PROBABILITY

(KEY CONCEPTS + SOLVED EXAMPLES)

PROBABILITY

1. *Definitions*
2. *Mathematical definition of probability*
3. *Odds for an event*
4. *Addition theorem of probability*
5. *Conditional probability*
6. *Multiplication theorem of probability*
7. *Binomial distribution for repeated trials*
8. *Boole's inequality*
9. *Some important results*

KEY CONCEPTS

1. Definitions

1.1 Trial and Event :

An experiment is called a **trial** if it results in anyone of the possible outcomes and all the possible outcomes are called **events**.

For Example :-

(i) Participation of player in the game to win a game, is a trial but winning or losing is an event.

(ii) Tossing of a fair coin is a trial and turning up head or tail are events.

(iii) Throwing of a dice is a trial and occurrence of number 1 or 2 or 3 or 4 or 5 or 6 are events.

(iv) Drawing a card from a pack of playing cards is a trial and getting an ace or a queen is an event.

1.2 Exhaustive Events :

Total possible outcomes of an experiment are called its **exhaustive events**.

For Example :-

(i) Tossing a coin has 2 exhaustive cases i.e. either head or tail may come upward.

(ii) Throwing of a die has 6 exhaustive cases because any one of six digits 1, 2, 3, 4, 5, 6 may come upward.

(iii) The drawing of one ball from a bag which contains 4 black and 3 white balls result in ${}^7C_1$ events. Thus ${}^7C_1 = 7$ events are exhaustive.

(iv) Throwing of a pair of dice has 36 exhaustive cases because any one of six digits 1, 2, 3, 4, 5, 6 may come upward on any dice so total number of exhaustive cases = $6 \times 6 = 36$.

(v) Tossing of two and three coins results in 4 and 8 exhaustive cases respectively because head or tail may come upward on any coin. So in case of two coins total number of cases = $2 \times 2 = 4$ and in case of three coins total number of cases = $2 \times 2 \times 2 = 8$

(vi) The drawing of three cards from a pack of 52 cards results in ${}^{52}C_3$ events. Thus ${}^{52}C_3 = 22100$ events are exhaustive.

(vii) The drawing of three balls from a bag containing 4 blue, 5 white and 4 red balls results in ${}^{13}C_3$ events.

Thus ${}^{13}C_3 = 286$ events are exhaustive

1.3 Favourable Events :

Those outcomes of a trial in which a given event may happen, are called **favourable cases** for that event.

For Example :-

(i) If a coin is tossed then favourable cases of getting H is 1.

(ii) If a dice is thrown then favourable case for getting 1 or 2 or 3 or 4 or 5 or 6, is 1.

(iii) If a ball is drawn from a bag containing 4 black and 3 white balls then favourable cases for drawn ball to be black are ${}^4C_1 = 4$

(iv) If two dice are thrown, then favourable cases of getting a sum of numbers as 9 are four i.e (4,5), (5,4), (3,6), (6,3).

(v) If three cards are drawn from a pack of 52 cards then favourable cases for all drawn cards be spade are ${}^{13}C_3$ i.e. 286 favourable events.

(vi) If three balls are drawn from a bag containing 3 blue, 4 white and 2 red balls then favourable cases for drawn balls to contain 2 white and 1 red ball are ${}^4C_2 \times {}^2C_1$ i.e. 12 favourable events. ($\because$ 2 white balls and 1 red ball will be drawn from 4 white balls and 2 red balls respectively.)

1.4 Equally likely events :

Two or more events are said to be **equally likely events** if they have same number of favourable cases.

For Example :-

(i) The result of drawing a card from a well shuffled pack of cards, any card may appear in a draw, so 52 different cases are equally likely.

(ii) In tossing of a coin, getting of 'H' or 'T' are two equally likely events.

(iii) In throwing of a dice, getting 1 or 2 or 3 or 4 or 5 or 6 are six equally likely events.

1.5 Mutually exclusive or disjoint events :

Two or more events are said to be **mutually exclusive**, if the occurrence of one prevents or precludes the occurrence of the others. In other words they cannot occur together.

For example :-

(i) In tossing of a coin, getting of 'H' or 'T' are two mutually exclusive events because then can not happen together.

(ii) In throwing of a dice, getting 1 or 2 or 3 or 4 or 5 or 6 are six mutually exclusive events.

(iii) In drawing a card from a pack of cards, getting a card of diamond or heart or club or spade are four mutually exclusive events.

1.6 Simple and Compound events :

If in any experiment only one event can happen at a time then it is called a **simple event**. If two or more events happen together then they constitute a **compound event.**

For Example :-

If we draw a card from a well shuffled pack of cards, then getting a queen of spade is a simple event and if two coins A and B are tossed together then getting 'H' from A and 'T' from B is a compound event.

