MATHEMATICS

Class-X

Topic-10

SURFACE AREA AND VOLUME

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CH-10

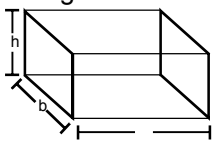
SURFACE AREA AND VOLUME

(A) SURFACE AREA AND VOLUME OF SOLID FIGURES

In class IX, we had learnt how to find the surface areas and volumes of some regular solids like cuboid, cube, cylinder, cone and sphere. In our day to day life we come across various solids which are combinations of two or more such solids. For example, a conical circus tent with cylinderical base is a combination of a right circular cylinder and a right circular cone. In this chapter, we shall learn how to find the surface area and volume of such combinations.

(a) Cuboid :

Total Surface Area (T.S.A.) : The area of surface from which cuboid is formed. There are six faces (rectangular), eight vertices and twelve edges in a cuboid.



(i) **Total Surface Area (T.S.A.)** = 2 [$\ell \times b + b \times h + h \times \ell$]

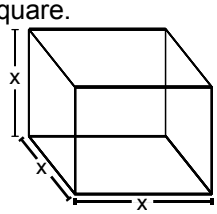
(ii) **Lateral Surface Area (L.S.A.)** = 2 [b × h + h × ℓ] (or **Area of 4 walls)** = 2 h [ℓ + b]

(iii) **Volume of Cuboid** = (Area of base) × height = ($\ell \times b$) × h

(iv) **Length of diagonal** = $\sqrt{\ell^2 + b^2 + h^2}$

(b) Cube :

Cube has six faces. Each face is a square.



(i) **T.S.A.** = 2 [x · x + x · x + x · x]
= 2 [$x^2 + x^2 + x^2$] = 2 ($3x^2$) = $6x^2$

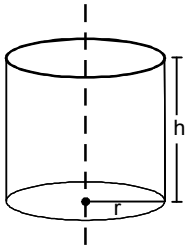
(ii) **L.S.A.** = 2 [$x^2 + x^2$] = $4x^2$

(iii) **Volume** = (Area of base) × Height) = (x^2) · x = x^3

(iv) **Length of diagonal** = $x\sqrt{3}$

(c) Cylinder :

Curved surface area of cylinder (C.S.A.) : It is the area of surface from which the cylinder is formed. When we cut this cylinder, we will find a rectangle with length 2πr and height h units.



(i) **C.S.A. of cylinder** = (2πr) × h = 2πrh.

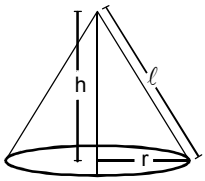
(ii) **Total Surface Area (T.S.A.) :**
T.S.A. = C.S.A. + circular top & bottom
= 2πrh + 2πr^2 = 2πr (h + r) sq. units.

(iii) Volume of cylinder :

Volume = Area of base × height

$= (\pi r^2) \times h = \pi r^2 h$ cubic units

(d) Cone :



(i) **C.S.A.** $= \pi r \ell$

(ii) **T.S.A.** = C.S.A. + other area

$= \pi r \ell + \pi r^2$

$= \pi r (\ell + r)$

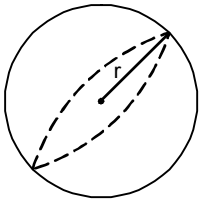
(iii) **Volume** $= \frac{1}{3} \pi r^2 h$

Where, h = height

r = radius of base

ℓ = slant height

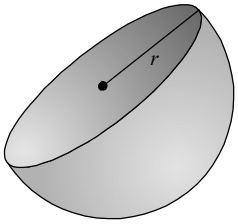
(e) Sphere :



T.S.A. = S.A. $= 4\pi r^2$

Volume $= \frac{4}{3} \pi r^3$

(f) Hemisphere :



C.S.A. $= 2\pi r^2$

T.S.A. = C.S.A. + area of circular top

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

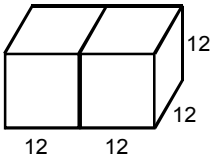
Volume $= \frac{2}{3} \pi r^3$

Solved Examples

Example. 1

When two cube of side 12 cm are joined end to end to form a cuboid then find the total surface area of cuboid.

Sol.



ℓ = 24, b = 12, h = 12

T.S.A.= $2(\ell b + bh + h\ell)$

= 2(24 × 12 + 12 × 12 + 12 × 24)

= 2[288 + 144 + 288] = 2[720] = 1440 cm^2

Example. 2

The height of a cylinder is 15 cm. Its curved surface area is 660 sq. cm. Find the diameter.

Sol. Curved surface area of a cylinder = $2\pi rh = 660\ cm^2$

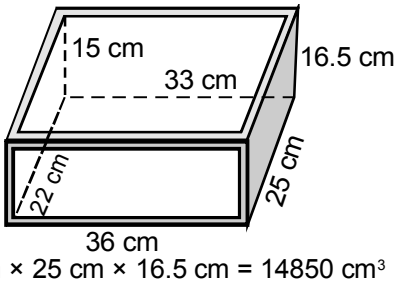
$\Rightarrow 2\times \frac{22}{7} \times r \times 15 = 660 \quad \Rightarrow \frac{660\times 7}{2\times 22\times 15}$ cm = 7 cm

$\therefore$ Diameter of the cylinder = 14 cm

Example. 3

How many cubic centimetres of iron are required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm. If one cubic cm of iron weighs 7.5 g, find the weight of the box.

Sol. External volume of the box



= 36 cm × 25 cm × 16.5 cm = 14850 cm^3

When the thickness of iron sheet is 1.5 cm,

the internal dimensions of the open iron box will be

(36 – 3)cm, (25 – 3)cm and (16.5 – 1.5) cm

or 33cm, 22 cm 15 cm

Internal volume of the box

= 33 cm × 22 cm × 15 cm = 10890 cm^3

Volume of iron used = External volume – Internal volume

= (14850 – 10890) cm^3 = 3960 cm^3

Weight of 1 cm^3 of iron = 7.5 g

Weight of 3960 cm^3 of iron = 7.5 × 3960 g = 29700 g

Hence, weight of the box = 29.7 kg.

Example. 4

The sides of cube are increased by 100%. Find the percent increase in the volume of the cube ?

Sol. Side = a

volume (v) = side = a^3

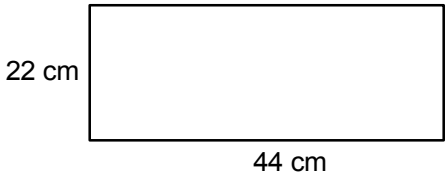
when side is increased by 100% then new side (a') become 2a and new volume

v' = $(2a)^3 = 8a^3$ = 8v % increase in volume = $\frac{8v - v}{v}\times 100\%$ = 700%

Example. 5

If a rectangular sheet of paper 44 cm × 22 cm is rolled along its length of form a cylinder, then find the volume of cylinder

Sol. Rolled across 44 cm, so $2\pi r$ = 44 cm.



$\Rightarrow \quad r = \frac{44}{2\pi} \Rightarrow \quad r = 7$ cm.

Height of the cylinder = 22 cm

Volume = $\pi \times r^2 \times h$

$= \frac{22}{7} \times 7 \times 7 \times 22 = 3388$ cm^3.

Example. 6

How many metres of 5 m wide cloth will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m ?

Sol. Slant height of the conical tent, $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$ m

Curved surface area of the conical tent = $\pi r l = \frac{22}{7} \times 7 \times 25$ m^2 = 550 m^2

Required length of 5 m wide cloth $= \frac{\text{Area}}{\text{Width}} = \frac{550}{5}$ m = 110 m

Example. 7

If the total surface area of a solid hemisphere is 462 cm^2, find its volume. (Take $\pi = \frac{22}{7}$)

Sol. Total surface area of the hemisphere = $3\pi r^2$ = 462 cm^2

$\Rightarrow \quad r^2 = \frac{462 \times 7}{3 \times 22} = 7 \times 7 \quad \Rightarrow r = 7$ cm

Hence, the volume of the hemisphere

$= \frac{2}{3} \pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$ cm^3 = 718.67 cm^3 (Approx)

Example. 8

The total surface area of a cube is numerically equal to the surface area of a sphere then find the ratio of their volume.

Sol. TSA of a cube = $6a^2$

SA of a sphere = $4\pi r^2$

TSA of a cube = S.A. of a sphere

$6a^2 = 4\pi r^2$

$\sqrt{3}\, a = \sqrt{2\pi}\, r$

So, $a = \sqrt{\frac{2\pi}{3}}\, r$

Volume of a cube = a^3

Volume of a sphere = $\frac{4}{3}\pi r^3$

$a^3 = \left(\sqrt{\frac{2}{3}\pi}\ \ r\right)^3 = r^3 \left(\frac{2\pi}{3}\right)^{3/2}$

$$\text{Ratio} = \frac{r^3 \left(\frac{2}{3}\pi\right)^{3/2}}{r^3 \frac{4}{3}\pi} = \frac{\left(\frac{8\pi^3}{27}\right)^{1/2}}{\left(\frac{16\pi^2}{9}\right)^{1/2}} = \sqrt{\frac{\pi}{6}}.$$

Example. 9

How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm ?

Sol. Volume of the spherical ball of radius 8 cm $= \frac{4}{3}\pi \times 8^3$ cm^3

Also, volume of each smaller spherical ball of radius 1 cm $= \frac{4}{3}\pi \times 1^3$ cm^3.

Let n be the number of smaller balls that can be made. Then, the volume of the larger ball is equal to the sum of all the volumes of n smaller balls.

Hence, $\frac{4}{3}\pi \times n = \frac{4}{3}\pi \times 8^3 \quad \Rightarrow \quad n = 8^3 = 512$

Hence, the required number of balls = 512.

Check Your Level

1. From the four corners of a rectangular sheet of paper 4 squares of side 5 cm are cut off. The length and breadth of the paper are 35 cm, 25 cm. The remaining part is folded is such a way to make a cuboid. Find the volume of the cuboid.

2. A right angled triangle has the sides containing the right angle, 6 cm and 8 cm. This triangle is revolved about its hypotenuse find the volume of the solid generated by the revolution.

3. The surface areas of 3 faces of a cuboid are 36 sq. cm, 49 sq. cm and 64 sq. cm. Then find the volume of the cuboid

4. A cone has volume of 297 cubic cm. The height is 14 cm then find the diameter of the base.

5. The curved surface area of a cone is 550 sq. cm. Diameter is 14 cm. Then find the height.

6. There are two cones. The curved surface area of the first one is twice the second. The slant height of the second one is twice that of the first. Then find the ratio of their radii.

