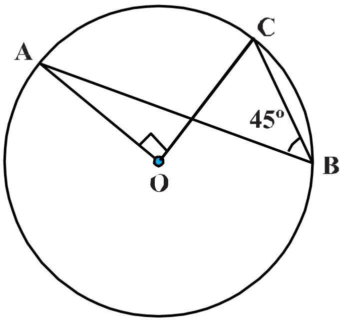


TYPE (II) : SHORT ANSWER TYPE QUESTIONS : **[02 MARKS EACH]**

10. In Figure, AOC is a diameter of the circle and arc AXB = $\frac{1}{2}$ arc BYC. Find ∠ BOC.



11. In Figure, $\angle ABC = 45^\circ$, prove that $OA \perp OC$.

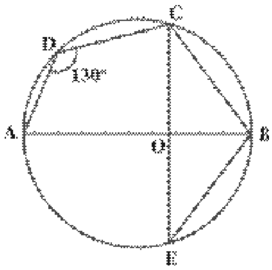


12. If a line segment joining mid-points of two chords of a circle passes through the centre of the circle, prove that the two chords are parallel.

13. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that $\angle CBD + \angle CDB = \frac{1}{2} \angle BAD$

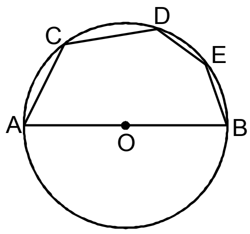
14. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.

15. In Figure, AB is a diameter, $\angle ADC = 130^\circ$ and chord BC = chord BE. Find $\angle CBE$.

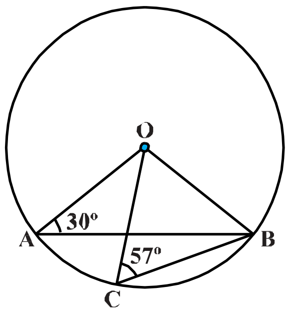


16. Two circles with centres O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through A(or B) intersecting the circles at P and Q. Prove that PQ = 2OO'.

17. In Figure, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of $\angle ACD + \angle BED$



18. In Figure, $\angle OAB = 30^\circ$ and $\angle OCB = 57^\circ$. Find $\angle BOC$ and $\angle AOC$.



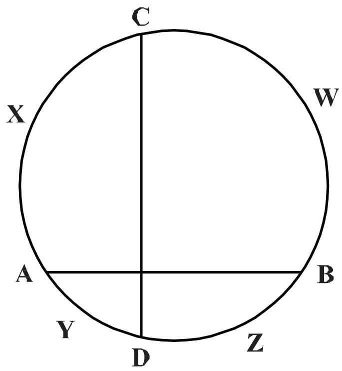
19. Prove that among all the chords of a circle passing through a given point inside the circle that one is smallest which is perpendicular to the diameter passing through the point.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: [03 MARK EACH]

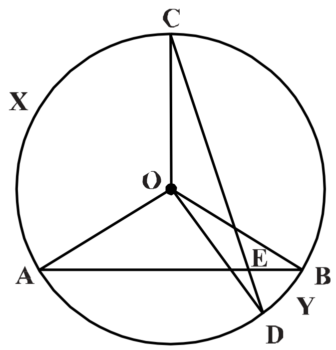
20. If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.

21. Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

22. If two chords AB and CD of a circle AYDZBWCX intersect at right angles (see Figure), prove that arc CXA + arc DZB = arc AYD + arc BWC = semi- circle.



23. In Figure, AB and CD are two chords of a circle intersecting each other at point E. Prove that $\angle$ AEC = 1/2 (Angle subtended by arc CXA at centre + angle subtended by arc DYB at the centre).



24. A circle has radius $\sqrt{2}$ cm. It is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by the chord at a point in major segment is 45º.

25. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.

Exercise-1

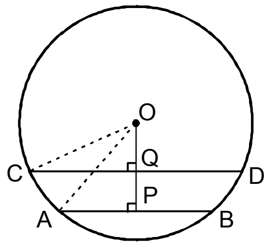
SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction and chord properties of Circles

A-1. The radius of a circle is 13 cm and the length of one of its chords is 10 cm. Find the distance of the chord from the centre.

