MATHEMATICS

Class-IX

Topic-12

SURFACE AREA AND VOLUME

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CH-12
SURFACE AREA AND VOLUME

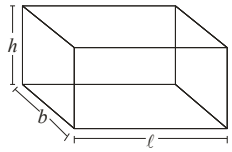
A. CUBE & CUBOID

(a) Cuboid

There are six faces (rectangular), eight vertices and twelve edges in a cuboid.

Total Surface Area (T.S.A.) : The area of surface from which cuboid is formed.

(i) Total Surface Area (T.S.A.) $= 2[\ell \times b + b \times h + h \times \ell]$ sq. units



(ii) Lateral Surface Area (L.S.A.) $= 2[b \times h + h \times \ell]$

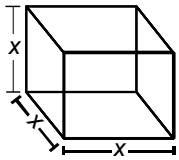
(or Area of 4 walls) $= 2h[\ell + b]$ sq. units

(iii) Volume of Cuboid = (Area of base) × height $= (\ell \times b) \times h$ cubic units

(iv) Length of diagonal = $\sqrt{\ell^2 + b^2 + h^2}$ unit

(b) Cube

Cube has six faces. Each face is a square.



(i) T.S.A. $= 2[x \cdot x + x \cdot x + x \cdot x] = 2[x^2 + x^2 + x^2] = 2(3x^2) = 6x^2$ sq. units
(ii) L.S.A. $= 2[x^2 + x^2] = 4x^2$ sq. units
(iii) Volume = (Area of base) × Height $= (x^2) \times x = x^3$ cubic units
(iv) Length of diagonal $= x\sqrt{3}$ unit

Solved Examples

Example. 1

Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of three cubes.

Sol. Let the side of each of the three equal cubes be a cm.

Then, surface area of one cube $= 6a^2$ cm^2

$\therefore$ Sum of the surface areas of three cubes $= 3 \times 6a^2 = 18a^2$ cm^2.

For new cuboid

length $(\ell) = 3a$ cm

breadth $(b) = a$ cm

height $(h) = a$ cm

$\therefore$ Total surface area of the new cuboid

$= 2(\ell \times b + b \times h + h \times \ell) = 2[3a \times a + a \times a + a \times 3a] = 2[3a^2 + a^2 + 3a^2] = 14a^2$ cm^2

$$\therefore \text{Required ratio} = \frac{\text{Total surface area of the new cuboid}}{\text{Sum of the surface areas of three cubes}} = \frac{14a^2}{18a^2} = \frac{7}{9} = 7:9.$$

Example.2

A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m × 1.4 m and three windows each measuring 2 m × 1 m The interior walls are to be colour-washed. The contractor charge Rs. 15 per sq.m. Find the cost of colour washing.

Sol. ℓ = 7 m, b = 6.5 m and h = 4 m

$\therefore$ Area of the 4 walls of room = $2(\ell + b)\, h = 2(7 + 6.5)\, 4 = 108\ m^2$

Area of 1 door = $3 \times 1.4 = 4.2\ m^2$

Area of 1 window = $2 \times 1 = 2\ m^2$

$\therefore$ Area of 3 windows = $3 \times 2 = 6\ m^2$

$\therefore$ Area of the walls of the room to be colour washed = $108 - (4.2 + 6) = 108 - 10.2 = 97.8\ m^2$

$\therefore$ Cost of colour washing @ Rs. 15 per square metre = Rs. 97.8×15 = Rs. 1467.

Example.3

The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many persons can sit in the hall, if each required 150 m^3 of air ?

Sol. ℓ = 100 m, b = 50 m & h = 18 m

$\therefore$ Volume of the cinema hall = $\ell bh = 100 \times 50 \times 18 = 90000\ m^3$

Volume occupied by 1 person = $150\ m^3$

$\therefore$ Number of persons who can sit in the hall = $\dfrac{\text{Volume of the hall}}{\text{Volume occupied by 1 person}} = \dfrac{90000}{150} = 600.$

Hence, 600 persons can sit in the hall.

Example. 4

The outer measurements of a closed wooden box are 42 cm, 30 cm and 27 cm. If the box is made of 1 cm thick wood, determine the capacity of the box.

Sol. **Outer dimensions**

L = 42 cm, B = 30 cm & H = 27 cm

Thickness of wood = 1 cm

Inner dimensions

$\ell = 42 - (1 + 1) = 40$ cm

$b = 30 - (1 + 1) = 28$ cm

$h = 27 - (1 + 1) = 25$ cm

$\therefore$ Capacity of the box = $\ell \times b \times h = 40 \times 28 \times 25 = 28000\ cm^3$.

Example. 5

If v is the volume of a cuboid of dimensions a, b, and c and s is its surface area, then prove that $\dfrac{1}{v} = \dfrac{2}{s}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$.

Sol. Volume of cuboid (v) = abc cubic unit.

Surface area of cuboid (s) = 2 (ab + bc + ca) sq. units

L.H.S. = $\dfrac{1}{v} = \dfrac{1}{abc}$ ... (i)

R.H.S. = $\dfrac{2}{s}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right) = \dfrac{2}{2(ab + bc + ca)}\left(\dfrac{bc + ca + ab}{abc}\right)$

$= \dfrac{1}{abc}$ .. (ii)

From (i) and (ii) . $\dfrac{1}{v} = \dfrac{2}{s}\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$. **Hence proved.**

Example. 6

Three cubes of metal with edges 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. Find the lateral surface area and volume of the new cube formed.

Sol. Volume of the new cube = $(3^3 + 4^3 + 5^3)$ cm^3 = 216 cm^3
Let, the edge of this cube = a cm.
Then, $a^3 = 216$.
$\therefore$ a = 6 cm.
Lateral surface area of the new cube = $4a^2 = 4(6)^2 = 4 \times 36 = 144$ cm^2.

Example. 7

An open wooden box 80 cm long, 65 cm wide and 45 cm high, is made of 2.5 cm thick wood. Find
(i) the capacity of the box
(ii) volume of the wood used
(iii) weight of the box, it is being given that 100 cm^3 of wood weighs 8 g.

Sol. External length of the box = 80 cm.
External breadth of the box = 65 cm.
External height of the box = 45 cm.
External volume of the box = (80 × 65 × 45) cu cm
= 234000 cm^3
Internal length of the box = [80 – (2.5 × 2)] cm
= (80 – 5) cm = 75 cm
Internal breadth of the box = [65 – (2.5 × 2)] cm
= (65 – 5) cm = 60 cm
Internal height of the box = (45 – 2.5) cm = 42.5 cm
(i) Capacity of the box = Internal Volume of the box
= (75 × 60 × 42.5) cm^3
= 191250 cm^3.
(ii) Volume of wood used
= (External Volume) – (Internal Volume)
= (234000 – 191250) cm^3 = 42750 cm^3.
(iii) Weight of 100 cm^3 of wood = 8 g

Weight of 42750 cm^3 of wood $= \left(\frac{8}{100} \times 42750\right)$ g
= 3420 g = 3 kg 420 g.

