# JEE MAIN + ADVANCED

# MATHEMATICS

## TOPIC NAME

# QUADRATIC EQUATION

# &

# EXPRESSION

## (PRACTICE SHEET)

# LEVEL # 1

## Question based on: Roots of Quadratic Equation

**Q.1** The roots of the equation $(x+2)^2 = 4(x+1) - 1$ are-
(A) $\pm 1$  (B) $\pm i$
(C) 1,2  (D) $-1, -2$

**Q.2** The roots of quadratic equation $x^2 + 14x + 45 = 0$ are -
(A) $-9, 5$  (B) 5, 9
(C) $-5, 9$  (D) $-5, -9$

**Q.3** The roots of the equation $x^4 - 8x^2 - 9 = 0$ are-
(A) $\pm 3, \pm 1$  (B) $\pm 3, \pm i$
(C) $\pm 2, \pm i$  (D) None of these

**Q.4** Which of the following equations has 1 and $-2$ as the roots -
(A) $x^2 - x - 2 = 0$  (B) $x^2 + x - 2 = 0$
(C) $x^2 - x + 2 = 0$  (D) $x^2 + x + 2 = 0$

**Q.5** Roots of $3^x + 3^{-x} = 10/3$ are-
(A) 0, 1  (B) 1, $-1$
(C) 0, $-1$  (D) None of these

**Q.6** If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2 and 3 are roots of the equations $f(x) = 0$, then values of m and n are-
(A) 5, 30  (B) $-5$, 30
(C) $-5, -30$  (D) 5, $-30$

**Q.7** The number of roots of the quadratic equation $8 \sec^2\theta - 6 \sec\theta + 1 = 0$ is -
(A) Infinite  (B) 1
(C) 2  (D) 0

## Question based on: Nature of roots

**Q.8** If roots of the equation $ax^2 + 2(a+b)x + (a + 2b + c) = 0$ are imaginary, then roots of the equation $ax^2 + 2bx + c = 0$ are -
(A) rational  (B) irrational
(C) equal  (D) complex

**Q.9** If a and b are the odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0$, $a \neq 0$, will be-
(A) rational  (B) irrational
(C) non-real  (D) equal

**Q.10** If the roots of the equation $6x^2 - 7x + k = 0$ are rational then k is equal to -
(A) $-1$  (B) $-1, -2$
(C) $-2$  (4) 1, 2

**Q.11** The roots of the equation $(a^2 + b^2)x^2 - 2(bc + ad)x + (c^2 + d^2) = 0$ are equal, if -
(A) ab = cd  (B) ac = bd
(C) ad + bc = 0  (4) None of these

**Q.12** For what value of m, the roots of the equation $x^2 - x + m = 0$ are not real-
(A) $]\frac{1}{4}, \infty[$  (B) $]-\infty, \frac{1}{4}[$
(C) $]-\frac{1}{4}, \frac{1}{4}[$  (4) None of these

**Q.13** Roots of the equation $(a + b - c)x^2 - 2ax + (a - b + c) = 0$, $(a,b,c \in Q)$ are –
(A) rational  (B) irrational
(C) complex  (D) none of these

**Q.14** The roots of the equation $x^2 - x - 3 = 0$ are-
(A) Imaginary  (B) Rational
(C) Irrational  (D) None of these

**Q.15** The roots of the equation $x^2 + 2\sqrt{3}x + 3 = 0$ are-
(A) Real and equal
(B) Rational and equal
(C) Irrational and equal
(D) Irrational and unequal

**Q.16** If the roots of the equation $ax^2 + x + b = 0$ be real, then the roots of the equation $x^2 - 4\sqrt{ab}\,x + 1 = 0$ will be -

(A) Rational (B) Irrational
(C) Real (D) Imaginary

**Q.17** If one root of equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots then the value of q is-
(A) 49/4 (B) 4/49
(C) 4 (D) None of these

**Q.18** If roots of the equation $(a - b)x^2 + (c - a)x + (b - c) = 0$ are equal, then a, b, c are in -
(A) A.P. (B) H.P.
(C) G.P. (D) None of these

**Q.19** If the roots of $x^2 - 4x - \log_2 a = 0$ are real, then-
(A) $a \geq \frac{1}{4}$ (B) $a \geq \frac{1}{8}$
(C) $a \geq \frac{1}{16}$ (D) None of these

**Q.20** If the roots of both the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}x + q = 0$ are real, then -
(A) $p = q$ , $r \neq 0$ (B) $2q = \pm\sqrt{pq}$
(C) $p/q = q/r$ (D) None of these

**Q.21** The roots of the equation $(p - 2)x^2 + 2(p - 2)x + 2 = 0$ are not real when-
(A) $p \in [1, 2]$ (B) $p \in [2, 3]$
(C) $p \in (2, 4)$ (D) $p \in [3, 4]$

**Q.22** If the roots of the equation $x^2 - 10x + 21 = m$ are equal then m is-
(A) 4 (B) 25
(C) – 4 (D) 0

**Question based on** **Sum and product of roots**

**Q.23** For what value of a, the difference of roots of the equation $(a - 2)x^2 - (a - 4)x - 2 = 0$ is equal to 3
(A) 3, 3/2 (B) 3, 1
(C) 1, 3/2 (D) None of these

**Q.24** If α, β are roots of the equation $x^2 + px - q = 0$ and γ, δ are roots of $x^2 + px + r = 0$, then the value of $(\alpha - \gamma)(\alpha - \delta)$ is-
(A) p + r (B) p – r
(C) q – r (D) q + r

**Q.25** If α, β are roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 . (2\beta - 35)^3$ is equal to-
(A) 1 (B) 8
(C) 64 (D) None of these

**Q.26** If α, β are roots of the equation $px^2 + qx - r = 0$, then the value of $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ is equal to-
(A) $-\frac{p}{qr^2}(3pr + q^2)$ (B) $-\frac{q}{pr^2}(3pr + q^2)$
(C) $-\frac{q}{pr^2}(3pr - q^2)$ (D) $\frac{p}{pr^2}(3pr + q)$

**Q.27** If product of roots of the equation $mx^2 + 6x + (2m - 1) = 0$ is – 1, then m equals-
(A) – 1 (B) 1 (C) 1/3 (D) – 1/3