7. The radius of a sphere is increase 4 times then find the increase in the volume.

8. Two cylinders have the radii in the ratio 1 : 2 and heights are in the ratio 2 : 1. Find the ratio of their volumes.

9. Find the length of longest stick that can be kept in a room of 3 m by 4 m by 5 m

Answers

1.	1875 cubic cm	**2.**	241.37	**3.**	336 cubic cm	**4.**	9 cm
5.	24 cm	**6.**	4 : 1	**7.**	64 times	**8.**	1 : 2
9.	$5\sqrt{2}$ m						

(B) CONVERSION OF SOLIDS

In this section, we shall discuss problems pertaining to conversion of a solid (discussed in the previous classes) into another solid of different shape. For example, a metallic sphere is melted and recast into a cylindrical wire, the earth taken out by digging a well and spreading it uniformly around the well to form an embankment in the form of a cylindrical shell from its original shape of right circular cylinder, etc.

Solved Examples

Example. 10

By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder and the ratio of the height of the cylinder and the height of the cone is 4 : 3, find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively. Then,

$3r = 2R$

and $H : h = 4 : 3$ ....(i)

$\Rightarrow \quad \frac{H}{h} = \frac{4}{3}$

$\Rightarrow \quad 3H = 4h$ ....(ii)

Let n be the required number of cones which can be made from the materials of the cylinder. Then, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$$\pi R^2 H = \frac{n}{3}\pi r^2 h \quad \Rightarrow \quad 3R^2H = nr^2h$$

$$\Rightarrow \quad n = \frac{3R^2H}{r^2h} = \frac{3 \times 9r^2 \times 4h}{r^2h \quad 4 \quad 3}$$ [From (i) and (ii), $R = \frac{3r}{2}$ and $H = \frac{4h}{3}$]

$$\Rightarrow \quad n = \frac{3 \times 9 \times 4}{3 \times 4} \quad \Rightarrow \quad n = 9.$$

Hence, the required number of cones is 9.

Example. 11

An iron rod of length 1 m and diameter 4 cm is melted and cast into thin wires of length 20 cm each. If the number of such wires be 2000, find the radius of each thin wire.

Sol. Let the radius of each thin wire be r cm. Then, the sum of the volumes of 2000 thin wires will be equal to the volume of the iron rod. Now, the shape of the iron rod and each thin wire is cylindrical.

Hence, the volume of the iron rod of radius 2 cm $= \pi \times 2^2 \times 100\ cm^3$

Again, the volume of each thin wire $= \pi r^2 \times 20$

Hence, we have $\pi \times 2^2 \times 100 = 2000 \times \pi r^2 \times 20$

$$\Rightarrow \quad 400\ r^2 = 4 \quad \Rightarrow \quad r^2 = \frac{1}{100} \quad \Rightarrow \quad r = \frac{1}{10}$$ [Taking positive square root only]

Hence, the required radius of each thin wire is $\frac{1}{10}$ cm or 0.1 cm.

Example. 12

The base diameter of a solid in the form of a cone is 6 cm and the height of the cone is 10 cm. It is melted and recast into spherical balls of diameter 1 cm. Find the number of balls, thus obtained.

Sol. Let the number of spherical balls be n. Then, the volume of the cone will be equal to the sum of the volumes of the spherical balls. The radius of the base of the cone $= \frac{6}{2}$ cm = 3 cm and the radius of the sphere $= \frac{1}{2}$ cm.

Now, the volume of the cone $= \frac{1}{3}\pi \times 3^2 \times 10\ cm^3 = 30\pi\ cm^3$

and, the volume of each sphere $= \frac{4}{3}\pi\left(\frac{1}{2}\right)^3\ cm^3 = \frac{\pi}{6}\ cm^3$

Hence, we have $n\frac{\pi}{6} = 30\pi \quad \Rightarrow\ n = 6 \times 30 = 180$

Hence, the required number of balls = 180.

Example. 13

A conical empty vessel is to be filled up completely by pouring water into it successively with the help of a cylindrical can of diameter 6 cm and height 12 cm. The radius of the conical vessel is 9 cm and its height is 72 cm. How many times will it require to pour water into the conical vessel to fill it completely, if, in each time, the cylindrical can is filled with water completely?

Sol. Let n be the required number of times. Then, the volume of the conical vessel will be equal to n times the volume of the cylindrical can.

Now, the volume of the conical vessel = $\frac{1}{3}\pi \times 9^2 \times 72\ cm^3 = 24 \times 81\pi\ cm^3$

And the volume of the cylindrical can $= \pi \times 3^2 \times 12\ cm^3 = 9 \times 12\pi\ cm^3$

Hence, $24 \times 81\pi = 9 \times 12\pi \times n \quad \Rightarrow \quad n = \frac{24 \times 81}{9 \times 12} = 18$

Hence, the required number of times = 18.

Example. 14

The height of a right circular cylinder is equal to its diameter. If it is melted and recast into a sphere of radius equal to the radius of the cylinder, find the part of the material that remained unused.

Sol. Let h be height of the cylinder. Then, its diameter is h and so its radius is $\frac{h}{2}$. Hence, its volume is

$V_1 = \pi\left(\frac{h}{2}\right)^2 .h = \frac{\pi h^3}{4}$.

Again, the radius of the sphere = $\frac{h}{2}$. Hence, the volume of the sphere is $V_2 = \frac{4}{3}\pi\left(\frac{h}{2}\right)^3 = \frac{\pi h^3}{6}$

The volume of the unused material = $V_1 - V_2 = \frac{\pi h^3}{4} - \frac{\pi h^3}{6} = \frac{\pi h^3(3-2)}{12} = \frac{\pi h^3}{12} = \frac{1}{3} \times \frac{\pi h^3}{4} = \frac{1}{3}V_1$

Hence, the required volume of the unused material is equal to $\frac{1}{3}$rd of the volume of the cylinder.

Example. 15

Water flows at the rate of 10 m per minute through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel whose diameter of the base is 40 cm and depth 24 cm?

Sol. Diameter of the pipe = 5 mm = $\frac{5}{10}$cm = $\frac{1}{2}$cm.

Radius of the pipe = $\frac{1}{2} \times \frac{1}{2}$cm = $\frac{1}{4}$ cm.

In 1 minute, the length of the water column in the cylindrical pipe = 10 m = 1000 cm.

Volume of water that flows out of the pipe in 1 minute = $\pi \times \frac{1}{4} \times \frac{1}{4} \times 1000\ cm^3$.

Also, volume of the cone = $\frac{1}{3} \times \pi \times 20 \times 20 \times 24\ cm^3$.

Hence, the time needed to fill up this conical vessel

$= \left(\frac{1}{3}\pi \times 20 \times 20 \times 24 \div \pi \times \frac{1}{4} \times \frac{1}{4} \times 1000\right)$ minutes $= \left(\frac{20 \times 20 \times 24}{3} \times \frac{4 \times 4}{1000}\right) = \frac{4 \times 24 \times 16}{30}$ minutes

$= \frac{256}{5}$ minutes = 51.2 minutes.

Hence, the required time is 51.2 minutes.

Example. 16

A hemispherical tank of radius 1 $\frac{3}{4}$m is full of water. It is connected with a pipe which empties it at the rate of 7 liters per second. How much time will it take to empty the tank completely?

Sol. Radius of the hemisphere = $\frac{7}{4}$ m = 175 cm

Volume of the hemisphere $= \frac{2}{3} \times \pi \times 175 \times 175 \times 175\ cm^3$

The cylindrical pipe empties it at the rate of 7 litres i.e., 7000 cm^3 of water per second.
Hence, the required time to empty the tank

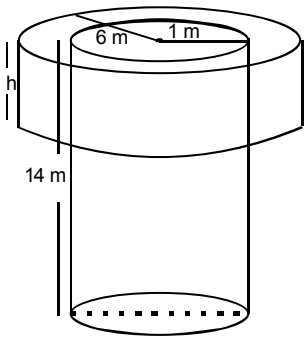
$= \left(\frac{2}{3}\times\frac{22}{7}\times 175\times 175\times 175\div 7000\right)$ s $= \frac{2}{3}\times\frac{22}{7}\times\frac{175\times 175\times 175}{7000\times 60}$ min

$= \frac{11\times 25\times 7}{3\times 2\times 12}$ min $= \frac{1925}{72}$ min = 26.74 min. (nearly)

Example. 17

A well of diameter 2 m is dug 14 m deep. The earth taken out of it is spread evenly all around it to a width of 5 m to form an embankment. Find the height of the embankment.

Sol. Let **h** be the required height of the embankment.
The shape of the embankment will be like the shape of a cylinder of internal radius 1 m and external radius (5 + 1) m = 6 m [figure].



The volume of the embankment will be equal to the volume of the earth dug out from the well. Now, the volume of the earth = volume of the cylindrical well

$= \pi \times 1^2 \times 14\ m^3 = 14\pi\ m^3$

Also, the volume of the embankment

$= \pi\,(6^2 - 1^2)\,h\ cm^3 = 35\,\pi\,h\ m^3$

Hence, we have

$35\pi h = 14\pi$ $\qquad h = \frac{14}{35} = \frac{2}{5} = 0.4$

Hence, the required height of the embankment = 0.4 m

Example. 18

Water in a canal, 30 dm wide and 12 dm deep, is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes, if 8 cm of standing water is required for irrigation.

Sol. Speed of water in the canal = 10 km/h = 10000 m/60 min = $\frac{500}{3}$ m/min.

The volume of the water flowing out of the canal in 1 minute = $\left(\frac{500}{3}\times\frac{30}{10}\times\frac{12}{10}\right)\ m^3 = 600\ m^3$

In 30 min, the amount of water flowing out of the canal = $(600 \times 30)\ m^3 = 18000\ m^3$
If the required area of the irrigated land is x m^2, then the volume of water to be needed to irrigate the land

$= \left(x\times\frac{8}{100}\right) m^3 = \frac{2x}{25}\ m^3$

Hence, $\frac{2x}{25} = 18000$ $\qquad x = 18000 \times \frac{25}{2} = 225000$

Hence, the required area is 225000 m^2.

