MATHEMATICS

Class-X

Topic-14

ARITHMETIC PROGRESSIONS

CH-14

ARITHMETIC PROGRESSIONS

(A) INTRODUCTION TO ARITHMETIC PROGRESSION AND ITS GENERAL TERM

(a) Difference between Sequences, Series and Progressions

(i) Sequences

Sequences are a set of numbers, which are arranged according to any specific rule. The set of numbers should have a definite, logical rule according to which they are arranged. It need not be a mathematical formula, but it should be logical. Such a set of numbers are called a sequence of numbers.

Eg.   2, 4, 6, 8, 10, ...   Rule : n^{th} term = 2n
Eg.   3, 5, 7, 9, 11, ...   Rule : n^{th} term = 2n + 1
Eg.   2, 3, 5, 7, 11, ...   Rule : Prime numbers.

(ii) Series

A series is a sequence of numbers that is added by '+' signs. The term 'series' is closely related to the total sum of sequence of numbers. However, the word 'series' is said to represented the sum of the numbers, and not the sum itself. There is only one difference directly visible between a series and a sequence. The numbers in a series are separated by plus (+) signs, whereas the numbers in a sequence are separated by commas (,).

Eg.   2 + 4 + 6 + 8 + ...
Eg.   3 + 5 + 7 + ...

A series is always based on a sequence.
A series of numbers is always associated with a sequence of numbers.

(iii) Progressions

Progressions are yet another type of number sets which are arranged according to some definite rule. The difference between a progression and a sequence is that a progression has a specific formula to calculate its nth term, whereas a sequence can be based on a logical rule like 'a group of prime numbers' which does not have a formula associated with it. Prime numbers cannot be predicted with the help of any formula. Till date, the formula for n^{th} prime number has not be found. This means that we can only calculate the nth prime number with the method of selecting each successive number and checking whether it is prime or not.

It can be concluded that all series are based on specific sequences, and all progressions are sequences but all sequences are not progressions (for example, prime numbers).

(b) General types of Progressions

Those sequence whose terms follow certain patterns are called **progressions**. Generally there are three types of progressions.

(i) Arithmetic Progression **(A.P.)**

(ii) Geometric Progression **(G.P.)**

(iii) Harmonic Progression **(H.P.)**

In this chapter, we will deal only with AP and rest of the progressions will be discussed later on.

(c)   Introduction to Arithmetic Progression

(i) Arithmetic Progression

A sequence is called an **A.P.**, if the difference of a term and the previous term is always same. i.e. $d = t_{n+1} - t_n = $ **Constant** for all $n \in N$. The constant difference, generally denoted by **'d'** is called the **common difference**.

Eg.			
(i)	2,3,4,5,6, ...	(An AP with common difference 1)
(ii)	$-90, -100, -110, -120, ...$	(An AP with common difference -10)
(iii)	$\frac{1}{2}, 1, \frac{3}{2}, 2, ...$	(An AP with common difference $\frac{1}{2}$)
(iv)	5,5,5,5, ...	(An AP with common difference 0)

Solved Examples

Example. 1

If $x + 1$, $3x$ and $4x + 2$ are in A.P., find the value of x.

Sol.   Since, $x + 1$, $3x$ and $4x + 2$ are in A.P.

$\therefore \quad 2(3x) = x + 1 + 4x + 2$

$\Rightarrow \quad 6x = 5x + 3 \quad \Rightarrow \quad x = 3.$

Example. 2

Find the common difference of the following A.P. :
1, 4, 7, 10, 13, 16, ...

Sol. $d = 4 - 1 = 7 - 4 = 10 - 7 = 13 - 10 = 16 - 13 = 3$ (constant).
Common difference (d) = 3.

(ii) General form of an A.P.

If we denote the starting number i.e. the **first number** by **'a'** and a fixed number to be added is **'d'** then

a, a + d, a + 2d, a + 3d, a + 4d,........... forms an **A.P.**

Example. 2

Find the A.P. whose 1st term is 10 & common difference is 5.

Sol.   Given : First term (a) = 10 & Common difference (d) = 5.
A.P. is 10, 15, 20, 25, 30,.......

(iii) n^{th} Term of an Arithmetic Progression

Let A.P. be a, a + d, a + 2d, a + 3d,...........

Then,   First term (a_1) = a + 0.d

Second term (a_2) = a + 1.d

Third term (a_3) = a + 2.d

.   .
.   .

n^{th} term (a_n) = a + (n – 1) d

$a_n = a + (n - 1) d$ is called the nth term.

Example. 3

Determine the A.P. whose third term is 16 and the difference of 5th term from 7th term is 12.

Sol.   Given : $a_3 = a + (3 - 1) d = a + 2d = 16$ ...... (i)

$a_7 - a_5 = 12$ ...... (ii)

$(a + 6d) - (a + 4d) = 12$

$a + 6d - a - 4d = 12$

2d = 12
d = 6
Put d = 6 in equation (i)
a = 16 – 12
a = 4
A.P. is 4, 10, 16, 22, 28,.......

Example. 4

Which term of the sequence 72, 70, 68, 66,....... is 40 ?

Sol. Here 1st term a = 72 and common difference   d = 70 – 72 = – 2
For finding the value of n
a_n = a + (n – 1)d $\Rightarrow$ 40 = 72 + (n – 1) (–2)
$\Rightarrow$ 40 – 72 = – 2n + 2 $\Rightarrow$ – 32 = – 2n + 2
$\Rightarrow$ – 34 = – 2n $\Rightarrow$ n = 17
17th term is 40.

Example. 5

Is 184, a term of the sequence 3, 7, 11,.......... ?

Sol. Here 1st term (a) = 3 and common difference (d) = 7 – 3 = 4
nth term (a_n) = a + (n – 1) d
$\Rightarrow$ 184 = 3 + (n – 1) 4 $\Rightarrow$ 181 = 4n – 4
$\Rightarrow$ 185 = 4n $\Rightarrow$ n = $\dfrac{185}{4}$

Since, n is not a natural number.
184 is not a term of the given sequence.

Example. 6

Which term of the sequence 20, $19\dfrac{1}{4}$, $18\dfrac{1}{2}$, $17\dfrac{3}{4}$ is the 1st negative term.