**Q.28** For what value of a the sum of roots of the equation $x^2 + 2(2 - a - a^2)x - a^2 = 0$ is zero -
(A) 1, 2 (B) 1, – 2
(C) – 1, 2 (D) – 1, – 2

**Q.29** The difference between the roots of the equation $x^2 - 7x - 9 = 0$ is -
(A) 7 (B) $\sqrt{85}$
(C) 9 (D) $2\sqrt{85}$

**Q.30** The HM of the roots of the equation $x^2 - 8x + 4 = 0$ is -
(A) 1 (B) 2
(C) 3 (D) None of these

**Q.31** If the sum of the roots of the equation $ax^2 + 4x + c = 0$ is half of their difference, then the value of ac is-
(A) 4 (B) 8 (C) 12 (D) – 12

**Q.32** If the sum of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is –1, then the product of the roots is -
(A) 0 (B) 1 (C) 2 (D) 3

**Q.33** Sum of roots is – 1 and sum of their reciprocals is $\frac{1}{6}$, then equation is –

(A) $x^2 + x - 6 = 0$ (B) $x^2 - x + 6 = 0$

(C) $6x^2 + x + 1 = 0$ (D) $x^2 - 6x + 1 = 0$

**Q.34** If $\alpha, \beta$ are roots of the equation $2x^2 - 5x + 3 = 0$, then $\alpha^2\beta + \beta^2\alpha$ is equal to-

(A) 15/2 (B) – 15/4

(C) 15/4 (D) – 15/2

**Q.35** If $\alpha, \beta$ be the roots of the equation $p(x^2 + n^2) + pnx + qn^2x^2 = 0$ then the value of $p(\alpha^2 + \beta^2) + p\alpha\beta + q\alpha^2\beta^2$ is -

(A) $\alpha + \beta$ (B) 0

(C) $p + q$ (D) $\alpha + \beta + p + q$

**Q.36** If $\alpha$ and $\beta$ are roots of $ax^2 - bx + c = 0$, then $(\alpha + 1)(\beta + 1)$ is equal to -

(A) $\frac{a-b+c}{a}$ (B) $\frac{a+b-c}{a}$

(C) $\frac{a+b+c}{a}$ (D) $\frac{b-a+c}{a}$

**Q.37** If difference of roots of the equation $x^2 - px + q = 0$ is 1, then $p^2 + 4q^2$ equals-

(A) $2q + 3$ (B) $(1 - 2q)^2$

(C) $(1 + 2q)^2$ (D) $2q - 3$

**Q.38** If $\alpha$ and $\beta$ are the roots of the equation $x^2 + (\sqrt{\alpha})x + \beta = 0$ then the values of $\alpha$ and $\beta$ are

(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 2, \beta = -2$

(C) $\alpha = 1, \beta = -1$ (D) $\alpha = -1, \beta = 1$

**Q.39** If roots $\alpha$ and $\beta$ of the equation $x^2 + px + q = 0$ are such that $3\alpha + 4\beta = 7$ and $5\alpha - \beta = 4$, then (p, q) is equal to -

(A) (1, 1) (B) (– 1, 1)

(C) (– 2, 1) (D) (2, 1)

**Q.40** If one root of the equation $x^2 - 30x + p = 0$ is square of the other, then p is equal to-

(A) 125, 216 (B) 125, – 216

(C) Only 125 (D) Only – 216

**Q.41** If $\alpha, \beta$ are roots of the equation $x^2 - mx + n = 0$, then value of $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$ is -

(A) $1 + (m + n) + (m^2 - mn + n^2)$

(B) $1 + (m + n) + (m^2 + mn + n^2)$

(C) $1 - (m - n) + (m^2 + mn + n^2)$

(D) None of these

**Q.42** If the equation $\frac{a}{x-a} + \frac{b}{x-b} = 1$ has roots equal in magnitude but opposite in sign, then the value of a + b is -

(A) – 1 (B) 0

(C) 1 (D) None of these

**Q.43** If $\alpha$ and $\beta$ are the root of $ax^2 + bx + c = 0$, then the value of $\left\{\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}\right\}$ is-

(A) $\frac{a}{bc}$ (B) $\frac{b}{ca}$

(C) $\frac{c}{ab}$ (D) None of these

**Q.44** If roots of the equations $2x^2 - 3x + 5 = 0$ and $ax^2 + bx + 2 = 0$ are reciprocals of the roots of the other then (a, b) equals -

(A) ( –5, 3) (B) (5, 3)

(C) (5, –3) (D) ( –5, –3)

**Q.45** If the sum of the roots of $ax^2 + bx + c = 0$ be equal to sum of the squares, then -

(A) $2ac = ab + b^2$ (B) $2ab = bc + c^2$

(C) $2bc = ac + c^2$ (D) None of these

**Q.46** If one root of $ax^2 + bx + c = 0$ be square of the other, then the value of $b^3 + ac^2 + a^2c$ is-

(A) 3 abc (B) – 3abc

(C) 0 (D) None of these

**Question based on** **Formation of Quadratic Equation with given roots**

**Q.47** The quadratic equation with one root 2i is-

(A) $x^2 + 4 = 0$ (B) $x^2 - 4 = 0$

(C) $x^2 + 2 = 0$ (D) $x^2 - 2 = 0$

**Q.48** The sum of the roots of a equation is 2 and sum of their cubes is 98, then the equation is -

(A) $x^2 + 2x + 15 = 0$
(B) $x^2 + 15x + 2 = 0$
(C) $2x^2 - 2x + 15 = 0$
(D) $x^2 - 2x - 15 = 0$

**Q.49** If $\alpha$ and $\beta$ are roots of $2x^2 - 3x - 6 = 0$, then the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$ will be -
(A) $4x^2 + 49x - 118 = 0$
(B) $4x^2 - 49x - 118 = 0$
(C) $4x^2 - 49x + 118 = 0$
(D) $4x^2 + 49x + 118 = 0$

**Q.50** If $\alpha$ and $\beta$ are roots of $2x^2 - 7x + 6 = 0$, then the quadratic equation whose roots are $-\frac{2}{\alpha}, -\frac{2}{\beta}$ is-
(A) $3x^2 + 7x + 4 = 0$
(B) $3x^2 - 7x + 4 = 0$
(C) $6x^2 + 7x + 2 = 0$
(D) $6x^2 - 7x + 2 = 0$