A-2. In figure, O is the centre of the circle of radius 5 cm. OP $\perp$ AB, OQ $\perp$ CD, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ.



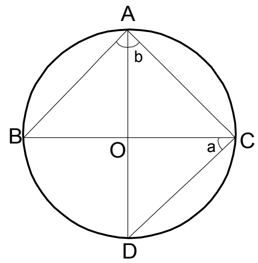
A-3. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite side of the centre and the distance between them is 17 cm, find the radius of the circle.

A-4. In a circle of radius 5 cm, AB and AC are two chords such that AB = AC = 6 cm. Find the length of the chord BC.

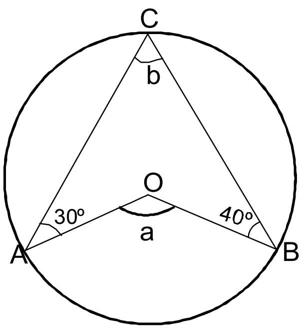
A-5. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distances on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Section (B) : Results on angles subtended by arcs and Cyclic quadrilateral

B-1. In the given figure, BC is diameter bisecting $\angle$ACD, find the values of a, b (O is centre of circle).

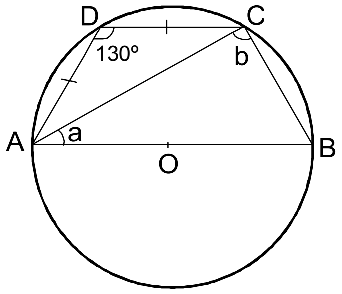


B-2. Find the value of a and b.

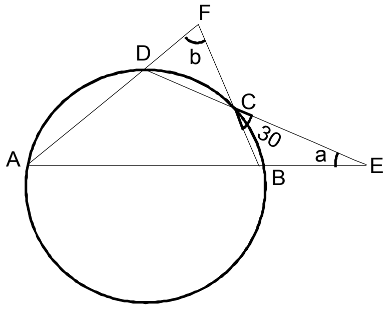


B-3. A chord of a circle is equal to the radius of the circle, find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

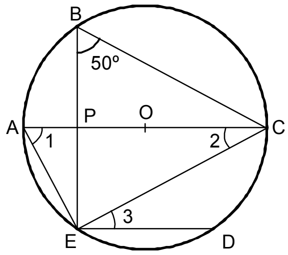
B-4. Find the value of a and b.



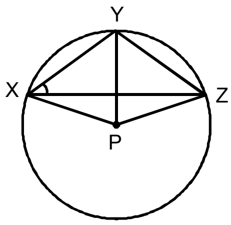
B-5. Find the value of a and b, if b = 2a.



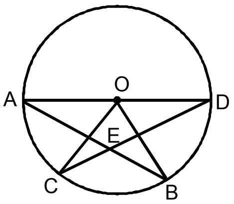
B-6. In the given figure, the chord ED is parallel to the diameter AC. Find ∠CED.



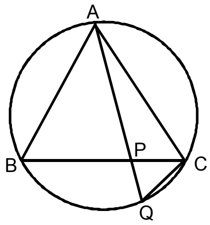
B-7. In the given figure, P is the centre of the circle. Prove that : ∠XPZ = 2 (∠XZY + ∠YXZ).



B-8. In a circle with centre O, chords AB and CD intersects inside the circumference at E. Prove that ∠AOC + ∠BOD = 2∠AEC.

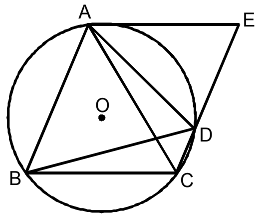


B-9. In figure, P is any point on the chord BC of a circle such that AB = AP. Prove that CP = CQ.

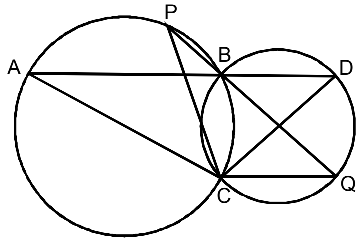


B-10. If two sides of a cyclic quadrilateral are parallel, prove that the remaining two sides are equal and the diagonals are also equal.