Check Your Level

1. Find the volume and total surface area of the cuboid with the following data :
length = 80 cm breadth = 50 cm and height = 40 cm

2. Find the volume and total surface area of a cube of.
(i) side 40 cm **(ii)** side 120 cm **(iii)** side 15 cm

3. The perimeter of a rectangular hall is 250 m. If the cost of painting the walls at the rate of Rs. 10 per square meter is Rs. 15000, find the height of the hall.

4. A village has a population of 8000. Each person needs 150 litres of water per day. The village has a tank measuring 20 m × 15 m × 6 m. The tank is full. How many days will the water last?

Answers

1. Volume = 160000 cm^3 , Total surface area = 18400 cm^2

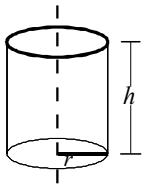
2. **(i)** Volume = 64000 cm^3 , Total surface area = 9600 cm^2
(ii) Volume = 1728000 cm^3 , Total surface area = 86400 cm^2
(iii) Volume = 3375 cm^3 , Total surface area = 1350 cm^2

3. 6 meter. **4.** 1.5 days

B. CYLINDER & CONE

(a) Cylinder

Curved surface area of cylinder (C.S.A.) : It is the area of surface from which the cylinder is formed. When we cut this cylinder along its height, we will find a rectangle with length $2\pi r$ and breadth h units.

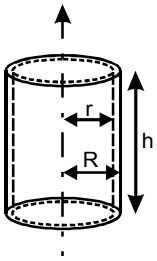


(i) C.S.A. of cylinder = $(2\pi r) \times h = 2\pi rh$ sq. units

(ii) T.S.A. = C.S.A. + Area of circular top & bottom
$= 2\pi rh + (\pi r^2) + (\pi r^2)$
$= 2\pi rh + 2\pi r^2$
$= 2\pi r\,(h + r)$ sq. units

(iii) Volume of cylinder = Area of base × height
$= (\pi r^2) \times h$
$= \pi r^2 h$ cubic units

Hollow cylinder :



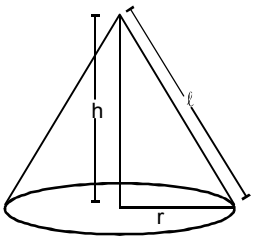
(i) C.S.A. of hollow cylinder = $2\pi\,(R + r)\,h$ sq. units

(ii) T.S.A. of hollow cylinder $= 2\pi\,(R + r)h + 2\pi(R^2 - r^2)$ sq. units

(iii) Volume of hollow cylinder = $\pi\,(R^2 - r^2)h$ cubic units

where, r = Inner radius of Cylinder
R = Outer radius of Cylinder
h = Height of the Cylinder

(b) Cone



(i) C.S.A. of cone = $\pi r\ell$

(ii) T.S.A. of cone = C.S.A. + Base area $= \pi r\ell + r^2$

$= \pi r(\ell + r)$

(iii) Volume of cone $= \frac{1}{3}\pi r^2 h$

Where, h = height
r = radius of base
ℓ = slant height

Solved Examples

Example. 8

A cylindrical vessel, without lid, has to be tin coated both of its surface (inner and outer). If the radius of its base is $\frac{1}{2}$ m and its height is 1.4 m, calculate the cost of tin-coating at the rate of Rs 50 per 1000 cm^2.

Sol. Radius of the base (r) = $\frac{1}{2}$m = $\frac{1}{2}$ × 100 cm = 50 cm

Height (h) = 1.4 m = 1.4 × 100 cm = 140 cm.

Surface area of cylinder to be tin-coated

$= 2(2\pi rh + \pi r^2)$

$= 2[2 \times 3.14 \times 50 \times 140 + 3.14 \times (50)^2]$

$= 2[43960 + 7850] = 2(51810) = 103620\ cm^2$

∴ Cost of tin-coating at the rate of Rs 50 per 1000 cm^2

$= \frac{50}{1000} \times 103620$ = Rs 5181.

Example. 9

The diameter of a roller 120 cm long is 84 cm. If it takes 500 complete revolutions to level a playground, determine the cost of levelling at the rate of Rs 25 per square metre. [Use $\pi = \frac{22}{7}$]

Sol. 2r = 84 cm

∴ r = $\frac{84}{2}$ cm = 42 cm

h = 120 cm.

Area of the playground levelled in one complete revolution = $2\pi rh$

$= 2 \times \frac{22}{7} \times 42 \times 120 = 31680\ cm^2$

∴ Area of the playground levelled in 500 revolutions = 31680 × 500 cm^2

$= \frac{31680 \times 500}{100 \times 100}\ m^2 = 1584\ m^2$.

∴ Cost of levelling @ Rs 25 per square metre

= Rs. 1584 × 25 = Rs. 39600.

Example. 10

A lead pencil consists of a cylinder of wood with a solid cylinder of graphite fitted into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 14 cm. Calculate

(i) Volume of the graphite

(ii) Volume of the wood

(iii) The weight of the whole pencil, if the specific gravity of the wood is 0.7 g/ cm^3 and that of the graphite is 2.1 g/cm^3.

Sol. **(i)** We have,

Diameter of the graphite cylinder = 1 mm = $\frac{1}{10}$ cm

$\therefore$ Radius of the graphite cylinder = $\frac{1}{20}$ cm

Length of the graphite cylinder = 14 cm

V_1 = Volume of the graphite cylinder

= $\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14\ cm^3 = 0.11\ cm^3$

(ii) We, have,

Diameter of pencil = 7 mm = $\frac{7}{10}$ cm

$\therefore$ Radius of pencil = $\frac{7}{20}$ cm

and, Length of pencil = 14 cm

$\therefore$ V_2 = Volume of pencil = $\frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14\ cm^3 = 5.39\ cm^3$

Volume of wood $V_2 - V_1 = (5.39 - 0.11)\ cm^3 = 5.28\ cm^3$

(iii) we have,

Specific gravity of wood = 0.7 gm/cm^3

and Specific gravity of graphite = 2.1 gm/cm^3

$\therefore$ Weight of the pencil = Volume of wood × specific gravity of wood + Volume of graphite × specific gravity of graphite

= (5.28 × 0.7 + 0.11 × 2.1) gm = 3.927 gm.

Example. 11

How many metres of cloth of 1.1 m width will be required to make a conical tent whose vertical height is 12 m and base radius is 16 m ? Find also the cost of the cloth used at the rate of Rs 14 per metre.

Sol. h = 12 m

r = 16 m

$\therefore$ $\ell = \sqrt{r^2 + h^2}$

= $\sqrt{(16)^2 + (12)^2} = \sqrt{256 + 144}$

= $\sqrt{400} = 20$ m.

$\therefore$ Curved surface area = $\pi r \ell$

= $\frac{22}{7} \times 16 \times 20$.