**Q.51** If roots of quadratic equation $ax^2 + bx + c = 0$ are $\alpha$ and $\beta$ then symmetric expression of its roots is -
(A) $\frac{\alpha}{\beta} + \frac{\beta^2}{\alpha}$ (B) $\alpha^2\beta^{-2} + \alpha^{-2}\beta^2$
(C) $\alpha^2\beta + 2\alpha\beta^2$ (D) $\left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\alpha}\right)$

**Q.52** The quadratic equation with one root $\frac{1}{2}(1 + \sqrt{-3})$ is-
(A) $x^2 - x - 1 = 0$ (B) $x^2 + x - 1 = 0$
(C) $x^2 + x + 1 = 0$ (D) $x^2 - x + 1 = 0$

**Q.53** The quadratic equation with one root $\frac{1}{1+i}$ is-
(A) $2x^2 + 2x + 1 = 0$ (B) $2x^2 - 2x + 1 = 0$
(C) $2x^2 + 2x - 1 = 0$ (D) $2x^2 - 2x - 1 = 0$

**Q.54** If $\alpha$ and $\beta$ are roots of $x^2 - 2x + 3 = 0$, then the equation whose roots are $\frac{\alpha-1}{\alpha+1}$ and $\frac{\beta-1}{\beta+1}$ will be -
(A) $3x^2 - 2x + 1 = 0$ (B) $3x^2 + 2x + 1 = 0$
(C) $3x^2 - 2x - 1 = 0$ (D) $x^2 - 3x + 1 = 0$

**Q.55** If $\alpha$ and $\beta$ be the roots of the equation $2x^2 + 2(a + b)x + a^2 + b^2 = 0$, then the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is-
(A) $x^2 - 2abx - (a^2 - b^2)^2 = 0$
(B) $x^2 - 4abx - (a^2 - b^2)^2 = 0$
(C) $x^2 - 4abx + (a^2 - b^2)^2 = 0$
(D) None of these

**Q.56** If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation whose roots are $\alpha/\beta$ and $\beta/\alpha$ is-
(A) $x^2 - 5x - 3 = 0$
(B) $3x^2 + 12x + 3 = 0$
(C) $3x^2 - 19x + 3 = 0$
(D) None of these

Question based on **Roots under particular cases**

**Q.57** For the roots of the equation $a - bx - x^2 = 0$ $(a > 0, b > 0)$ which statement is true -
(A) positive and same sign
(B) negative and same sign
(C) greater root in magnitude is negative and opposite in signs
(D) greater root is positive in magnitude and opposite in signs

**Q.58** If p and q are positive then the roots of the equation $x^2 - px - q = 0$ are-
(A) imaginary
(B) real & of opposite sign
(C) real & both negative
(D) real & both positive

**Q.59** If $a > 0$, $b > 0$, $c > 0$, then both the roots of the equation $ax^2 + bx + c = 0$ -
(A) Are real and negative
(B) Have negative real parts
(C) are rational numbers

(D) None of these

**Q.60** The roots of the equation $ax^2 + bx + c = 0$ will be imaginary if -
(A) $a > 0, b = 0, c < 0$
(B) $a > 0, b = 0, c > 0$
(C) $a = 0, b > 0, c > 0$
(D) $a > 0, b > 0, c = 0$

**Q.61** If roots of the equation $\ell x^2 + mx - 2 = 0$ are reciprocal of each other, then-
(A) $\ell = 2$ (B) $\ell = -2$
(C) $m = 2$ (D) $m = -2$

**Q.62** If one of the roots of $x(x + 2) = 4 - (1 - ax^2)$ tends $\infty$, then a will tend to-
(A) 0 (B) –1
(C) 1 (D) 2

## Question based on Condition for common roots

**Q.63** If the equation $x^2 - ax + b = 0$ and $x^2 + bx - a = 0$ have a common root, then-
(A) $a = b$ (B) $a + b = 0$
(C) $a - b = 1$ (D) $a - b + 1 = 0$

**Q.64** If $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$ have one common root then a is equal to-
(A) 0, – 24 (B) 0, 1
(C) 0, 24 (D) 1, 24

**Q.65** If one of the roots of $x^2 + ax + bc = 0$ and $x^2 + bx + ca = 0$ is common, then their other roots are -
(A) a, b (B) b, a (C) b, c (D) c, a

**Q.66** The equation $ax^2 + bx + a = 0$ & $x^3 - 2x^2 + 2x - 1 = 0$ have two root in common, then (a + b) is equal to -
(A) 1 (B) 0 (C) –1 (D) 2

**Q.67** If $f(x) = 4x^2 + 3x - 7$ and $\alpha$ is a common root of the equation $x^2 - 3x + 2 = 0$ and $x^2 + 2x - 3 = 0$ then the value of $f(\alpha)$ is -
(A) 3 (B) 2 (C) 1 (D) 0

**Q.68** If the two equations $x^2 - cx + d = 0$ and $x^2 - ax + b = 0$ have one common root and the second has equal roots, then $2(b + d) =$
(A) 0 (B) a + c (C) ac (D) – ac

**Q.69** If both the roots of the equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ & $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then $2r - p$ is equal to -
(A) 1 (B) – 1 (C) 2 (D) 0

## Question based on Quadratic Expression

**Q.70** For all real values of x, the maximum value of the expression $\frac{x}{x^2 - 5x + 9}$ is-
(A) 1 (B) 45
(C) 90 (D) None of these

**Q.71** If x is real, then the value of the expression $\frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ does not exist between-
(A) –5 and 9 (B) 5 and –9
(C) –5 and –9 (D) 5 and 9

**Q.72** The factors of $2x^2 - x + p$ are rational if -
(A) p = 3 (B) p = – 8
(C) p = 6 (D) p = – 6

**Q.73** If one of the factors of $ax^2 + bx + c$ and $bx^2 + cx + a$ is common, then -
(A) $a = 0$
(B) $a^3 + b^3 + c^3 = 3abc$
(C) $a = 0$ or $a^3 + b^3 + c^3 = 3abc$
(D) None of these

**Q.74** $x^2 + k(2x + 3) + 4(x + 2) + 3k - 5$ is a perfect square, if k equals -
(A) 2 (B) – 2 (C) 1 (D) – 1

**Q.75** If $\alpha - x$ is a factor of $x^2 - ax + b$, then $\alpha(a - \alpha)$ is equal to-
(A) –b (B) b (C) a (D) –a

**Q.76** If x + 1 is a factor of the expression $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$ then the value of p is-
(A) 1 (B) 2 (C) 3 (D) 4