Sol. Here 1st term = a = 20, common difference = d = $19\dfrac{1}{4} - 20 = -\dfrac{3}{4}$.

Let nth term of the given A.P. be 1st negative term
a_n < 0 ,i.e., a + (n – 1) d < 0

$\Rightarrow 20 + (n-1)\left(-\dfrac{3}{4}\right) < 0$ $\Rightarrow \dfrac{83}{4} - \dfrac{3n}{4} < 0 \Rightarrow 3n > 83 \Rightarrow n > \dfrac{83}{3} \Rightarrow n > 27\dfrac{2}{3}$

Since, 28 is the natural number just greater then $27\dfrac{2}{3}$.

1st negative term is 28th.

Example. 7

If pth, qth and rth term of an A.P. are a, b, c respectively, then show that
a (q – r) + b (r – p) + c (p – q) = 0.

Sol. a_p = a $\Rightarrow$ A + (p – 1) D = a   ...(1)
a_q = b $\Rightarrow$ A + (q – 1) D = b   ...(2)
a_r = c $\Rightarrow$ A + (r – 1) D = c   ...(3)
Now, L.H.S.
= a (q – r) + b (r – p) + c (p – q)
= {A + (p – 1) D} (q – r) + {A + (q – 1) D}(r – p) + {A + (r – 1) D} (p – q) = 0.   **R.H.S.**

Example. 8

If m times the mth term of an A.P. is equal to n times its nth term. Show that the $(m + n)^{th}$ term of the A.P. is zero.

Sol. Let A be the 1st term and D be the common difference of the given A.P.

Then, $ma_m = na_n$

$\Rightarrow \quad m[A + (m - 1)D] = n[A + (n - 1)D]$

$\Rightarrow \quad A(m - n) + D[(m + n)(m - n) - (m - n)] = 0$

$\Rightarrow \quad A + (m + n - 1)D = 0$

$\Rightarrow \quad a_{m+n} = 0.$

Example. 9

If the pth term of an A.P. is q and the qth term is p, prove that its nth term is $(p + q - n)$.

Sol. Let A be the 1st term and D be the common difference of the given A.P.

$a_p = q \Rightarrow A + (p - 1)D = q \qquad \qquad \text{......(i)}$

$\& \qquad a_q = p \Rightarrow A + (q - 1)D = p \qquad \text{......(ii)}$

Solve (i) & (ii) to get D = –1 & A = p + q – 1

$a_n = A + (n - 1)D$

$a_n = (p + q - 1) + (n - 1)(-1)$

$a_n = p + q - n.$

Example. 10

If the mth term of an A.P. be $\dfrac{1}{n}$ and n^{th} term be $\dfrac{1}{m}$, then show that its $(mn)^{th}$ term is 1.

Sol. Let A be the 1st term and D be the common difference of the given A.P.

$a_m = \dfrac{1}{n} \Rightarrow A + (m - 1)D = \dfrac{1}{n} \qquad \qquad \text{......(i)}$

$\& \qquad a_n = \dfrac{1}{m} \Rightarrow A + (n - 1)D = \dfrac{1}{m} \qquad \text{......(ii)}$

By solving (i) & (ii) $D = \dfrac{1}{mn}$ & $A = \dfrac{1}{mn}$

$a_{mn} = A + (mn - 1)D = 1.$

(iv) m^{th} term of an A.P. from the end

Let **'a'** be the 1st term and **'d'** be the common difference of an **A.P.** having **n** terms. Then **m^{th}** term from the end is **$(n - m + 1)^{th}$** term from beginning or **$\{n - (m - 1)\}^{th}$** term from beginning.

Example. 11

Find 20th term from the end of an A.P. 3, 7, 11........407.

Sol. Let a be the 1st term and d be the common difference of the given A.P.

$a = 3$, $d = 4$ and $a_n = 407$

$407 = 3 + (n - 1)4 \qquad \Rightarrow n = 102$

20th term from end $\qquad \Rightarrow m = 20$

$a_{102 - (20 - 1)} = a_{102 - 19} = a_{83}$ from the beginning.

$a_{83} = 3 + (83 - 1)4 = 331.$

Check Your Level

1.   In an AP the first term is 2, the last term is 29 and the sum is 155; Find the common difference.

2.   What is the value of k for which the numbers k, k – 2, 3k will be in AP?

3.   Find the 15th term of the A.P x - 7, x - 2, x + 3, …

4.   If the 8th and 12th term of an A.P are 32 and 52 respectively, then find the 20th term of the same A.P.

5.   Find whether –150 is a term of the AP 17,12,7,2,.....

Answers

1.   3   **2.**   k = – 2   **3.**   x + 63   **4.**   92   **5.**   No

---

(B) SELECTION OF TERMS AND SUM OF FIRST 'n' TERMS OF AN ARITHMETIC PROGRESSION

(a)   Selection of Terms of an Arithmetic Progression

Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

No. of Terms	Terms	Common Difference
For 3 terms	a – d, a, a + d	d
For 4 terms	a – 3d, a – d, a + d, a + 3d	2d
For 5 terms	a – 2d, a – d, a, a + d, a + 2d	d
For 6 terms	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

(b)   Sum of First 'n' Terms of an Arithmetic Progression

Let A.P. be **a, a + d, a + 2d, a + 3d,............., a + (n – 1)d**

Then, $S_n = a + (a + d) + ... + \{a + (n – 2) d\} + \{a + (n – 1) d\}$ ...(i)

$S_n = \{a + (n – 1) d\} + \{a + (n – 2) d\} +...+ (a + d) + a$ ...(ii)

Add (i) & (ii)

$\Rightarrow$ $2S_n = 2a + (n – 1)d + 2a + (n – 1)d +...+ 2a + (n – 1)d$

$\Rightarrow$ $2S_n = n [2a + (n – 1) d]$

$\Rightarrow$ $S_n = \dfrac{n}{2}[2a + \overline{n – 1}\ d]$

$\Rightarrow$ $S_n = \dfrac{n}{2} [a + a + (n – 1)d] = \dfrac{n}{2} [a + \ell]$

$\therefore$ $S_n = \dfrac{n}{2}[a + \ell]$ where, ℓ is the last term.