1.7 Independent and Dependent events :

Two or more events are said to be **independent** if happening of one does not affect other events. On the other hand if happening of one event affects (partially or totally) other event, then they are said to be **dependent events.**

For Example :-

(i) If we toss two coins, then the occurrence of head on one coin does not influence the occurrence of head or tail on the other coin in any way. Hence these events are independent.

(ii) Suppose a bag contains 5 white and 4 black balls. Two balls are drawn one by one. Then two events that first ball is white and second ball is black are independent if the first ball is replaced before drawing the second ball. If the first ball is not replaced then these two events will be dependent because second draw will have only 8 exhaustive cases.

Note :

Generally students find themselves in problem to distinguish between Independent and mutually exclusive events and get confused. These events have the following differences-

(i) Independent events are always taken from different experiments, while mutually exclusive events are from only one experiment.

(ii) Independent events can happen together but in mutually exclusive events one event may happen at one time.

(iii) Independent events are represented by the word "and" but mutually exclusive events are represented by the word "or".

1.8 Sample Space :

The set of all possible outcomes of a trial is called its **sample space**. It is generally denoted by S and each outcome of the trial is said to be a point of sample of S.

For example :-

(i) If a dice is thrown once, then its sample space S = {1, 2, 3, 4, 5, 6}

(ii) If two coins are tossed together then its sample space S = {HT, TH, HH, TT}.

2. Mathematical Definition of Probability

Let there are n exhaustive, mutually exclusive and equally likely cases for an event A and m of those are favourable to it, then probability of happening of the event A is defined by the ratio m/n which is denoted by P(A). Thus

$$P(A) = \frac{m}{n} = \frac{\text{No. of favourable cases to A}}{\text{No. of exhaustive cases to A}}$$

Note :

It is obvious that $0 \le m \le n$. If an event A is certain to happen, then m = n thus P (A) = 1.

If A is impossible to happen then m = 0 and so P (A) = 0. Hence we conclude that

$$0 \le P(A) \le 1$$

Further, if $\overline{A}$ denotes negative of A i.e. event that A doesn't happen, then for above cases m, n ; we shall have

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$\therefore P(A) + P(\overline{A}) = 1$$

3. Odds for an Event

If an event A happens in m number of cases and if total number of exhaustive cases are n then we can say that -

The probability of event A, $P(A) = \frac{m}{n}$

and $P(\overline{A}) = 1 - \frac{m}{n} = \frac{n-m}{n}$

$\therefore$ Odds in favour of

$$A = \frac{P(A)}{P(\overline{A})} = \frac{m/n}{(n-m)/n} = \frac{m}{n-m}$$

$\therefore$ Odds in against of

$$A = \frac{P(\overline{A})}{P(A)} = \frac{(n-m)/n}{m/n} = \frac{n-m}{m}$$

So Odds in favour of A = m : (n – m)

Odds in against of A= (n – m) : m

Notations :

(i) $P(A + B)$ or $P(A \cup B)$

= Probability of happening of A or B

= Probability of happening of the events A or B or both

= Probability of occurrence of at least one event A or B

(ii) $P(AB)$ or $P(A \cap B)$ = Probability of happening of events A and B together.

(iii) $P(A/B)$ = Conditional Probability of A when B has happened.

4. Addition theorem of Probability

Case I : When events are mutually exclusive:

If A and B are mutually exclusive events then $n(A \cap B) = 0 \Rightarrow P(A \cap B) = 0$

$\therefore \quad P(A \cup B) = P(A) + P(B)$

For any three events A, B, C which are mutually exclusive then $P(A \cap B) = 0$, $P(B \cap C) = 0$, $P(C \cap A) = 0$ and $P(A \cap B \cap C) = 0$

$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$

The probability of happening of any one of several mutually exclusive events is equal to the sum of their probabilities, i.e. if $A_1, A_2 .... A_n$ are mutually exclusive events then

$P(A_1 + A_2 + ... + A_n) = P(A_1) + P(A_2) + ... + P(A_n)$

i.e. $P(\Sigma A_i) = \Sigma P(A_i)$

Case II : When events are not mutually exclusive.

If A & B are two events which are not mutually exclusive then.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or $P(A + B) = P(A) + P(B) - P(AB)$

For any three events A, B, C

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

or $P(A + B + C) = P(A) + P(B) + P(C)$

$- P(AB) - P(BC) - P(CA) + P(ABC)$

5. Conditional Probability

If A and B are dependent events, then the probability of B when A has happened is called **conditional probability** of B with respect to A and it is denoted by P (B/A). It may be seen that

$$P\left(\frac{B}{A}\right) = \frac{P(AB)}{P(A)}$$

6. Multiplication theorem of Probability

6.1 Case I : When events are independent :

If $A_1, A_2, \ldots, A_n$ are independent events, then

$P(A_1. A_2. \ldots A_n) = P(A_1) P(A_2) \ldots P(A_n)$.