**Q.77** If x be real then the minimum value of $40 - 12x + x^2$ is -
(A) 28 (B) 4 (C) –4 (D) 0

**Q.78** If x be real then the value of $\frac{x^2-2x+1}{x+1}$ will not lie between-

(A) 0 and 8 (B) – 8 and 8

(C) – 8 and 0 (D) None of these

## Question based on Inequality

**Q.79** If x be real then $2x^2 + 5x - 3 > 0$ if -

(A) $x < -2$ (B) $x > 0$

(C) $x > 1$ (D) $-3 < x < 1/2$

**Q.80** The solution of the equation $2x^2 + 3x - 9 \le 0$ is given by-

(A) $3/2 \le x \le 3$ (B) $-3 \le x \le 3/2$

(C) $-3 \le x \le 3$ (D) $3/2 \le x \le 2$

**Q.81** If for real values of x, $x^2 - 3x + 2 > 0$ and $x^2 - 3x - 4 \le 0$, then-

(A) $-1 \le x < 1$

(B) $-1 \le x < 4$

(C) $-1 \le x < 1$ and $2 < x \le 4$

(D) $2 < x \le 4$

## Question based on Quadratic Expression in two variables

**Q.82** If $x^2 + 2xy + 2x + my - 3$ have two rational factors then m is equal to -

(A) 6, 2 (B) – 6, 2

(C) 6, –2 (D) –6, –2

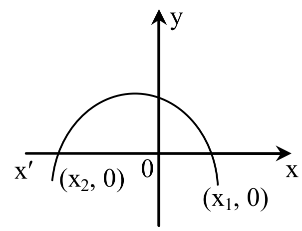
**Q.83** If $2x^2 + mxy + 3y^2 - 5y - 2$ have two rational factors then m is equal to-

(A) ± 7 (B) ± 6

(C) ± 5 (D) None of these

## Question based on Sign of Quadratic Expression

**Q.84** The diagram shows the graph of $y = ax^2 + bx + c$. Then-




(A) $a > 0$ (B) $b^2 - 4ac < 0$

(C) $c > 0$ (D) $b^2 - 4ac = 0$

**Q.85** The maximum value of the function $y = \frac{1}{4x^2+2x+1}$ is-

(A) $\frac{4}{3}$ (B) $\frac{5}{2}$

(C) $\frac{13}{4}$ (D) None of these

# LEVEL- 2

**Q.1** If roots the equation
$x^2(1+m^2) + 2mcx + c^2 - a^2 = 0$ are equal, then value of c is-
(A) $a\sqrt{(1+m^2)}$ (B) $a\sqrt{(1-m^2)}$
(C) $m\sqrt{(1+a^2)}$ (D) $m\sqrt{(1-a^2)}$

**Q.2** If the roots of the equation $\frac{x-a}{ax-1} = \frac{x-b}{bx+1}$ are reciprocal to each other, then -
(A) a = 1 (B) b = 2
(C) a = 2b (D) b = 0

**Q.3** The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has -
(A) no root
(B) one root
(C) two equal root
(D) infinitely many roots

**Q.4** The roots of the equation $|x|^2 + |x| - 6 = 0$ are-
(A) only one real number
(B) real and sum = 1
(C) real and sum = 0
(D) real and product = 0

**Q.5** The roots of the equation
$x^2 - 2px + p^2 + q^2 + 2qr + r^2 = 0$ ($p, q, r \in Z$) are
(A) rational and different
(B) rational and equal
(C) irrational
(D) imaginary

**Q.6** If a, b, c are positive real numbers, then the number of real roots of the equations
$ax^2 + b|x| + c = 0$ is-
(A) 0 (B) 1
(C) 2 (D) None of these

**Q.7** If product of roots of the equation
$x^2 - 3kx + 2e^{\log k} - 1 = 0$ is 7, then-
(A) roots are integers and positive
(B) roots are integers and negative
(C) roots are rational not integers
(D) roots are irrational

**Q.8** If roots of the equation $3x^2 + 2(a^2+1)x + (a^2 - 3a + 2) = 0$ are of opposite signs, then a lies in the interval -
(A) $(-\infty, 1)$ (B) $(-\infty, 0)$
(C) (1, 2) (D) (3/2, 2)

**Q.9** For what values of p, the roots of the equation $12(p+2)x^2 - 12(2p-1)x - 38p - 11 = 0$ are imaginary-
(A) $p = R^-$
(B) $p \in (-\infty, -1) \cup \left(-\frac{1}{2}, \infty\right)$
(C) $p \in \left(-1, -\frac{1}{2}\right)$
(D) p = – 1

**Q.10** The equation whose roots are $\frac{q}{p+q}$, $\frac{-p}{p+q}$ is-
(A) $(p+q)^2 x^2 + (p^2 - q^2)x + pq = 0$
(B) $x^2 - \left(\frac{q-p}{q+p}\right)x - \frac{pq}{(q+p)^2} = 0$
(C) $(p+q)x^2 + (p^2 - q^2)x - pq = 0$
(D) None of these

**Q.11** If one root of the equations $ax^2 + bx + c = 0$ and $x^2 + x + 1 = 0$ is common, then-
(A) a + b + c = 0
(B) a = b = c
(C) a = b or b = c or c = a
(D) None of these

**Q.12** The imaginary roots of the equation
$(x^2+2)^2 + 8x^2 = 6x(x^2+2)$ are -
(A) $1 \pm i$ (B) $2 \pm i$
(C) $-1 \pm i$ (D) None of these

**Q.13** If one root of the equation $2x^2 - 6x + c = 0$ is $\frac{3+5i}{2}$, then the value of c will be -

(A) 7 (B) – 7 (C) 17 (D) – 17

**Q.14** If $\alpha$, $\beta$ are roots of the equation $ax^2 + bx + c = 0$ and $\alpha - \beta = \alpha\beta$, then –

(A) $b^2 - 4ac = c^2$ (B) $b^2 - 4ac = a^2$

(C) $a(b^2 + 4ac) = 2c$ (D) $b^2 + 4ac = a$

**Q.15** If x – 2 is a common factor of $x^2 + ax + b$ and $x^2 + cx + d$, then -

(A) $d - b = 2(c - a)$ (B) $b - d = (c - a)$

(C) $4 + 2c + b = 0$ (D) $b - d = 2(c - a)$

**Q.16** If $x = \sqrt{6+\sqrt{6+\sqrt{6+...}}}$ , then -

(A) $-2 < x < 3$ (B) $2 < x < 3$

(C) $x = 3$ (D) $x > 3$

**Q.17** If $x^{2/3} + x^{1/3} - 2 = 0$ then x-

(A) –2, 1 (B) –8, –2

(C) –8, 1 (D) None of these

**Q.18** If 8, 2 are roots of the equation $x^2 + ax + \beta = 0$ and 3, 3 are roots of $x^2 + \alpha x + b = 0$ then roots of the equation $x^2 + ax + b = 0$ are -