Solved Examples

Example. 12

The sum of three numbers in A.P. is – 3 and their product is 8. Find the numbers.

Sol. Three no.'s in A.P. be $a - d$, a, $a + d$

$$a - d + a + a + d = -3$$
$$3a = -3 \Rightarrow a = -1 \qquad \& \qquad (a - d)\, a\, (a + d) = 8$$
$$\Rightarrow \quad a(a^2 - d^2) = 8 \qquad\qquad \Rightarrow \quad (-1)(1 - d^2) = 8$$
$$\Rightarrow \quad 1 - d^2 = -8 \qquad\qquad \Rightarrow \quad d^2 = 9$$
$$\Rightarrow \quad d = 3$$

If $a = -1$ & $d = 3$ numbers are $-4, -1, 2$.

If $a = -1$ & $d = -3$ numbers are $2, -1, -4$

Example. 13

Find the sum of 20 terms of the A.P. 1, 4, 7, 10....

Sol. $a = 1$, $d = 3$

$$S_n = \frac{n}{2}\,[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}\,[2(1) + (20 - 1)3] = 590.$$

Example. 14

Find the sum of all three digit natural numbers, which are divisible by 7.

Sol. The first and the last three digit number which is divisible by 7 is 105 and 994 respectively

105, 112, ... , 994 in AP with $a = 105$, $d = 112 - 105 = 7$ and $a_n = 994$

$$994 = 105 + (n - 1)7$$
$$n = 128$$

Sum, $S_{128} = \dfrac{128}{2}\,[105 + 994] = 70336.$

Example. 15

The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the mth and n^{th} terms is $(2m - 1) : (2n - 1)$.

Sol. **Given :**

$$S_m : S_n = m^2 : n^2 \qquad \Rightarrow \qquad \frac{\dfrac{m}{2}\big[2a + (m-1)d\big]}{\dfrac{n}{2}\big[2a + (n-1)d\big]} = \frac{m^2}{n^2}$$

$$\Rightarrow \quad \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n} \qquad\qquad \Rightarrow \quad 2an + n(m-1)\,d = 2am + m(n-1)\,d$$

$$\Rightarrow \quad 2a(n - m) + d\,[nm - n - mn + m] = 0$$

$$\Rightarrow \quad 2a(n - m) = d(n - m) \qquad\qquad \Rightarrow \quad 2a = d$$

So, $\dfrac{a_m}{a_n} = \dfrac{a + (m-1)d}{a + (n-1)d} \qquad\qquad \Rightarrow \qquad \dfrac{a_m}{a_n} = \dfrac{a + (m-1)\,2a}{a + (n-1)\,2a} = \dfrac{a[1 + (m-1)\,2]}{a[1 + (n-1)\,2]} = \dfrac{2m-1}{2n-1}.$

Example. 16

In an A.P., the sum of first n terms is $4n^2 - 3n$. Find its nth term.

Sol. $a_n = S_n - S_{n-1}$

$$= 4n^2 - 3n - [\,4(n - 1)^2 - 3(n - 1)\,]$$
$$= 4n^2 - 3n - [\,4(n^2 - 2n + 1) - 3n + 3\,]$$
$$= 4n^2 - 3n - 4n^2 + 8n - 4 + 3n - 3$$
$$= 8n - 7.$$

Check Your Level

1. Find three numbers in A.P. whose sum is 15 and the sum of the squares of the two extremes is 58.

2. The sum of 15 terms of an AP is 600, and the common difference is 5; find the first term.

3. Find the sum of 15 terms of the series whose n^{th} term is 4n + 1.

4. If the sum of n terms of an AP is $n^2 + 7n$, find the common difference.

5. Find the sum of all two digit numbers which are divisible by 4.

Answers

1. 3, 5, 7   **2.**   5   **3.**   495   **4.**   2   **5.**   1188

(C) WORD PROBLEMS

Solved Examples

Example. 17

A manufacturer of TV sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in (i) the first year (ii) the 10th year (iii) 7 years.

Sol. Since the production increases uniformly by a fixed number every year. Therefore the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e. 'a' denotes the production in the first year and 'd' denotes the number of units by which production increases every year.

a_3 = 600 and a_7 = 700

So,   a + 2d = 600   .... (i)

        a + 6d = 700   .... (ii)

Solving equation (i) and (ii)

a = 550 and d = 25

(i)   Production in the first year is 550 TV sets.

(ii)   Production in the 10th year = a_{10} = a + 9d = 550 + 9(25) = 550 + 225 = 775.

(iii)   Total production in 7 years = S_7 = $\frac{7}{2}\left[2(550) + (7-1)25\right]$ = 4375.

Thus, the total production in 7 years is of 4375 TV sets.

Example. 18

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceeding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x.

Sol. Let there be a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it.

House : H_1 H_2 H_3 .... H_{x-1} H_x H_{x+1}............ H_{49}

House No. :   1   2   3 .... x – 1   x   x +1 49

House number will form an A.P. whose first term and common difference is 1.

ATP   : $S_{x-1} = S_{49} - S_x$

$$\Rightarrow \quad \frac{(x-1)}{2}\big[2(1)+(x-1-1)1\big] = \frac{49}{2}[2+48] - \frac{x}{2}\big[2(1)+(x-1)1\big]$$

$$\Rightarrow \quad \frac{(x-1)}{2}\big[2+(x-2)\big] = \frac{49}{2}[50] - \frac{x}{2}\big[2+x-1\big]$$

$$\Rightarrow \quad \frac{(x-1)}{2}\big[x\big] = \frac{49}{2}[50] - \frac{x}{2}\big[x+1\big]$$

$$\Rightarrow \quad \frac{x}{2}\big[x-1+x+1\big] = \frac{49}{2}[50]$$

$$\Rightarrow \quad \frac{x}{2}\big[2x\big] = 49 \times 25$$

$$\Rightarrow \quad x^2 = 49 \times 25$$

$$\Rightarrow \quad x = 7 \times 5 = 35.$$

Since, x is not a fraction. Hence, the value of x satisfying the given condition exists and is equal to 35.

Example. 19

A sum of Rs. 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize. Find the value of each prize.

