JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

DIFFERENTIAL EQUATION

(PRACTICE SHEET)

LEVEL- 1

Question based on **Order and Degree of Differential Equation**

Q.1 A differential equation of first order and first degree is-

(A) $x\left(\frac{dy}{dx}\right)^2 - x + a = 0$ (B) $\frac{d^2y}{dx^2} + xy = 0$

(C) $dy + dx = 0$ (D) None of these

Q.2 The order and degree of differential equation $\sqrt{1-y^2}\,dx + y\,\sqrt{1-x^2}\,dy = 0$ are respectively-

(A) 1, 2 (B) 1, 1 (C) 2, 1 (D) 2, 2

Q.3 The order and degree of the differential equation $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$ is-

(A) 1, 2 (B) 2, 1 (C) 1, 1 (D) 2, 2

Q.4 The order and degree of the differential equation $\left[4 + \left(\frac{dy}{dx}\right)^2\right]^{2/3} = \frac{d^2y}{dx^2}$ are-

(A) 2, 2 (B) 3, 3 (C) 2, 3 (D) 3, 2

Q.5 The order and the degree of differential equation $\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 8\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = 0$ are respectively-

(A) 4, 1 (B) 1, 4

(C) 1, 1 (D) None of these

Q.6 The order and degree of differential equation $(xy^2 + x)\,dx + (y - x^2 y)\,dy = 0$ are-

(A) 1, 2 (B) 2, 1

(C) 1, 1 (D) 2, 2

Q.7 The degree of the differential equation $\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$ is -

(A) 1 (B) 2

(C) 3 (D) 6

Q.8 The order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x}$ is-

(A) 3 (B) 2

(C) 1 (D) None of these

Q.9 The differential equation of all circles of radius a is of order-

(A) 2 (B) 3

(C) 4 (D) None of these

Q.10 The order of the differential equation of all circles of radius r, having centre on y-axis and passing through the origin is-

(A) 1 (B) 2

(C) 3 (D) 4

Q.11 The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$ is-

(A) 1 (B) 2

(C) 3 (D) None of these

Q.12 The differential equation $x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = x^2$ is of -

(A) degree 2 and order 2

(B) degree 1 and order 1

(C) degree 4 and order 3

(D) degree 4 and order 4

Question based on **Linear and Non-linear Differential Equation**

Q.13 Which of the following equation is linear?

(A) $\frac{dy}{dx} + xy^2 = 1$

(B) $x^2\frac{dy}{dx} + y = e^x$

(C) $\frac{dy}{dx} + 3y = xy^2$

(D) $x\frac{dy}{dx} + y^2 = \sin x$

Q.14 Which of the following equation is non- linear-

(A) $\frac{dy}{dx} = \cos x$ (B) $\frac{d^2y}{dx^2} + y = 0$

(C) $dx + dy = 0$ (D) $x\frac{dy}{dx} + \frac{3}{dy/dx} = y^2$

Q.15 Which of the following equation is linear ?

(A) $\left(\frac{d^2y}{dx^2}\right)^2 + x^2\left(\frac{dy}{dx}\right)^2 = 0$

(B) $y = \frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

(C) $\frac{dy}{dx} + \frac{y}{x} = \log x$

(D) $y\frac{dy}{dx} - 4 = x$

Question based on Formation of Differential Equation

Q.16 $y = 4 \sin 3x$ is a solution of the differential equation-

(A) $\frac{dy}{dx} + 8y = 0$ (B) $\frac{dy}{dx} - 8y = 0$

(C) $\frac{d^2y}{dx^2} + 9y = 0$ (D) $\frac{d^2y}{dx^2} - 9y = 0$

Q.17 The differential equation of the family of curves represented by the equation $x^2 + y^2 = a^2$ is-

(A) $x + y\frac{dy}{dx} = 0$ (B) $y\frac{dy}{dx} = x$

(C) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (D) None of these

Q.18 The differential equation of the family of curves $y^2 = 4a (x + a)$, where a is an arbitrary constant, is-

(A) $y\left[1+\left(\frac{dy}{dx}\right)^2\right] = 2x\frac{dy}{dx}$

(B) $y\left[1-\left(\frac{dy}{dx}\right)^2\right] = 2x\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

(D) $\left(\frac{dy}{dx}\right)^3 + 3\frac{dy}{dx} + y = 0$

Q.19 The differential equation of all the non-vertical lines in the xy- plane is-

(A) $\frac{dy}{dx} - x = 0$ (B) $\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

(C) $\frac{d^2y}{dx^2} = 0$ (D) $\frac{d^2y}{dx^2} + x = 0$

Q.20 The differential equation of the family of curves represented by the equation $(x - a)^2 + y^2 = a^2$ is-

(A) $2xy\frac{dy}{dx} + x^2 = y^2$

(B) $2xy\frac{dy}{dx} + x^2 + y^2 = 0$

(C) $xy\frac{dy}{dx} + x^2 = y^2$

(D) None of these

Q.21 The differential equation of all parabolas whose axes are parallel to y- axis is-

(A) $\frac{d^3y}{dx^3} = 0$ (B) $\frac{d^2x}{dy^2} = c$

(C) $\frac{d^3y}{dx^3} + \frac{d^2x}{dy^2} = 0$ (D) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = c$

Q.22 The differential equation of family of curve $y = Ae^{x} + Be^{-x}$, where A and B are arbitrarily constants, is

(A) $\frac{d^2y}{dx^2} + y = 0$

(B) $\frac{d^2y}{dx^2} = y$

(C) $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0$

(D) None of these

Q.23 The differential equation for the line $y = mx + c$ is (where c is arbitrary constant)-

(A) $\frac{dy}{dx} = m$ (B) $\frac{dy}{dx} + m = 0$

(C) $\frac{dy}{dx} = 0$ (D) None of these

Q.24 The differential equation of the family of curves v $= \frac{A}{r} + B$, where A & B are arbitrary constants, is-

(A) $\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} = 0$

(B) $\frac{d^2v}{dr^2} - \frac{2}{r}\frac{dv}{dr} = 0$

(C) $\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$

(D) None of these

Question based on **Variable separable method**

Q.25 The general solution of the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ is-

(A) $x^3 - y^3 = c$ (B) $x^3 + y^3 = c$

(C) $x^2 + y^2 = c$ (D) $x^2 - y^2 = c$

Q.26 The general solution of the equation $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$ is-

(A) $(e^y + 1) \cos x = c$ (B) $(e^y - 1) \sin x = c$

(C) $(e^y + 1) \sin x = c$ (D) None of these

Q.27 The solution of the differential equation $dy = \sec^2 x \, dx$ is-

(A) $y = \sec x \tan x + c$ (B) $y = 2 \sec x + c$

(C) $y = \frac{1}{2} \tan x + c$ (D) None of these

Q.28 The solution of the differential equation $(1 + x^2)\frac{dy}{dx} = x$ is-

(A) $y = \tan^{-1} x + c$

(B) $y = -\tan^{-1} x + c$

(C) $y = \frac{1}{2} \log_e (1 + x^2) + c$

(D) $y = -\frac{1}{2} \log_e(1 + x^2) + c$

Q.29 The solution of $\frac{dy}{dx} = e^x (\sin x + \cos x)$ is-

(A) $y = e^x (\sin x - \cos x) + c$

(B) $y = e^x (\cos x - \sin x) + c$

(C) $y = e^x \sin x + c$

(D) $y = e^x \cos x + c$

Q.30 The solution of $\frac{dy}{dx} = x \log x$ is-

(A) $y = x^2 \log x - \frac{x^2}{2} + c$

(B) $y = \frac{x^2}{2} \log x - x^2 + c$

(C) $y = \frac{1}{2} x^2 + \frac{1}{2} x^2 \log x + c$

(D) None of these

Q.31 The solution of the differential equation $\frac{dy}{dx} = (1 + x)(1 + y^2)$ is-