(A) 1, 9 (B) –1, 8 (C) 2, – 9 (D) –2, 8

**Q.19** If the difference of the roots is equal to the product of the roots of the equation $2x^2 - (a + 1)x + (a - 1) = 0$ then the value of a is-

(A) 2 (B) 3 (C) 4 (D) 5

**Q.20** If one root of the equation $x^2 - x - k = 0$ is square of the other, then k equals to -

(A) $2 \pm \sqrt{5}$ (B) $3 \pm \sqrt{2}$

(C) $2 \pm \sqrt{3}$ (D) $5 \pm \sqrt{2}$

**Q.21** The roots of $a_1x^2 + b_1x + c_1 = 0$ are reciprocal of the roots of the equation $a_2x^2 + b_2x + c_2 = 0$, if-

(A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (B) $\frac{b_1}{b_2} = \frac{c_1}{a_2} = \frac{a_1}{c_2}$

(C) $\frac{a_1}{a_2} = \frac{b_1}{c_2} = \frac{c_1}{b_2}$

(D) $a_1 = \frac{1}{a_2}, b_1 = \frac{1}{b_2}, c_1 = \frac{1}{c_2}$

**Q.22** If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocal, then-

(A) $c^2b, a^2c, b^2a$ are in A.P.

(B) $c^2b, a^2c, b^2a$ are in G.P.

(C) $\frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in H.P.

(D) $\frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in G.P.

**Q.23** If the quadratic equations $3x^2 + ax + 1 = 0$ and $2x^2 + bx + 1 = 0$ have a common root, then the value of the expression $5ab - 2a^2 - 3b^2$ is-

(A) 0 (B) 1

(C) –1 (D) None of these

**Q.24** For the roots of the equations $2x^2 - 5x + 1 = 0$ and $x^2 + 5x + 2 = 0$ , which of the following statement is true-

(A) reciprocal of roots of one another

(B) reciprocal of roots of one another and opposite signs

(C) roots are of opposite signs of each other

(D) equal in product

**Q.25** If x is real, then the values of the expression $\frac{(x+m)^2 - 4mn}{2(x-n)}$ are not -

(A) greater than (m + n)

(B) greater than (m + 2n)

(C) between 2m and 2n

(D) between m and m + n

**Q.26** If x is the real, then the value of the expression $\frac{2x^2 + 4x + 1}{x^2 + 4x + 2}$ is -

(A) any number

(B) only positive number

(C) only negative number

(D) only 1

**Q.27** If one root of the equations $ax^2 + bx + c = 0$ is equal to $n^{th}$ power of the other root, then $(ac^n)^{1/(n+1)} + (a^nc)^{1/(n+1)}$ equals -

(A) – b (B) b

(C) $(-b)^{1/(n+1)}$ (D) $(b)^{1/(n+1)}$

**Q.28** The number of real roots of the equation $|x^2 + 4x + 3| + 2x + 5 = 0$ is-

(A) 2 (B) 3

(C) 4 (D) 1

**Q.29** If product of roots of the equation $x^2 - 4mx + 3e^{2\log m} - 4 = 0$ is 8, then its roots are real, when m equals-

(A) 1 (B) 2

(C) 2 or –2 (D) –2

**Q.30** For what value of c, the root of $(c-2)x^2 + 2(c-2)x + 2 = 0$ are not real -

(A) ]1,2[ (B) ]2,3[

(C) ]3,4[ (D) ]2,4[

**Q.31** For $x^3 + 1 \geq x^2 + x$-

(A) $x \leq 0$ (B) $x \geq 0$

(C) $x \geq -1$ (D) $-1 \leq x \leq 1$

**Q.32** If roots of the equation $ax^2 + bx + c = 0$ are $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then $(a + b + c)^2$ equals-

(A) $2b^2 - ac$ (B) $b^2 - ac$

(C) $b^2 - 4ac$ (D) $4b^2 - 2ac$

**Q.33** If the product of the roots of the equation $x^2 - 3kx + 2e^{\sin k} - 1 = 0$ is 7 then its roots will be real if -

(A) $|k| \leq 2\sqrt{7/9}$ (B) $|k| \geq 2\sqrt{7/9}$

(C) $|k| > 2\sqrt{7/9}$ (D) Never

**Q.34** If x > 1, then the minimum value of the expression $2\log_{10} x - \log_x (0.01)$ is -

(A) 2 (B) 4

(C) 1 (D) None of these

**Q.35** If $7^{\log_7(x^2-4x+5)} = x - 1$, x may have values -

(A) 2, 3 (B) 7

(C) – 2, –3 (D) 2, – 3

**Q.36** If α, β are roots of the equation $(3x + 2)^2 + p(3x + 2) + q = 0$, then roots of $x^2 + px + q = 0$ are -

(A) α, β (B) $3\alpha + 2, 3\beta + 2$

(C) $\frac{1}{3}(\alpha - 2), \frac{1}{3}(\beta - 2)$ (D) $\alpha - 2, \beta - 2$

**Q.37** For what value of a the curve $y = x^2 + ax + 25$ touches the x-axis-

(A) 0 (B) ±5

(C) ±10 (D) None of these

**Q.38** If roots of the equation $2x^2 - (a^2 + 8a + 1)x + a^2 - 4a = 0$ are in opposite sign, then -

(A) $0 < a < 4$ (B) $a > 0$

(C) $a < 8$ (D) $-4 < a < 0$

**Q.39** If the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ are equal in magnitude but opposite in sign, then their product is -