Sol. Let the first prize is Rs. a

Then, second prize = a – 20, ...

So, this arrangement forms an A.P. with common difference equal to – 20.

Given :

$S_7 = 700$

$$\frac{7}{2}\big[2a+(6)(-20)\big] = 700$$

2a – 120 = 200

2a = 320

a = 160

So, the Ist prize = 160, 2nd prize = 140, 3rd prize = 120, 4th prize = 100, 5th prize = 80, 6th prize = 60, 7th prize =20.

Check Your Level

1. From your pocket money, you save Re. 1 on day 1, Rs. 2 on day 2, Rs. 3 on day 3, and so on. How much money you will save in the month of march 2018

2. Shubham saves Rs. 32 during the first month, Rs. 36 in the second month and Rs. 40 in third month. If he continue to save in this manner, in how many month will he save Rs. 2000

3. Yash started work in 2017 at an annual salay of Rs.25000 and received Rs. 2000 raise in each year. In what year did his income reach Rs. 45,000

4. Kanishk saves Rs. 12 in the first week of a year and then increased her weekly saving by Rs. 1.75. If in n^{th} the week, her weekly saving become Rs. 20.75, find n.

Answers

| 1. | Rs.496 | 2. | 25 months | 3. | 2027 | 4. | 6 |

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS :   **[01 MARK EACH]**

1.   Find the 11th term of the AP : -5 , $\dfrac{-5}{2}$, 0 , $\dfrac{5}{2}$, ...

2.   If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?

3.   Which term of the AP : 21, 42, 63, 84,... is 210 ?

4.   What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

5.   Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

6.   If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be

7.   For the AP : -3 , -7 , -11 , ... , can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20} ? Give reasons for your answer.

8.   Verify that each of the following is an AP, and then write its next three terms.

(i) 5 , $\dfrac{14}{3}$, $\dfrac{13}{3}$, 4,...   **(ii)** $\sqrt{3}$, $2\sqrt{3}$,$3\sqrt{3}$,...   **(iii)** a, $2a + 1$, $3a + 2$, $4a + 3$,...

9.   Write the first three terms of the APs when $a = -5$ and $d = -3$

TYPE (II) : SHORT ANSWER TYPE QUESTIONS :   **[02 MARKS EACH]**

10.   Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.

11.   The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the A P.

12.   If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.

13.   Determine k so that $k^2+ 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.

14.   The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

15.   Which term of the AP: 53, 48, 43,... is the first negative term ?

16.   Which term of the AP: -2, -7, -12,... will be -77?. Find the sum of this AP upto the term -77.

17.   If $a_n = 3 - 4n$, show that a_1, a_2 , a_3 ,... form an AP. Also find S_{20} .

18.   Solve the equation: $-4 + (-1) + 2 +...+ x = 437$

19.   In an AP, if $S_n = n (4n + 1)$, find the AP.

20.   In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

TYPE (III) : LONG ANSWER TYPE QUESTIONS: **[04 MARK EACH]**

21. How many terms of the AP: -15, -13, – 11,......... are needed to make the sum -55?
    Explain the reason for double answer.

22. Kanika was given her pocket money on Jan 1st, 2008. She puts Rs 1 on Day 1, Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?

23. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

24. Find the sum :
    (i) $1 + (-2) + (-5) + (-8) + ... + (-236)$

    (ii) $\left(4 - \dfrac{1}{n}\right) + \left(4 - \dfrac{2}{n}\right) + \left(4 - \dfrac{3}{n}\right) + ...$ upto n terms

    (iii) $\dfrac{a-b}{a+b} + \dfrac{3a-2b}{a+b} + \dfrac{5a-3b}{a+b} ...$ to 11 terms.

25. Find the
    (i)   sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.
    (ii)   sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .
    (iii)   sum of those integers from 1 to 500 which are multiples of 2 or 5.

26. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.

27. Find the sum of the integers between 10 and 200 that are
    (i)   divisible by 9   **(ii)**   not divisible by 9

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS   **[05 MARK EACH]**

28. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

29. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$

30. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2n terms of another AP whose first term is – 30 and the common difference is 8. Find n.

31. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

32. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

33. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

Previous Year Problems

1. The common difference of the AP $\dfrac{1}{2b}, \dfrac{1-6b}{2b}, \dfrac{1-12b}{2b}$ is : **[1 MARK/CBSE 10TH BOARD: 2013]**

    (A) 2b   (B) – 2b   (C) 3   (D) – 3

2. How many three – digit natural numbers are divisible by 7 ?
    [1 MARK /CBSE 10TH BOARD: 2013]

3. The 24th term of an AP is twice its 10th term . Show that its 72nd term is 4 times its 15th term .
    [4 MARKS/CBSE 10TH BOARD: 2012]

4. The nth term of an AP is given by (– 4n + 15) . Find the sum of first 20 terms of this AP .
    [3 MARKS/CBSE 10TH BOARD: 2013]

5. The next term of the $\sqrt{7}, \sqrt{28}$, $\sqrt{63}$ . . . is. **[1 MARK /CBSE 10TH BOARD: 2013]**

    (A) $\sqrt{70}$ (B) $\sqrt{84}$ (C) $\sqrt{97}$ (D) $\sqrt{112}$

6. If S_n denotes the sum of the first n terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$.
    [4 MARKS /CBSE 10TH BOARD: 2013]

7. The sum of the first n terms of an A.P. is $5n - n^2$, Find the nth term of this A.P.
    [2 MARKS /CBSE 10TH BOARD: 2014]

8. The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P. **[3 MARKS/CBSE 10TH BOARD: 2014]**

9. In an A.P., $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms. **[2 MARKS /CBSE 10TH BOARD: 2014]**

10. The 14th term of an AP is twice its 8th term. If its 6th term is -8, then find the sum of its first 20 terms. **[3 MARKS /CBSE 10TH BOARD: 2014]**

11. Find the 60th term of the AP 8, 10, 12, ..., if it has a total of 60 terms and hence find the sum of its last 10 terms. **[4 MARKS /CBSE 10TH BOARD: 2014]**

12. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.
    [2 MARKS /CBSE 10TH BOARD: 2015]