Check Your Level

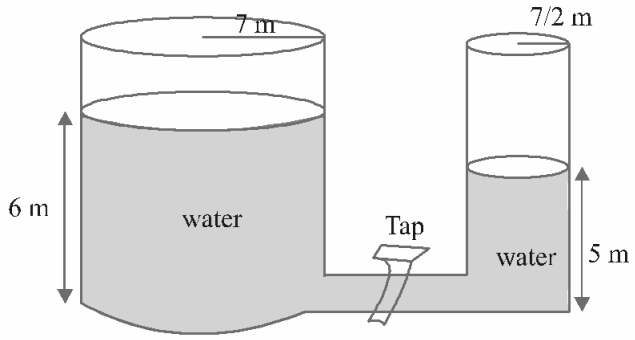
1. The radius of a metal sphere is 9 cm. It is melted and drawn into a wire of diameter 2 m. Find the length of the wire in metre.

2. A metallic sphere of radius 21 cm is melted and then recast into small cones each of radius 7 cm and height 6 cm. Find the number of cones made.

3. A vessel is in the form of a cone. Its height is 16 cm and radius of the open top is 10 cm. It is filled with water completely. Some lead shots which have 1 cm radius are dropped into the vessel. One fourth of the water flow out. Find the number shots dropped into the vessel.

4. A well of diameter 6 m is dug 28 m deep. The earth taken out has been spread evenly all around it in the shape of a circular ring of width 5 m to form an embankment. Find the height of the embankment.

5. Two cylindrical tanks are connected by a tube with a tap. The radii of the tanks are 7 m and $\frac{7}{2}$ m. The water stands to a height 6 m and 5 m respectively as shown in the figure. If the tap is opened find the height to which the water will stand in both the tanks.



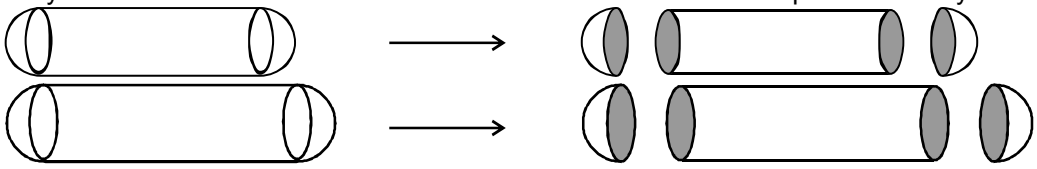
6. A rectangular swimming bath 60 m long and 40 m broad. It can be filled by a supply pipe in 5 days but if 6000 cubic metres of water is put in first the rest can be filled in 3 days and 18 hours. Find the depth of the bath.

Answers

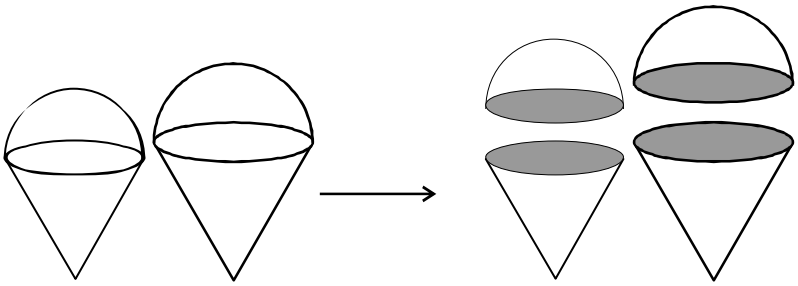
1.	972×10^{-6} metres	**2.**	126	**3.**	100	**4.**	4.58 m
5.	5.8 m	**6.**	10 m				

(C) SURFACE AREA AND VOLUME OF A COMBINATION OF SOLIDS

Consider a solid as shown in figure. The solid can be divided into two hemisphere and a cylinder. The surface of the solid is sum of curved surface area of two hemisphere and curved surface area of cylinder. The volume of the solid is sum of volumes of two hemisphere and a cylinder.



Similarly a playing top (lattu) is divided into a hemisphere and a cone. The surface area of playing top
= curved surface area of hemisphere + curved surface area of cone
volume of playing top = volume of hemisphere + volume of a cone.

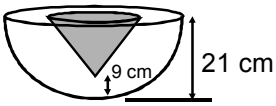


The following solved examples will illustrate the method of finding out total surface area and volume of combination of solids.

Solved Examples

Example. 19

A conical depression is carried out of a hemisphere as shown in figure. Find the surface area of the solid obtained if the radius of cone is 5 cm.

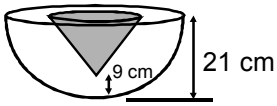


Sol. Radius of cone, $r_1 = 5$ cm

Radius of hemisphere, r = 21 cm

Height of cone, h = 21 – 9 = 12 cm

Slant height of cone, l $= \sqrt{r_1^2 + h^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = 13$ cm



Surface area of solid = TSA of hemisphere + CSA of cone – Area of base of cone

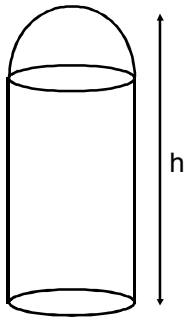
$= 3\pi r^2 + \pi r_1 l - \pi r_1^2$

$= 3 \times \frac{22}{7} \times 21 \times 21 + \frac{22}{7} \times 5 \times 13 - \frac{22}{7} \times 5 \times 5$

$= \frac{22}{7}(1323 + 65 - 25) = \frac{22}{7} \times 1363 = 4283.7 \text{ cm}^2$

Example. 20

A building is in the form of a cylinder surmounted by a hemispherical dome as shown in figure. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m³ of air.



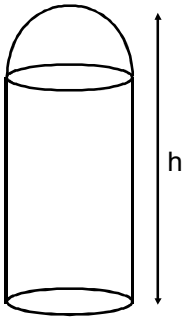
Sol. Let r be the common radius of dome and cylinder and h be the height of the building.

$\therefore$ base diameter of dome = 2r m

According to question $2r = \frac{2}{3}h$

$\therefore \quad r = \frac{h}{3}$ m

Height of cylinder, $h_1 = h - r$



Volume of solid = Volume of cylinder + Volume of hemisphere

$$67\frac{1}{21} = \pi r^2 h_1 + \frac{2}{3}\pi r^3 \quad [\because \text{Volume of solid = volume of air} = 67\frac{1}{21}\ m^3]$$

$$\frac{1408}{21} = \pi r^2 (h - r) + \frac{2}{3}\pi r^3$$

$$\frac{1408}{21} = \pi \times \frac{h^2}{9}\left(h - \frac{h}{3}\right) + \frac{2}{3}\pi \times \frac{h^3}{27}$$

$$\frac{1408}{21} = \pi \times \frac{h^2}{9} \times \frac{2h}{3} + \frac{2}{3}\pi \times \frac{h^3}{27}$$

$$\frac{1408}{21} = \frac{\pi h^3}{27}\left(2 + \frac{2}{3}\right)$$

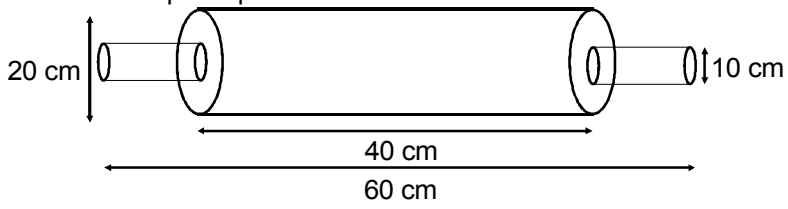
$$\frac{1408}{21} = \frac{\pi h^3}{27} \times \frac{8}{3}$$

$$\frac{1408 \times 27 \times 3}{21 \times 8 \times \pi} = h^3 \quad \Rightarrow \quad h^3 = \frac{1408 \times 27 \times 3 \times 7}{21 \times 8 \times 22} \quad \Rightarrow h^3 = 27 \times 8$$

$h = 6$m

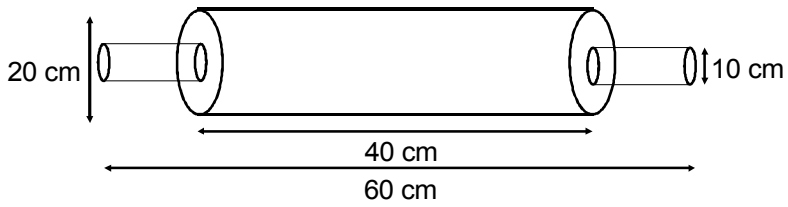
Example. 21

A roller pin is made by joining 3 cylindrical pieces of wood as shown in figure. Find the cost of painting it at the rate of 14 paisa per cm^2



Sol. Radius of bigger cylinder, $r = 10$ cm
Radius of smaller cylinder, $r_1 = 5$ cm
Height of larger cylinder, $h = 40$ cm
Height of smaller cylinder, $h_1 = 10$ cm

TSA of the roller pin = TSA of bigger cylinder + 2 × CSA of smaller cylinder – 2 × area of base of smaller cylinder + 2 × area of top of smaller cylinder



$= 2\pi r(h + r) + 2 \times 2\pi r_1 h_1 - 2 \times \pi r_1^2 + 2\pi r_1^2$

$= 2\pi r\,(h + r) + 4\pi r_1 h_1$

$= 2 \times \frac{22}{7} \times 10(40 + 10) + 4 \times \frac{22}{7} \times 5 \times 10$

$= 2 \times \frac{22}{7}\,(500 + 100)$

$= 2 \times \frac{22}{7} \times 600 = 3771.43\ cm^2$

cost of painting = 0.14×3771.43 = Rs 528.0002.

Example. 22

A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also find the total surface area of toy.

Sol. Radius of hemisphere and cone,

r = 4 cm

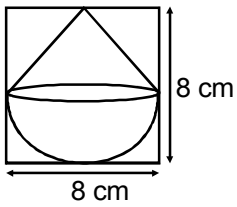
Height of cone, h = 4 cm

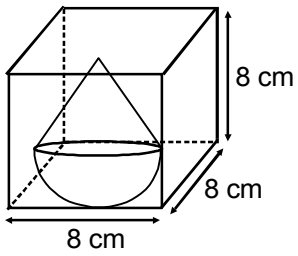
Volume of toy = Volume of hemisphere + Volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h.$

$= \frac{\pi}{3} r^2(2r + h) = \frac{22}{7} \times \frac{1}{3} \times 4 \times 4\,(2 \times 4 + 4)$

$= \frac{22}{21} \times 16 \times 12 = 201.14\ cm^3$





A cube circumscribes the toy, so the edge of the cube = Diameter of hemisphere = 8 cm

Volume of cube = $8^3 = 512\ cm^3$

Difference of volumes of cube and cone

$= 512 - 201.14 = 310.86\ cm^3$

Slant height of cone, $\ell = \sqrt{r^2 + h^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ cm

Surface area of toy = CSA of cone + CSA of hemisphere

$= \pi r\ell + 2\pi r^2 = \pi r\,(\ell + 2r)$

$= \frac{22}{7} \times 4 \times (4\sqrt{2} + 2 \times 4)$

$= \frac{88}{7} \times (5.656 + 8)$

$= \frac{88}{7} \times 13.656 = 171.67\ cm^3$

Check Your Level

1. A biggest sphere is made from a cube of side 14 cm. Find the volume of solid in cubic centimeter.

2. A solid 21 cm long, 20 cm broad and 10 cm high has hemisphere fixed on the top The radius of the hemisphere is 7 cm. Find the total surface area of the combined object.