(A) $\frac{1}{2}(a^2 + b^2)$ (B) $\frac{-1}{2}(a^2 + b^2)$

(C) $\frac{1}{2}ab$ (D) $\frac{-1}{2}ab$

**Q.40** If both roots of the equation $x^2 - (m + 1)x + (m + 4) = 0$ are negative, then m equals -

(A) $-7 < m < -5$ (B) $-4 < m \leq -3$

(C) $2 < m < 5$ (D) None of these

**Q.41** If $\frac{x^2 + 2x + 7}{2x + 3} < 6$, $x \in R$, then -

(A) $x > 11$ or $x < \frac{-3}{2}$ (B) $x > 11$ or $x < -1$

(C) $\frac{-3}{2} < x < -1$ (D) $-1 < x < 11$ or $x < \frac{-3}{2}$

**Q.42** If roots of the equation $x^2 - bx + c = 0$ are two successive integers, then $b^2 - 4c$ equals -

(A) 1 (B) 2 (C) 3 (D) 4

**Q.43** The numbers of real roots of $3^{2x^2-7x+7} = 9$ is-

(A) 0 (B) 2

(C) 1 (D) 4

**Q.44** If $a(p + q)^2 + 2apq + c = 0$ and $a(p + r)^2 + 2apr + c = 0$, then qr equals -

(A) $p^2 + c/a$ (B) $p^2 + a/c$

(C) $p^2 + a/b$ (D) $p^2 + b/a$

**Q.45** If a, b are roots of the equation $x^2 + qx + 1 = 0$ and c, d are roots of $x^2 + px + 1 = 0$, then the value of $(a - c)(b - c)(a + d)(b + d)$ will be-

(A) $q^2 - p^2$ (B) $p^2 - q^2$

(C) $-p^2 - q^2$ (D) $p^2 + q^2$

**Q.46** If one root of equation $Ax^2 + Bx + C = 0$ is $i(a - b)$ then $\frac{AB}{C}$ equals-

(A) $\frac{1}{(a-b)^2}$ (B) 0

(C) $\frac{1}{(a-b)}$ (D) None of these

**Q.47** Two students solve a quadratic equation $x^2 + bx + c = 0$. One student solves the equation by taking wrong value of b and gets the roots as 2 and 5, while second student solves it by taking wrong value of c and gets the roots as – 3 and – 4. The correct roots of the equation are -

(A) – 2, – 5 (B) 2, – 5
(C) 2, 10 (D) None of these

**Q.48** If in the equation $ax^2 + bx + c = 0$, the sum of roots is equal to sum of squares of their reciprocals, then $\frac{b^2}{ac} + \frac{bc}{a^2}$ equals -

(A) 1 (B) –1 (C) 2 (D) –2

**Q.49** If ratio of roots of the equations $x^2 + ax + b = 0$ and $x^2 + px + q = 0$ are equal, then -

(A) $aq = bp$ (B) $a^2q = bp^2$
(C) $a^2p = b^2q$ (D) $aq^2 = bp^2$

**Q.50** Let $\alpha$, $\beta$ be the roots of the equation $ax^2 + 2bx + c = 0$ and $\gamma$, $\delta$ be the roots of the equation $px^2 + 2qx + r = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then -

(A) $q^2 ac = b^2 pr$ (B) $qac = bpr$
(C) $c^2 pq = r^2 ab$ (D) $p^2 ab = a^2 qr$

**Q.51** If real value of x and y satisfies the equation $x^2 + 4y^2 - 8x + 12 = 0$, then -

(A) $0 < y < 1$ (B) $2 < y < 6$
(C) $-1 \le y \le 1$ (D) $-2 < y < 6$

**Q.52** If roots of $x^2 - (a-3)x + a = 0$ are such that both of them is greater than 2, then-

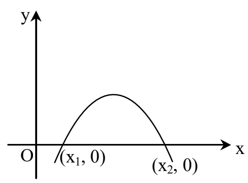
(A) $a \in [7, 9]$ (B) $a \in [9, 10)$
(C) $a \in [9, 7]$ (D) $a \in [9, 12]$

**Q.53** The real roots of the equation $x^2 + 5|x| + 4 = 0$ are-

(A) –1, –4 (B) 1, 4
(C) – 4, 4 (D) None of these

# LEVEL- 3

**Q.1** The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then -




(A) $a < 0$  (B) $b^2 < 4ac$
(C) $c > 0$
(D) a and b are of opposite signs

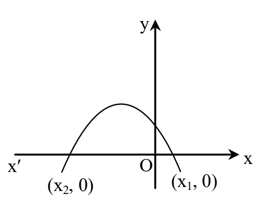
**Q.2** The expression $y = ax^2 + bx + c$ has always the same sign as c if -
(A) $4ac < b^2$  (B) $4ac > b^2$
(C) $ac < b^2$  (D) $ac > b^2$

**Q.3** If the roots of the equation $(x - a)(x - b) - k = 0$ be c & d then find the equation whose roots are a & b-
(A) $(x - c)(x - d) + k = 0$
(B) $(x + c)(x - a) + k = 0$
(C) $(x - c) + (x - a) = 0$
(D) None of these

**Q.4** Given that $ax^2 + bx + c = 0$ has no real roots and $a + b + c < 0$, then -
(A) $c = 0$  (B) $c > 0$
(C) $c < 0$  (D) None of these

**Q.5** The quadratic equation whose roots are reciprocal of the roots of the equation $ax^2 + bx + c = 0$ is -
(A) $cx^2 + bx + a = 0$  (B) $bx^2 + cx + a = 0$
(C) $cx^2 + ax + b = 0$  (D) $bx^2 + ax + c = 0$

**Q.6** The diagram shows the graph of $y = ax^2 + bx + c$. Then -




(A) $a > 0$  (B) $b < 0$
(C) $c > 0$  (D) $b^2 - 4ac = 0$

**Q.7** If the roots of the equation
$a(b - c)x^2 + b(c - a)x + c(a - b) = 0$
are equal, then a, b, c are in -
(A) HP  (B) GP
(C) AP  (D) None of these

**Q.8** If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1$ for all $x \in R$, then $\lambda$ belong to interval.
(A) $\left(-2, \frac{2}{5}\right)$  (B) $(-2, 1)$
(C) $\left(\frac{2}{5}, 1\right)$  (D) None of these

**Q.9** The roots of the equation
$\log_2(x^2 - 4x + 5) = (x - 2)$ are -
(A) 4, 5  (B) 2, –3  (C) 2, 3  (D) 3, 5

**Q.10** If $f(x) = ax^2 + bx + c$, $g(x) = -ax^2 + bx + c$, where $ac \neq 0$, then $f(x)\,g(x) = 0$ has -
(A) At least three real roots
(B) No real roots
(C) At least two real roots
(D) Two real roots and two imaginary roots