(A) $y = \tan (x^2 + x + c)$

(B) $y = \tan (2x^2 + x + c)$

(C) $y = \tan (x^2 - x + c)$

(D) $y = \tan \left(\frac{x^2}{2} + x + c\right)$

Q.32 The solution of the differential equation $x \sec y \frac{dy}{dx} = 1$ is-

(A) $x \sec y \tan y = c$

(B) $cx = \sec y + \tan y$

(C) $cy = \sec x \tan x$

(D) $cy = \sec x + \tan x$

Q.33 The solution of the equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is-

(A) $x \sqrt{1-y^2} - y \sqrt{1-x^2} = c$

(B) $x \sqrt{1-y^2} + y \sqrt{1-x^2} = c$

(C) $x \sqrt{1+y^2} + y \sqrt{1+x^2} = c$

(D) None of these

Q.34 Solution of the equation $\cos x \cos y \frac{dy}{dx} = -\sin x \sin y$ is-

(A) $\sin y + \cos x = c$

(B) $\sin y - \cos x = c$

(C) $\sin y. \cos x = c$

(D) $\sin y = c \cos x$

Q.35 The general solution of the equation $\frac{d^2y}{dx^2} = -\frac{1}{x^2}$ is-

(A) $y = \log x + c_1x + c_2$

(B) $y = -\log x + c_1x + c_2$

(C) $y = -\frac{1}{x} + c_1x + c_2$

(D) None of these

Q.36 The general solution of the differential equation $e^y \frac{dy}{dx} + (e^y + 1)\cot x = 0$ is-

(A) $(e^y + 1)\cos x = K$ (B) $(e^y + 1)\operatorname{cosec} x = K$

(C) $(e^y + 1)\sin x = K$ (D) None of these

Q.37 The solution of the equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is-

(A) $e^y = e^x + \frac{x^3}{3} + c$ (B) $e^y = e^x + 2x + c$

(C) $e^y = e^x + x^3 + c$ (D) None of these

Q.38 The solution of the differential equation $\frac{dy}{dx} = e^x + \cos x + x + \tan x$ is-

(A) $y = e^x + \sin x + \frac{x^2}{2} + \log \cos x + c$

(B) $y = e^x + \sin x + \frac{x^2}{2} + \log \sec x + c$

(C) $y = e^x - \sin x + \frac{x^2}{2} + \log \cos x + c$

(D) $y = e^x - \sin x + \frac{x^2}{2} + \log \sec x + c$

Q.39 The solution of the differential equation $(1 + \cos x)\,dy = (1 - \cos x)\,dx$ is-

(A) $y = 2\tan\frac{x}{2} - x + c$

(B) $y = 2\tan x + x + c$

(C) $y = 2\tan\frac{x}{2} + x + c$

(D) $y = x - 2\tan\frac{x}{2} + c$

Q.40 If $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$, then-

(A) $y + \sin^{-1} x = c$

(B) $y^2 + 2\sin^{-1} x + c = 0$

(C) $x + \sin^{-1} y = 0$

(D) $x^2 + 2\sin^{-1} y = 1$

Q.41 The solution of the differential equation $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$ is-

(A) $\tan x = c \tan y$ (B) $\tan x = c \tan (x + y)$

(C) $\tan x = c \cot y$ (D) $\tan x \sec y = c$

Q.42 The solution of $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$ is-

(A) $y = e^{3x} + c$

(B) $-y = e^{3x} - c$

(C) $y = -e^{3x} + c$

(D) None of these

Q.43 The general solution of differential equation $(4 + 5\sin x)\frac{dy}{dx} = \cos x$ is-

(A) $y = \frac{1}{5}\log|4 + 5\sin x| + c$

(B) $y = \frac{1}{5}\log|4 + 5\cos x| + c$

(C) $y = -\frac{1}{5}\log|4 - 5\sec x| + c$

(D) None of these

Q.44 The general solution of differential equation $\frac{dy}{dx} = \log x$ is -

(A) $y = x(\log x + 1) + c$

(B) $y + x(\log x + 1) = c$

(C) $y = x(\log x - 1) + c$

(D) None of these

Q.45 The general solution of differential equation $\frac{dy}{dx} = \sin^3 x \cos^2 x + x e^x$ is-

(A) $y = \frac{1}{5}\cos^5 x + \frac{1}{3}\operatorname{cosec}^3 x + (x + 1)e^x + c$

(B) $y = \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + (x - 1)e^x + c$

(C) $y = -\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x - (x - 1)e^x - c$

(D) None of these

Q.46 The solution of the differential equation $x(e^{2y} - 1)\,dy + (x^2 - 1)e^y dx = 0$ is-

(A) $e^y + e^{-y} = \log x - \frac{x^2}{2} + c$

(B) $e^y - e^{-y} = \log x - \frac{x^2}{2} + c$

(C) $e^y + e^{-y} = \log x + \frac{x^2}{2} + c$

(D) None of these

Q.47 The solution of the differential equation $(1 + x^2)(1+ y)\,dy + (1 + x)(1+ y^2)\,dx = 0$ is-

(A) $\tan^{-1} x + \log(1 + x^2) + \tan^{-1} y + \log(1 + y^2) = c$

(B) $\tan^{-1} x - \frac{1}{2}\log(1 + x^2) + \tan^{-1} y - \frac{1}{2}\log(1 + y^2) = c$

(C) $\tan^{-1} x + \frac{1}{2}\log(1 + x^2) + \tan^{-1} y + \frac{1}{2}\log(1 + y^2) = c$

(D) None of these

Q.48 The solution of the differential equation $y\,dx - x\,dy = 0$ is-

(A) $x = cy$ (B) $xy = c$
(C) $x = c \log x$ (D) None of these

Q.49 The solution of differential equation $x\frac{dy}{dx} + y = y^2$ is-

(A) $y = 1 + cxy$ (B) $y = \log(cxy)$
(C) $y + 1 = cxy$ (D) $y = c + xy$

Q.50 The solution of the differential equation $\frac{dy}{dx} = e^{-y}\cos x$, given that $y(0) = 0$ is–

(A) $e^y = \sin x + 1$ (B) $e^y = \text{cosec } x + 1$
(C) $e^y = \cos x + 1$ (D) $e^y = -\sin x - 1$

Q.51 The solution of the differential equation $(1 + x^2)\frac{dy}{dx} = x(1 + y^2)$ is-