13. For what value of k will k + 9, 2k – 1 and 2k + 7 are the consecutive terms of an A.P.?
    [1 MARK /CBSE 10TH BOARD: 2015]

14. If the ratio of the sum of first n terms of two A.P's is (7n + 1) : (4n + 27), find the ratio of their m^{th} terms. **[3 MARKS /CBSE 10TH BOARD: 2015]**

15. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceding the house numbered X is equal to sum of the numbers of houses following X. **[4 MARKS/CBSE 10TH BOARD: 2015]**

16. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P. **[3 MARKS/CBSE 10TH BOARD: 2016]**

17. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?
    [1 MARKS/CBSE 10TH BOARD: 2017]

18. Which term of the progression 20, $19\dfrac{1}{4}, 18\dfrac{1}{2}, 17\dfrac{3}{4}, ...$ is the first negative term ?
    [2 MARKS /CBSE 10TH BOARD: 2017]

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction to Arithmetic Progression and its General term

A-1. Is 51 a term of the A.P. 5, 8, 11, 14,.............?

A-2. Which term of the arithmetic progression 8, 14, 20, 26,......... will be 72 more than its 41st term ?

A-3. Write the next term of the $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, ...

A-4. If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a.

A-5. Find the 10th term from end of the A.P. 4, 9, 14,...., 254.

A-6. The fourth term of an A.P. is equal to three times the first term and the seventh term exceeds twice the third by one. Find the first term and the common difference.

A-7. Which term of the sequence 17, $16\frac{1}{5}$, $15\frac{2}{5}$, $14\frac{3}{5}$,...... is the first negative term.

A-8. Which term of the A.P. 3, 15, 27, 39,... will be 120 more than its 21st term ?

Section (B) : Selection of terms and Sum of 'n' terms of an Arithmetic Progressions

B-1. Find the common difference of an A.P. whose first term is 100 and the sum of whose first six terms is five times the sum of the next six terms.

B-2. Find three numbers in A.P. whose sum is 21 and their product is 336.

B-3. A student purchased a pen for Rs. 100. At the end of 8 years, it was valued at Rs. 20. Assuming the yearly depreciation is a constant amount, find the annual depreciation.

B-4. Find the sum of all the three digit numbers which leave remainder 2 when divided by 5.

B-5. Find the sum of all two digit odd positive numbers.

B-6. The sum of the first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Determine the A.P. and its 12th term.

B-7. Find the sum of the first 25 terms of an A.P. whose nth term is given by $t_n = 2 - 3n$.

B-8. In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$. Find its 25th term.

B-9. If $S_n = n^2p$ and $S_m = m^2p$, (mn), in an A.P. Prove that $S_p = p^3$.

B-10. Find the number of terms of A.P. 54, 51, 48,........ so that their sum is 513.

B-11. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively. Find the sum of all terms of this A.P.

B-12. The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

B-13. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P.

B-14. If the sum of the first q terms of an A.P. is $2q + 3q^2$, what is its common difference ?

B-15. Find the common difference of an A.P. whose first term is 4, the last term is 49 and the sum of all its terms is 265.

B-16. In an AP., the sum of its first ten terms is -80 and the sum of its next ten terms is -280. Find the A.P.

Section (C) : Word problems

C-1. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows the 200 logs are placed and how many logs are in the top row ?

C-2. Jaspal Singh repays his loan of Rs. 118000 by paying every month starting with the first instalment of Rs. 1000. If he increases the instalment by Rs. 100 every month, what amount will be paid by him in the 30th instalment ? What loan does he still have to pay after the 30th instalment ?

C-3. 150 workers were engaged to finish a work in a certain number of days. Four workers were dropped the second day, four more workers were dropped the third day and so on. It takes 8 more days to finish the work now. Find the total number of days to finish the work now.

C-4. Two cars start simultaneously from the same place and in the same direction. The first car moves with a uniform speed of 10 km/h. The second car moves at a speed of 8 km/h in the first hour and then increases its speed by 0.5 km/h in each succeeding hour. If both the cars move non-stop, then how long will it take for the second car to overtake the first car ?

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Introduction to Arithmetic Progression and its General term

A-1. pth term of the series $\left(3-\dfrac{1}{n}\right)+\left(3-\dfrac{2}{n}\right)+\left(3-\dfrac{3}{n}\right)+........$ will be :

(A) $3 + \dfrac{p}{n}$ 　　　(B) $3 - \dfrac{p}{n}$ 　　　(C) $3 + \dfrac{n}{p}$ 　　　(D) $3 - \dfrac{n}{p}$

A-2. 8th term of the series $2\sqrt{2} + \sqrt{2} + 0 +.........$ will be :

(A) $-5\sqrt{2}$ 　　　(B) $5\sqrt{2}$ 　　　(C) $10\sqrt{2}$ 　　　(D) $-10\sqrt{2}$

A-3. If 9th term of an A.P. be zero then the ratio of its 29th and 19th term is :

(A) 1 : 2 　　　(B) 2 : 1 　　　(C) 1 : 3 　　　(D) 3 : 1

A-4. Which term of the sequence 3, 8, 13, 18,....... is 498
(A) 95th (B) 100th (C) 102th (D) 101th

A-5. Which of the following sequence is an A.P.
(A) $f(n) = an + b, n \in N$
(B) $f(n) = kr^n, n \in N$
(C) $f(n) = (an + b)kr^n, n \in N$
(D) $f(n) = \dfrac{1}{a\left(n + \dfrac{b}{n}\right)}, n \in N$

A-6. The 4th term from the end of the AP $-11, -8, -5, ...49$ is :
(A) 37 (B) 40 (C) 43 (D) 58

A-7. The 16th term of the AP : 15, $\dfrac{25}{2}$, 10, $\dfrac{15}{2}$, 5, ... is :
(A) $\dfrac{45}{2}$ (B) $-\dfrac{45}{2}$ (C) $\dfrac{105}{2}$ (D) $-\dfrac{105}{2}$

A-8. The 31st term of the AP whose first two terms are respectively -2 and -7 is :
(A) -152 (B) 150 (C) 148 (D) -148

A-9. The 11th term of AP whose 3rd term is 11 and 8th term is 26 is :
(A) 25 (B) -2 (C) -8 (D) 35