**Q.11** The equation
$$2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2},\ 0 \le x \le \frac{\pi}{2} \text{ has -}$$
(A) No real solution
(B) One real solution
(C) More than one real solution
(D) None of these

**Q.12** The number of solutions of the equation
$2\sin(e^x) = 5^x + 5^{-x}$ is -
(A) 0  (B) 1
(C) 2  (D) Infinitely

**Q.13** The number of real solutions of the equation

$(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$ is -

(A) 2 (B) 4

(C) 6 (D) None of these

**Q.14** If the equation $ax^2 + 2bx - 3c = 0$ has no real roots and $\left(\frac{3c}{4}\right) < a+b$, then-

(A) $c < 0$ (B) $c > 0$

(C) $c \geq 0$ (D) $c = 0$

**Q.15** The product of all the solutions of the equation $(x-2)^2 - 3|x-2| + 2 = 0$ is

(A) 0 (B) 2

(C) –4 (D) None of these

**Q.16** If a, b, c are all positive and in H.P., then the roots of $ax^2 + 2bx + c = 0$ are -

(A) Real (B) Imaginary

(C) Rational (D) Equal

**Q.17** The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ is -

(A) 1 (B) 2

(C) 3 (D) None of these

**Q.18** If $\alpha$, $\beta$ are the roots of $ax^2 + bx + c = 0$; $\alpha + h$, $\beta + h$ are the roots of $px^2 + qx + r = 0$, and $D_1$, $D_2$ the respective discriminants of these equations, then $D_1 : D_2$ -

(A) $\frac{a^2}{p^2}$ (B) $\frac{b^2}{q^2}$

(C) $\frac{c^2}{r^2}$ (D) None of these

**Q.19** If $\alpha$, $\beta$ are the roots of $ax^2 + bx + c = 0$ and $\alpha + h$, $\beta + h$ are the roots of $px^2 + qx + r = 0$, then h =

(A) $\left(\frac{b}{a}-\frac{q}{p}\right)$ (B) $\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$

(C) $-\frac{1}{2}\left(\frac{a}{b}-\frac{p}{q}\right)$ (D) None of these

**Q.20** a, b, c $\in$ R, $a \neq 0$ and the quadratic equation $ax^2 + bx + c = 0$ has no real roots, then -

(A) $a + b + c > 0$ (B) $a(a + b + c) > 0$

(C) $b(a + b + c) > 0$ (D) $c(a + b + c) > 0$

**Q.21** If the product of the roots of the equation $x^2 - 2\sqrt{2}\,kx + 2e^{2\log k} - 1 = 0$ is 31, then the roots of the equation are real for k equal to -

(A) 1 (B) 2 (C) 3 (D) 4

**Q.22** The number of real solutions of the equation $\left(\frac{9}{10}\right)^x = -3 + x - x^2$ is -

(A) 0 (B) 1

(C) 2 (D) None of these

**Q.23** If the roots of the equation $ax^2 + bx + c = 0$ are real and distinct, then -

(A) Both roots are greater than $\frac{-b}{2a}$

(B) Both roots are less than $\frac{-b}{2a}$

(C) One of the roots exceeds $\frac{-b}{2a}$

(D) None of these

**Q.24** The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ -

(A) 0 (B) –2

(C) 2 (D) None of these

**Q.25** If $ax^2 + bx + 6 = 0$ does not have two distinct real roots, where $a \in R$, $b \in R$, then the least value of $3a + b$ is-

(A) –2 (B) –1

(C) 4 (D) 1

**Passage Based Questions (Q. 26-28)**

Consider the expression $y = ax^2 + bx + c$, $a \neq 0$ and a, b, c $\in$ R then the graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upward and if $a < 0$ then the shape of the parabola is concave down ward. If $y > 0$ or $y < 0$ then discriminant $D < 0$.

**Q.26** Let $x^2 + 2ax + 10 - 3a > 0$ for every real value of x, then –

(A) $a > 5$ (B) $a < -5$

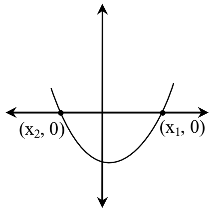
(C) $-5 < a < 2$ (D) $2 < a < 5$

**Q.27** The value of $x^2 + 2bx + c$ is positive if –

(A) $b^2 - 4c > 0$ (B) $b^2 - 4c < 0$

(C) $c^2 < b$ (D) $b^2 < c$

**Q.28** The diagram show the graph of $y = ax^2 + bx + c$ then –




(A) $a < 0$ (B) $c < 0$
(C) $b^2 - 4ac < 0$ (D) $b^2 - 4ac = 0$

**Questions based on statements (Q. 29 - 33)**

**Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.**

**(A)** **If both Statement- I and Statement- II are true, and Statement - II is the correct explanation of Statement– I.**

**(B)** **If both Statement - I and Statement - II are true but Statement - II is not the correct explanation of Statement – I.**

**(C)** **If Statement - I is true but Statement - II is false.**

**(D)** **If Statement - I is false but Statement - II is true.**

**Q.29** **Statement I :** $x^2 + 4x + 7 > 0 \ \forall\ x \in R$

**Statement II :** $ax^2 + bx + c > 0 \ \forall\ x \in R$ if $b^2 - 4ac < 0$ and $a > 0$.

**Q.30** **Statement I :** The remainder obtained on dividing the polynomial P(x) by $(x - 3)$ is equal to P(3).

**Statement II :** $f(x) : (x - 8)^3 (x + 4) \Rightarrow f'(x)$ may not be divisible by $(x^2 - 16x + 64)$.

**Q.31** **Statement I:** $f(x) = ax^2 + bx + c$, then $f(x) = 0$ has integral roots when $a = 1$, $b, c \in I$ and $b^2 - 4ac$ is a perfect square of integer.

**Statement II :** $x^3 + 1 = 0$ has only one integral root.

**Q.32** **Statement I :** $x^2 + bx + c = 0$ has distinct roots and both greater than 2 if $b^2 - 4c > 0$, $b < -4$ and $2b + c + 4 > 0$.