(A) $2\tan^{-1} y = \log(1 + x^2) + c$
(B) $\tan^{-1} y = \log(1 + x^2) + c$
(C) $2\tan^{-1} y + \log(1 + x^2) + c$
(D) None of these

Q.52 Solution of the equation $(e^x + 1)\,y\,dy = (y + 1)\,e^x\,dx$ is-

(A) $c(y + 1)(e^x + 1) + e^y = 0$
(B) $c(y + 1)(e^x - 1) + e^y = 0$
(C) $c(y + 1)(e^x - 1) - e^y = 0$
(D) $c(y + 1)(e^x + 1) = e^y$

Q.53 The solution of the given differential equation $\frac{dy}{dx} + 2xy = y$ is-

(A) $y = ce^{x-x^2}$ (B) $y = ce^{x^2-x}$
(C) $y = ce^x$ (D) $y = ce^{-x^2}$

Q.54 The solution of $\frac{d^2y}{dx^2} = \sec^2 x + xe^x$ is-

(A) $y = \log(\sec x) + (x - 2)e^x + c_1x + c_2$
(B) $y = \log(\sec x) + (x + 2)e^x + c_1x + c_2$
(C) $y = \log(\sec x) - (x + 2)e^x + c_1x + c_2$
(D) None of these

Q.55 The equation of the curve which passes through the point (1, 1) & whose slope is given by $\frac{2y}{x}$, is -

(A) $y = x^2$ (B) $x^2 - y^2 = 0$
(C) $2x^2 + y^2 = 3$ (D) None of these

Q.56 Equation of curve passing through (3, 9) which satisfies the differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$, is-

(A) $6xy = 3x^2 - 6x + 29$
(B) $6xy = 3x^3 - 29x + 6$
(C) $6xy = 3x^3 + 29x - 6$
(D) None of these

Q.57 The equation of the curve through the point (1, 0) and whose slope is $\frac{y-1}{x^2+x}$ is-

(A) $(y - 1)(x + 1) + 2x = 0$
(B) $2x(y - 1) + x + 1 = 0$
(C) $x(y - 1)(x + 1) + 2 = 0$
(D) None of these

Q.58 If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$, the value of x for $y = 3$ is-

(A) e^5 (B) $e^6 + 1$
(C) $\frac{e^6+9}{2}$ (D) $\log_e 6$

Q.59 The differential equation $y\frac{dy}{dx} + x = a$ (a is any constant) represents-

(A) a set of circles having centre on the y-axis
(B) a set of circles centre on the x-axis
(C) a set of ellipses
(D) None of these

Question based on: Differential Equation of the form dy/dx = f(ax + by + c)

Q.60 The solution of the equation $\frac{dy}{dx} = (x + y)^2$ is-

(A) $x + y + \tan (x + c) = 0$
(B) $x - y + \tan (x + c) = 0$
(C) $x + y - \tan (x + c) = 0$
(D) None of these

Q.61 The solution of the differential equation $\frac{dy}{dx} = \cot^2 (x + y)$ is-

(A) $y = x + 1/2 \sin 2 (x + y) + c$
(B) $y = x - 1/2 \sin 2 (x + y) + c$
(C) $y = x + 1/2 \cos 2 (x + y) - c$
(D) None of these

Q.62 The solution of $\frac{dy}{dx} = \sin (x + y) + \cos (x + y)$ is-

(A) $\log \left[1 + \tan\left(\frac{x+y}{2}\right)\right] + c = 0$
(B) $\log \left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c$
(C) $\log \left[1 - \tan\left(\frac{x+y}{2}\right)\right] = x + c$
(D) None of these

Q.63 The solution of $\frac{dy}{dx} = (4x + y + 1)^2$ is-

(A) $4x - y + 1 = 2 \tan (2x - 2c)$
(B) $4x - y - 1 = 2 \tan (2x - 2c)$
(C) $4x + y + 1 = 2 \tan (2x + 2c)$
(D) None of these

Question based on: Differential Equation of homogeneous type

Q.64 The solution of the differential equation $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ is-

(A) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$
(B) $\tan^{-1}\left(\frac{y}{x}\right) = -\log x + c$
(C) $\sin^{-1}\left(\frac{y}{x}\right) = \log x + c$
(D) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

Q.65 The solution of the differential equation $x \frac{dy}{dx} = y (\log y - \log x + 1)$ is-

(A) $y = xe^{cx}$ (B) $y + xe^{cx} = 0$
(C) $y + e^x = 0$ (D) None of these

Q.66 The solution of the equation $\frac{dy}{dx} = \frac{x+y}{x-y}$ is-

(A) $c\,(x^2 + y^2)^{1/2} + e^{\tan^{-1}(y/x)}$
(B) $c\,(x^2 + y^2)^{1/2} = e^{\tan^{-1}(y/x)}$
(C) $c\,(x^2 - y^2) = e^{\tan^{-1}(y/x)}$
(D) None of these

Q.67 The solution of the differential equation $(x^2 + y^2)dx = 2\,xy\,dy$ is-

(A) $x = c\,(x^2 + y^2)$
(B) $x = c\,(x^2 - y^2)$
(C) $x + c\,(x^2 + y^2) = 0$
(D) None of these

Q.68 The solution of the equation $x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$ is-

(A) $x \sin\left(\frac{x}{y}\right) + c = 0$ (B) $x \sin y + c = 0$
(C) $x \sin\left(\frac{y}{x}\right) = c$ (D) None of these

Question based on: Linear Differential Equation

Q.69 The solution of the differential equation, $\frac{dy}{dx} + \frac{y}{x} = x^2$ is-

(A) $4xy = x^4 + c$ (B) $xy = x^4 + c$
(C) $\frac{1}{4} xy = x^4 + c$ (D) $xy = 4x^4 + c$

Q.70 The solution of the differential equation $\frac{dy}{dx} + y = \cos x$ is-

(A) $y = \frac{1}{2} (\cos x + \sin x) + ce^{-x}$
(B) $y = \frac{1}{2} (\cos x - \sin x) + ce^{-x}$
(C) $y = \cos x + \sin x + ce^{-x}$
(D) None of these

Q.71 The solution of the differential equation

$x \log x \dfrac{dy}{dx} + y = 2 \log x$ is-

(A) $y = \log x + c$

(B) $y = \log x^2 + c$

(C) $y \log x = (\log x)^2 + c$

(D) $y = x \log x + c$

Q.72 The solution of the equation

$x \dfrac{dy}{dx} + 3y = x$ is-

(A) $x^3y + \dfrac{x^4}{4} + c = 0$ (B) $x^3y = \dfrac{x^4}{4} + c$

(C) $x^3 y + \dfrac{x^4}{4} = 0$ (D) None of these

Q.73 The solution of the differential equation $\dfrac{dy}{dx} + y \cot x = 2 \cos x$ is-

(A) $y \sin x + \cos 2x = 2c$

(B) $2 y \sin x + \cos x = c$

(C) $y \sin x + \cos x = c$

(D) $2y \sin x + \cos 2x = c$

Q.74 The solution of $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$ is -