A-10. If the common difference of an AP is -6, then what is $a_{16} - a_{12}$?
(A) -24 (B) 24 (C) -30 (D) 30

A-11. Which term of the AP, 8, 16, 24, 32 .... is 256?
(A) 31st (B) 32nd (C) 33rd (D) 34th

A-12. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is :
(A) -1 (B) -8 (C) 7 (D) -9

A-13. If $\dfrac{6}{5}$, a, 4 are in AP, the value of a is :
(A) 1 (B) 13 (C) $\dfrac{13}{5}$ (D) $\dfrac{26}{5}$

A-14. The 9th term of an AP is 499 and 499th term is 9. The term which is equal to zero is :
(A) 501th (B) 502th (C) 508th (D) None of these

Section (B) : Selection of terms and Sum of 'n' terms of an Arithmetic Progressions

B-1. If the nth term of an A.P. be $(2n - 1)$, then the sum of its first n terms will be :
(A) $n^2 - 1$ (B) $(2n - 1)^2$ (C) n^2 (D) $n^2 + 1$

B-2. If the first, second and last terms of an A.P. be a, b, 2a respectively, then its sum will be :
(A) $\dfrac{ab}{-a + b}$ (B) $\dfrac{ab}{2(b - a)}$ (C) $\dfrac{3ab}{2(b - a)}$ (D) $\dfrac{3ab}{4(b - a)}$

B-3. The sum of the 'p' terms of an AP is 'q' and the sum of 'q' terms is 'p', then the sum of (p + q) terms will be :
(A) 0 (B) $p - q$ (C) $p + q$ (D) $-(p + q)$

B-4. If the sum of n terms of an AP is $2n^2 + 5n$, then its nth term will be :
(A) 4n – 3   (B) 3n – 4   (C) 4n + 3   (D) 3n + 4

B-5. If the sum of n terms of an AP is $3n^2 + 5n$ then which of its terms is 164 ?
(A) 26th (B) 27th (C) 28th (D) None of these

Section (C) : Word problems

C-1. The interior angles of a polygon are in A.P. If the smallest angle be 120^0 and the common difference be 5, then the number of sides is :
(A) 8   (B) 10   (C) 9   (D) 6

C-2. In a garden bed there are 23 rose plants in the first row, twently one in the second row, nineteen in the third row and so on. There are five plants in the last row. How many rows are there of rose plants ?
(A) 9   (B) 10   (C) 11   (D) 12

C-3. Suba Rao started work in 1995 at an annual salary of Rs 5000 and received a Rs 200 raise each year. In what year did his annual salary will reach Rs 7000?
(A) 2004   (B) 2005   (C) 2006   (D) None of these

C-4. Jasleen saved Rs 5 in the first week of the year and then increased her weekly savings by Rs1.75 each week. In what week will her weekly savings be Rs 20.75?
(A) 7th (B) 8th (C) 9th (D) 10th

C-5. Along a road lie an odd number of stones placed at intervals of 10 metre. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. If a man starts from one of the end stones, and by carrying them in succession he covers 3 km to pile all stones at the centre. The number of stones is then :
(A) 12   (B) 15   (C) 30   (D) 25

Exercise-2

OBJECTIVE QUESTIONS

1. The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero ?
(A) 1st (B) 9th (C) 12th (D) None of the above

2. The sides of a right angled triangle are in A.P. The ratio of sides is :
(A) 1 : 2 : 3   (B) 2 : 3 : 4   (C) 3 : 4 : 5   (D) 5 : 8 : 3

3. The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers.
(A) 3, 4, 5   (B) 2, 4, 6   (C) 2, 5, 8   (D) 3, 6, 9

4. The value of $1^2 – 2^2 + 3^2 – 4^2 + 5^2 – 6^2 + ... + 99^2 – 100^2$ is :
(A) – 100   (B) – 5050   (C) – 2500   (D) – 2520

5. Find the common difference of an A.P. whose first term is 100 and the sum of whose first six terms is five times the sum of the next six terms.
(A) 10   (B) – 10   (C) 5   (D) – 5

6. The sum of n terms of two A.Ps. are in ratio $\dfrac{7n+1}{4n+27}$. Find the ratio of their 11th terms.

(A)　　　　　　(B)　　　　　　(C)　　　　　　(D)

7. If sum of n terms of a sequence is given by $S_n = 2n^2 + 3n$, find its 50th term.

(A) 250　　　　(B) 225　　　　(C) 201　　　　(D) 205

8. Sum of n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$ is :

(A) $\dfrac{n(n+1)}{2}$　　(B) $2n(n+1)$　　(C) $\dfrac{n(n+1)}{\sqrt{2}}$　　(D) 1

9. If S_r denotes the sum of the first r terms of an A.P. Then, $S_{3n} : (S_{2n} - S_n)$ is :

(A) n　　　　　(B) 3n　　　　　(C) 3　　　　　(D) None of these

10. Consider the sequence $1, -2, 3-4, 5, -6, \ldots, n(-1)^{n+1}$. What is the average of the first 300 terms of the sequence ?

(A) -1　　　　(B) 0.5　　　　(C) 0　　　　　(D) -0.5

11. Let S_n denote the sum of the first 'n' terms of an A.P. and $S_{2n} = 3S_n$. Then, the ratio $S_{3n} : S_n$ is equal to :

(A) 4 : 1　　　　(B) 6 : 1　　　　(C) 8 : 1　　　　(D) 10 : 1

12. The sum of all natural numbers less than 400 which are not divisible by 6, is :

(A) 13266　　　(B) 66534　　　(C) 79800　　　(D) 93066

13. If $\dfrac{b+c-a}{a}$, $\dfrac{c+a-b}{b}$ and $\dfrac{a+b-c}{c}$ are in A.P. and $a + b + c \neq 0$, then :

(A) $b = \dfrac{ac}{a+c}$　　(B) $b = \dfrac{2ac}{a+c}$　　(C) $b = \dfrac{a+c}{2}$　　(D) $b = \sqrt{ac}$

14. If S_n denotes the sum of n terms of an A.P., then $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is equal to

(A) 0　　　　　(B) 1　　　　　(C) $\dfrac{1}{2}$　　　　(D) none of these

15. The sum of first n terms of an AP is given by $S_n = (1 + T_n)(n + 2)$, where T_n is the nth term of A.P. Then T_2 is

(A) $-\dfrac{11}{6}$　　(B) $\dfrac{-5}{3}$　　(C) $\dfrac{5}{3}$　　(D) 2

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. If t_{11} and t_{16} of an A.P. are respectively 38 and 73, then t_{31} is _____.