So if A and B are two independent events then happening of B will have no effect on A. So $P(A/B) = P(A)$ and $P(B/A) = P(B)$, then

$P(A \cap B) = P(A) . P(B)$ OR

$P(AB) = P(A) . P(B)$

Case II : When events are not independent :

The probability of simultaneous happening of two events A and B is equal to the probability of A multiplied by the conditional probability of B with respect to A (or probability of B multiplied by the conditional probability of A with respect to B).i.e.

$P(A \cap B) = P(A) . P(B/A)$ or $P(B) . P(A/B)$

OR

$P(AB) = P(A) . P(B/A)$ or $P(B) . P(A/B)$

6.2 Probability of at least one of the n Independent events :

If $p_1, p_2, p_3, \ldots p_n$ are the probabilities of n independent events $A_1, A_2, A_3 \ldots A_n$ then the probability of happening of at least one of these event is

$$1 - [(1 - p_1)(1 - p_2) \ldots (1 - p_n)]$$

$$P(A_1 + A_2 + A_3 + \ldots + A_n) = 1 - P(\overline{A}_1)$$

$$P(\overline{A}_2) P(\overline{A}_3) \ldots P(\overline{A}_n)$$

7. Binomial distribution for Repeated Trials

Let an experiment is repeated **n** times and probability of happening of any event called success is **p** and not happening the event called failure is $q = 1 - p$ then by binomial theorem.

$$(q + p)^n = q^n + {}^nC_1 q^{n-1} p + \ldots + {}^nC_r q^{n-r} p^r + \ldots + p^n$$

Now probability of

(a) Occurrence of the event exactly r times

$$= {}^nC_r q^{n-r} p^r$$

(b) Occurrence of the event at least r times

$$= {}^nC_r q^{n-r} p^r + \ldots + p^n$$

(c) Occurrence of the event at the most r times

$$= q^n + {}^nC_1 q^{n-1} p + \ldots + {}^nC_r q^{n-r} p^r$$

8. Boole's Inequality

(a) For any two events A and B.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad P(A \cup B) \le P(A) + P(B)$

$\{\because P(A \cap B) \ge 0\}$

(b) For any three events A,B,C

$P(A \cup B \cup C) \le P(A) + P(B) + P(C)$

(c) In general for any n events $A_1, A_2, \ldots A_n$

$P(A_1 \cup A_2 \cup \ldots \cup A_n) \le P(A_1) + P(A_2) + \ldots + P(A_n)$

9. Some Important Results

(a) Let A and B be two events, then

(i) $P(A) + P(\overline{A}) = 1$

(ii) $P(A + B) = 1 - P(\overline{A}\,\overline{B})$

(iii) $P(A/B) = \dfrac{P(AB)}{P(B)}$

(iv) $P(A + B) = P(AB) + P(\overline{A}B) + P(A\overline{B})$

(v) $A \subset B \Rightarrow P(A) \le P(B)$

(vi) $P(\overline{A}B) = P(B) - P(AB)$

(vii) $P(AB) \le P(A) P(B) \le P(A + B) \le P(A) + P(B)$

(viii) $P(AB) = P(A) + P(B) - P(A + B)$

(ix) P (Exactly one event)

$= P(A\overline{B}) + P(\overline{A}B) = P(A) + P(B) - 2p(AB)$

$= P(A + B) - P(AB)$

(x) P(neither A nor B) $= P(\overline{A}\,\overline{B}) = 1 - P(A+B)$

(xi) $P(\overline{A} + \overline{B}) = 1 - P(AB)$

(b) Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) = $\mathbf{2^n}$

(c) Number of exhaustive cases of throwing n dice simultaneously (or throwing one dice n times) = $\mathbf{6^n}$

(d) Playing Cards :

(i) Total : 52 (26 red, 26 black)

(ii) Four suits : Heart, Diamond, Spade, Club - 13 cards each

(iii) Court Cards : 12 (4 Kings, 4 queens, 4 jacks)

(iv) Honour Cards : 16 (4 aces, 4 kings, 4 queens, 4 jacks)

(e) Probability regarding n letters and their envelopes:

If n letters corresponding to n envelopes are placed in the envelopes at random, then

(i) Probability that all letters are in right envelopes = $\frac{1}{n!}$

(ii) Probability that all letters are not in right envelopes = $1 - \frac{1}{n!}$

(iii) Probability that no letter is in right envelope

$$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ..... + (-1)^n \frac{1}{n!}$$