3. A top is of the form of hemisphere mounted on a cone. The diameter of the conical portion is 12 cm and height 8 cm. Find the surface area and volume of the top.

4. 6 cylindrical milk powder tins are packed in a rectangular box, 3 on longer side and 2 on breadth side. Find the volume of the empty space if the radius of the tin is 7 cm and height is 10 cm.

5. A cylinder has hemisphere on one end and cone on the other end with same diameter. Height of the cylindrical part is 10 cm. Common diameter is 8 cm. The height of the conical part is 3 cm. Find the total surface area of the solid formed.

6. A circus tent is cylindrical to a height of 8 m surrounded by a conical part. If the total height is 13 m and the diameter of the tent is 24 m calculate
(i) total surface area of the tent
(ii) volume of the air contained in it

7. A cone is surmounted on a cylinder of diameter is 10 cm. Total height is 19.5 cm ,and height of cylindrical part is 7.5 cm. Then find curved surface area.

8. A cylinder has diameter 7 cm and 8 cm vertical height. From the cylinder a cone of height 3 cm with the same diameter of the cylinder is curved out. Find the volume of remaining solid.

Answers

1. 1437.33 cubic cm **2.** 1814 cm^2 **3.** 414.48 sq. cm, 753.6 c.c

4. 2520 cc **5.** 415 cm^2 (apporox)

6. **(i)** $1093\frac{5}{7}\ m^2$ **(ii)** $4374\frac{6}{7}$ cubic metre **7.** 440 sq. cm

8. 269.5 cubic cm

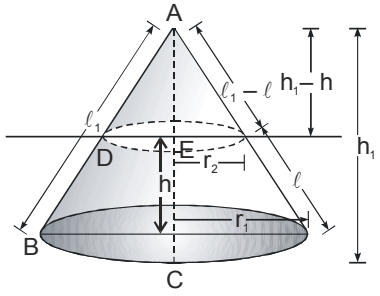
(D) FRUSTUM OF A CONE :

When a cone is cut by a plane parallel to base, a small cone is obtained at top and other part is obtained at bottom. This is known as **'Frustum of Cone'.**

$\Delta ABC \sim \Delta ADE$

$\therefore \quad \frac{AC}{AE} = \frac{AB}{AD} = \frac{BC}{DE}$

$\frac{h_1}{h_1 - h} = \frac{\ell_1}{\ell_1 - \ell} = \frac{r_1}{r_2}$ Or $\frac{h_1}{h} = \frac{\ell_1}{\ell} = \frac{r_1}{r_1 - r_2}$



Volume of Frustum

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h) = \frac{1}{3}\pi [r_1^2 h_1 - r_2^2 (h_1 - h)]$$

$$= \frac{1}{3}\pi\left[r_1^2\left(\frac{r_1 h}{r_1 - r_2}\right) - r_2^2\left(\frac{r_1 h}{r_1 - r_2} - h\right)\right] = \frac{1}{3}\pi h\left[\frac{r_1^3 - r_2^3}{r_1 - r_2}\right] = \frac{1}{3}\pi h \left[r_1^2 + r_2^2 + r_1 r_2\right]$$

Curved Surface Area of Frustum $= \pi r_1 \ell_1 - \pi r_2(\ell_1 - \ell)$

$$= \pi\left[r_1\left(\frac{r_1 \ell}{r_1 - r_2}\right) - r_2\left(\frac{r_1 \ell}{r_1 - r_2} - \ell\right)\right] = \pi\ell\left[\frac{r_1^2}{r_1 - r_2} - \frac{r_2^2}{r_1 - r_2}\right] = \pi\ell\,(r_1 + r_2)$$

Total Surface Area of a Frustum

$= \text{CSA of frustum} + r_1^2 + r_2^2 = \pi\ell\,(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

Slant height of a Frustum $= \sqrt{h^2 + (r_1 - r_2)^2}$

where, h = height of the frustum

r_1 = radius of larger circular end

r_2 = radius of smaller circular end

Solved Examples

Example. 23

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol. Circumference of one circular end $= 2\pi r_1 = 18$ cm $\Rightarrow$ $\pi r_1 = 9$ cm.

Circumference of other circular end $= 2\pi r_2 = 6$ cm $\Rightarrow$ $\pi r_2 = 3$ cm.

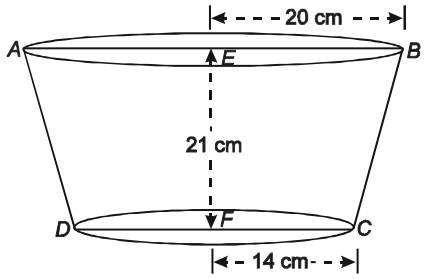
Slant height of the frustum, $\ell = 4$ cm.

CSA of the frustum $= \pi(r_1 + r_2)\ell = (\pi r_1 + \pi r_2)\ell = (9 + 3) \times 4\ \text{cm}^2 = 48\ \text{cm}^2$.

Example. 24

A bucket is 40 cm in diameter at the top and 28 cm in diameter at the bottom. Find the capacity of the bucket in litres, if it is 21 cm deep. Also, find the cost of tin sheet used in making the bucket, if the cost of tin is Rs 1.50 per sq dm.

Sol. Given : $r_1 = 20$ cm, $r_2 = 14$ cm and $h = 21$ cm



Now, the required capacity (i.e., volume) of the bucket $= \frac{\pi h}{3}(r_1^2 + r_1 r_2 + r_2^2)$

$= \frac{22 \times 21}{7 \times 3}(20^2 + 20 \times 14 + 14^2)\ \text{cm}^3$

$= 22 \times 876\ \text{cm}^3 = 19272\ \text{cm}^3 = \frac{19272}{1000}$ litres $= 19.272$ litres.

Now, $\ell = \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(20 - 14)^2 + 21^2}$ cm $= \sqrt{6^2 + 21^2}$ cm

$= \sqrt{36 + 441}$ cm $= \sqrt{477}$ cm $\simeq 21.84$ cm.

$\therefore$ Total surface area of the bucket (which is open at the top)

$= \pi\ell\,(r_1 + r_2) + r_2^2 = \pi[(r_1 + r_2)\,\ell + r_2^2] = \frac{22}{7}\left[(20+14)\quad 21.84 + 14^2\right] = 2949.76\ \text{cm}^2.$

$\therefore$ Required cost of the tin sheet at the rate of Rs 1.50 per dm^2 i.e., per 100 cm^2

$= \text{Rs}\ \frac{1.50 \times 2949.76}{100} \simeq \text{Rs}\ 44.25.$

Example. 25

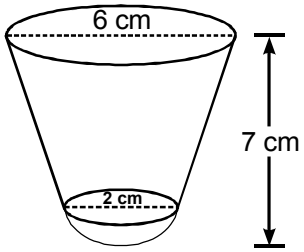
A shuttlecock used for playing badminton has the shape of a frustum of a cone maunted on a hemisphere. The external of the frustum are 6 cm and 2 cm and the height of the entire shuttlecock is 7 cm. Find its external surface area.

Sol. For the frustum of cone : $r_1 = \frac{6}{2}\text{cm} = 3\ \text{cm}.$

$r_2 = \frac{2}{2}\ \text{cm} = 1\ \text{cm}, h = (7 - 1)\ \text{cm} = 6\ \text{cm}$

Slant height, $\ell = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{6^2 + (3-1)^2}\ \text{cm} = \sqrt{40}\ \text{cm} = 6.32\ \text{cm}$

External surface area of the shuttlecock



= CSA of the frustum of cone + CSA of the base hemisphere

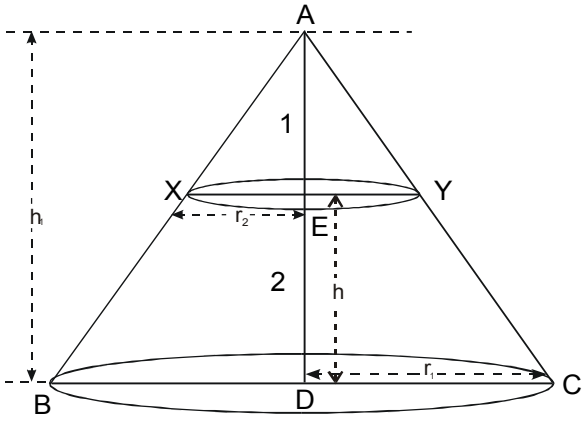
$= \pi\,(r_1 + r_2)\ell + 2\pi r_2^2 = \left[\frac{22}{7} \times (3+1) \times 6.32 + 2\frac{22}{7} \times 1 \times 1\right]\ \text{cm}^2$

$= \frac{44}{7}\ (12.64 + 1)\ \text{cm}^2 = \frac{44 \times 13.64}{7}\ \text{cm}^2 = 85.74\ \text{cm}^2\ (\text{Approx.}).$

Example. 26

A cone is divided into two parts by drawing a plane through a point which divides its height in the ratio 1 : 2 starting from the vertex and the plane is parallel to the base. Compare the volume of the two parts.

Sol. Let the plane XY divide the cone ABC in the ratio AE : ED = 1 : 2, where AED is the axis of the cone. Let r_2 and r_1 be the radii of the circular section XY and the base BC of the cone respectively and let $h_1 - h$ and h_1 be their heights **[figure]**.