[Orissa NTSE Stage-1 2012]

(A) 178　　　　(B) 177　　　　(C) 176　　　　(D) 175

2. If $f(n+1) = \dfrac{2f(n)+1}{2}$, n = 1, 2, _____ and f(1) = 2, then f(101) = _____.

[Orissa NTSE Stage-1 2012]

(A) 53　　　　(B) 52　　　　(C) 51　　　　(D) 50

3. The sum of all two digit numbers each of which leaves remainder 3 when divided by 5 is
   [Delhi NTSE Stage-1 2013]
   (A) 952   (B) 999   (C) 1064   (D) 1120

4. The sum of the third and seventh terms of an A.P. is 6 and their product is 8, then common difference is :   **[Raj. NTSE Stage-1 2013]**
   (A) 1   (B) 2   (C) $\pm \dfrac{1}{2}$ (D) $\pm \dfrac{1}{4}$

5. If $a_1, a_2 ...., a_{19}$ are the first 19 term of an AP and $a_1 + a_8 + a_{12} + a_{19} = 224$. Then $\displaystyle\sum_{i=1}^{19} a_i$ is equal to
   [Harayana NTSE Stage-1 2013]
   (A) 896   (B) 1064   (C) 1120   (D) 1164

6. If 9, a, b, – 6 are in Arithmetic progression, then a + b =   **[Raj. NTSE Stage-1 2014]**
   (A) 1   (B) 5   (C) 15   (D) 3

7. A person got a job with a fixed salary and a certain yearly increment. After 2 years his salary was Rs. 10,000 and after 4 years it was Rs. 15,000. Find his salary after 10 years.
   [Maharashtra NTSE Stage-1 2014]
   (A) Rs. 32,500   (B) Rs. 27,500   (C) Rs. 27,250   (D) Rs. 30,000

8. $\left[\left(1-\dfrac{1}{n+1}\right) + \left(1-\dfrac{2}{n+1}\right) + ... + \left(1-\dfrac{n}{n+1}\right) \right]$ is :   **[Delhi NTSE Stage-1 2014]**

   (A) n   (B) $\dfrac{n}{2}$ (C) n + 1   (D) 2n

9. In an A. P. 5 times the 5th term is equal to 8 times the 8th term, then its 13th term is :
   [Bihar NTSE Stage-1 2014]
   (A) 0   (B) –1   (C) –12   (D) –13

10. If an arithmetic progression sum of first n terms is $2n^2 + 3n$. Its common difference is :
    [Chattisgarh NTSE Stage-1 2014]
    (A) 6   (B) 3   (C) 2   (D) 4

11. The first and last terms of an A.P. of n terms is 1 and 31 respectively. The ratio of 8th term and $(n – 2)^{th}$ term is 5 : 9, the value of n is :   **[Delhi NTSE Stage-1 2014]**
    (A) 14   (B) 15   (C) 16   (D) 13

12. The sum of 18 consecutive natural numbers is a perfect square. The smallest possible value of this sum is :   **[Harayana NTSE Stage-1 2014]**
    (A) 144   (B) 169   (C) 225   (D) 289

13. If $a_1, a_2, a_3, ...$ is an arithmetic progression with common difference 1 and $\displaystyle\sum_{i=1}^{98} a_i = 137$, then the value of $a_2 + a_4 + a_6 + ...a_{98}$ is :   **[Harayana NTSE Stage-1 2014]**
    (A) 67   (B) 83   (C) 93   (D) 98

14. A club consists of members whose ages are in A.P. the common difference being 3 months. If the youngest members of the club is just 7 years old and the sum of the ages of all the members is 250 years, then the number of members in the club are :   **[Karnataka NTSE Stage-1 2014]**
    (A) 15   (B) 20   (C) 25   (D) 30

15. The total two-digit numbers which are divisible by 5, are **[Raj. NTSE Stage-1 2015]**
(A) 17 (B) 18 (C) 19 (D) 20.

16. Sum of series (1+2) + (3 + 5) + (6 +7) + (9 + 10) + ...+ (93 + 94) + (95 + 97) +(98 + 99) will be :
[Harayana NTSE Stage-1 2015]
(A) 5050 (B) 3750 (C) 1350 (D) 4250

17. In an A.P. the second and fifth terms are respectively (x-y) and (x + y) then thrice the first term is
[Bihar NTSE Stage-1 2015]
(A) 3x – y (B) 3x – 2y (C) 3x – 4y (D) 3x – 5y

18. If $1^3 + 2^3 + ... + 10^3 = 3025$ then 4 + 32 + 108 + ... + 4000 is equal to
[Bihar NTSE Stage-1 2015]
(A) 1200 (B) 12100 (C) 12200 (D) 12400

19. What is the value of $\dfrac{160}{2\times 7} + \dfrac{160}{7\times 12} + \dfrac{160}{12\times 17} + \dfrac{160}{17\times 22} + \dfrac{160}{22\times 27} + \dfrac{160}{27\times 32}$
[Delhi NTSE Stage-1 2015]
(A) 17 (B) 15 (C) 13 (D) 11

20. If $\dfrac{1}{p+q}, \dfrac{1}{q+r}, \dfrac{1}{r+p}$ are in A.P. then **[Jharkhand NTSE Stage-1 2015]**
(A) p,q,r are in A.P. (B) q^2, p^2, r^2 are in A.P.
(C) p^2, q^2, r^2 are in A.P. (D) q,p,r, are in A.P.