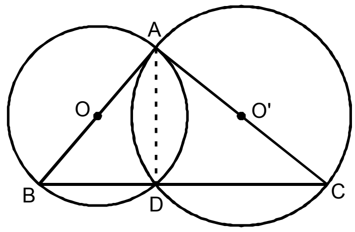
B-11. D is a point on the circumcircle of ΔABC in which AB = AC such that B and D are on the opposite side of line AC. If CD is produced to a point E such that CE = BD, prove that AD = AE.



B-12. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively as shown in figure. Prove that $\angle$ACP = $\angle$QCD.



B-13. In the figure given below, two circles intersect at A and D, and AC, AB are respectively the diameters of the circles. Prove that the points C, D, B are collinear.



B-14. ABCD is a parallelogram. The circle through A, B, C intersects CD (produced if necessary) at E. Prove that AD = AE.

B-15. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^\circ - \frac{1}{2}\angle A$, $90^\circ - \frac{1}{2}\angle B$ and $90^\circ - \frac{\angle C}{2}$.

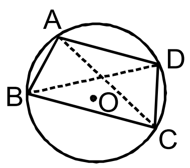
OBJECTIVE QUESTIONS

<u>Single Choice Objective, straight concept/formula oriented</u>

Section (A) : Introduction and chord properties of Circles

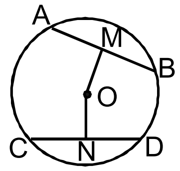
A-1. In a circle of radius 10 cm, the length of chord whose distance is 6 cm from the centre is :
(A) 4 cm (B) 5 cm (C) 8 cm (D) 16 cm

A-2. If $\overset{\frown}{BAD} = \overset{\frown}{ADC}$, then



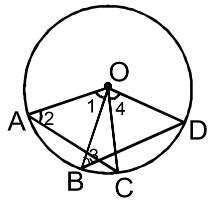
(A) AB = CD (B) AB $\neq$ CD (C) AC $\neq$ BD (D) AD = BC

A-3. O is the centre of a circle, AB and CD are two chords of the circle. OM $\perp$ AB and ON$\perp$CD. If OM = ON = 3 cm and AM = BM = 4.5 cm, then CD =



(A) 9 cm (B) 10 cm (C) 8 cm (D) 6 cm

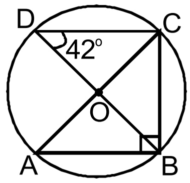
A-4. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then :



(A) $\angle 1 = \angle 2$ (B) $\angle 3 = \angle 4$ (C) $\angle 2 = \angle 3$ (D) None of these.

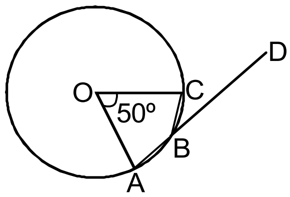
Section (B) : Results on angles subtended by arcs and Cyclic quadrilateral

B-1. In the given circle ABCD, O is the centre and $\angle$BDC = 42^o. The $\angle$ACB is equal to :



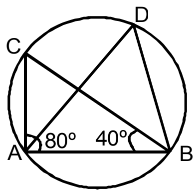
(A) 48^o (B) 45^o (C) 42^o (D) 60^o

B-2. In the diagram, O is the centre of the circle. The angles CBD is equal to :



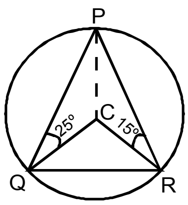
(A) 25^o (B) 50^o (C) 40^o (D) 130^o

B-3. In the given figure, $\angle$CAB = 80^o, $\angle$ABC = 40^o. The sum of $\angle$DAB + $\angle$ABD is equal to :



(A) 80^o (B) 100^o (C) 120^o (D) 140^o

B-4. In the given figure, if C is the centre of the circle and $\angle$PQC = 25^o and $\angle$PRC = 15^o, then $\angle$QCR is equal to :