= $\frac{7040}{7}\ m^2$

Width of cloth = 1.1 m

$\therefore$ Length of cloth = $\frac{7040/7}{1.1} = \frac{70400}{77} = \frac{6400}{7}$ m

$\therefore$ Cost of the cloth used @ Rs 14 per metre

= Rs $\frac{6400}{7} \times 14$ = Rs 12800.

Example. 12

The ratio of the volumes of the two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. Find the ratio of their vertical heights.

Sol. Let the radii of bases, vertical heights and volumes of the two cones be r_1, h_1, v_1 and r_2, h_2, v_2 respectively.

According to the question,

$$\frac{v_1}{v_2} = \frac{4}{5} \quad \text{... (i)} \qquad \frac{r_1}{r_2} = \frac{2}{3} \quad \text{... (ii)}$$

From (i), we have $\dfrac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{4}{5}$

$$\Rightarrow \quad \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{4}{5} \quad \Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 \frac{h_1}{h_2} = \frac{4}{5} \quad \Rightarrow \quad \left(\frac{2}{3}\right)^2 \frac{h_1}{h_2} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{4}{5}\left(\frac{3}{2}\right)^2 \quad \text{[Using (ii)]}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{9}{5}$$

Hence the ratio of their vertical height is 9 : 5.

Example. 13

If h, c and v be the height, curved surface and volume of a cone, show that $3\pi vh^3 - c^2h^2 + 9v^2 = 0$.

Sol. Let the radius of the base and slant height of the cone be r and ℓ respectively. Then,

$$c = \text{curved surface} = \pi r\ell = \pi r\sqrt{r^2 + h^2} \quad \text{... (i)}$$

$$v = \text{volume} = \frac{1}{3}\pi r^2 h \quad \text{... (ii)}$$

LHS

$3\pi vh^3 - c^2h^2 + 9v^2 = 3\pi vh^3 - c^2h^2 + 9v^2$

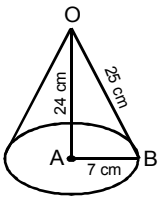
$= 3\pi\left(\frac{1}{3}\pi r^2 h\right)h^3 - \pi^2 r^2(r^2 + h^2)\,h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2$ [Using (i) and (ii)]

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0.$ **Hence proved.**

Example. 14

How many meters of the cloth, 5 m wide, will be required to make a conical tent, the radius of whose base is 7 m and the height is 24 m ?

Sol.



Radius of the tent, r = 7 m and its height, h = 24 m.

$\therefore$ Slant height, $\ell = \sqrt{r^2 + h^2} = \sqrt{(7)^2 + (24)^2}$ m $= \sqrt{625}$ m = 25 m.

Area of the curved surface = $(\pi r\ell)$ sq m

$$= \left(\frac{22}{7} \times 7 \times 25\right) m^2 = 550\ m^2.$$

Thus, the area of the cloth = 550 m^2

Length of the both required = $\left(\frac{\text{area}}{\text{width}}\right) = \left(\frac{550}{5}\right)$ m = 110 m.

Hence, length of the cloth required is 110 m.

Example. 15

A bus stop is barricated from the remaining part of the road, by using 50 hollow cones made of recycled card - board. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted is Rs. 12 per m^2, what will be the cost of painting all these cones.

[Use $\pi = 3.14$ and $\sqrt{1.04} = 1.02$]

Sol. We have,

r = radius of base = 20 cm = 0.2 m,

h = height of cone = 1 m.

$\therefore \quad \ell = \sqrt{r^2 + h^2} = \sqrt{0.04 + 1}$ m = $\sqrt{1.04}$ m = 1.02 m

Curved surface area of a cone $= \pi r\ell$

$= 3.14 \times 0.2 \times 1.02\ m^2$

Cost of painting = Rs. [(3.14 × 0.2 × 1.02 × 12) × 50] = Rs. 384.34.

Example. 16

A conical tent is 9 m high and radius of its base is 12 m.

(i) What is the cost of the canvas required to make it, if a square meter canvas costs Rs. 10 ?

(ii) How many persons can be accommodated in the tent, if each person requires 2 square meter on the ground and 15 m^3 of space to breathe in ?

Sol. We have

r = Radius of the base of the conical tent = 12 m

h = Height of the conical tent = 9 m

ℓ = Slant height of the conical tent = $\sqrt{r^2 + h^2} = \sqrt{12^2 + 9^2}$ m = $\sqrt{225}$ m = 15 m.

(i) Area of lateral surface = $\pi r\ell = \frac{22}{7} \times 12 \times 15\ m^2 = 565.7\ m^2$

$\therefore$ Total cost of canvas = Rs (565.7 × 10) = Rs.5657

(ii) Area of the base of the conical tent

$\pi r^2 = \frac{22}{7} \times 12 \times 12\ m^2 = 452.57\ m^2$

Since each person requires 2 sq. metres of floor area.

$\therefore$ Max. no. of persons who will have enough space on the ground = $\frac{452.57}{2} = 226$

Again, Volume of the conical tent

$= \frac{1}{3} \times$ Area of the base × Height

$= \frac{1}{3} \times 452.57 \times 9\ m^3 = 1357.71\ m^3$

Volume of the conical tent = 1357.71 m^3.

We have, Air space required for each person = 15 m^3

$\therefore$ No. of persons who will have enough air space to breathe in = $\frac{1357.71}{15} = 90.$

Between 226 and 90, the smaller number is 90.

Hence 90 persons can be accommodated.

Check Your Level

1. Find the volume of cone having radius 7m and height 12 m.

2. Find the total surface area of cylinder having radius 7 m and height 15 m

3. The volume of cone is 4928 cm^3 if its height is 24 m find its radius.

4. A right triangle with side 3,4,5 is revolved about the side 4 find the volume of solid so formed.

5. Find the volume of hollow cylinder having inner and outer radius is 7cm and 21 cm respectively and have height 14 cm. then find its volume.

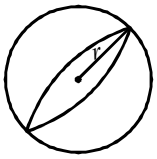
Answers

1. 616 m^3 **2.** 968m^2 **3.** 140 cm **4.** 37.71m^3

5. 17248 cm^3

C. SPHERE & HEMISPHERE

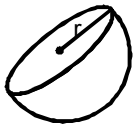
(a) Sphere



(i) T.S.A. of sphere $= 4\pi r^2$

(ii) Volume of sphere $= \frac{4}{3}\pi r^3$

(b) Hemisphere



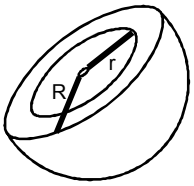
(i) C.S.A. $= 2\pi r^2$

(ii) T.S.A. = C.S.A. + Base circular area

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

(iii) Volume $= \frac{2}{3}\pi r^3$

Hollow Hemisphere :



(i) C.S.A. $= 2\pi (R^2 + r^2)$

(ii) T.S.A. $= 2\pi (R^2 + r^2) + \pi(R^2 - r^2)$

(iii) Volume $= \frac{2}{3}\pi (R^3 - r^3)$

Solved Examples

Example. 17

Find volume and surface area of a sphere of diameter 14 cm.