(A) $xe^{\tan^{-1} y} = e^{\tan^{-1} y} (\tan^{-1} y - 1) + c$

(B) $xe^{\tan^{-1} y} = (\tan^{-1} y + 1) - c$

(C) $xe^{\tan^{-1} y} = (\tan^{-1} y - 1) + c$

(D) None of these

Q.75 The integrating factor of the differential equation

$(x \log x) \dfrac{dy}{dx} + y = 2 \log x$ is-

(A) $\log x$ (B) $\log (\log x)$

(C) e^x (D) x

Q.76 The equation of the curve passing through the origin and satisfying the differential equation

$(1 + x^2) \dfrac{dy}{dx} + 2xy = 4x^2$ is-

(A) $(1 + x^2)y = x^3$

(B) $2 (1 + x^2)y = 3x^3$

(C) $3 (1 + x^2)y = 4x^3$

(D) None of these

Q.77 The solution of the differential equation $x \dfrac{dy}{dx} + y = x^2 + 3x + 2$ is-

(A) $xy = \dfrac{x^3}{3} + \dfrac{3}{2}x^2 + 2x + c$

(B) $xy = \dfrac{x^4}{4} + x^3 + x^2 + c$

(C) $xy = \dfrac{x^4}{4} + \dfrac{x^3}{3} + x^2 + c$

(D) $xy = \dfrac{x^4}{4} + x^3 + x^2 + cx$

Q.78 The solution of the equation $\dfrac{dy}{dx} = \dfrac{1}{x+y+1}$ is-

(A) $x = ce^y - y - 2$

(B) $y = x + ce^y - 2$

(C) $x + ce^y - y - 2 = 0$

(D) None of these

Q.79 Integrating factor of the differential equation $\dfrac{dy}{dx} + y \tan x - \sec x = 0$ is-

(A) $e^{\sin x}$ (B) $\dfrac{1}{\sin x}$

(C) $\dfrac{1}{\cos x}$ (D) $e^{\cos x}$

Question based on **Equation reducible to linear form**

Q.80 The solution of differential equation

$\dfrac{dy}{dx} + 1 = e^{x-y}$ is-

(A) $e^y = e^x/ 2 + ce^{-x}$

(B) $e^y = e^x + ce^{-x}$

(C) $3e^y = e^x/ 2 + ce^{-x}$

(D) None of these

LEVEL- 2

Q.1 The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 . e^{x+C_5}$, where C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is-

(A) 5 (B) 4

(C) 3 (D) 2

Q.2 The differential equation of all circles in the first quadrant which touch the coordinate axes is of order-

(A) 1 (B) 2

(C) 3 (D) None of these

Q.3 The differential equations of all circles passing through origin and having their centres on the x-axis is

(A) $\frac{dy}{dx} = \frac{y^2 + x^2}{2xy}$ (B) $\frac{dy}{dx} = \frac{y^2 + x^2}{2x}$

(C) $\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{2xy}$ (D) $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Q.4 The solution of the equation

$\frac{dy}{dx} + y \tan x = x^m \cos x$ is-

(A) $(m+1)\, y = x^{m+1} \cos x + c\,(m+1) \cos x$

(B) $my = (x^m + c) \cos x$

(C) $y = (x^{m+1} + c) \cos x$

(D) None of these

Q.5 The solution of the differential equation $(\sin x + \cos x)\, dy + (\cos x - \sin x) dx = 0$ is-

(A) $e^x (\sin x + \cos x) + c = 0$

(B) $e^y (\sin x + \cos x) = c$

(C) $e^y (\cos x - \sin x) = c$

(D) $e^y (\sin x - \cos x) = c$

Q.6 The solution of the differential equation

$\frac{dy}{dx} + \frac{3x^2}{1+x^3} y = \frac{\sin^2 x}{1+x^3}$ is -

(A) $y (1 + x^3) = x + \frac{1}{2} \sin 2x + c$

(B) $y (1 + x^3) = cx + \frac{1}{2} \sin 2x$

(C) $y (1 + x^3) = cx - \frac{1}{2} \sin 2x$

(D) $y (1 + x^3) = \frac{x}{2} - \frac{1}{4} \sin 2x + c$

Q.7 The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is-

(A) $y = \tan^{-1}\left(\log\left(\frac{e}{x}\right)\right)$

(B) $y = x \tan^{-1}\left(\log\left(\frac{x}{e}\right)\right)$

(C) $y = x \tan^{-1}\left(\log\left(\frac{e}{x}\right)\right)$

(D) None of these

Q.8 The solution of the equation $(1 - x^2)\, dy + xy\, dx = xy^2\, dx$ is-

(A) $(y - 1)^2 (1 - x^2) = 0$

(B) $(y - 1)^2 (1 - x^2) = c^2y^2$

(C) $(y - 1)^2 (1 + x^2) = c^2y^2$

(D) None of these

Q.9 The solution of $\frac{dy}{dx} = \frac{e^x (\sin^2 x + \sin 2x)}{y(2 \log y + 1)}$ is-

(A) $y^2 (\log y) - e^x \sin^2 x + c = 0$

(B) $y^2 (\log y) - e^x \cos^2 x + c = 0$

(C) $y^2 (\log y) + e^x \cos^2 x + c = 0$

(D) None of these

Q.10 The solution of $(\operatorname{cosec} x \log y)dy + (x^2y)dx = 0$ is-

(A) $\frac{\log y}{2} + (2 - x^2) \cos x + 2 \sin x = c$

(B) $\left(\frac{\log y}{2}\right)^2 + (2 - x^2) \cos x + 2x \sin x = c$

(C) $\frac{(\log y)^2}{2} + (2 - x^2) \cos x + 2x \sin x = c$

(D) None of these

Q.11 The solution of the differential equation

$xy \frac{dy}{dx} = \frac{(1+y^2)(1+x+x^2)}{(1+x^2)}$ is-

(A) $\frac{1}{2} \log (1 + y^2) = \log x - \tan^{-1} x + c$

(B) $\frac{1}{2} \log (1 + y^2) = \log x + \tan^{-1} x + c$

(C) $\log (1 + y^2) = \log x - \tan^{-1} x + c$

(D) $\log (1 + y^2) = \log x + \tan^{-1} x + c$

Q.12 The solution of

$(x\sqrt{1+y^2})dx + (y\sqrt{1+x^2})dy = 0$ is -

(A) $\sqrt{1+x^2} + \sqrt{1+y^2} = c$

(B) $\sqrt{1+x^2} - \sqrt{1+y^2} = c$

(C) $(1 + x^2)^{3/2} + (1 + y^2)^{3/2} = c$

(D) None of these

Q.13 The solution of the differential equation $\cos y \log(\sec x + \tan x)dx$ $= \cos x \log(\sec y + \tan y)\, dy$ is -

(A) $\sec^2 x + \sec^2 y = c$ (B) $\sec x + \sec y = c$

(C) $\sec x - \sec y = c$ (D) None of these

Q.14 The slope of a curve at any point is the reciprocal of twice the ordinate at the point and it passes through the point (4, 3). The equation of the curve is-

(A) $x^2 = y + 5$ (B) $y^2 = x - 5$

(C) $y^2 = x + 5$ (D) $x^2 = y - 5$

Q.15 The solution of the differential equation

$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(A) $y = e^{x-y} + x^2e^{-y} + c$

(B) $e^y - e^x = \frac{1}{3}x^3 + c$

(C) $e^x + e^y = \frac{1}{3}x^3 + c$

(D) $e^x - e^y = \frac{1}{3}x^3 + c$

Q.16 The solution of the equation

$\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$ is-

(A) $\log\left(\frac{y}{x}\right) = cx$ (B) $\frac{y}{x} = \log y + c$

(C) $y = \log y + 1$ (D) $y = xy + c$

Q.17 The solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ is-

(A) $ay^2 = e^{x^2/y^2}$ (B) $ay = e^{x/y}$

(C) $y = + + c$ (D) $y = + y^2 + c$

Q.18 Solution of the differential equation $\sin\frac{dy}{dx} = a$ with y (0) = 1 is-

(A) $\sin^{-1}\frac{(y-1)}{x} = a$ (B) $\sin\frac{(y-1)}{x} = a$

(C) $\sin\frac{(1-y)}{(1+x)} = a$ (D) $\sin\frac{y}{(x+1)} = a$

Q.19 Which of the following differential equations has the same order and degree?