Then, $\frac{h_1}{h} = \frac{3}{2} \Rightarrow h_1 = \frac{3}{2}h$

And $\frac{r_1}{r_2} = \frac{h_1}{h_1 - h} = \frac{\frac{3}{2}h}{\frac{1}{2}h} = 3$

$\therefore \quad r_1 = 3r_2$

Volume of cone AXY

$= \frac{1}{3}\pi r_2^2 (h_1 - h)$

$= \frac{1}{3}\pi r_2^2 (\frac{3}{2}h - h)$

$= \frac{1}{6}\pi r_2^2 h$

Volume of frustum XYBC

$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3}\pi h(9r_2^2 + r_2^2 + 3r_2^2)$

$= \frac{1}{3}\pi h(13r_2^2)$

So, $\frac{\text{Volume of cone AXY}}{\text{Volume of frustum XYBC}} = \frac{\frac{1}{6}\pi r_2^2 h}{\frac{13}{3}\pi r_2^2 h}$

$\frac{\text{Volume of cone AXY}}{\text{Volume of frustum XYBC}} = \frac{1}{26}$.

i.e., the ratio between the volume of the cone AXY and the remaining portion BCYX is 1 : 26.

Check Your Level

1. A frustum of a right circular cone has diameter of the base 10 cm and top 6 cm and height 5 cm find the volume.

2. Radii of the ends of frustum of a right circular cone are 33 cm and 27 cm, its slant height is 10 cm. Find its volume and total surface area.

3. A bucket is in the form a frustum of a cone and holds 15.25 litres. The diameters of the top and bottom are 25 cm and 20 cm. Find the slant height and area of the tin used to its construction.

4. The circumference of the end of the frustum of a right circular cone is 48 cm and the other end is 34 cm. the height of the frustum is 10 cm. Find its volume.

Answers

1. V = 256.67 cubic centimetres **2.** 22704 cm^3, 7599.43 cm^2

3. Height = 38.18 cm (approx), Area = 3019.82 cm^2 (approx)

4. 1351 cm^3 (approx)

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]

1. Name the solid figures whose combination gives the shape of a gilli, in the gilli-danda game (see Figure)

2. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. Then name new part that is left over on the other side of the plane

3. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm, is melted to form a solid sphere. Find the radius of the sphere

4. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm, respectively. Find the curved surface area of the bucket

5. A right circular cylinder of radius r cm and height h cm (h > 2r) just encloses a sphere find the diameter of sphere.

6. During conversion of a solid from one shape to another, how will the volume of the new shape change?

7. Volumes of two spheres are in the ratio 64 : 27. Find the ratio of their surface areas

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

8. A hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then find the number of marbles that the cube can accomodate.

9. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form a cone of base diameter 8cm. Find the height of the cone.

10. A mason constructs a wall of dimensions 270cm× 300cm × 350cm with the bricks each of size 22.5cm × 11.25cm × 8.75cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then find the number of bricks used to construct the wall ?

11. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsules 2 cm. Find the capacity of the capsule.

12. The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. Find the capacity of the bucket.

13. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed.

14. A cone of radius 8 cm and height 12cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

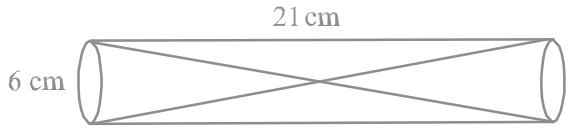
15. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid ?

16. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm.

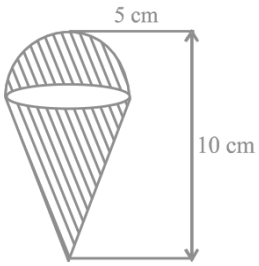
17. Find the number of metallic circular disc with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[03 MARK EACH]**

18. Two solid cones A and B are placed in a cylindrical tube as shown in the Figure. The ratio of their capacities is 2:1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.



19. An ice cream cone full of ice cream having radius 5 cm and height 10 cm as shown in the Figure. Calculate the volume of ice cream, provided that its $\frac{1}{6}$ part is left unfilled with ice cream.



20. Marbles of diameter 1.4 cm are dropped into a cylinical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm.

21. A wall 24 m long, 0.4 m thick and 6 m high is instructed with the bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies $\frac{1}{10}$ th of the volume of the wall, then find the number bricks used in constructing the wall.

22. A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height the cone.

23. A rectangular water tank of base 11 m × 6 m contains water upto a height of 5 m. If the water in the tank is transferred to a cylindrical tank of radius 3.5 m, find the height of the water level in the tank.

24. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one fifth of a litre ?

25. Water flows at the rate of 10m/minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm ?

26. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap ?

27. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at Rs 0.05 per dm^2.

28. 500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is $0.04m^3$?

29. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm?

30. A milk container of height 16 cm made of metal sheet in the form of a frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of Rs. 22 per litre which the container can hold.

31. A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

32. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the the slant height of the conical portion is 5 cm, find the total surface area and volume of the rocket [Use $\pi = 3.14$].

33. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building ?

34. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone.

Previous Year Problems

1. A bucket open at the top , and made up of a metal sheet is in the form of a frustum of a cone . The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are 30 cm and 10 cm respectively . Find the cost of metal sheet used in it at the rate of Rs 10 per 100 cm^2 .
[Use $\pi = 3.14$] **[4 MARKS /CBSE 10TH BOARD: 2012]**

2. Water is flowing through a cylindrical pipe , of internal diameter 2cm , into a cylindrical tank of base radius 40cm , at the rate of 0.4 m/s . Determine the rise in level of water in the tank in half an hour .
[4 MARKS /CBSE 10TH BOARD: 2012]

3. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder .If the height of the cylinder is10 cm , and its base is of radius 3.5 cm , find the volume of wood in the toy. [using $\pi = \frac{22}{7}$]. **[3 MARKS /CBSE 10TH BOARD: 2012]**

4. A vessel is in the form of hemispherical bowl surmounted by a hollow cylinder of same diameter . The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm . Find the total surface area of the vessel . [using $\pi = \frac{22}{7}$] . **[3 MARKS /CBSE 10TH BOARD: 2013]**

5. A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid. [Use $\pi = \frac{22}{7}$] . **[4 MARKS /CBSE 10TH BOARD: 2013]**

6. A metallic bucket, open at the top, of height 24 cm is in the form of the frustum of a cone, the radii of whose lower and upper circular ends are 7 cm and 14 cm respectively. Find:
(i) the volume of water which can completely fill the bucket. (ii) the area of the metal sheet used to make the bucket. [Use $\pi = \frac{22}{7}$] **[4 MARKS /CBSE 10TH BOARD: 2014]**

7. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is. **[1 MARK /CBSE 10TH BOARD: 2014]**

8. A 5m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs 25 per metre. [Use $\pi = \frac{22}{7}$] .

[3 MARKS /CBSE 10TH BOARD: 2014]

9. A girl empties a cylindrical bucket, full of sand, of base radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct up to one place of decimal. **[3 MARKS /CBSE 10TH BOARD: 2014]**

10. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m, and the canvas to be used costs Rs. 100 per sq. m, find the amount, the associations will have to pay. What values are shown by these associations? [Use $\pi = \frac{22}{7}$]

[3 MARKS /CBSE 10TH BOARD: 2014]

11. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

[4 MARKS /CBSE 10TH BOARD: 2014]

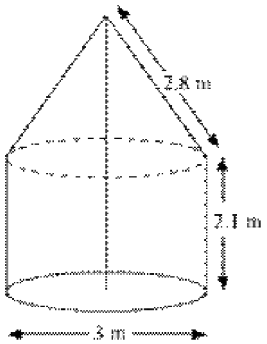
12. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs 5 per 100 sq. cm. [Use $\pi = 3.14$] . **[3 MARKS /CBSE 10TH BOARD: 2015]**

13. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. [Use $\pi = \frac{22}{7}$].

[3 MARKS /CBSE 10TH BOARD: 2015]

14. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. [Use $\pi = \frac{22}{7}$] **[4 MARKS/CBSE 10TH BOARD: 2015]**

15. In the figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m, respectively, and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs 500/sq. metre. (Use $\pi = \frac{22}{7}$). **[3 MARKS/ CBSE 10TH BOARD: 2016]**



16. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is empied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. (Use $\pi = \frac{22}{7}$) . **[3 MARKS/ CBSE 10TH BOARD: 2016]**

17. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.? **[3 MARKS/ CBSE 10TH BOARD: 2016]**

18. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs 120 per sq.m, find the amount shared by each school to set up the tents. What value is generated by the above problem? (Use $\pi = \frac{22}{7}$).
[4 MARKS/ CBSE 10TH BOARD: 2016]

19. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.
[3 MARKS/ CBSE 10TH BOARD: 2017]

20. The dimensions of a solid iron cuboid are 4·4 m × 2·6 m × 1·0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.
[3 MARKS/ CBSE 10TH BOARD: 2017]

21. In a rain water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3·5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation. **[4 MARKS/ CBSE 10TH BOARD: 2017]**

22. Water in a canal, 5·4 m wide and 1·8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?
[4 MARKS/ CBSE 10TH BOARD: 2017]

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Surface area and volume of solid figures

A-1. Find the volume of a cube whose surface area is 150 m^2.

A-2. Three cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

A-3. The difference between the outer and inner curved surface areas of a hollow right circular cylinder, 14 cm long, is 88 cm^2. If the volume of metal used in making the cylinder is 176 cm^3, find the outer and inner diameters of the cylinder. [Use $\pi = \frac{22}{7}$]

A-4. Two cylinders of same volume have their heights in the ratio 1 : 3, find the ratio of their radii.

A-5. If the diameter of a cylinder jar is increased by 25% without altering the volume then by what %its height must decrease by :

A-6. A right-angled triangle whose sides are 15 cm and 20 cm, is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. [Take $\pi \simeq 3.14$]

(B) Conversion of solids

B-1. Three cubes with sides in the ratio 3 : 4 : 5 are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. Then find sides of the cubes

B-2. A wall 10 m long, 1 m broad and 4 m high is to be constructed with bricks 20 cm long 10 cm broad and 10 cm high. How many bricks are required to build the wall ?

B-3. 500 persons took dip in a rectangular tank which is 80 m long and 50 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3?

B-4. Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in the tank in half an hour ?

B-5. A solid sphere of radius 3 cm is melted and then cast into small spherical balls each of diameter 0.6 cm. Find the number of balls thus obtained.

B-6. The diameters of external and internal surfaces of a hollow spherical shell are 10 cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

B-7. A hemispherical bowl of internal radius 36 cm is full of liquid. The liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 6 cm. How many bottles are needed to empty the bowl ?

B-8. The rain-water collected on the roof of a building, of dimensions 22 m × 20 m, is drained into a cylindrical vessel having base diameter 2 m and height 3.5 m. If the vessel is full up to the brim, find the height of rain-water on the roof. [Use $\pi = \frac{22}{7}$]

B-9. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed. [Use $\pi = \frac{22}{7}$]

B-10. A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the volume of water displaced out of the cylindrical vessel.