Sol. Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\frac{14}{2}\times\frac{14}{2}\times\frac{14}{2} = 1437\frac{1}{3}$ cubic centimetre.

Surface area = $4\pi r^2 = 4\times\frac{22}{7}\times\frac{14}{2}\times\frac{14}{2}$ = 616 square centimetre.

Example. 18

Find the volume of the metal used to make a hollow sphere of inner radius 3.5 cm and outer radius 4.2 cm.

Sol. Volume of metal used

$= \frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3}\times\frac{22}{7}\left[(4.2)^3 - (3.5)^3\right]$

$= \frac{4}{3}\times\frac{22}{7}\times(74.088 - 42.875)$

$= \frac{4}{3}\times\frac{22}{7}\times 31.213$ = 130.7973 cubic centimetre.

Example. 19

If the surface area of a sphere is 616 square centimetre, find its volume.

Sol. Let the radius of the sphere be 'r'

Surface area = $4\pi r^2 = 616$

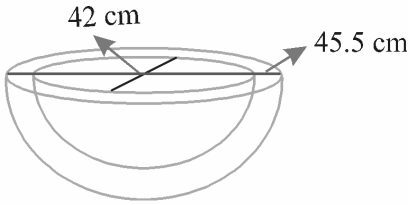
$r^2 = \frac{616}{4}\times\frac{7}{22} = 49$ cm

$\therefore$ r = 7 cm

$\therefore$ The volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 7\times 7\times 7 = 1437\frac{1}{3}$ cubic centimetre

Example. 20

The internal and external diameters of a hollow hemispherical vessel are 42 cm and 45.5 cm respectively. Find its capacity and the outer curved area.



Sol. Internal radius $r = \frac{42}{2} = 21$ cm

External radius $R = \frac{45.5}{2} = 22.75$ cm

Capacity of the vessel

= $\frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times 21\times 21\times 21$ = 19404 cm^3

Outer surface area = $2\times\frac{22}{7}\times(22.75)^2$ = 3253. 25 cm^2

Example. 21

Find the diameter of a sphere whose volume is $179\frac{2}{3}$ cubic metres

Sol. Volume = $\frac{4}{3}\pi r^3 = 179\frac{2}{3} = \frac{539}{3}$

$r^3 = \frac{539}{3} \times \frac{3}{4} \times \frac{7}{22}$

$r^3 = \frac{343}{8}$

$\therefore\ r = \frac{7}{2}$ metres

Diameter, 2r = 7 metres.

Check Your Level

1. Find the surface area and volume of the sphere having diameter 7 cm.

2. Surface areas of spheres is 154 cm^2. Find its volumes

3. How much water will a hemispherical bowl, whose radius is 2 metres, contain?

4. A hollow hemisphere has inner radius of 5 cm and thickness 1 cm. Find the volume of the hollow hemisphere.

5. A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is Rs. 5 per 100 cm^2.

Answers

1. S=154 cm^2 V=179.66m^3 **2.** V = 179.66m^3 **3.** 16761.90 liter

4. 190.66 cm^3 **5.** Rs.24640

D. INTER CONVERSION AND MIXED PROBLEMS

In this section, we shall discuss problems pertaining to conversion of a solid (discussed in the previous classes) into another solid of different shape. For example, a metallic sphere is melted and recast into a cylindrical wire, the earth taken out by digging a well and spreading it uniformly around the well to form an embankment in the form of a cylindrical shell from its original shape of right circular cylinder, etc.

Solved Examples

Example. 22

The base diameter of a solid in the form of a cone is 6 cm and the height of the cone is 10 cm. It is melted and recast into spherical balls of diameter 1 cm. Find the number of balls, thus obtained.

Sol. Let the number of spherical balls be n. Then, the volume of the cone will be equal to the sum of the volumes of the spherical balls. The radius of the base of the cone = cm = 3 cm

and the radius of the sphere = $\frac{1}{2}$ cm.

Now, the volume of the cone = $\frac{1}{3}\pi \times 3^2 \times 10\ cm^3 = 30\ cm^3$

and, the volume of each sphere = $\frac{4}{3}\pi\left(\frac{1}{2}\right)^3 \text{ cm}^3 = \frac{\pi}{6}\text{cm}^3$

Hence, we have

$n\frac{\pi}{6} = 30$ $\qquad$ $n = 6 \times 30 = 180$

Hence, the required number of balls = 180.

Example. 23

The diameters of external and internal surfaces of a hollow spherical shell are 10 cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

Sol. Let the radius of base of the cylinder be 'r' cm.

So, volume of metallic solid cylinder of height $2\frac{2}{3}$ cm $(\frac{8}{3}\text{cm})$ = Volume of metal in the spherical shell

$\pi r^2 \times \frac{8}{3} = \frac{4}{3}\pi(5^3 - 3^3) \Rightarrow r^2 = \frac{4}{3} \times \frac{3}{8}(98) \Rightarrow r^2 = 49 \Rightarrow r = 7$ cm

Hence, diameter of the cylinder = 14 cm.

Example. 24

By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder and the ratio of the height of the cylinder and the height of the cone is 4 : 3, find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively. Then,

$3r = 2R$ and $H : h = 4 : 3$ ....(i)

$\frac{H}{h} = \frac{4}{3}$

$3H = 4h$ ....(ii)

Let n be the required number of cones which can be made from the materials of the cylinder. Then, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$\pi R^2H = \frac{n}{3}\pi r^2h$ $\qquad$ $3R^2H = nr^2h$

$n = \frac{3R^2H}{r^2h} = \frac{3 \times \frac{9r^2}{4} \times \frac{4h}{3}}{r^2h}$ $\qquad$ [From (i) and (ii), $R = \frac{3r}{2}$ and $H = \frac{4h}{3}$]

$= \frac{3 \times 9 \times 4}{3 \times 4} = 9$

Hence, the required number of cones is 9.

Example. 25

Water is flowing at the rate of 5 km per hour through a pipe of diameter 14 cm into a rectangular tank, which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 7 cm

Sol. Let the level of water in tank will rise by 7cm in x hrs.

As rate of flow of water is 5km/hr.

$\therefore$ Its length in x hrs = 5x = 5000 x metres Water column forms a cylinder whose radius is

$r = \frac{14}{2}\text{ cm} = \frac{7}{100}\text{m}$

Volume of water flowing through the cylindrical pipe in x hrs.

$= \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 5000 \text{ x m} = 77x \text{ m}^3.$

Vol. of water that falls into tank $= 50 \times 40 \times \frac{7}{100} \text{ m}^3 = 154 \text{ m}^3$

But, volume of water flowing through the cylindrical pipe in x hrs = Vol. of water that falls in tank in x - hrs.

$77x = 154$

$\therefore \quad x = \frac{154}{77} = 2 \text{ hr.}$

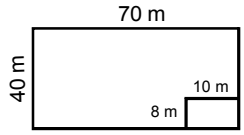
Example. 26

A field is 70 m long and 40 m broad. In one corner of the field, a pit which is 10 m long, 8 m broad and 5 m deep, has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Sol. Area of the field = $(70 \times 40) \text{ m}^2 = 2800 \text{ m}^2$.