(A) $\frac{d^4y}{dx^4} + 8\left(\frac{dy}{dx}\right)^6 + 5y = e^x$

(B) $5\left(\frac{d^3y}{dx^3}\right)^4 + 8\left(1+\frac{dy}{dx}\right)^2 + 5y = x^8$

(C) $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{2/3} = 4\frac{d^3y}{dx^3}$

(D) $y = x^2\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Q.20 The general solution of the differential equation $\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2x} = 0$, is given by-

(A) $\tan y + \cot x = c$

(B) $\tan y - \cot x = c$

(C) $\tan x - \cot y = c$

(D) $\tan x + \cot y = c$

Q.21 The solution of $\frac{dy}{dx} + \sqrt{\left(\frac{1-y^2}{1-x^2}\right)} = 0$, is-

(A) $\tan^{-1} x + \cot^{-1} x = c$

(B) $\sin^{-1} x + \sin^{-1} y = c$

(C) $\sec^{-1} x + \text{cosec}^{-1} x = c$

(D) None of these

Q.22 The solution of $\frac{dy}{dx} + y \tan x = \sec x$ is-

(A) $y \sec x = \tan x + c$

(B) $y \tan x = \sec x + c$

(C) $\tan x = y \tan x + c$

(D) $x \sec x = y \tan y + c$

LEVEL- 3

Q.1 The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ is-
(A) 1 (B) 2
(C) 3 (D) none of these

Q.2 The differential equation of all non-horizontal lines in a plane is-
(A) $\frac{d^2y}{dx^2} = 0$ (B) $\frac{d^2x}{dy^2} = 0$
(C) $\frac{dy}{dx} = 0$ (D) $\frac{dx}{dy} = 0$

Q.3 The degree of the differential equation
$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$ is-
(A) 1 (B) 2
(C) 3 (D) none of these

Q.4 The degree of the differential equation satisfying the relation
$\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda(x\sqrt{1+y^2} - y\sqrt{1+x^2})$ is-
(A) 1 (B) 2
(C) 3 (D) none of these

Q.5 The order and degree the differential equation of all tangent lines to the parabola $x^2 = 4y$ is-
(A) 1, 2 (B) 2, 2 (C) 3, 1 (D) 4, 1

Q.6 The order of the differential equation of all circles of radius r, having centre on y-axis and passing through the origin is-
(A) 1 (B) 2 (C) 3 (D) 4

Q.7 Solution of differential equation
$(2x \cos y + y^2 \cos x)\, dx + (2y \sin x - x^2 \sin y)\, dy = 0$, is-
(A) $x^2 \cos y + y^2 \sin x = c$
(B) $x \cos y - y \sin x = c$
(C) $x^2 \cos^2 y + y^2 \sin^2 x = c$
(D) None of the above

Q.8 The solution of the differential equation
$\frac{dy}{dx} - \frac{\tan y}{x} = \frac{\tan y \sin y}{x^2}$, is -
(A) $\frac{x}{\sin y} + \ell n\, x = c$ (B) $\frac{y}{\sin x} + \ell n\, x = c$
(C) $\ell n\, y + x = c$ (D) $\ell n\, x + y = c$

Q.9 If $f(x)$, $g(x)$ be twice differentiable functions on [0, 2] satisfying $f''(x) = g''(x)$, $f'(1) = 2g'(1) = 4$ and $f(2) = 3\, g(2) = 9$, then $f(x) - g(x)$ at $x = 4$ equals-
(A) 0 (B) 10 (C) 8 (D) 2.

Q.10 The equation of the curve whose subnormal is constant is-
(A) $y = ax + b$ (B) $y^2 = 2ax + b$
(C) $ay^2 - x^2 = a$ (D) none of these

Q.11 The solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represents a parabola when-
(A) $a = 0, b = 0$
(B) $a = 1, b = 2$
(C) $a = 0, b \neq 0$
(D) $a = 2, b = 1$

Q.12 The differential equation of all parabolas having their axis of symmetry coinciding with the axis of X is-
(A) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$
(B) $x\frac{d^2x}{dy^2} + \left(\frac{dy}{dx}\right)^2 = 0$
(C) $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
(D) none of these.

Q.13 Solution of the differential equation
$x = 1 + xy\frac{dy}{dx} + \frac{(xy)^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{(xy)^3}{3!}\left(\frac{dy}{dx}\right)^3 + ...$
is–
(A) $y = \log_e(x) + C$
(B) $y = (\log_e x)^2 + C$
(C) $y = \pm\sqrt{(\log_e x)^2 + 2C}$
(D) $xy = x^y + k$

Q.14 The solution of $ydx - x\, dy + 3x^2 e^{x^3} y^2 dx = 0$ is-
(A) $\frac{x}{y} + e^{x^3} = C$ (B) $\frac{x}{y} - e^{x^3} = 0$
(C) $-\frac{x}{y} + e^{x^3} = C$ (D) none of these

Q.15 The solution of the differential equation
$xdy - ydx = \sqrt{x^2+y^2}\, dx$ is-
(A) $x + \sqrt{x^2+y^2} = cx^2$
(B) $y - \sqrt{x^2+y^2} = cx$
(C) $x - \sqrt{x^2+y^2} = cx$
(D) $y + \sqrt{x^2+y^2} = cx^2$

Q.16 The solution of the differential equation

$(1 + y^2) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$, is-

(A) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + K$

(B) $(x - 2) = K\, e^{\tan^{-1} y}$

(C) $2x\, e^{\tan^{-1} y} = e^{2\tan^{-1} y} + K$

(D) $xe^{\tan^{-1} y} = \tan^{-1}y + K$

Q.17 The degree of the differential equation $y_3^{2/3} + 2 + 3y_2 + y_1 = 0$ is-

(A) 1 (B) 2
(C) 3 (D) none of these

Q.18 The differential equation of system of concentric circles with centre (1, 2) is-

(A) $(x - 2) + (y - 2)\frac{dy}{dx} = 0$

(B) $(x - 1) + (y - 2)\frac{dy}{dx} = 0$

(C) $(x + 1)\frac{dy}{dx} + (y - 2) = 0$

(D) $(x + 2)\frac{dy}{dx} + (y - 1) = 0$

Q.19 If sinx is an integrating factor of the differential equation $dy/dx + Py = Q$, then P can be-