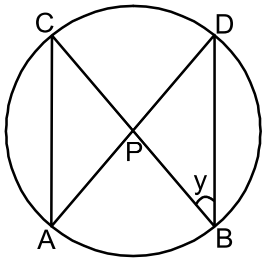


(A) 40^o (B) 60^o (C) 80^o (D) 120^o

B-5. The sides BA and CD of a cyclic quadrilateral ABCD are produced to meet at P, the sides DA and CB are produced to meet at Q. If $\angle ADC = 85^\circ$ and $\angle BPC = 40^\circ$, then $\angle CQD$ equals :

(A) 50° (B) 45° (C) 30° (D) 75°

B-6. In the given figure, if $\angle ACB = 40^\circ$, $\angle DPB = 120^\circ$, then y will be :



(A) 40° (B) 20° (C) 0° (D) 60°

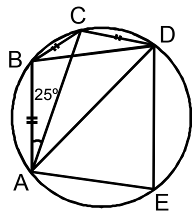
B-7. In a cyclic quadrilateral if $\angle A - \angle C = 70^\circ$, then the greater of the angles A and C is equal to :

(A) 95° (B) 105° (C) 125° (D) 115°

B-8. The length of a chord of a circle is equal to the radius of the circle. The angle which this chord subtends on the longer segment of the circle is equal to :

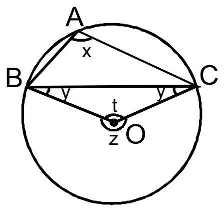
(A) 30° (B) 45° (C) 60° (D) 90°

B-9. In the given figure, AB = BC = CD, If $\angle BAC = 25^\circ$, then value of $\angle AED$ is :



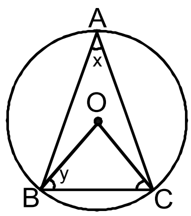
(A) 50° (B) 60° (C) 65° (D) 75°

B-10. A, B and C are three points on the circle whose centre is O. If $\angle BAC$ = x, $\angle CBO$ = $\angle CBO$ = y, $\angle BOC$ = t, reflex $\angle BOC$ = z, then :



(A) $x + y = 90^\circ$ (B) $x - y = 90^\circ$ (C) $t + 2y^\circ = 90^\circ$ (D) None of these

B-11. O is the centre of the circle. BC is a chord of the circle and point A lies on the circle. If $\angle BAC$ = x, $\angle OBC$ = y, then x + y :



(A) $> 90^\circ$ (B) $= 90^\circ$ (C) $< 90^\circ$ (D) $> 180^\circ$

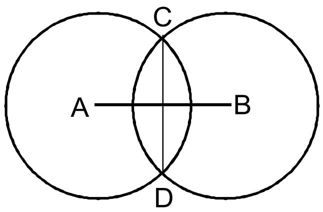
Exercise-2

OBJECTIVE QUESTIONS

1. Let P be a point on the circumference of a circle. Perpendiculars PA and PB are drawn to points A and B on two mutually perpendicular diameters. If AB = 36 cm, the diameter of the circle is :
(A) 16 cm (B) 24 cm (C) 36 cm (D) 72 cm

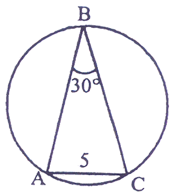
2. A semicircle is drawn with AB as its diameter. From C, a point on AB, a line perpendicular to AB is drawn meeting the circumference of the semicircle at D. Given that AC = 2 cm and CD = 6 cm, the area of the semicircle is:
(A) 32π (B) 50π (C) 40π (D) 36π

3. In the figure given below, A and B are the centers of the two congruent circles with radius 17 units. If AB = 30 units, the length of the common chord DC is :



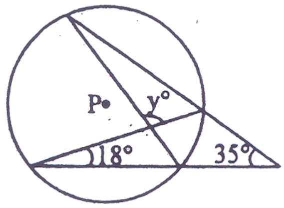
(A) 25 units (B) 18 units (C) 10 units (D) 16 units