Area of the pit = $(10 \times 8) \text{ m}^2 = 80 \text{ m}^2$

Area over which the earth is spread over = $(2800 - 80) \text{ m}^2 = 2720 \text{ m}^2$



Volume of the earth dug out = $(10 \times 8 \times 5) \text{ m}^3 = 400 \text{ m}^3$.

Rise in level of the field $= \left(\frac{\text{Volume of the earth dug out}}{\text{Area on which the earth is spread}}\right)$

$= \left(\frac{400}{2720}\right) \text{m} = \left(\frac{400 \times 100}{2720}\right) \text{cm} = \left(\frac{250}{17}\right) \text{cm} = 14.70 \text{ cm}$

Example. 27

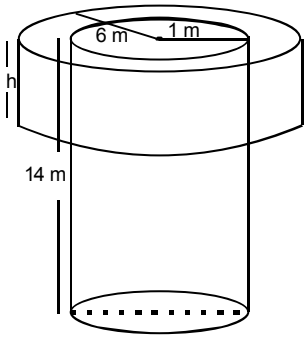
A well of diameter 2 m is dug 14 m deep. The earth taken out of it is spread evenly all around it to a width of 5 m to form an embankment. Find the height of the embankment.

Sol. Let **h** be the required height of the embankment.

The shape of the embankment will be like the shape of a cylinder of internal radius 1 m and external radius (5 + 1) m = 6 m.

The volume of the embankment will be equal to the volume of the earth dug out from the well. Now, the volume of the earth dug out = volume of the cylindrical well = $\pi \times 1^2 \times 14 \text{ m}^3 = 14 \pi \text{ m}^3$

Also, the volume of the embankment $= \pi (6^2 - 1^2) h \text{ cm}^3 = 35 \pi h \text{ m}^3$



Hence, we have

$35 \pi h = 14 \pi \qquad h = \frac{14}{35} = \frac{2}{5} = 0.4$

Hence, the required height of the embankment = 0.4 m.

Check Your Level

1. How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm ?

2. The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height of the cone.

3. What is the surface area and volume of the biggest sphere made out of cube of side 42 cm ?

4. How many spherical balls of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm ?

5. A hemispherical tank full of water and radius $1\frac{3}{4}$ metres is connected with a pipe which empties at the rate of 7 litres per second. How long will it take to empty the tank?

Answers

1. 512 **2.** 3 cm **3.** $S = 1764\ \pi\ cm^2$, $V = 12348\ \pi\ cm^3$

4. 64 **5.** 1604.16 second

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]

1. If the radius of a sphere is 2r, Find its volume.

2. If the total surface area of a cube is 96 cm^2. Find the volume of the cube.

3. Find the total surface area of a cone whose radius is $\frac{r}{2}$ and slant height is 2ℓ .

4. Find the length of the longest pole that can be put in a room of dimensions (10 m × 10 m × 5m).

5. If the length of the diagonal of a cube is $6\sqrt{3}$ cm, then find the length of the edge of the cube.

6. If a sphere is inscribed in a cube, then find the ratio of the volume of the cube to the volume of the sphere will be ?

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

7. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. Find the radius of the sphere.

8. The radii of two cylinders are in the ratio of 2:3 and their heights are in the ratio of 5:3. Find the ratio of their volumes.

9. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. Find the ratios of the surface areas of the balloon in the two cases.

10. An edge of a cube measures r cm. If the largest possible right circular cone is cut out of this cube, then find the volume of the cone (in cm^3)

11. Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely immersed in water.

12. How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m ?

13. The volumes of the two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.

14. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: [04 MARK EACH]

15. The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and the volume of the cone (taking $\pi = \frac{22}{7}$).

16. The radius of a sphere is increased by 10%. Prove that the volume will be increased by 33.1% approximately

17. Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer. [Use $\pi = 3.14$]

18. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface of the solid so formed.

19. Rain water which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if the rain fall is 1 cm. Give your answer to the nearest integer. (Take $\pi = 3.14$)

20. A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cubic centimeters of iron has been used in making the tube?

21. A semi-circular sheet of metal of diameter 28cm is bent to form an open conical cup. Find the capacity of the cup.

22. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Calculate the volume of water pumped into the tank.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[05 MARK EACH]**

23. A small village, having a population of 5000, requires 75 litres of water per head per day. The village has got an overhead tank of measurement 40 m × 25 m × 15 m. For how many days will the water of this tank last?

24. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m

(i) How many students can sit in the tent if a student, on an average, occupies $\frac{5}{7}$ m^2 on the ground ?

(ii) Find the volume of the cone.

25. 30 circular plates, each of radius 14 cm and thickness 3cm are placed one above the another to form a cylindrical solid. Find :

(i) the total surface area

(ii) volume of the cylinder so formed.

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Cube and Cuboid

A-1. Find the total surface area of a box whose length, breadth and height are 12 cm, 8 cm and 5 cm, respectively.

A-2. An open box is made of wood 3 cm thick. Its external length, breadth and height are 1.48 m, 1.16 m and 8.3 dm. Find the cost of painting the inner surface at Rs 5 per m^2.

A-3. A room 8 m long, 6 m broad and 3 m high has two windows $1\frac{1}{2}$ m × 1 m and a door 2 m × $1\frac{1}{2}$ m. Find the cost of papering the walls with paper 50 cm wide at Rs. 40 per meter.

A-4. The dimensions of a rectangular box are in the ratio 2 : 3 : 4 and the difference between the cost of covering it with the sheet of paper at the rate of Rs. 8 and Rs. 9.50 per m^2 is Rs. 1248. Find the dimensions of box.

A-5. Find the number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the cement and sand mixture occupies 1/20 of the volume of the wall.

Section (B) : Cylinder and Cone

B-1. The circumference of the base of 9 m high wooden solid cone is 44 m. Find the slant height of the cone.

B-2. The curved surface area of a right circular cylinder is 4400 cm^2. If the circumference of the base is 110cm, then find the height of the cylinder.

B-3. 50 circular plates, each of radius 7 cm and thickness $\frac{1}{2}$ cm, are placed one above the other to form a solid right circular cylinder. Find the total surface area.

B-4. The total surface area of the solid cylinder is 462 cm^2 and its curved surface area is of the total surface area. Find the volume of the cylinder.

B-5. A tent is in the shape of a right circular cylinder surmounted by a right circular cone. The heights of the cylindrical and the conical parts are 40 m and 21 m respectively. If the base diameter of the tent is 56 m, find the area of the required canvas to make this tent, if 20% of the area of canvas is consumed in folding and sewing.

B-6. A cylindrical metallic pipe is 14 cm long. The difference between the outside and inside surfaces is 44 cm^2. If the pipe is made up of 99 cubic cm of metal, find the outer and inner radii of the pipe.

B-7. A right triangle ABC with its sides 5 cm, 12 cm and 13 cm is revolved about side 12 cm. Find the volume of the solid so formed. If the triangle ABC is revolved about side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the two volumes obtained.