(A) log sinx (B) cot x
(C) sin x (D) log cos x

Q.20 The curve in which the slope of the tangent at any point equals the ratio of the abscissa to the ordinate of the point is –

(A) an ellipse
(B) a parabola
(C) a rectangular hyperbola
(D) a circle

Q.21 The solution of the differential equation

$x \sin x \frac{dy}{dx} + (x \cos x + \sin x)\, y = \sin x,$

When y(0) = 0 is -

(A) xy sin x = 1 – cos x
(B) xy sin x + cos x = 0
(C) x sin x + y cos x = 0
(D) x sin x + y cos x = 1

Q.22 Differential equation for $y = A \cos \alpha x + \beta \sin \alpha x$, where A and B are arbitrary constants, is-

(A) $\frac{d^2y}{dx^2} - \alpha y = 0$ (B) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$

(C) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ (D) $\frac{d^2y}{dx^2} + \alpha y = 0$

Q.23 If $x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x\right] dx$ and $y(1) = \frac{\pi}{2}$, then the value of $\cos\left(\frac{y}{x}\right)$ is equal to

(A) x (B) $\frac{1}{x}$ (C) log x (D) e^x

Q.24 The function $f(\theta) = \frac{d}{d\theta}\int_0^{\theta} \frac{dx}{1 - \cos\theta \cos x}$ satisfies the differential equation

(A) $\frac{df}{d\theta} + 2f(\theta) \cot \theta = 0$

(B) $\frac{df}{d\theta} - 2f(\theta) \cot \theta = 0$

(C) $\frac{df}{d\theta} + 2f(\theta) = 0$

(D) $\frac{df}{d\theta} - 2f(\theta) = 0$

Statement type Questions

Each of the questions given below consists of Statement-I (Assertion) and Statement-II (Reason). Use the following key to choose the appropriate answer.

(A) If both Statement-I, Statement-II are true, and Statement-II is the correct explanation of Statement-I.

(B) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I

(C) If Statement-I is true but Statement-II is false

(D) If Statement-I is false but Statement-II is true.

Q.25 **Statement-I:** The differential equation whose solution is

$y = c_1 e^{2x+c_2} + c_3 e^{x+c_4}$, $c_1, c_2, c_3, c_4 \in \mathbf{R}$ is of order 4.

Statement-II:
Order of the differential equation is equal to the number of independent arbitrary constants in the solution of differential equation.

Q.26 **Statement-I:** $y = c_1 \sin(x + c_2)$ is a general solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

Statement-II:
y = asin x + bcos x is a trigonometric function.

Q.27 Observe the following statements:

Statement-I: Integrating factor of $\frac{dy}{dx} + y = x^2$ is e^x

Statement-II: Integrating factor of $\frac{dy}{dx} + P(x)\, y = Q(x)$ is $e^{\int P(x)dx}$

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 The solution of the differential equation $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is- **[AIEEE 2002]**

(A) $x^2 + y^2 = cx$

(B) $x^2 - y^2 + cx = 0$

(C) $x^2 + 2xy = y^2 + cx$

(D) $x^2 + y^2 = 2xy + cx^2$

Q.2 The differential equation, which represents the family of plane curves $y = e^{cx}$, is-

[AIEEE 2002]

(A) $y' = cy$

(B) $xy' - \log y = 0$

(C) $x \log y = yy'$

(D) $y \log y = xy'$

Q.3 The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively-

[AIEEE 2003]

(A) 2, 3 (B) 2, 1

(C) 1, 2 (D) 3, 2

Q.4 The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\dfrac{dy}{dx} = 0$, is-

[AIEEE 2003]

(A) $x e^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

(B) $(x - 2) = k e^{-\tan^{-1} y}$

(C) $2x e^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

(D) $x e^{\tan^{-1} y} = \tan^{-1} y + k$

Q.5 The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is- **[AIEEE 2004]**

(A) $2(x^2 - y^2)\, y' = xy$

(B) $2(x^2 + y^2)\, y' = xy$

(C) $(x^2 - y^2)\, y' = 2xy$

(D) $(x^2 + y^2)\, y' = 2xy$

Q.6 The solution of the differential equation $y\,dx + (x + x^2 y)\,dy = 0$ is- **[AIEEE 2004]**

(A) $-\dfrac{1}{xy} = C$ (B) $-\dfrac{1}{xy} + \log y = C$

(C) $\dfrac{1}{xy} + \log y = C$ (D) $\log y = Cx$

Q.7 The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows-

[AIEEE- 2005, IIT- 1999]

(A) order 1, degree 2

(B) order 1, degree 1

(C) order 1, degree 3

(D) order 2, degree 2

Q.8 If $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is -

[AIEEE-2005]

(A) $y \log\left(\dfrac{x}{y}\right) = cx$ (B) $x \log\left(\dfrac{y}{x}\right) = cy$

(C) $\log\left(\dfrac{y}{x}\right) = cx$ (D) $\log\left(\dfrac{x}{y}\right) = cy$

Q.9 The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of – **[AIEEE 2006]**

(A) first order and second degree

(B) first order and first degree

(C) second order and first degree

(D) second order and second degree

Q.10 The differential equation of all circles passing through the origin and having their centres on the x-axis is- **[AIEEE 2007]**

(A) $x^2 = y^2 + xy\dfrac{dy}{dx}$ (B) $x^2 = y^2 + 3xy\dfrac{dy}{dx}$

(C) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$ (D) $y^2 = x^2 - 2xy\dfrac{dy}{dx}$

Q.11 The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is- **[AIEEE 2008]**

(A) $(y-2)\,y'^2 = 25-(y-2)^2$

(B) $(y-2)^2\,y'^2 = 25-(y-2)^2$

(C) $(x-2)^2\,y'^2 = 25-(y-2)^2$

(D) $(x-2)\,y'^2 = 25-(y-2)^2$

Q.12 The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition y (1) = 1 is **[AIEEE 2008]**

(A) $y = x\,\ell n\,x + x^2$ (B) $y = xe^{(x-1)}$

(C) $y = x\,\ell n\,x + x$ (D) $y = \ell n\,x + x$

Q.13 The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$ where c_1 and c_2 are arbitrary constants, is - **[AIEEE 2009]**

(A) $y' = y^2$ (B) $y'' = y'\,y$

(C) $yy'' = y'$ (D) $yy'' = (y')^2$

Q.14 Solution of the differential equation

$\cos x\,dy = y(\sin x - y)\,dx,\ 0 < x < \frac{\pi}{2}$ is - **[AIEEE 2010]**

(A) $\sec x = (\tan x + c)\,y$

(B) $y\sec x = \tan x + c$

(C) $y\tan x = \sec x + c$

(D) $\tan x = (\sec x + c)\,y$

Q.15 Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is : **[AIEEE 2011]**

(A) $T^2 - \frac{I}{k}$ (B) $I - \frac{kT^2}{2}$

(C) $I - \frac{k(T-t)^2}{2}$ (D) e–kT

Q.16 If $\frac{dy}{dx} = y + 3 > 0$ and y (0) =2, then y (*ln* 2) is equal to : **[AIEEE 2011]**