21. The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 149. The middle term of the three terms is **[Jharkhand NTSE Stage-1 2015]**
(A) 10 (B) 4 (C) 6 (D) 7

22. If the sum of the first "p" terms of an AP is the same as the sum of its first "q" terms (where pq) then sum of (p+q)th term is : **[Jharkhand NTSE Stage-1 2015]**
(A) 1 (B) 0 (C) p + q – 1 (D) p + q + 1

23. In an A.P. the sum of the first ten terms is 210 and the difference between the first and the last term is 36. Find the first term in the A.P. **[Maharashtra NTSE Stage-1 2015]**
(A) 2 (B) 3 (C) 4 (D) 5

24. The sum of first n natural numbers is........ **[MP NTSE Stage-1 2015]**
(A) $\dfrac{n}{2}$ (B) $\dfrac{n(n+1)}{2}$ (C) $\dfrac{n+1}{2}$ (D) n + 1

25. In an A.P $t_4 = 11$ and $t_{10} = 16$, then the sum of the first 40 terms is **[Orissa NTSE Stage -1_2015]**
(A) 550 (B) 660 (C) 880 (D) 990

26. In the year 2013, Pavin saves Rs. 1 on the first day, Rs. 3 on the second day, Rs. 5 on the third and so on. Find the total amount of his saving in that year. **[Maharashtra NTSE Stage-1 2016]**
(A) Rs. 133225 (B) Rs. 132225 (C) Rs. 123225 (D) Rs. 134225

27. Sum of last two terms of an A.P. is 60. If first term is 11 and common difference is 2, then the number of terms in the A.P. is : **[Rajasthan NTSE Stage-1 2016]**
(A) 22 (B) 20 (C) 11 (D) 19

28. The value of the following expression is $\left[\dfrac{1}{(2^2-1)}\right] + \left[\dfrac{1}{(4^2-1)}\right] + \left[\dfrac{1}{(6^2-1)}\right] + \ldots\ldots\ldots + \left[\dfrac{1}{(20^2-1)}\right]$ is

[Delhi NTSE Stage-1 2016]

(A) $\dfrac{10}{21}$ (B) $\dfrac{13}{27}$ (C) $\dfrac{15}{22}$ (D) $\dfrac{8}{33}$

29. The total number of squares on a chessboard is   **[Delhi NTSE Stage-1 2016]**
(A) 206   (B) 205   (C) 204   (D) 202

30. In an A.P. the sum of 'n' terms is . Find the 10th term of the A.P.?

[Maharashtra NTSE Stage-1 2017]

(A) 80   (B) 90   (C) 100   (D) 110

31. Sum of odd numbers between 0 and 50 is   **[UP NTSE Stage-1 2017]**
(A) 625   (B) 600   (C) 900   (D) 1200

Answer Key

Exercise Board Level

TYPE (I)

1. 20	**2.** 33	**3.** 10th	**4.** 8	**5.** 7

6. 0   **7.** Yes

8. **(i)** It is an AP, $\dfrac{11}{3}, \dfrac{10}{3}, 3$ **(ii)** It is an AP, $4\sqrt{3}, 5\sqrt{3}, 6\sqrt{3}$

    (iii) It is an A.P., 5a + 4, 6a + 5, 7a + 6   **9.** $- 5, - 8, - 11$

TYPE (II)

10. a = – 1, b = 15, c = 31   **11.** 1, 6, 11, 16, ...   **13.** k = 0

14. 80°, 60° and 40°   **15.** 12th   **16.** 16th, Sum = - 632

17. – 780   **18.** 50   **19.** 5, 13, 21, ...

20. 27

TYPE (III)

21. 5 or 11   **22.** 800   **23.** 67, 69, 71

24. **(i)** – 9400   **(ii)** $\dfrac{7n-1}{2}$ **(iii)** $\dfrac{11(11a-6b)}{a+b}$

25. **(i)** 12250   **(ii)** 12750   **(iii)** 75250

26. 3   **27.** **(i)** 2268   **(ii)** 17577

TYPE (IV)

28. 970   **30.** 11   **31.** 3, 7, 11, 15, ...

32. 1 : 3, 5 : 49   **33.** 728 m, 26 m

Previous Year Problems

1. (D)   **2.** 128   **4.** – 540   **5.** (D)

7. $T_n = 6 - 2n$ **8.** 57   **9.** 1, 6, 11........   **10.** $S_{20} = - 340$

11. $t_{60} = 126$, 1170   **13.** k = 18   **14.** $\dfrac{14m-6}{8m+23}$ **15.** X = 35

16. n = 16, d = 8/3   **17.** d = 6   **18.** 28th term

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A-1. No   **A-2.** 53rd term   **A-3.** .   **A-4.** the value of a is.

A-5. 209   **A-6.** First term is 3 and common difference is 2

A-7. 23rd term   **A-8.** 31st term

Section (B)

B-1.	−10	**B-2.**	8, 7, 6.	**B-3.**	Rs. 10	**B-4.**	98910

B-5. 2475   **B-6.** AP is −1,511,... and a_{12} = 65   **B-7.** −925

B-8. 76   **B-10.** 19 or 18   **B-11.** 5412   **B-12.** − 13,−8 & − 3

B-13. a_1 = 2 and a_{13} = 26.   **B-14.** 6   **B-15.** 5

B-16. AP is 1, −1, − 3, − 5, ...

Section (C)

C-1. number of rows=16, No. of logs in 16th row =5

C-2. loan paid in 30 instalments is Rs.73500,Loan left to be paid after 30th instalment is Rs. 44500

C-3. 25 days.   **C-4.** 9 hours.

OBJECTIVE QUESTIONS

Section (A)

A-1. (B)   **A-2.** (A)   **A-3.** (B)   **A-4.** (B)   **A-5.** (A)

A-6. (B)   **A-7.** (B)   **A-8.** (A)   **A-9.** (D)   **A-10.** (A)

A-11. (B)   **A-12.** (C)   **A-13.** (C)   **A-14.** (C)

Section (B)

B-1. (C)   **B-2.** (C)   **B-3.** (D)   **B-4.** (C)   **B-5.** (B)

Section (C)

C-1. (C)   **C-2.** (B)   **C-3.** (B)   **C-4.** (D)   **C-5.** (D)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	B	B	B	A	C	C	C	D	B	B	B	A	A

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	B	C	B	D	D	B	A	D	C	C	C	C	B	B	D	B	B	B
Ques.	21	22	23	24	25	26	27	28	29	30	31									
Ans.	D	B	B	B	D	A	C	A	C	B	A									