Section (C) : Sphere and Hemisphere

C-1. The internal and external diameters of a hollow hemispherical Vessal are 24 cm and 25 cm respectively. The cost to paint 1 cm^2 the surface is Rs. 0.05. Find the total cost to paint the vessel all over. [Use $\pi = \frac{22}{7}$]

C-2 A dome of the building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs. 498.96. If the cost of white - washing is Rs. 2.00 per square meter, find the inside surface area of the dome and volume of the air inside the dome.

C-3. Assuming the earth to be a sphere of radius 6370 km, how many square kilometers is area of the land, if three fourth of the earth's surface is covered by water ?

C-4. A hemispherical bowl is made of steel 0.5 cm thick. The inside radius of the bowl is 4 cm. Find the volume of steel used in making the bowl.

Section (D) : Inter conversion and mixed problems

D-1. A solid metallic sphere of diameter 28 cm is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

D-2. A toy is in the form of a right circular cylinder closed at one end and with a hemisphere on the other end. The height and the radius of the base are 15 cm and 6 cm respectively. The radius of the hemisphere and cylinder are same. Calculate the total surface area and the volume of the toy. If the toy is painted at the rate of Rs. 2.50 per 10 cm^2, find the cost of painting the toy.

D-3. An iron pillar has some portion in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm. The cylindrical portion is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar, if one cubic cm of iron weighs 7.8 g.

D-4. A spherical canon ball, 28 cm in diameter is melted and cast into a right circular conical mould, the base of which is 35 cm in diameter. Find the height of the cone, correct to one place of decimal.

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Cube and Cuboid

A-1. The percentage increase in the surface area of a cube, when each side is increased to $\frac{3}{2}$ times the original length is :

(A) 225 (B) 200 (C) 175 (D) 125

A-2. If 'l', 'b' and 'h' of a cuboid are increased, decreased and increased by 1%, 3% and 2% respectively, then the volume of the cuboid :

(A) increases
(B) decreases
(C) increases or decreases depending on original dimensions
(D) can't be calculated with given data

A-3. If the volume of a cube is 216 cm^3, then find its diagonals.

(A) $6\sqrt{3}$ cm (B) 6 cm (C) $\frac{6}{\sqrt{3}}$ cm (D) None of these

A-4. The whole surface of a rectangular block is 846 cm^2. Find the length, breadth and height, if these dimensions are in the ratio 5 : 4 : 3.

(A) 20 cm, 16 cm, 12 cm. (B) 15 cm, 12 cm, 9 cm.
(C) 10 cm, 8 cm, 6 cm. (D) None of these

Section (B) : Cylinder and Cone

B-1. The height of a conical tent at the centre is 5 m. The distance of any point on its circular base from the top of the tent is 13 m. The area of the slant surface is :

(A) 144 π sq m (B) 130 π sq m (C) 156 π sq m (D) 169 π sq m

B-2. A rectangular sheet of paper 22 cm long and 12 cm broad can be curved to form the lateral surface of a right circular cylinder in two ways by rolling along its length and breadth. Difference between the volumes of the two cylinders thus formed is : [Use $\pi = \frac{22}{7}$]

(A) 200 c.c. (B) 210 c.c. (C) 250 c.c. (D) 252 c.c.

B-3. The radius and height of a cone are each increased by 20%, then the volume of the cone is increased by :

(A) 20% (B) 40% (C) 60% (D) 72.8%

B-4. Consider a hollow cylinder of inner radius r and thickness of wall t and length ℓ. The volume of the above cylinder is given by :

(A) $2\pi\ell(r^2 - \ell^2)$ (B) $2\pi r\,\ell t\left(\frac{t}{2r}+1\right)$ (C) $2\pi\ell(r^2 + t^2)$ (D) $2\pi r\ell(r + t)$

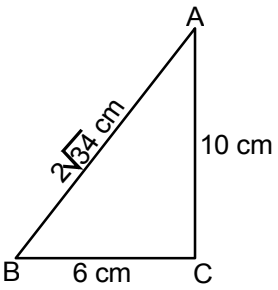
B-5. How many meters of cloth, 5 m wide, will be required to make a conical tent, the radius of whose base is 7 m and height is 24 m ?

(A) 550 m (B) 168 m (C) 110 m (D) 33.6 m

B-6. Find the total surface area of a cone whose radius is $\frac{r}{2}$ and slant height is 2ℓ

(A) $2\pi r(\ell + r)$ (B) $\pi r\left(\ell + \frac{r}{4}\right)$ (C) $\pi r(\ell + r)$ (D) $2\pi r\ell$

B-7. In the given figure, ΔABC is revolved about AC and a solid in the shape of cone is formed. The volume of the cone is :



(A) $24\sqrt{34}\ \pi$ cm^3 (B) $120\ \pi$ cm^3 (C) $12\sqrt{34}$ cm^3 (D) $240\ \pi$ cm^3

Section (C) : Sphere and Hemisphere

C-1. A sphere of radius 3 cm is dropped into a cylindrical vessel of radius 4 cm. If the sphere is submerged completely, then the height (in cm) to which the water rises, is :

(A) 2.35 (B) 2.30 (C) 2.25 (D) 2.15

C-2. The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon ?

(A) $\frac{1}{8}$ (B) $\frac{1}{64}$ (C) $\frac{1}{512}$ (D) None of these

C-3. Find the volume of the sphere whose surface area is 154 square cm.

(A) $\frac{77}{3}$ cm^3 (B) $\frac{49}{3}$ cm^3 (C) $\frac{539}{3}$ cm^3 (D) None of these

C-4 A hemispherical bowl has inner diameter 11.2 cm. Find the volume of milk it can hold.

(A) 367.96 ml (B) 347.96 ml (C) 337.96 ml (D) 397.96 ml

Section (D) : Inter conversion and mixed problems

D-1. There is a cylinder circumscribing the hemisphere such that their bases are common. The ratio of their volume is :

(A) 1 : 3 (B) 1 : 2 (C) 3 : 2 (D) 3 : 4

D-2. A cone and a cylinder have the same base area. They also have the same curved surface area. If the height of the cylinder is 3m, then the slant height of the cone (in m) is :

(A) 3 (B) 4 (C) 6 (D) 7

D-3. 27 metal balls each of radius r are melted together to form one big sphere of radius R. Then the ratio of surface area of the big sphere to that of a ball is :

(A) $\sqrt{27}$: 1 (B) $\sqrt{3}$: 1 (C) 3 : 1 (D) 9 : 1

D-4. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is :

(A) 10 : 17 (B) 20 : 27 (C) 17 : 27 (D) 20 : 37

Exercise-2

OBJECTIVE QUESTIONS

1. The length, breadth and height of a room are in the ratio 3: 2: 1. If the breadth and height are halved while the length is doubled, then the total area of the four walls of the room will :
(A) remain the same (B) decrease by 30%
(C) decrease by 15% (D) decrease by 18.75%