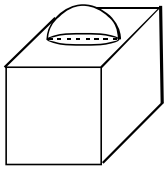
(C) Combination of solids

C-1. A tent consists of a frustum of a cone, surmounted by a cone. If the diameters of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of surmounted conical portion are equal)

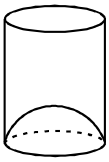
C-2. A hemi-spherical depression is cutout from one face of the cubical wooden block such that the diameter ℓ of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

C-3. A circus tent is cylindrical to a height of 3 m and conical above it. If its base radius is 52.5 m and slant height of the conical portion is 53 m, find the area of the canvas needed to make the tent.

C-4. Figure shows a decorative block which is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere, fixed on the top, has a diameter of 4.2 cm. Find the total surface area of the block. [Take $\pi = \frac{22}{7}$]



C-5. A juice seller serves his customers using a glass as shown in figure. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity.



(D) Frustum of a cone

D-1. If the radii of the ends of a bucket, 45 cm high are 28 cm and 7 cm, determine the capacity and total surface area of the bucket.

D-2. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum.

D-3. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?

D-4. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.

D-5. A bucket of height 8 cm and made up of copper sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 3 cm and 9 cm respectively. Calculate

(i) the height of the cone of which the bucket is a part.
(ii) the volume of water which can be filled in the bucket.
(iii) the area of copper sheet required to make the bucket.

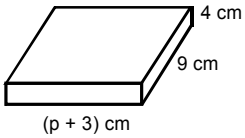
(Leave the answer in terms of π).

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

(A) Surface area and volume of solid figures

A-1. The figure shows a cuboid with a volume of 180 cm^3.



What is the value of "p" ?

(A) 2 (B) 18 (C) 36 (D) 72

A-2. The volume of a cube (in cubic cm) plus three times the total length of its edges (in cms) is equal to twice its surface area (in sq. cm). The length of its diagonal is :
(A) 6 (B) $6\sqrt{3}$ (C) $3\sqrt{6}$ (D) $6\sqrt{6}$

A-3. An aquarium has a rectangular base that measure 100 cm by 40 cm and has height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. The water level (in cm) rises by
(A) 0.5 (B) 1 (C) 1.5 (D) 2

A-4. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. The ratio of its diameter to its height is :
(A) 7 : 6 (B) 6 : 7 (C) 3 : 7 (D) 7 : 3

A-5. A cylindrical tank has base radius 7 m and height 9 m. If 2/3 of the tank is filled with water find the volume of water in the tank. [Use $\pi = \frac{22}{7}$]
(A) 1386 cu m (B) 462 cu m (C) 824 cu m (D) 924 cu m

A-6. Two steel sheets each of length a_1 and breadth a_2 are used to prepare the surface of two right circular cylinders-one having volume V_1 and height a_2 and the other having volume V_2 and height a_1. Then.
(A) $V_1 = V_2$ (B) $a_1V_1 = a_2V_2$ (C) $a_2V_1 = a_1V_2$ (D) $\frac{V_1^2}{a_1} = \frac{V_2^2}{a_2}$

A-7. The height of a conical tent at the centre is 5m. The distance of any point on its circular base from the top of the tent is 13m. The area of the slant surface is :
(A) 144 π sq m (B) 130 π sq m (C) 156 π sq m (D) 169 π sq m

A-8. The slant height of a cone is increased by P%. If radius remains same, the curved surface area is increased by :
(A) P % (B) P^2 % (C) 2 P % (D) None of these

A-9. It is required to construct a conical circus tent of radius 21 m and 35 m slant height. The width of the canvas cloth is 3 meters. What will be the length of the cloth which shall do the needful ?
(A) 700 m (B) 1250 m (C) 776.5 m (D) 770 m

A-10. If a hemi-spherical dome has an inner diameter of 28 m, then its volume (in m^3) is :
(A) 6186.60 (B) 5749.33 (C) 7099.33 (D) 7459.33

A-11. The radius of a sphere is increases by P%. Its surface area increase by :
(A) P % (B) P^2 % (C) $\left(2P + \frac{P^2}{100}\right)\%$ (D) $\frac{P^2}{2}$ %

(B) Conversion of solids

B-1. Number of cubes of volume 4 cubic units which can be cut from a cube with a surface area of 96 square units is
(A) 4 (B) 8 (C) 12 (D) 16

B-2. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is
(A) 142296 (B) 142396 (C) 142496 (D) 142596

B-3. A lead ball of radius 24 cm is melted down and recast into smaller balls of radius 6 cm. Assuming that no metal is lost in this process, number of complete smaller balls that can be made, is :
(A) 4 (B) 16 (C) 36 (D) 64

B-4. A sphere of radius 3 cms is dropped into a cylindrical vessel of radius 4 cms. If the sphere is submerged completely, then the height (in cm) to which the water rises, is :
(A) 2.35 (B) 2.30 (C) 2.25 (D) 2.15

B-5. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is :
(A) 4 cm (B) 3 cm (C) 2 cm (D) 6 cm

B-6. A sphere of radius 6 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 8 cm. If the sphere is submerged completely, then the surface of the water rises by :
(A) 4.5 cm (B) 4 cm (C) 3 cm (D) 2 cm

B-7. A metallic spherical shell of internal and external diameters 8 cm and 12 cm, respectively is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is :
(A) 114 cm (B) 76 cm (C) 38 cm (D) 19 cm

B-8. A solid piece of iron of dimensions 66 cm $\times$ 49 cm $\times$ 12 cm is moulded into a sphere. The radius of the sphere is :
(A) 7 cm (B) 3 cm (C) 21 cm (D) 14 cm

B-9. The volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 3 cm and height 7 cm is :
(A) 108 π cm^3 (B) 36 π cm^3 (C) 12 π cm^3 (D) $\frac{4}{3}\pi$ cm^3

B-10. The number of solid spheres, each of diametres 6 cm, that could be moulded to form a solid metal cylinder of height 45 cm and diameter 4 cm is :
(A) 3 (B) 4 (C) 5 (D) 6

(C) Combination of solids

C-1. There is a cylinder circumscribing the hemisphere such that their bases are common. The ratio of their volume is :
(A) 1 : 3 (B) 1 : 2 (C) 2 : 3 (D) 3 : 4

C-2. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemisphere stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is :
(A) 0.36 cm^3 (B) 0.40 cm^3 (C) 0.45 cm^3 (D) 0.30 cm^3

C-3. A solid consists of a circular cylinder with an exact fitting right circular cone placed at the top. If the height of the cone is h and the total volume of the solid is 3 times the volume of the cone, then the height of the circular cylinder is :
(A) 2h (B) $\frac{2h}{3}$ (C) $\frac{3h}{2}$ (D) 4h

C-4. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere. Find the total height of the toy if height of the conical part is 3 times its radius.
(A) 9 cm (B) 12 cm (C) 15 cm (D) 18 cm

C-5. A tent is in the form of a cylinder of diameter 4.2 m and height 4 m, surmounted by a cone of equal base and height 2.8 m. Find the cost of canvas for making the tent at Rs. 100 per sq. m. ?
(A) Rs. 7590 (B) Rs. 7950 (C) Rs. 7580 (D) Rs. 7850

(D) Frustum of a cone

D-1. A cone is divided into two parts by drawing a plane through the mid point of its axis parallel to its base then the ratio of the volume of two parts is :
(A) 1 : 3 (B) 1 : 7 (C) 1 : 8 (D) 1 : 9

D-2. If the radii of the circular ends of a bucket of height 40 cm are 35 cm and 14 cm respectively, then the volume of the bucket in cubic centimetres is :
(A) 60060 (B) 80080 (C) 70070 (D) 80160

D-3. A right circular cone is cut off at the middle of its height and parallel to the base. Call the smaller cone so formed as A and the remaining part as B, then:
(A) Vol. A < Vol. B (B) Vol. A = Vol. B (C) Vol A > Vol. B (D) Vol. A = $\frac{1}{2}$ (Vol. B)

D-4. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm (r < 14 cm). If the volume of bucket is 5390 cm^3, then find the value of r. (Use $\pi = \frac{22}{7}$).
(A) 4 cm (B) 5 cm (C) 6 cm (D) 7 cm

Exercise-2

OBJECTIVE QUESTIONS

1. The length, breadth and height of a room are in the ratio 3: 2: 1. If the breadth and height are halved while the length is doubled, then the total area of the four walls of the room will :
(A) remain the same (B) decrease by 30%
(C) decrease by 15% (D) decrease by 18.75%

2. If the surface area of cube A is 64% of the surface area of cube B, then the volume of cube A is 'k' percent of the volume of cube B. The value of 'k' is
(A) 0.64 (B) 0.512 (C) 51.2 (D) 64

3. If the radius of a right circular cylinder is increased by 50% and height is decreased by 20% then the percentage change in volume of cylinder is
(A) 40% (B) 50% (C) 60% (D) 80%

4. John's birthday cake is a delightful cylinder of radius 6 inches and height 3 inches, If these friends cut the cake into 8 equal sectors, the total surface area of a piece of birthday cake in sq. inches, is :
(A) $9(\pi + 4)$ (B) $\frac{27\pi}{2}$ (C) $4(\pi + 4)$ (D) $\frac{9}{2}(3\pi + 8)$

5. A rectangular tin sheet is 12 cm long and 5 cm broad. It is rolled along its length to form a cylinder by making the opposite edges just to touch each other. The volume of the cylinder (in cm^3) is :
(A) $\frac{180}{\pi}$ (B) $\frac{120}{\pi}$ (C) $\frac{100}{\pi}$ (D) $\frac{60}{\pi}$

6. If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original one ?
(A) 1 : 4 (B) 1 : 8 (C) 1 : 2 (D) 8 : 1

7. The radius of a wire is decreased to one-third. If volume remains the same, the length will become :
(A) 1 time (B) 3 times (C) 6 times (D) 9 times

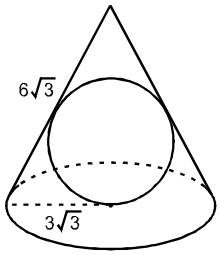
8. If a cone is cut into two parts by a horizontal plane passing through the mid-point of its axis, the ratio of the volumes of the upper part and the cone is:
(A) 1 : 2 (B) 1 : 4 (C) 1 : 6 (D) 1 : 8

9. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. If it is 6 m deep its capacity is :
(A) 176 m^3 (B) 196 m^3 (C) 200 m^3 (D) 110 m^3

10. Three solid spherical beads of radii 3 cm, 4 cm and 5 cm are melted into a spherical bead. Its radius is :
(A) 6 cm (B) 7 cm (C) 8 cm (D) 9 cm

11. If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the surface area of each ball (in sq. cm) is :
(A) 100π (B) 75π (C) 60π (D) 50π

12. A sphere is inscribed in a cone of radius $3\sqrt{3}$ and slant height $6\sqrt{3}$. The radius of the sphere, is :

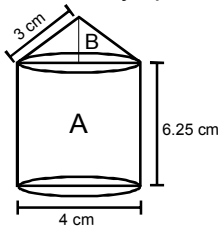


(A) 3 (B) $3\sqrt{3}$ (C) $6\sqrt{3}$ (D) $\frac{3\sqrt{3}}{2}$

13. If the height and diameter of a right circular cylinder are 32 cm and 6 cm respectively, then the radius of the sphere whose volume is equal to the volume of the cylinder is :
(A) 3 cm (B) 4 cm (C) 6 cm (D) None

14. A hollow spherical ball whose inner radius is 4 cm is full of water. Half of the water is transferred to a conical cup and it completely filled the cup. If the height of the cup is 2 cm, then the radius of the base of cone in cm is :
(A) 4 (B) 10 (C) 8 (D) 16

15. A is a right circular cylinder on which a cone B is placed. The entire structure is melted and spheres are formed each having radius 1 cm. How many spheres can be formed ?