(iv) Probability that exactly r letters are in right envelopes

$$= \frac{1}{r!}\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ....+ (-1)^{n-r}\frac{1}{(n-r)!}\right]$$

SOLVED EXAMPLE

Ex.1 One card is drawn from a pack of playing cards, then the probability that it is a card of king is-

(A) $\frac{1}{12}$ (B) $\frac{1}{13}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Sol. Probability of one card to be king

$p = \frac{4}{52} = \frac{1}{13}$

($\because$ favourable cases = 4, Total cases = 52)

Ans. [B]

Ex.2 If P (A) $= \frac{3}{8}$, then find the odds in against of A -

(A) 3 : 5 (B) 4 : 5

(C) 3 : 4 (D) 5 : 3

Sol. $P(A) = \frac{3}{8} \Rightarrow P(\overline{A}) = 1 - \frac{3}{8} = \frac{5}{8}$

$\therefore$ odds in against of A $= \frac{P(\overline{A})}{P(A)} = \frac{5}{3} = 5:3$

Ans.[D]

Ex.3 If the probability for A to fail in an examination is 0.2 and that of B to fail is 0.3, then the probability that either A or B fails is-

(A) 0.5 (B) 0.44

(C) 0.56 (D) None of these

Sol. Let A be event for A to fail and B be the event for B to fail, then

P (A) = 0.2 and P(B) = 0.3

Since A and B are independent events,

$\therefore$ P (AB) = P(A) P (B)

$\therefore$ Required probability

= P(A+ B)

= P(A) + P (B) – P (AB)

= P(A) + P (B) – P (A) P(B)

= 0.2 + 0.3 – 0.2 × 0.3

= 0.5 – 0.06 = 0.44 **Ans.[B]**

Ex.4 If two dice are thrown together then what is the probability that the sum of their numbers is greater than 9.

(A) 1/2 (B) 1/4

(C) 1/6 (D) 2/6

Sol. The sum of the numbers greater than 9 may be 10, 11 and 12. If these events be A, B, C respectively, then

P (A) = 3/36

[$\because$ favourable cases are (6, 4), (5, 5), (4, 6)]

P (B) = 2/36

[$\because$ favourable cases are (6, 5), (5, 6)]

P (C) = 1/36

[$\because$ favourable case is (6, 6)]

Now since A, B, C are mutually exclusive,

so P (A + B + C) = P(A) + P(B) + P (C)

$= \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{1}{6}$ **Ans.[C]**

Ex.5 Two cards are drawn one by one from a pack of 52 cards. If the first card is not replaced in the pack, then what is the probability that first card is that of a king and second card is that of a queen?

(A) 4/664 (B) 5/663

(C) 6/663 (D) 4/663

Sol. Let A ≡ first card is that of a king

B ≡ second card is that of a queen

that $P(A) = \frac{4}{52} = \frac{1}{13}$, $P(B/A) = \frac{4}{51}$;

$\therefore P(AB) = P(A)\,P(B/A) = \frac{1}{13} \cdot \frac{4}{51} = \frac{4}{663}$

Ans. [D]

Ex.6 Three coins are tossed together. What is the probability of getting tail on first, head on second and tail on third coin?

(A) 1/8 (B) 1/6 (C) 1/4 (D) 1/3

Sol. Let the three events be denoted by A, B and C respectively, then

P(A) = P(B) = P(C) = 1/2

since the events A, B and C are independent,

$\therefore$ P(ABC) = P(A) P(B) P(C) = 1/8

Ans.[A]

Ex.7 One person can kill a bird twice in 3 shots, second once in 3 shots and third thrice in 4 shots. If they shot together then what is the probability that the bird will be killed?

(A) 18/19 (B) 17/18

(C) 18/17 (D) 20/17

Sol. If A, B, C denote events of killing the bird by first second and third person respectively, then

P (A) = 2/3, P (B) = 1/3, P (C) = 3/4

The bird will be killed if atleast one of these three independent events happens. So

Required probability

$= 1 - P(\overline{A}_1)\, P(\overline{A}_2)\;\; P(\overline{A}_3)$

= 1 – (1 – 2/3) (1 – 1/3) (1 – 3/4)

= 1 – 1/3. 2/3. 1/4 = 17/18 **Ans. [B]**

Ex.8 A coin is tossed thrice. If E be the event of showing atleast two heads and F the event of showing head in the first throw, then P (E/F) is equal to-

(A) 3/4 (B)3/8

(C) 1/2 (D) 1/8

Sol. There are following 8 outcomes of three throws:

HHH,HHT, HTH, HTT, THH, THT, TTH, TTT

Also $P(E \cap F) = 3/8$ and P(F) = 4/8

$\therefore$ required probability $= P(E/F) = \dfrac{P(E \cap F)}{P(F)}$

$$= \frac{3/8}{4/8} = \frac{3}{4}$$ **Ans.[A]**

Ex.9 Two dice are thrown together 4 times. The probability that both dice will show same numbers twice is-

(A) 1/3 (B) 25/36

(C) 25/216 (D) None of these

Sol. The probability of showing same number by both dice p = 6/36 = 1/6

In binomial distribution here n = 4, r = 2, p = 1/6, q = 5/6

required probability $= {}^nC_r\, q^{n-r} p^r$

$= {}^4C_2\, (5/6)^2\, (1/6)^2$

$$= 6\left(\frac{25}{36}\right)\left(\frac{1}{36}\right) = \frac{25}{216}$$ **Ans.[C]**

Ex.10 For any two events A and B, $P\left(\dfrac{A}{A \cup B}\right)$ equals-

(A) $\dfrac{P(\overline{A})}{P(A \cup B)}$ (B) $\dfrac{P(\overline{B})}{P(A \cup B)}$

(C) $\dfrac{P(A)}{P(A \cup B)}$ (D) $\dfrac{P(A \cap B)}{P(A \cup B)}$

Sol. Here

$$P\left(\frac{A}{A \cup B}\right) = \frac{P(A \cap A \cup B)}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)}$$

Ans.[C]

Ex.11 There are four letters and four envelopes. The letters are placed into the envelopes randomly. The probability that all letters are not placed in the correct envelope is-

(A) 1/24 (B) 23/24 (C) 19/24 (D) 9/24

Sol. P (all not correct) = 1 – P (all correct)

$$= 1 - \frac{1}{4!} = \frac{23}{24}$$ **Ans.[B]**

Ex.12 A bag contains 5 brown and 4 white socks. A man pulls out 2 socks. The probability that they are of the same colour is-

(A) 5/108 (B) 1/6

(C) 5/18 (D) 4/9

Sol. Let A ≡ event of two socks being brown

B ≡ event of two socks being white

Then $P(A) = \dfrac{{}^5C_2}{{}^9C_2} = \dfrac{5.4}{9.8} = \dfrac{5}{18}$,

$$P(B) = \frac{{}^4C_2}{{}^9C_2} = \frac{4.3}{9.8} = \frac{3}{18}$$

Now since A and B are mutually exclusive events, so required probability = P(A + B)

= P(A) + P(B)

$$= \frac{5}{18} + \frac{3}{18} = \frac{4}{9}$$ **Ans.[D]**

Ex.13 One number is selected at random from first two hundred positive integers. The probability that it is divisible by 6 or 8 is-

(A) 1/3 (B) 2/3

(C) 3/4 (D) 1/4

Sol. Among first two hundred positive integers, 33 integers are divisible by 6, 25 integers are divisible by 8 but 8 integers are divisible by both 6 and 8. So

Required Probability $= \frac{33+25-8}{200} = \frac{50}{200} = \frac{1}{4}$

Ans.[D]

Ex.14 One card is drawn from a pack of 52 cards. The probability that it is a king or spade is-

(A) 1/26 (B) 3/26

(C) 4/13 (D) 3/13

Sol. One card can be drawn in 52 ways, Therefore total number of exhaustive cases = 52.

Now number of favourable cases

= 13 + 4 – 1 = 16

$\therefore$ Required probability $= \frac{16}{52} = \frac{4}{13}$ **Ans.[C]**

Ex.15 The probability that atleast one of the events A and B happens is 0.6. If probability of their simultaneous happening is 0.2, then

$P(\bar{A}) + P(\bar{B})$ is-

(A) 0.4 (B) 0.8

(C) 1.2 (D) 1.4

Sol. As given P (A+B) = 0.6 and P (AB) = 0.2

$\therefore P(A + B) = P(A) + P(B) – P(AB)$

$\Rightarrow 0.6 = P(A) + P(B) – 0.2$

$\Rightarrow –P(A) – P(B) = –0.8$

$\Rightarrow [1– P(A)] + [1–P(B)] = 2 – 0.8 = 1.2$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.2$ **Ans.[C]**

Ex.16 Two dice are thrown together. The probability of getting the sum of digits as a multiple of 4 is-

(A) 1/9 (B) 1/3

(C) 1/4 (D) 5/9

Sol. Total exhaustive cases = 6^2 = 36

Following 9 pairs are favourable as the sum of their digits are multiple of 4 i.e. 4 or 8 or 12.

(1, 3), (2, 2), (3,1), (2, 6), (3, 5), (4, 4), (5,3), (6, 2), (6, 6)

$\therefore$ required probability = 9/36 = 1/4.