2. If the surface area of cube A is 64% of the surface area of cube B, then the volume of cube A is 'k' percent of the volume of cube B. The value of 'k' is
(A) 0.64 (B) 0.512 (C) 51.2 (D) 64

3. If the radius of a right circular cylinder is increased by 50% and height is decreased by 20% then the percentage change in volume of cylinder is
(A) 40% (B) 50% (C) 60% (D) 80%

4. John's birthday cake is a delightful cylinder of radius 6 inches and height 3 inches, If these friends cut the cake into 8 equal sectors, the total surface area of a piece of birthday cake in sq. inches, is :
(A) $9(\pi + 4)$ (B) $\frac{27\pi}{2}$ (C) $4(\pi + 4)$ (D) $\frac{9}{2}(3\pi + 8)$

5. A rectangular tin sheet is 12 cm long and 5 cm broad. It is rolled along its length to form a cylinder by making the opposite edges just to touch each other. The volume of the cylinder (in cm^3) is :
(A) $\frac{180}{\pi}$ (B) $\frac{120}{\pi}$ (C) $\frac{100}{\pi}$ (D) $\frac{60}{\pi}$

6. If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the volume of the reduced cylinder to that of the original one ?
(A) 1 : 4 (B) 1 : 8 (C) 1 : 2 (D) 8 : 1

7. The radius of a wire is decreased to one-third. If volume remains the same, the length will become :
(A) 1 time (B) 3 times (C) 6 times (D) 9 times

8. Three solid spherical beads of radii 3 cm, 4 cm and 5 cm are melted into a spherical bead. Its radius is
(A) 6 cm (B) 7 cm (C) 8 cm (D) 9 cm

9. If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the surface area of each ball (in sq. cm) is :
(A) 100π (B) 75π (C) 60π (D) 50π

10. If the height and diameter of a right circular cylinder are 32 cm and 6 cm respectively, then the radius of the sphere whose volume is equal to the volume of the cylinder is :
(A) 3 cm (B) 4 cm (C) 6 cm (D) None

11. A hollow spherical ball whose inner radius is 4 cm is full of water. Half of the water is transferred to a conical cup and it completely filled the cup. If the height of the cup is 2 cm, then the radius of the base of cone in cm is :
(A) 4 (B) 10 (C) 8 (D) 16

12. Radius and height of a cylinder are 7 cm and 14 cm respectively. It is divided along its axis vertically into equal parts. The increase in its total surface area is :
(A) 392 sq. cm (B) 308 sq. cm (C) 196 sq. cm (D) 154 sq. cm

13. The moon diameter is approximately one fourth of the diameter of the earth. Volume of moon is what fraction the volume of earth :

(A) $\frac{1}{64}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) None of these

14. The volume of a box is given by $v = x^3 - 2x^2 - 24x$. If the depth of the box be the smallest, then the depth is :

(A) (x – 3) (B) (x – 6) (C) (x – 4) (D) (x – 8)

15. A cone, a right cylinder and a hemisphere stand on equal bases and have the same height. Their volumes are in the ratio :

(A) 1 : 2 : 3 (B) 1 : 3 : 2 (C) 2 : 3 : 1 (D) 2 : 1 : 3

16. If a sphere and a rectangular cylinder having equal radius, the height of the cylinder is double of its radius, then the ratio of volumes of sphere and cylinder is :

(A) 1 : 2 (B) 2 : 1 (C) 2 : 3 (D) 3 : 2

17. The largest sphere is cut off from a cube of side 5 cm. The volume of the sphere will be :

(A) 27 π cm^3 (B) $\frac{125}{6}\pi$ cm^3 (C) 108 π cm^3 (D) 30 π cm^3

18. Find the volume of a cube whose surface area is 600 cm^2.

(A) 900 cm^3 (B) 1000 cm^3 (C) 1500 cm^3 (D) 810 cm^3

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. The dimensions of a cuboid are 18 cm × 12 cm × 9 cm. How many cubes of side 3 cm can be made by melting the cuboid ? **[Raj. NTSE Stage-1 2006]**

(A) 72 (B) 69 (C) 60 (D) 55

2. If radii of two cones are in the ratio 3 : 1 and heights are in the ratio 1:3, then the ratio of their volumes is : **[Raj. NTSE Stage-1 2007]**

(A) 3 : 1 (B) 2 : 1 (C) 1 : 3 (D) 1 : 2

3. Areas of six surfaces of a cuboid are 12, 12, 20, 20, and 15, 15 sq. cm respectively. Volume of this cuboid in cm^3 is: **[Raj NTSE Stage - I 2007]**

(A) 12 (B) 15 (C) 60 (D) 94

4. On increasing each of the radius of the base and the height of a cone by 20%, its volume will be increased by _____. **[M.P. NTSE Stage-1 2012]**

(A) 20% (B) 40% (C) 72.2% (D) 72.8%

5. If the radius of a cylinder is decreased by 50% and the height increased by 50% to form a new cylinder, then the volume will be decreased by : **[M.P. NTSE Stage-1 2013]**

(A) 50% (B) 55% (C) 62.5% (D) 63%

6. If the volume and surface area of a sphere are numerically the same, then its radius is : **[M.P. NTSE Stage-1 2013]**

(A) 4 (B) 3 (C) 2 (D) 1

7. A cone has radius r and height h. It is melted and 3 identical cones are formed each having the ame radius as the original cone and height H. Then the value of $\frac{H}{h}$ is :**[M.P. NTSE Stage-1 2013]**

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1

8. From a 25 cm × 35 cm rectangular cardboard, an open box is to be made by cutting out identical squares of area 25 cm^2 from each corner and turning up the sides. The volume of the box is : **[Delhi NTSE Stage-1 2013]**

(A) 3000 cm^3 (B) 1875 cm^3 (C) 21875 cm^3 (D) 1250 cm^3

9. The surface water in a swimming pool forms a rectangle of length 40 m and breadth 15m. The depth of water increases uniformly from 1.2m at one end to 2.4m at the other end. The volume (in m^3) of water in the pool is : **[Harayana NTSE Stage-1 2013]**

(A)1080 (B) 720 (C) 600 (D) 540

10. A sphere of diameter 12.6 cm is melted and cast into a right circular cone of height 25.2 cm. The diameter of the base of the cone is : **[M.P. NTSE Stage-1 2013]**

(A) 12.6 cm (B) 79.38 cm (C) 39.69 cm (D) 69.39 cm

11. The area of adjacent faces and surface area of a cuboid with volume v and sides a,b,c are respectively x,y,z and s. Then which of the following is false. **[M.P. NTSE Stage-1 2014]**

(A) $\frac{1}{v} = \frac{2}{s}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ (B) $v^2 = x.y.z$ (C) $s = 2(x + y + z)$ (D) $s = x + y + z$

12. If a right circular cone, with slant height ℓ, and a right circular cylinder have the same radius r, same total surface area and heights h and h' respectively, then $\sqrt{\frac{\ell - r}{\ell + r}}$ = **[Haryana NTSE Stage-1 2014]**