(A) 18 (B) 20 (C) 21 (D) 23

16. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.
(A) $102\frac{2}{3}\text{cm}^3$ (B) $103\frac{2}{3}\text{cm}^3$ (C) $104\frac{2}{3}\text{cm}^3$ (D) $105\frac{2}{3}\text{cm}^3$

17. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm, respectively. Find the height of the bucket.
(A) 12 cm (B) 15 cm (C) 18 cm (D) 21 cm

18. The height of a cone is 40 cm. A small cone is cut off at the top by a plane parallel to the base. If the volume of the small cone is $\frac{1}{64}$ of the volume of the given cone, at what height above the base is the section made?
(A) 10 cm (B) 20 cm (C) 25 cm (D) 30 cm

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. The radius of a hemisphere is 3 cm. The total surface area is - **[Raj NTSE Stage-1 2005]**

(A) $\frac{9}{4}\pi$ (B) 9π (C) 18π (D) 27π

2. The radius of a metallic cylinder is 2 cm and is of height 6 cm. If it is melted to form a cone whose radius is 3 cm, the height of the cone is - **[Raj. NTSE Stage-1 2005]**

(A) 8 cm (B) 9 cm (C) 12 cm (D) 24 cm

3. If the radii of two spheres are in the ratio 2 : 3, then their volumes are in the ratio : **[Raj. NTSE Stage-1 2006]**

(A) 7 : 8 (B) 8 : 27 (C) 4 : 9 (D) 1 : 27

4. The dimension of a cuboid is 18 cm × 12 cm × 9 cm. How many cubes of side 3 cm can be made by melting the cuboid ? **[Raj. NTSE Stage-1 2006]**

(A) 72 (B) 69 (C) 60 (D) 55

5. The dimension of a cuboid is 10 m × 8m × 6 m. Then the diagonal of the cuboid is : **[Raj. NTSE Stage-1 2007]**

(A) 41.44 m (B) 14.14 m (C) 41.14 m (D) 14.41 m

6. If radii of two cones are in the ratio 3 : 1 and heights are in the ratio 1:3, then the ratio of their volumes is : **[Raj. NTSE Stage-1 2007]**

(A) 3 : 1 (B) 2 : 1 (C) 1 : 3 (D) 1 : 2

7. Area of six surfaces of a cuboid are 12, 12, 20, 20, 15 and 15 sq. cm respectively. Volume of this cuboid in cm^3 is: **[NTSE Stage - I/Raj. /2007]**

(A) 12 (B) 15 (C) 60 (D) 94

8. Radius and height of a cylinder are 7 cm and 14 cm respectively. It is divided along its axis vertically into equal parts. The increase in its total surface area is : **(NTSE Stage - I/Raj. /2008)**

(A) 392 sq. cm (B) 308 sq. cm (C) 196 sq. cm (D) 154 sq. cm

9. The moon diameter is approximately one fourth of the diameter of the earth. Volume of moon is what fraction the volume of earth : **(NTSE-StageI/Himachal/2008)**

(A) $\frac{1}{64}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) None of these

10. The volume of a box is given by $v = x^3 - 2x^2 - 24x$. If the depth of the box be the smallest, then the depth is : **(NTSE Stage -I/Dehli/2009)**

(A) (x – 3) (B) (x – 6) (C) (x – 4) (D) (x – 8)

11. A cone, a right cylinder and a hemi sphere stand on equal bases and have the same height. Their volumes are in the ratio : **(NTSE Stage -I/Bihar/2009)**

(A) 1 : 2 : 3 (B) 1 : 3 : 2 (C) 2 : 3 : 1 (D) 2 : 1 : 3

12. If a sphere and a rectangular cylinder having equal radius, the height of the cylinder is double of its radius, then the ratio of volumes of sphere and cylinder is : **(NTSE Stage -I/Raj. 2010)**

(A) 1 : 2 (B) 2 : 1 (C) 2 : 3 (D) 3 : 2

13. The largest sphere is cut off from a cube of side 5 cm. The volume of the sphere will be : **(NTSE Stage -I/Raj. 2011)**

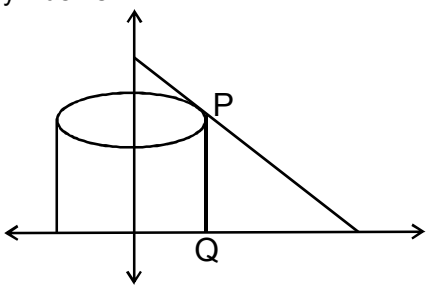
(A) $27\ \pi\ cm^3$ (B) $\frac{125}{6}\ \pi\ cm^3$ (C) $108\ \pi\ cm^3$ (D) $30\ \pi\ cm^3$

14. Find the volume of a cube whose surface area is 600 cm^2. **(NTSE Stage -I/Raj. 2012)**
(A) 900 cm^3 (B) 1000 cm^3 (C) 1500 cm^3 (D) 810 cm^3

15. On increasing each of the radius of the base and the height of a cone by 20% its volume will be increased by _____. **[UP NTSE Stage-1 2012]**
(A) 20% (B) 40% (C) 72.2% (D) 72.8%

16. From a 25 cm × 35 cm rectangular cardboard, an open box is to be made by cutting out identical squares of area 25 cm^2 from each corner and turning up the sides. The volume of the box is : **[Delhi NTSE Stage-1 2013]**
(A) 3000 cm^3 (B) 1875 cm^3 (C) 21875 cm^3 (D) 1250 cm^3

17. Let P (4, k) be any point on the line y = 6 –x. If the vertical segment PQ is rotated about y – axis, the volume of the resulting cylinder is : **[Delhi NTSE Stage-1 2013]**



(A) 32 π (B) 16π (C) $\frac{32}{3}\pi$ (D) 8π

18. A sphere of diameter 12.6 cm is melted and cast into a right circular cone of height 25.2 cm. The diameter of the base of the cone is : **[MP NTSE Stage-1 2013]**
(A) 12.6 cm (B) 79.38 cm (C) 39.69 cm (D) 69.39 cm

19. If the radius of a cylinder is decreased by 50% and the height increased by 50% to form a new cylinder, then the volume will be decreased by : **[UP NTSE Stage-1 2013]**
(A) 50% (B) 55% (C) 62.5% (D) 63%

20. If the volume and surface area of a sphere are numerically the same, then its radius is : **[UP NTSE Stage-1 2013]**
(A) 4 (B) 3 (C) 2 (D) 1

21. A cone has radius r and height h. It is melted and 3 identical cones are formed each having the same radius as the original cone and height H. Then the value of $\frac{H}{h}$ is : **[UP NTSE Stage-1 2013]**
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1

22. The surface water in a swimming pool forms a rectangle of length 40 m and breadth 15m. The depth of water increases uniformly from 1.2m at one end to 2.4m at the other end. The volume (in m^3) of water in the pool is **[Harayana NTSE Stage-1 2013]**
(A)1080 (B) 720 (C) 600 (D) 540

23. Volume and total surface area of a solid hemisphere are equal. Find the radius of hemisphere. **[Maharashtra NTSE Stage-1 2013]**
(A) 3 cm (B) 4 cm (C) 4.5 cm (D) 5.5 cm

24. A drinking glass is in the shape of frusturm of a cone of height 14 cm. The diameter of its two circular ends are 4 cm and 2 cm. Then the capacity of glass is : **[Raj. NTSE Stage-1 2013]**
(A) $102\frac{2}{3}\ cm^3$ (B) $102\frac{1}{3}\ cm^3$ (C) $101\frac{2}{3}\ cm^3$ (D) $101\frac{1}{3}\ cm^3$

25. Curved surface of right circular cylinder is 4.4 m^2, radius of base is 0.7 m. then the height is (Take $\pi = \frac{22}{7}$) **[Raj. NTSE Stage-1 2013]**

(A) 1 m (B) 2 m (C) 3 m (D) 4 m

26. If the diameter of a sphere is decreased by 25%, by what per cent does its curved surface area decrease ? **[Raj. NTSE Stage-1 2014]**

(A) 43.75% (B) 21.88% (C) 50% (D) 25%

27. The surface area of a cylindrical pipe, open at both ends is 628 sq. m. The difference between it radius and length is 15 m the length being larger. If the pipe was closed at one end, the amount of water that it can hold is : **[Bihar NTSE Stage-1 2014]**

(A) 500 π cu. m (B) 525 π cu. m (C) 550 π cu. m (D) None of these

28. A cone of height 7 cm and base radius 3 cm. is carved from a rectangular block of wood of dimension 10 cm. x 5 cm. x 2 cm. The percentage of wood wasted is :**[Bihar NTSE Stage-1 2014]**

(A) 34% (B) 46% (C) 54% (D) 66%

29. The radius of a sphere is r and radius of base of a cylinder is r and height is 2r. The ratio of their volumes will be– **[UP NTSE Stage-1 2014]**

(A) 2 : 1 (B) $\sqrt{3}$: 2 (C) 2 : 3 (D) 3 : 4

30. The area of adjacent faces and surface area of a cuboid with volume v and sides a,b,c are respectively x,y,z and s. Then which of the following is false. **[MP NTSE Stage-1 2014]**

(A) $\frac{1}{v} = \frac{2}{s}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ (B) $v^2 = x.y.z$

(C) s = 2(x + y + z) (D) s = x + y + z

31. How many surfaces are there in solid cylinder : **[Chattisgarh NTSE Stage-1 2014]**

(A) 1 (B) 2 (C) 3 (D) 4

32. The area of three adjoining faces of cuboid are A, B and C respectively then its volume will be : **[Chattisgarh NTSE Stage-1 2014]**

(A) ABC (B) $\sqrt{ABC}$ (C) $A^2B^2C^2$ (D) none of these

33. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm. in diameter. The time taken to fill up a conical vessel, whose diameter at the base is 40 cm and depth 24 cm., is : **[Jharkhand NTSE Stage-1 2014]**

(A) 55 minute (B) 52 minute 1 sec. (C) 51 minutes 12 sec. (D) 48 minutes 15 sec.