Ans.[C]

Ex.17 If from a factory a labourer is chosen, randomly. the probability that he is a male is 0.6 and is married is 0.7. The probability that the chosen labourer is a married woman is-

(A) 0.42 (B) 0.28

(C) 0.12 (D) None of these

Sol. Let A and B respectively be two events that a chosen labourer is a man and is married, then required probability

$= P(\bar{A}B) = \{1 – P(A)\}\, P(B)$

$= (1 – 0.6)\,(0.7) = 0.28$ **Ans.[B]**

Ex.18 A speaks truth in 75% cases and B in 80% cases. What is the probability that they contradict each other in stating the same fact?

(A) 7/20 (B) 13/20

(C) 3/20 (D) 1/5

Sol. There are two mutually exclusive cases in which they contradict each other i.e. $\bar{A}B$ and $A\bar{B}$. Hence

Required probability

$= P(A\bar{B} + \bar{A}B) = P(A\bar{B}) + P(\bar{A}B)$

$= (A\bar{B} + \bar{A}B) = P(A\bar{B}) + P(\bar{A}B)$

$= P(A)\,P(\bar{B}) + P(\bar{A}).P(B)$

$= \frac{3}{4}.\frac{1}{5} + \frac{1}{4}.\frac{4}{5} = \frac{7}{20}$ **Ans. [A]**

Ex.19 The letters of the word HIRDESH are written in a row randomly. The probability of the words starting with H and ending with H is-

(A) 1/21 (B) 2/21

(C) 1/7 (D) None of these

Sol. Total no. of cases $= \frac{7!}{2!}$

Since H is written at first and last places, therefore at the remaining 5 places, 5 letters can be written in 5! ways.

Required probability $= \frac{5!}{7!/2!} = \frac{1}{21}$

Ans.[A]

Ex.20 If a dice is thrown twice, then the probability of getting 1 in the first throw only is-

(A) 1/36 (B) 3/36

(C) 5/36 (D) 1/6

Sol. Probability of getting 1 in first throw = $\frac{1}{6}$

Probability of not getting 1 in second throw = $\frac{5}{6}$

Both are independent events, so the required probability = $\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$ **Ans.[C]**

Ex.21 Let A and B be two independent events. The probability that both A and B occur together is 1/6 and the probability that neither of them occurs is 1/3. The probability of occurrence of A is-

(A) P(A) = 1/4, P(B) = 1/3

(B) P(A) = 1/2, P(B) = 1/6

(C) P(A) = 1/3, P(B) = 1/2

(D) None of these

Sol. $\because$ A and B are independent events

$\therefore$ P(AB) = 1/6 P(A) P(B) = 1/6 ...(1)

Further $P(A + B) = 1 - P(\overline{A}\ \overline{B}) = 1 - 1/3 = 2/3$

$\Rightarrow P(A + B) = P(A) + P(B) - P(AB) = 2/3$

$\Rightarrow P(A) + P(B) = 2/3 + 1/6 = 5/6$ ...(2)

(1), (2) $\Rightarrow$ P(A) = 1/3, P(B) = 1/2

Ans.[C]

Ex.22 Two dice are thrown simultaneously. What is the probability of obtaining a multiple of 2 on one of them and a multiple of 3 on the other-

(A) 5/36 (B) 11/36

(C) 1/6 (D) 1/3

Sol. Favourable cases for one are three i.e. 2,4 and 6 and for other are two i.e. 3,6.

Hence required probability =

$$\left[\left(\frac{3\times 2}{36}\right)2 - \frac{1}{36}\right] = \frac{11}{36}$$

[As same way happen when dice changes numbers among themselves] **Ans.[B]**

Ex.23 A target is hit by A, 4 times out of 5 attempts; by B, 3 times out of 4 attempts and by C, 2 times out of 3 attempts. The probability that the target is hit by two of them is-

(A) 25/60 (B) 26/60

(C) 1/2 (D) 5/6

Sol. The following mutually exclusive cases are possible.

(i) $AB\overline{C}$ (ii) $A\overline{B}C$ (iii) $\overline{A}BC$

Since A, B and C are independent event therefore $P(AB\overline{C}) = P(A)\,P(B)\,P(\overline{C})$

$$= \frac{4}{5} \cdot \frac{3}{4} = \left(1 - \frac{2}{3}\right) = \frac{12}{60}$$

Similarly $P(A\overline{B}C) = \frac{8}{60}$ and $P(\overline{A}BC) = \frac{6}{60}$

Thus the required probability

$$= P(AB\overline{C}) + P(A\overline{B}C) + P(\overline{A}BC)$$

$$= \frac{12}{60} + \frac{8}{60} + \frac{6}{60} = \frac{26}{60}$$

Ans.[B]

Ex.24 Three numbers are selected at random from whole numbers 1 to 20. The probability that they are consecutive integers is-

(A) 1/380 (B) 3/190

(C) 3/20 (D) None of these

Sol. Total number of sequences of 3 numbers selected one by one from whole numbers 1 to 20

$$= {}^{20}C_3 = \frac{20 \times 19 \times 18}{\lfloor 3}$$

Now sequences which will contain three consecutive integers are (1, 2, 3) (2, 3, 4), (3, 4, 5)......, (18,19,20).