(A) h/h' (B) 2h/h' (C) h/2h' (D) 2h'/h

13. The diameter of a right circular cylinder is decreased by 10%. The volume of cylinder remains the same then the percentage increase in height is : **[Delhi NTSE Stage-1 2014]**

(A) 20% (B) 23.45% (C) 5% (D) 20.5%

14. The surface area of a cylindrical pipe, open at both ends is 628 sq. m. The difference between its radius and length is 15 m (the length being larger). If the pipe was closed at one end, the amount of water that it can hold is : **[Bihar NTSE Stage-1 2014]**

(A) 500 π cu. m (B) 525 π cu. m (C) 550 π cu. m (D) None of these

15. How many surfaces are there in solid cylinder : **[Chattisgarh NTSE Stage-1 2014]**

(A) 1 (B) 2 (C) 3 (D) 4

16. The area of three adjoining faces of cuboid are A, B and C respectively then its volume will be : **[Chattisgarh NTSE Stage-1 2014]**

(A) ABC (B) $\sqrt{ABC}$ (C) $A^2B^2C^2$ (D) none of these

17. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm. in diameter. The time taken to fill up a conical vessel, whose diameter at the base is 40 cm and depth 24 cm., is : **[Jharkhand NTSE Stage-1 2014]**

(A) 55 minute (B) 52 minute 1 sec (C) 51 minutes 12 sec (D) 48 minutes 15 sec

18. The radius of a sphere is r and radius of base of a cylinder is r and height is 2r. The ratio of their volumes will be– **[U.P. NTSE Stage-1 2014]**

(A) 2 : 1 (B) 3 : 2 (C) 3 : 1 (D) 2 : 3

19. A metallic spherical shell of internal and external diameters 4 cm. and 8 cm. respectively, is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is **[Jharkhand NTSE Stage-1 2015]**

(A) 12 cm. (B) 14 cm. (C) 15 cm. (D) 18 cm.

20. An empty pool being filled with water at a constant rate takes 8 hours to fill $\frac{3}{5}$th of its capacity. How much more time will it take to finish filling the pool? **[Delhi NTSE Stage-1 2016]**

(A) 5 hours 30 minutes (B) 5 hours 20 minutes
(C) 4 hours 48 minutes (D) 4 hours 50 minutes

21. If the volume of two cubes are in the ratio 27 : 64, then the ratio of their total surface area is **[Bihar NTSE Stage-1 2016]**

(A) 27 : 64 (B) 3 : 4 (C) 9 : 16 (D) 3 : 8

22. The volume and whole surface area of a cylindrical solid of radius 'r' units are v and s respectively. If the height of cylinder is 1 unit then $\frac{v}{s}$ is equal to **[Delhi NTSE Stage-1 2016]**

(A) $\frac{1}{2}\left(1-\frac{1}{r+1}\right)$ (B) $\frac{1}{2}\left(1+\frac{1}{r+1}\right)$ (C) $\frac{1}{2}\left(1-\frac{1}{r}\right)$ (D) $\frac{1}{2}\left(1+\frac{1}{r}\right)$

23. If the height of right circular cylinder is increased by 10% while the radius of base is decreased by 10% then curved surface area of cylinder **[Delhi NTSE Stage-1 2016]**

(A) Remains same (B) Decreases by 1% (C) Increases by 1% (D) Increases by 0.1%

24. The volume of a cube is 2744 cm^3, its surface area is : **[M.P. NTSE Stage-1 2016]**

(A) 196 cm^2 (B) 1176 cm^2 (C) 784 cm^2 (D) 588 cm^2

25. If r is the radius of the base of a cylinder and h is the height of cylinder, then total surface area will be : **[M.P. NTSE Stage-1 2016]**

(A) $2\pi rh$ (B) $2\pi rh + 2\pi r^2$ (C) $\pi r^2 h$ (D) None of these

26. The length of the longest pole that can be kept in a room of size 12 m × 9 m × 8 m is : **[M.P. NTSE Stage-1 2017]**

(A) 29 m (B) 17 m (C) 21 m (D) 19 m

27. The height of a cylinder is 14 cm and its curved surface area is 264 cm^2, the volume of cylinder is : **[M.P. NTSE Stage-1 2017]**

(A) 308 cm^3 (B) 396 cm^3 (C) 1232 cm^3 (D) 1848 cm^3

Answer Key

Exercise Board Level

TYPE (I)

1. $\frac{32}{3}\pi r^3$ **2.** 64 cm^3 **3.** $\pi r\left(\ell+\frac{r}{4}\right)$ **4.** 15 m

5. 6 cm **6.** 6 : π.

TYPE (II)

7. 2.1 cm **8.** 20 : 27 **9.** 1 : 4 **10.** $\frac{\pi r^3}{12}$

11. 38.81 cm **12.** 471.42 m^2 **13.** 16 : 9 **14.** 30.48 cm^3

TYPE (III)

15. 3 cm , 50.29 cm^2 **17.** 488 cm^3 **18.** 301.7 cm^3,188.5 cm^2

19. 191 cm **20.** 8800 cm^3 **21.** 622.38 cm^3 **22.** 668.66 m^3

TYPE (IV)

23. 40 days **24.** **(i)** 110 **(ii)** 241.7 m^3

25. **(i)** 9152 cm^2 **(ii)** 55440 cm^3

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-1. 392 cm^2 **A-2.** 6 cm **A-3.** 15 cm, 12 cm, 9 cm. **A-4.** Rs. 27.97

A-5. Rs. 6240 **A-6.** 16 m. **A-7.** 6080.

Section (B)

B-1. $\sqrt{130}$ m **B-2.** 40 cm **B-3.** 1408 cm^2 **B-4.** 539 cm^3

B-5. $10120 \times \frac{80}{100} = 12650\ m^2$. **B-6.** Outer radius = 2.5 cm, Inner radius = 2 cm.

B-7. 5 : 12.

Section (C)

C-1. Rs. 96.29 **C-2** 523.908 m^3. **C-3.** 127527400 km^2. **C-4.** 56.83 cm^3.

Section (D)

D-1. n = 672 **D-2.** Rs. 169.715 ≈ Rs. 170 **D-3.** 395.37 kg

D-4. 35.84 cm

OBJECTIVE QUESTIONS

Section (A)

A-1. (D) **A-2.** (B) **A-3.** (A) **A-4.** (B)

Section (B)

B-1. (C) **B-2.** (B) **B-3.** (D) **B-4.** (B) **B-5.** (C)

B-6. (B) **B-7.** (B)

Section (C)

C-1. (C) **C-2.** (B) **C-3.** (C) **C-4** (A)

Section (D)

D-1. (C) **D-2.** (C) **D-3.** (D) **D-4.** (B)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	B	C	D	D	A	A	B	A	A	D	C	A	A	B	B	C	B	B

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	C	D	C	B	B	B	A	A	D	D	B	D	B	B	C	D	B	B
Ques.	21	22	23	24	25	26	27													
Ans.	C	A	B	B	B	B	B													