(A) 7 (B) 5 (C) 13 (D) –2

Q.17 The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by – **[AIEEE 2011]**

(A) $2y - 3x = 0$ (B) $y = \frac{6}{x}$

(C) $x^2 + y^2 = 13$ (D) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$

Q.18 The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\,p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is : **[AIEEE 2012]**

(A) ln 9 (B) $\frac{1}{2}$ ln 18

(C) ln 18 (D) 2 ln 18

Q.19 At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is – **[JEE Main - 2013]**

(A) 3500 (B) 4500 (C) 2500 (D) 3000

SECTION-B

Q.1 The differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$ is (where a is a constant)- **[IIT 1992]**

(A) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\,\frac{d^2y}{dx^2}$

(B) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(C) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(D) None of these

Q.2 The solution of the differential equation

$(2x - 10y^3)\,\frac{dy}{dx} + y = 0$ is- **[IIT 1993]**

(A) $x + y = ce^{2x}$ (B) $y^2 = 2x^3 + c$

(C) $xy^2 = 2y^5 + c$ (D) $x(y^2 + xy) = 0$

Q.3 A curve C has the property that if the tangent drawn at any point P on C meets the coordinate axis at A and B, then P is the midpoint of AB. If the curve passes through the point (1, 1) then the equation of the curve is- **[IIT-1998]**

(A) xy = 2 (B) xy = 3

(C) xy = 1 (D) None of these

Q.4 The order of the differential equation whose general solution is given by
$y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$,
where c_1, c_2, c_3, c_4, c_5 are arbitrary constant is – **[IIT- 1998]**

(A) 5 (B) 4 (C) 3 (D) 2

Q.5 Let $(1 + t)\frac{dy}{dt} - ty = 1$, $y(0) = -1$, find $y(t)$ at $t = 1$? **[IIT Scr.2003]**

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $e-\frac{1}{2}$ (D) $e+\frac{1}{2}$

Q.6 If $y = y(x)$ satisfies $\frac{2+\sin x}{1+y}\left(\frac{dy}{dx}\right) = -\cos x$ such that $y(0) = 1$ then $y(\pi/2)$ is equal to- **[IIT Scr.2004]**

(A) 3/2 (B) 5/2 (C) 1/3 (D) 1

Q.7 $(x^2 + y^2)\,dy = xy\,dx$ (initial value problem), $y > 0$, $x > 0$, $y(1) = 1$, $y(x_0) = e$ then find $x_0 = ?$ **[IIT Scr.2005]**

(A) $\sqrt{\frac{e^2-1}{2}}$ (B) $\sqrt{2e^2-1}$

(C) $\sqrt{e^2-2}$ (D) $\sqrt{3}\,e$

Q.8 $xdy - ydx = y^2dy$, $y > 0$ & $y(1) = 1$ then find $y(-3) = ?$ **[IIT Scr.2005]**

(A) 3 (B) 2 (C) 4 (D) 5

Q.9 The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with **[IIT-2007]**

(A) variable radii and a fixed centre at (0, 1)
(B) variable radii and a fixed centre at (0, –1)
(C) fixed radius 1 and variable centers along the x–axis
(D) fixed radius 1 and variable centers along the y–axis

Q.10 Let f(x) be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x} = 1$ for each $x > 0$. Then f(x) is- **[IIT- 2007]**

(A) $\frac{1}{3x}+\frac{2x^2}{3}$ (B) $\frac{-1}{3x}+\frac{4x^2}{3}$

(C) $\frac{-1}{x}+\frac{2}{x^2}$ (D) $\frac{1}{x}$

Q.11 Let a solution $y = y(x)$ of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$

STATEMENT -1

$y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$ and

STATEMENT-2

y(x) is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$ **[IIT 2008]**

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 False, Statement-2 is True

Q.12 If $y' = y + 1$ and $y(0) = 1$, then value (s) of $y(\ell n\,2)$ **[IIT 2009]**

(A) 2 (B) 3 (C) 4 (D) 5

Q.13 Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)\,dt = 3x f(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is **[IIT 2011]**

Q.14 If y (x) satisfies the differential equations $y' - y\tan x = 2x\sec x$ and $y(0) = 0$, then **[IIT 2012]**

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Q.15 A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$. Then the equation of the curve is – **[JEE - Advance 2013]**

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (B) $\text{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	A	C	A	C	B	A	A	A	D	A	B	D	C	C	A	B	C	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	A	B	A	C	A	C	D	C	C	D	D	B	B	D	A	C	A	B	A	A
Q.No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	C	A	A	C	B	A	C	A	A	A	A	D	A	A	A	C	A	C	B	C
Q.No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans.	A	B	C	A	A	B	B	C	A	A	C	B	D	A	A	C	A	A	C	A

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	D	A	B	D	C	B	A	C
Q.No.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	B	A	D	C	B	A	A	B	C	B
Q.No.	**21**	**22**								
Ans.	B	A								

LEVEL- 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	D	A	A	A	A	A	B	B	C	A	C	A	D	C	B	B	B	C
Ques.	**21**	**22**	**23**	**24**	**25**	**26**	**27**													
Ans.	A	B	C	A	D	B	A													

LEVEL- 4

SECTION-A

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	A	D	C	C	C	B	C	C	C	C	B	C	D	A	B	A	B	D	A

SECTION-B

1.[B] $(x-h)^2 + (y-k)^2 = a^2$

diff. w.r.t. to x

$$(x-h) + (y-k)\frac{dy}{dx} = 0$$

$$1 + (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

Solving we get

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$$

2.[C] $y\frac{dx}{dy} + 2x = 10y^3$

$$\Rightarrow \frac{dx}{dy} + \frac{2x}{y} = 10y^2$$

$$\text{I.F.} = e^{\int \frac{2}{y}dy} = y^2$$

$$x.y^2 = \int 10y^4 dy$$

$$\Rightarrow \quad xy^2 = 2y^5 + c$$

3.[C] Let P(x, y) then equation of tangent is

$(Y - y) = \frac{dy}{dx}(X - x)$

According to question

$A\left(x - y\frac{dx}{dy}, 0\right)$ and $B\left(0, x\frac{dy}{dx} - y\right)$

$\because \quad 2x = x - y\frac{dx}{dy}$

$\Rightarrow \quad \int \frac{dy}{y} = -\int \frac{dx}{x}$

$\ell n\,|y| = -\ell n\,|x| + \ell n\, c$

$\Rightarrow \quad xy = c$

It is passes through (1, 1) $\Rightarrow c = 1$

$\Rightarrow \quad xy = 1$

4.[C] $y = a \cos (x + c_3) - be^x$

where $c_1 + c_2 = a$ and $c_4 e^{c_5} = b$

order = 3

5.[A] $\frac{dy}{dt} - \frac{t}{1+t}y = \frac{1}{1+t}$

I.F. $= e^{-\int \frac{t}{1+t}dt} = e^{\ell n(1+t) - t} = (1+t)e^{-t}$

$y\,(1 + t)e^{-t} = \int e^{-t}dt$

$\Rightarrow \quad y(1 + t)e^{-t} = -e^{-t} + c$

$\because \quad y(0) = -1 \quad \Rightarrow \quad c = 0$

$\Rightarrow \quad y = -\frac{1}{1+t}$

y(t) at t = 1

$y = -1/2$

6.[C] $\frac{2+\sin x}{1+y}\frac{dy}{dx} = -\cos x$

$\Rightarrow \quad \int \frac{1}{1+y}dy = -\int \frac{\cos x}{2+\sin x}dx$

$\Rightarrow \quad \log|1 + y| = -\log|2 + \sin x| + \log C$

$\because \quad y(0) = 1 \quad \Rightarrow \quad C = 4$

$\Rightarrow \quad (1 + y)(2 + \sin x) = 4$

$\Rightarrow \quad y = \frac{4}{2+\sin x} - 1$

$y(\pi/2) = \frac{4}{3} - 1 = \frac{1}{3}$

7.[D] $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \int\left(\frac{1}{v^3} + \frac{1}{v}\right)dv = -\int \frac{dx}{x}$