These are 18 sequences. Hence

$\therefore$ required probability

$$= \frac{18}{\frac{20 \times 19 \times 18}{\lfloor 3}} = \frac{6}{380} = \frac{3}{190}$$

Ans.[B]

Ex.25 There are three urns A, B and C. Urn A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls. Urn C contains 2 white balls and 4 blue balls. One ball is drawn from each of these urns. What is the probability that out of these three balls drawn, two are white balls and one is a blue ball?

(A) 64/190 (B) 63/189

(C) 64/189 (D) 65/189

Sol. Consider the following events:

E_1 = ball drawn from urn A is white, E_2 = ball drawn from urn B is white, E_3 = ball drawn from urn C is white.

Then $P(E_1)=\frac{4}{9}$, $P(E_2) = \frac{4}{7}$ and $P(E_3)=\frac{2}{6} = \frac{1}{3}$.

$\therefore$ P= $(\bar{E}_1)$ ball drawn from urn A is black

$$= 1 - P(E_1) = 1 - \frac{4}{9} = \frac{5}{9},$$

$P(\bar{E}_2)$ = ball drawn from urn B is black

$$= 1 - P(\bar{E}_2) = 1 - \frac{4}{7} = \frac{3}{7}$$

and $P(\bar{E}_3)$ = ball drawn from urn C is black

$$= 1 - P(\bar{E}_3) = 1 - \frac{1}{3} = \frac{2}{3}.$$

Now, two white balls and one black ball can be drawn in the following mutually exclusive ways :

(I) White from urn A, white from urn B and black from urn C i.e. $E_1 \cap E_2 \cap \bar{E}_3$

(II) White from urn A, black from urn B and white from urn C i.e. $E_1 \cap \bar{E}_2 \cap E_3$

(III) Black from urn A, white from urn B and white from urn C i.e. $\bar{E}_1 \cap E_2 \cap E_3$

Required probability = P(I) + P(II) + P(III)

$= P(E_1 \cap E_2 \cap \bar{E}_3) + P(E_1 \cap \bar{E}_2 \cap E_3)$
$+ P(\bar{E}_1 \cap E_2 \cap E_3)$

$= P(E_1). P(E_2) P(\bar{E}_3) + P(E_1) P(\bar{E}_2) P(E_3)$
$+ P(\bar{E}_1) P(E_2) P(E_3)$

[$\because$ E_1, E_2, E_3 are independent events]

$$= \frac{4}{9} \times \frac{4}{7} \times \frac{2}{3} + \frac{4}{9} \times \frac{3}{7} \times \frac{1}{3} + \frac{5}{9} \times \frac{4}{7} \times \frac{1}{3}$$

$$= \frac{64}{189}$$ **Ans.[C]**

Ex.26 A box contains 20 cards. The letter I is written on 10 cards and T is written on other 10 cards, 3 cards are chosen randomly and are kept in the same order. The probability of making the word IIT is-

(A) 9/80 (B) 1/80

(C) 4/27 (D) 5/38

Sol. Required probability = P (I on first card) × P (I on second card) × P (T on third card)

$$= \frac{10}{20} \times \frac{9}{19} \times \frac{10}{18} = \frac{5}{38}$$ **Ans.[D]**

Ex.27 A coin is tossed twice and the four possible outcomes are assumed to be equally likely. If A is the event, ' both head and tail have appeared', and B be the event, ' at most one tail is observed', then the value of P(B/A) is-

(A) 1 (B) 3

(C) 4 (D) None of these

Sol. Here S = {HH, HT, TH, TT}, A = {HT, TH} and B = {HH, HT, TH}.

$\therefore$ $A \cap B$ = {HT, TH}.