4. In the diagram the circle contains the vertices A, B, C of triangle ABC. Now $\angle ABC$ is 30° and the length of AC is 5. The diameter of the circle is :



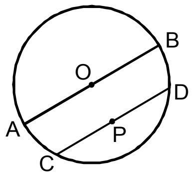
(A) $5\sqrt{3}$ (B) 8 (C) 10 (D) $5\sqrt{5}$

5. Find the measure of angle y in the figure if P is the centre of the circle :



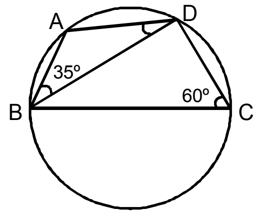
(A) 99° (B) 105° (C) 90° (D) 109°

6. The centre of a circle is at O. AB and CD are two chords of length d and ℓ respectively. If P is the mid point of CD, then the length OP is :



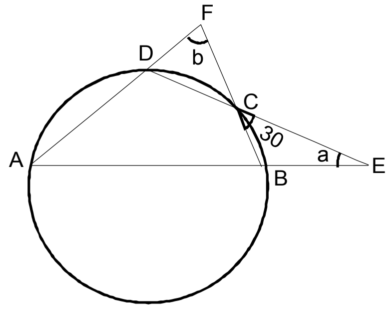
(A) $\sqrt{d^2 + \ell^2}$ (B) $\sqrt{d^2 - \ell^2}$ (C) $\frac{1}{2}\sqrt{d^2 + \ell^2}$ (D) $\frac{1}{2}\sqrt{d^2 - \ell^2}$

7. BC is the diameter of a circle. Points A and D are situated on the circumference of the semi circle $\angle ABD = 35^\circ$ and $\angle BCD = 60^\circ$, $\angle ADB$ equals to :



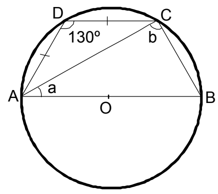
(A) 20º (B) 25º (C) 30º (D) 115º

8. Find the value of a + b, if b = 2a.



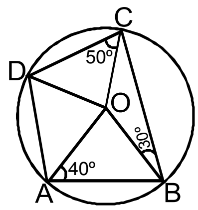
(A) 40º (B) 80º (C) 120º (D) 160º

9. In the given figure value of 'a' is :



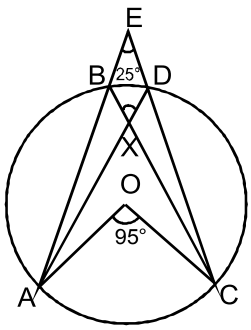
(A) 30º (B) 40º (C) 60º (D) 90º

10. ABCD is a cyclic quadrilateral inscribed in a circle with the centre O. Then $\angle OAD$ is equal to :



(A) 30º (B) 40º (C) 50º (D) 60º

11. In figure, O is centre, then $\angle BXD$ =



(A) 65° (B) 60° (C) 70° (D) 55°

12. Which of the following shapes of equal perimeter, the one having the largest area is :

(A) circle (B) equilateral triangle (C) square (D) regular pentagon

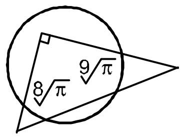
13. Let ΔXYZ be right angle triangle, with right angle at Z. Let A_x denotes the area of the circle with diameter YZ. Let A_y denote the area of the circle with diameter XZ and let A_z denotes the area of the circle diameter XY. Which of the following relations is true ?

(A) $A_z = A_x + A_y$ (B) $A_z = A_x^2 + A_y^2$ (C) $A_z^2 = A_x^2 + A_y^2$ (D) $A_z^2 = A_x^2 - A_y^2$

14. A triangle with side lengths in the ratio 3 : 4 : 5 is inscribed in a circle of radius 3. The area of the triangle is equal to :