34. One solid hemisphere melted into solid sphere, then what is the ratio of radius of solid sphere to radius of solid hemisphere ? **[Maharashtra NTSE Stage - 1 2014]**

(A) $\sqrt[3]{2}$: 1 (B) 8 : 1 (C) 1 : $3\sqrt{2}$ (D) 1 : 8

35. The diameter of a right circular cylinder is decreased by 10%. The volume of cylinder remains the same then the percentage increase in height is : **[Delhi NTSE Stage-1 2014]**

(A) 20% (B) 23.45% (C) 5% (D) 20.5%

36. The ratio of the volume of a cube to that of a sphere which exactly fits inside the cube is **[Raj. NTSE Stage-1 2014]**

(A) 6 : π (B) π : 6 (C) π : 12 (D) 12 : π

37. If the heights and radii of a cone and a hemisphere are same then the ratio of their volumes is **[Raj. NTSE Stage-1 2015]**

(A) 1 : 2 (B) 2 : 3 (C) 1 : 3 (D) 1 : 1

38. If a cylinder of radius 3 cm and height of 10 cm is melted and recast into the shapes of small spheres of diameter 1 cm, then the number of spheres so formed is **[Raj. NTSE Stage-1 2015]**
(A) 135 (B) 270 (C) 540 (D) 1080

39. A metallic spherical shell of internal and external diameters 4 cm. and 8 cm. respectively, is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is **[Jharkhand NTSE Stage-1 2015]**
(A) 12 cm. (B) 14 cm. (C) 15 cm. (D) 18 cm.

40. A rectangular sheet of dimensions 1.57 m × 4.16 m was brought to form an ice-cream cone of radius 2.5 cm and height 6cm. Then find out how many ice-cream cones can be prepared from that ? **[MP NTSE Stage - 1 2015]**
(A) 320 (B) 1280 (C) 640 (D) 512

41. If the areas of three adjoining faces a cuboid are a^2, b^2 and c^2 respectively, then the volume of the cuboid is **[Raj. NTSE Stage-1 2016]**
(A) $a^2b^2c^2$ (B) abc (C) $a^3b^3c^3$ (D) $\sqrt{abc}$

42. An empty pool being filled with water at a constant rate takes 8 hours to fill $\frac{3}{5}$th of its capacity. How much more time will it take to finish filling the pool? **[Delhi NTSE Stage-1 2016]**
(A) 5 hours 30 minutes (B) 5 hours 20 minutes
(C) 4 hours 48 minutes (D) 4 hours 50 minutes

43. If the volume of two cubes are the ratio 27 : 64, then the ratio of their total surface area is **[Bihar NTSE Stage-1 2016]**
(A) 27 : 64 (B) 3 : 4 (C) 9 : 16 (4) 3 : 8

44. The volume and whole surface area of a cylindrical solid of radius 'r' units are v and s respectively. If the height of cylinder is 1 unit then $\frac{v}{s}$ is equal to **[Delhi NTSE Stage-1 2016]**
(A) $\frac{1}{2}\left(1-\frac{1}{r+1}\right)$ (B) $\frac{1}{2}\left(1+\frac{1}{r+1}\right)$ (C) $\frac{1}{2}\left(1-\frac{1}{r}\right)$ (D) $\frac{1}{2}\left(1+\frac{1}{r}\right)$

45. If the height of right circular cylinder is increased by 10% while the radius of base is decreased by 10% then curved surface area of cylinder **[Delhi NTSE Stage-1 2016]**
(A) Remains same (B) Decreases by 1% (C) Increases by 1% (D) Increases by 0.1%

46. A roller of diameter 1.4 m and length 1.4 m is used to press the ground having area 3080 sq.m. Find the number of revolutions that the roller will make to press the ground. **[Maharashtra NTSE Stage-1 2016]**
(A) 7000 (B) 500 (C) 1000 (D) 800

47. The volume of a cube is 2744 cm^3, its surface area is : **[MP NTSE Stage-1 2016]**
(A) 196 cm^2 (B) 1176 cm^2 (C) 784 cm^2 (D) 588 cm^2

48. If r is the radius of the base of a cylinder and h is the height of cylinder, then total surface area will be : **[MP NTSE Stage-1 2016]**
(A) $2\pi rh$ (B) $2\pi rh + 2\pi r^2$ (C) $\pi r^2 h$ (D) None of these

49. Side of a cube is increased by 50%, then what percent increase will be in the area of the vertical faces of the cube? **[Maharashtra NTSE Stage-1 2017]**
(A) 125% (B) 150% (C) 100% (D) 50%

50. The radius of a cylindrical vessel is 7 cm and its height is 12 cm. $\frac{2}{3}$ of the vessel is filled with water. A sphere having radius 6 cm is dropped into the water. Find the volume of the water that will come out of the vessel. **[Maharashtra NTSE Stage-1 2017]**

(A) 196 πcm^3 (B) 92 πcm^3 (C) 288 πcm^3 (D) 588 πcm^3

51. The length of the longest pole that can be kept in a room of size 12 m × 9 m × 8 m is : **[MP NTSE Stage-1 2017]**

(A) 29 m (B) 17 m (C) 21 m (D) 19 m

52. The height of a cylinder is 14 cm and its curved surface area is 264 cm^2, the volume of cylinder is : **[MP NTSE Stage-1 2017]**

(A) 308 cm^3 (B) 396 cm^3 (C) 1232 cm^3 (D) 1848 cm^3

Answer Key

Exercise Board Level

TYPE (I)

1.	Two cones and a cylinder	**2.**	A frustum of a cone	**3.**	21cm
4.	4950 cm^2	**5.**	2r cm	**6.**	remain unaltered
7.	16 : 9				

TYPE (II)

8.	142296	**9.**	14cm	**10.**	11200	**11.**	0.36 cm^3
12.	32.7 litres	**13.**	6 cm	**14.**	1 : 7	**15.**	160 cm^2
16.	2541	**17.**	450				

TYPE (III)

18.	14 cm, 7 cm and 396 cm^3			**19.**	327.4 cm^3	**20.**	150
21.	12960	**22.**	28.44 cm	**23.**	8.6 m	**24.**	480000
25.	51 min. 12 sec.			**26.**	Volume = 74.25 m^3 , Canvas required = 80.61 m^2		
27.	Rs 2250	**28.**	0.5 cm				

TYPE (IV)

29.	2 hours	**30.**	Rs 230.12	**31.**	Radius = 36 cm , Height = 43.267 cm		
32.	TSA = 301.44 cm^2, Volume = 376.8 cm^3			**33.**	4 m	**34.**	1.584 m^3

Previous Year Problems

1.	Rs. 171.13	**2.**	45 cm	**3.**	205.33 cm^3	**4.**	572 cm^2
5.	332.5 cm^2	**6.**	**(i)** 8624 cm^3	**(ii)**	1804 cm^2	**7.**	(A)
8.	Rs. 2750	**9.**	43.3 cm	**10.**	Rs. 379500	**11.**	5.4 cm
12.	Rs. 33.93	**13.**	21 cm & 1386 cm^2			**14.**	158.4 cm
15.	Rs. 16,500	**16.**	2 cm	**17.**	18 cm	**18.**	Rs. 332640
19.	48 cm^2	**20.**	112 m	**21.**	2.5 cm	**22.**	16240 m^2

Exercise-1

SUBJECTIVE QUESTIONS

Section(A)

A-1.	125 cubic meters.	**A-2.**	350 cm^2	**A-3.**	5 cm, 3 cm	**A-4.**	$\sqrt{3}$: 1
A-5.	36%	**A-6.**	V=3771.42 cm^3. S = 1320 cm^2.				

Section(B)

B-1.	6,8,10 cm	**B-2.**	20000	**B-3.**	0.5 cm	**B-4.**	90 cm.
B-5.	1000	**B-6.**	14 cm.	**B-7.**	2304.	**B-8.**	2.5 cm.
B-9.	14 cm.	**B-10.**	77 cm^3.				

Section(C)

C-1. 892.57 m^2 **C-2.** $\frac{\ell^2}{4}$ [24 + π) sq. units. **C-3.** 9735 m^2.

C-4. 163.86 cm^2. **C-5.** 196.43 cm^3, 163.54 cm^3

Section(D)

D-1. V = 48510 cm^3, S = 5616 cm^2 **D-2.** 3 cm. **D-3.** 20 cm. **D-4.** $\frac{1}{2}$.

D-5. **(i)** 12 cm. **(ii)** 312 π cm^3 **(iii)** 129 π cm^3.

OBJECTIVE QUESTIONS

Section(A)

A-1. (A) **A-2.** (B) **A-3.** (D) **A-4.** (D) **A-5.** (D)

A-6. (C) **A-7.** (C) **A-8.** (A) **A-9.** (D) **A-10.** (B)

A-11. (C)

Section(B)

B-1. (D) **B-2.** (A) **B-3.**___(D) **B-4.** (C) **B-5.** (C)

B-6. (A) **B-7.** (C) **B-8.** (C) **B-9.** (B) **B-10.** (C)

Section(C)

C-1. (C) **C-2.** (A) **C-3.** (B) **C-4.** (B) **C-5.** (A)

Section(D)

D-1. (B) **D-2.** (B) **D-3.** (A) **D-4.** (D)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	B	C	D	D	A	A	D	D	A	A	A	A	C	C	C	B	B	D

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	A	B	A	B	A	C	A	A	B	B	C	B	B	D	B	A	A	C	B
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	B	A	C	A	A	A	D	A	C	D	C	B	C	A	B	A	A	C	B	B
Ques.	41	42	43	44	45	46	47	48	49	50	51	52								
Ans.	B	B	C	A	B	B	B	B	A	B	B	B								