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$,

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$$

and, $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$,

$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/2}{3/4} = \frac{2}{3}$ and,

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/2}{1/2} = 1.$$ **Ans.[A]**

Ex.28 If two events A and B are such that $P(\bar{A}) = 0.3$, P(B) = 0.4 and $P(A\bar{B}) = 0.5$, then $P\left(\frac{B}{A \cup \bar{B}}\right)$ equals-

(A) 1/2 (B) 1/3

(C) 1/4 (D) 1/5

Sol. We have

$$P\left(\frac{B}{A \cup \bar{B}}\right) = \frac{P[B \cap (A \cup \bar{B})]}{P(A \cup \bar{B})}$$

$$= \frac{P[(B \cap A)(B \cap \bar{B})]}{P(A) + P(\bar{B}) - P(A \cap \bar{B})}$$

$$= \frac{P(AB)}{P(A) + P(\bar{B}) - P(A\bar{B})}$$

$$= \frac{P(A) - P(A\bar{B})}{P(A) + P(\bar{B}) - P(A\bar{B})}$$

$$= \frac{0.7 - 0.5}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$ **Ans.[C]**

Ex.29 Two numbers are selected at random from 40 consecutive natural numbers. The probability that the sum of the selected numbers is odd will be-

(A) 14/29 (B) 20/39

(C) 1/2 (D) None of these

Sol. Total number of selection of 2 numbers from 40 natural numbers = ${}^{40}C_2$.

Now, since the sum of two natural numbers is odd if one of them is even and the other is odd. Also among 40 consecutive natural numbers 20 are even and 20 are odd. Hence number of ways of selection of one even and one odd number = ${}^{20}C_1 \times {}^{20}C_1$

$\therefore$ required probability = $\frac{{}^{20}C_1 \times {}^{20}C_1}{{}^{40}C_2}$

$$= \frac{20 \times 20 \times 2}{40 \times 39} = \frac{20}{39}$$ **Ans.[B]**

Ex.30 The letter of the word 'ASSASSIN' are written down at random in a row. The probability that no two S occur together is-

(A) 1/35 (B) 1/14

(C) 1/15 (D) None of these

Sol. Total ways of arrangements = $\frac{8!}{2!.4!}$

• w • x • y • z •

Now 'S' can have places at dot's and in places of w, x, y, z we have to put 2 A's, one I and one N.

Therefore favourable ways = $5\left(\frac{4!}{2!}\right)$

Hence required probability

$$= \frac{5.4!\ 2!\ 4!}{2!\ 8!} = \frac{1}{14}$$ **Ans.[B]**

Ex.31 Assuming that for a husband-wife couple the chances of their child being a boy or a girl are the same, the probability of their two children being a boy and a girl is-

(A) 1/4 (B) 1

(C) 1/2 (D) 1/8

Sol. Following four mutually exclusive cases are possible:

(i) both children are boys

(ii) both children are girls

(iii) first is girl and second is boy

(iv) first is boy and second is girl

Out of these the last two are favourable cases. Hence the required probability

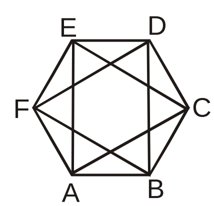
$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$ **Ans.[C]**

Ex.32 Three vertices out of six vertices of a regular hexagon are chosen randomly. The probability of getting a equilateral triangle after joining three vertices is-

(A) 1/5 (B) 1/20

(C) 1/10 (D) 1/2

Sol. The total no. of cases = ${}^{6}C_3 = 20$



As shown in the figure only two triangles ACE and BDF are equilateral. So number of favourable cases is 2.

Hence the required probability = $\frac{2}{20} = \frac{1}{10}$

Ans.[C]

Ex.33 Three numbers are chosen at random without replacement from {1, 2, 3,...., 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7, is-

(A) 7/40 (B) 3/10

(C) 13/60 (D) None of these

Sol. Let A ≡ the event that minimum number selected is 3

B ≡ the event that maximum number selected is 7 then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ...(1)

Total number of ways in which 3 numbers can be chosen from the given 10 numbers

$$= {}^{10}C_3 = 120.$$

Now minimum of the chosen three numbers is 3 if one of them is 3 and the remaining two from 4, 5, 6, 7, 8, 9, 10. This can be obtained in

${}^{7}C_2 = 21$ ways.

$$\therefore \qquad P(A) = \frac{21}{120} \qquad ...(2)$$

Also maximum of the chosen three numbers is 7 if one of them is 7 and the remaining two from 1, 2, 3, 4, 5, 6. This can be obtained in ${}^{6}C_2 = 15$ ways

$$\therefore \qquad P(B) = \frac{15}{120} \qquad ...(3)$$

Further favourable cases for $A \cap B = {}^{3}C_1 = 3$ because AB is possible if numbers are chosen from 4, 5, 6.

$$\therefore \qquad P(A \cap B) = \frac{3}{120} \qquad ...(4)$$

So from (1), (2), (3), (4), we have.

$$\text{required probability} = \frac{21}{120} + \frac{15}{120} - \frac{3}{120}$$

$$= \frac{33}{120} = \frac{11}{40} \quad \textbf{Ans.[C]}$$