$\Rightarrow -\frac{1}{2v^2} + \log v = -\log x + c$

$\Rightarrow -\frac{x^2}{2y^2} + \log y = c$

$\because y(1) = 1 \Rightarrow c = -\frac{1}{2}$

$\Rightarrow -\frac{x^2}{2y^2} + \log y = -\frac{1}{2}$

$\because y(x_0) = e$

$\Rightarrow -\frac{x_0^2}{2e^2} + 1 = -\frac{1}{2} \Rightarrow x_0 = \sqrt{3}\, e$

8.[A] $xdy - ydx = y^2dy$

$\Rightarrow \frac{ydx - xdy}{y^2} = -dy$

$\Rightarrow \int \frac{d}{dy}\left(\frac{x}{y}\right)dy = -\int dy$

$\Rightarrow \frac{x}{y} = -y + c$

$y(1) = 1 \Rightarrow c = 2$

$\Rightarrow x = -y^2 + 2y$

Put $x = -3$

$\Rightarrow y^2 - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$\because y > 0$

$y = 3$

9.[C] $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

$\Rightarrow \int \frac{y}{\sqrt{1-y^2}}dy = \int dx \quad -\sqrt{1-y^2} = x \pm c$

$(1 - y^2) = (x \pm c)^2$

$x^2 + y \pm 2cx + c^2 - 1 = 0$

radius = 1, center $(\pm c, 0)$

10.[A] $\lim\limits_{t\to x}\dfrac{t^2f(x)-x^2f(t)}{t-x}=1 \quad \left(\dfrac{0}{0}\right)$

According to L'Hospital rule

Limit = $\lim\limits_{t\to x}\dfrac{2t\,f(x)-x^2f'(t)}{1}$

or $2x\,f(x) - x^2 f'(x) = 1$

If f(x) = y then $f'(x) = \dfrac{dy}{dx}$

$\therefore x^2\dfrac{dy}{dx} - 2xy = -1$

or $\dfrac{dy}{dx}-\dfrac{2}{x}y=-\dfrac{1}{x^2}$ (Linear)

I.F. = $e^{\int -\frac{2}{x}dx} = e^{\log 1/x^2} = 1/x^2$

Both side multiplied by I.F. and integrating

$y.\dfrac{1}{x^2}=\int\left(-\dfrac{1}{x^2}\right).\dfrac{1}{x^2}dx + c = \dfrac{1}{3x^3}+c$

$\therefore y = \dfrac{1}{3x} + cx^2$. given : x = 1, y = f(1) = 1

$\therefore 1 = \dfrac{1}{3}+c \qquad \therefore c = \dfrac{2}{3}$

$\therefore y = \dfrac{1}{3x}+\dfrac{2}{3}x^2$

11.[C] $\int\dfrac{dy}{y\sqrt{y^2-1}}-\int\dfrac{dx}{x\sqrt{x^2-1}}=c$

$\sec^{-1}y - \sec^{-1}x = c$

$\therefore y(2) = \dfrac{2}{\sqrt{3}} \Rightarrow c = \dfrac{-\pi}{6}$

$\Rightarrow y = \sec\left(\sec^{-1}x - \dfrac{\pi}{6}\right)$

Statement –I is true,
Statement–II is false

12.[B] $y' = y + 1$

$\Rightarrow \int\dfrac{dy}{y+1}=\int dx$

$\ln(y+1) = x + c$

$\because y(0) = 1 \Rightarrow c = \ln 2 \Rightarrow \ln(y+1) = x + \ln 2$

at x = ln2 we have

$\ln(y+1) = 2\ln 2 \Rightarrow y + 1 = 4 \Rightarrow y = 3$

13.[6] $6\int_1^x f(t)\,dt = 3xf(x) - x^3$

$6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$3f(x) = 3x f'(x) - 3x^2$

$3y = 3x\dfrac{dy}{dx} - 3x^2$

$x\dfrac{dy}{dx} - y = x^2$

$\dfrac{dy}{dx} - \dfrac{y}{x} = x$

I.F. = $e^{\int\frac{-1}{x}dx} = e^{-\ell n\,x} = \dfrac{1}{x}$

$y.\dfrac{1}{x} = \int x.\dfrac{1}{x}dx \Rightarrow \dfrac{y}{x} = x + c \Rightarrow y = x^2 + cx$

$\because f(1) = 2 \Rightarrow c = 1$

$y = x^2 + x$

$f(2) = 4 + 2 = 6$

14.[A,D]

$\dfrac{dy}{dx} - y\tan x = 2x\sec x$

P = tan x

I.F. = $e^{-\int\tan x\,dx} = \cos x$

$y.\cos x = \int 2x.dx$

$y\cos x = x^2 + C$

$\because y(0) = 0 \Rightarrow C = 0$

$y = x^2\sec x$

$y\left(\dfrac{\pi}{4}\right)=\dfrac{\pi^2}{8\sqrt{2}} \quad y\left(\dfrac{\pi}{3}\right)=\dfrac{2\pi^2}{9}$

$y'(\pi) = 2x\sec x + x^2\sec x\tan x$

$y'\left(\dfrac{\pi}{4}\right)=\dfrac{\pi}{\sqrt{2}}+\dfrac{\pi^2}{16}\sqrt{2}=\dfrac{\pi}{\sqrt{2}}+\dfrac{\pi^2}{8\sqrt{2}}=\dfrac{9\pi^2}{8\sqrt{2}}$

$y'\left(\dfrac{\pi}{3}\right)=\dfrac{2\pi}{3}\times 2+\dfrac{\pi^2}{9}2.\sqrt{3}$

15.[A] $\dfrac{dy}{dx}=\dfrac{y}{x}+\sec\dfrac{y}{x}$

y = vx

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$v + x\dfrac{dv}{dx} = v + \sec v$

$\int\cos v\,dv = \int\dfrac{dx}{x}$

$\sin v = \ell n|x| + c \quad \Rightarrow \sin\dfrac{y}{x} = \ell n|x| + c$

As curve passes through $\left(1,\dfrac{\pi}{6}\right)$

So $\sin\dfrac{\pi}{6} = 0 + c \Rightarrow c = \dfrac{1}{2}$

So $\sin\dfrac{y}{x} = \log x + \dfrac{1}{2}$