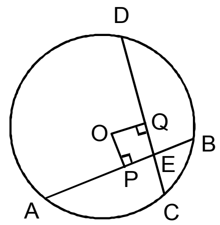
(A) 8.64 (B) 12 (C) 6 (D) 10.28

15. Two legs of a right triangle are $8\sqrt{\pi}$ and $9\sqrt{\pi}$ as shown. A circle is drawn so that the area inside the circle but outside the triangle equals the area inside the triangle but outside the circle. The radius of the circle is (Use π = 22/7)



(A) $6\sqrt{\pi}$ (B) 6 (C) 5 (D) 8

16. In the diagram O is the centre of a circle. AE + EB = CE + ED. OP $\perp$ AB and OQ $\perp$ CD, then true relation between OP and OQ is :

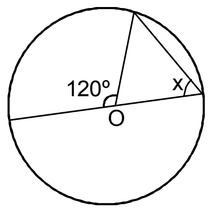


(A) OP > OQ (B) OP < OQ (C) OP = $\frac{1}{2}$ OQ (D) OP = OQ

Exercise-3

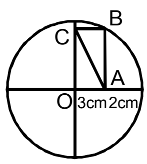
NTSE PROBLEMS (PREVIOUS YEARS)

1. In the following figure, O is the centre of the circle. The value of x is :
[RAJASTHAN NTSE Stage-1 2006]



(A) 60° (B) 45° (C) 40° (D) 80°

2. In the figure, O is the centre of the circle and OABC is rectangle : **[KERALA NTSE Stage-1 2007]**



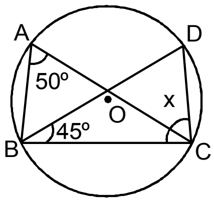
What is the length of AC ?

(A) 4 cm (B) 4.5 cm (C) 5 cm (D) 5.5 cm

3. In a circle, a 16 unit long chord is at a distance 6 units away from the center. Find the distance of a 12 unit long chord from the centre. **[GUJRAT NTSE Stage-1 2007]**

(A) 5 (B) 6 (C) 7 (D) 8

4. In the following figure. O is the centre of the circle . The value of x is **[Rajasthan NTSE Stage-1 2007]**

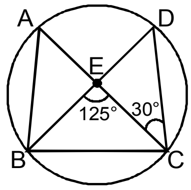


(A) 45° (B) 65° (C) 85° (D) 95°

5. The chord of maximum length in a circle is called : **[RAJASTHAN NTSE Stage-1 2013]**

(A) Radius (B) Arc (C) Diameter (D) Point

6. In figure, A, B, C and D are four point on a cirlcle. AC and BD intersect at a point E such that $\angle BEC = 125°$ and $\angle ECD = 30°$. Then $\angle BAC =$ **[RAJASTHAN NTSE Stage-1 2013]**



(A) 95° (B) 110° (C) 85° (D) 105°

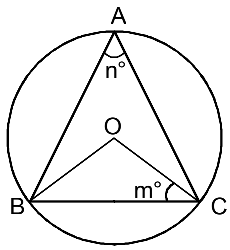
7. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of the centre and the distance between them is 17 cm, the radius of the circle is : **[DELHI NTSE Stage-1 2013]**

(A) 14 cm (B) 10 cm (C) 13 cm (D) 15 cm

8. If two equal circles of radius r passes through centre of the other then the length of their common chord is **[U.P. NTSE Stage-1 2014]**

(A) $\frac{r}{\sqrt{3}}$ (B) $r\sqrt{3}$ (C) $\frac{\sqrt{3}}{4}r$ (D) $r\sqrt{2}$

9. In the following figure O is the centre of circle and $\angle BAC = n°$, $\angle OCB = m°$ then **[U.P. NTSE Stage-1 2014]**

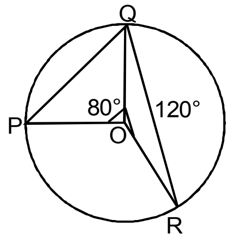


(A) $m° + n° = 90°$ (B) $m° + n° = 180°$ (C) $m° + n° = 120°$ (D) $m° + n° = 150°$

10. AB and AC are equal chord of a circle with centre O. Then by which angle OA bisects BC. **[M.P. NTSE Stage-1 2014]**

(A) 30° (B) 60° (C) 90° (D) 120°

11. In the given figure find $\angle PQR$ (where O is centre of the circle) **[U.P. NTSE Stage-1 2014]**

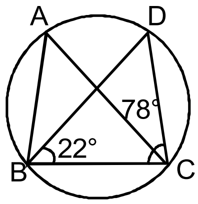


(A) 60° (B) 80° (C) 100° (D) 120°

12. If two chords of a circle are equidistance from the centre of the circle, then they are............ **[M.P. NTSE Stage-1 2015]**

(A) Equal to each other
(B) Not equal to each other.
(C) Intersect each other.
(D) None of these

13. In the given figure, $\angle DBC = 22°$ and $\angle DCB = 78°$ then $\angle BAC$ is equal to **[Rajasthan NTSE Stage-1 2015]**

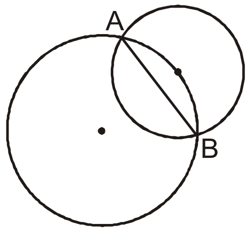


(A) 90°
(B) 80°
(C) 78°
(D) 22°

14. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, then the distance of the other chord from the centre is **[Rajasthan NTSE Stage-1 2015]**

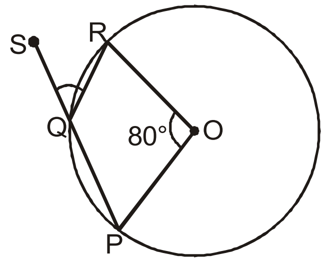
(A) 5 cm
(B) 4 cm
(C) 3 cm
(D) 2 cm

15. In the figure, the radius of the larger circle is 2 cm and the radius of the smaller circle is 1 cm and the larger circle passes through the centre of the smaller circle. The length (in cm) of the chord AB is **[HARYANA NTSE Stage-1 2016]**



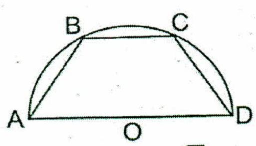
(A) $\frac{\sqrt{15}}{2}$
(B) 2
(C) $\frac{3}{2}$
(D) $\frac{5\sqrt{34}}{17}$

16. In the figure O is the centre of the circle and $\angle$ POR = 80º. Then $\angle$ RQS is **[Rajasthan NTSE Stage-1 2016]**



(A) 30º
(B) 40º
(C) 1400
(D) 50º

17. In the figure, the semicirlce centered at O has a diameter 6 cm. The chord BC is parallel to AD and $BC = \frac{1}{3}$ AD. The area of the trapezium ABCD in cm², is : **[HARYANA NTSE Stage-1 2016]**



(A) 4
(B) $4\sqrt{2}$
(C) 8
(D) $8\sqrt{2}$

Answer Key

Exercise Board Level

TYPE (I)

1. 3 : 1 **2.** 75° **3.** 2 cm **4.** 10 cm **5.** 40°

6. 45° **7.** 50° **8.** 70° **9.** 60°

TYPE (II)

10. 120° **17.** 270° **18.** $\angle$ BOC = 66° and $\angle$ AOC = 54°

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-1. 12 cm. **A-2.** 1 cm. **A-3.** 13 cm. **A-4.** 9.6 cm. **A-5.** $20\sqrt{3}$ m.

Section (B)

B-1. a = 45º & b = 90º. **B-2.** a = 140º and b = 70º **B-3.** 300º

B-4. a = 40º and b = 90º. **B-5.** 80º. **B-6.** 150º.

OBJECTIVE QUESTIONS

Section (A)

A-1. (D) **A-2.** (A) **A-3.** (A) **A-4.** (C)

Section (B)

B-1. (A) **B-2.** (A) **B-3.** (C) **B-4.** (C) **B-5.** (A)

B-6. (B) **B-7.** (C) **B-8.** (A) **B-9.** (D) **B-10.** (B)

B-11. (B)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	D	B	D	C	D	D	B	B	B	C	C	A	A	A	B	D

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Ans.	A	C	D	C	C	A	C	B	A	C	B	A	B	C	A	B	D