**Statement II :** $x^2 + 2x + c = 0$ has distinct roots and both less than 1 iff $c \in (-3, 1)$.

**Q.33** **Statement I :** We can get the equation whose roots are 2 more than the roots of equation $ax^2 + bx + c = 0$ by replacing x by $(x + 2)$.

**Statement II :** $x^2 + |x| + 5 = 0$ has no real roots.

# LEVEL- 4

## (Question asked in previous AIEEE and IIT-JEE)

### SECTION –A

**Q.1** If the roots of the equation $x^2 - 5x + 16 = 0$ are $\alpha, \beta$ and the roots of the equation $x^2 + px + q = 0$ are $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$, then- **[AIEEE-2002]**

(A) p = 1 and q = 56

(B) p = 1 and q = – 56

(C) p = – 1 and q = 56

(D) p = – 1 and q = – 56

**Q.2** If $\alpha$ and $\beta$ be the roots of the equation $(x - a)(x - b) = c$ and $c \neq 0$, then roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are - **[AIEEE-2002]**

(A) a and c (B) b and c

(C) a and b (D) a+ b and b + c

**Q.3** If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is **[AIEEE-2002]**

(A) 19/3 (B) 25/3

(C) – 19/3 (D) None of these

**Q.4** If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in- **[AIEEE-2003]**

(A) Arithmetic Geometric Progression

(B) Arithmetic Progression

(C) Geometric Progression

(D) Harmonic Progression

**Q.5** The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is- **[AIEEE-2003]**

(A) $-\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $-\frac{2}{3}$ (D) $\frac{1}{3}$

**Q.6** The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is **[AIEEE-2003]**

(A) 3 (B) 2 (C) 4 (D) 1

**Q.7** If (1– p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are- **[AIEEE-2004]**

(A) 0,1 (B) – 1, 1

(C) 0, – 1 (D) – 1, 2

**Q.8** If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is- **[AIEEE-2004]**

(A) 49/4 (B) 12 (C) 3 (D) 4

**Q.9** The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is - **[AIEEE-2005]**

(A) 1 (B) 0 (C) 3 (D) 2

**Q.10** If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals - **[AIEEE-2005]**

(A) –2 (B) 3 (C) 2 (D) 1

**Q.11** In a triangle PQR, $\angle R = \frac{\pi}{2}$, If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then - **[AIEEE-2005]**

(A) a = b + c (B) c = a + b

(C) b = c (D) b = a + c

**Q.12** If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[AIEEE-2005]**

(A) (5, 6] (B) $(6, \infty)$

(C) $(-\infty, 4)$ (D) [4, 5]

**Q.13** If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30º and tan 15º, respectively then the value of 2 + q – p is – **[AIEEE-2006]**

(A) 3 (B) 0 (C) 1 (D) 2

**Q.14** All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than $-2$ but less than 4, lie in the interval – **[AIEEE-2006]**

(A) $m > 3$ (B) $-1 < m < 3$

(C) $1 < m < 4$ (D) $-2 < m < 0$

**Q.15** If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is – **[AIEEE-2006]**

(A) 41 (B) 1

(C) $\frac{17}{7}$ (D) $\frac{1}{4}$

**Q.16** If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is- **[AIEEE-2007]**

(A) $(-3, 3)$ (B) $(-3, \infty)$

(C) $(3, \infty)$ (D) $(-\infty, -3)$

**Q.17** The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE-2008]**

(A) 4 (B) 3 (C) 2 (D) 1

**Q.18** How many real solution does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have ? **[AIEEE-2008]**

(A) 1 (B) 3 (C) 5 (D) 7

**Q.19** If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is- **[AIEEE-2009]**

(A) greater than 4 ab (B) less than 4ab

(C) greater than – 4ab (D) less than – 4ab

**Q.20** If $\alpha$ and $\beta$ are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE-2010]**

(A) $-2$ (B) $-1$ (C) 1 (D) 2

**Q.21** Let $\alpha, \beta$ be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that : **[AIEEE-2011]**

(A) $\beta \in (0,1)$ (B) $\beta \in (-1,0)$

(C) $|\beta| = 1$ (D) $\beta \in (1,\infty)$

**Q.22** If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a, b, c $\in$ R, have a common root, then a : b : c is – **[JEE Main - 2013]**

(A) 1 : 3 : 2 (B) 3 : 1 : 2

(C) 1 : 2 : 3 (D) 3 : 2 : 1

**SECTION – B**

**Q.1** If $e^{\{(\sin^2 x + \sin^4 x + \sin^6 x \ldots\ldots\infty)\,\ell n\, 2\}}$ satisfies the eqn. $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$ **[IIT-1991]**

(A) $\frac{1}{1+\sqrt{3}}$ (B) $\frac{1}{1-\sqrt{3}}$

(C) $\frac{2}{1-\sqrt{2}}$ (D) None of these

**Q.2** The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite sign is- **[IIT-1992]**

(A) $(-\infty, 0)$ (B) $(0, 1)$

(C) $(1, \infty)$ (D) $(0, \infty)$

**Q.3** Let p, q $\in$ {1, 2, 3, 4}. The number of equations of the form $px^2 + qx + 1 = 0$ having real roots is- **[IIT Sc. -1994]**

(A) 15 (B) 9 (C) 7 (D) 8

**Q.4** Let $\alpha$ and $\beta$ be the roots of the equation $x^2 + x + 1 = 0$. The equation whose roots are $\alpha^{19}, \beta^7$ is **[IIT-1994]**

(A) $x^2 - x - 1$ (B) $x^2 - x + 1 = 0$

(C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

**Q.5** If p,q are roots of the equation $x^2 + px + q = 0$, then- **[IIT Sc.-95]**

(A) $p = 1$ (B) $p = -2$

(C) $p = 1$ or 0 (D) $p = -2$ or 0

**Q.6** Let p and q are roots of the equation $x^2 - 2x + A = 0$ and r,s are roots of $x^2 - 18x + B = 0$ if $p < q < r < s$ are in A.P. then the value of A and B are - **[IIT-1997]**

(A) – 7, – 33 (B) – 7, – 37
(C) – 3, 77 (D) None of these

**Q.7** The equation $\sqrt{(x+1)} - \sqrt{(x-1)} = \sqrt{(4x-1)}$ has- **[IIT-1997 can.]**

(A) No Solution
(B) One solution
(C) Two solutions
(D) More than 2 solutions

**Q.8** The sum of all real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is **[IIT-1997]**

(A) 2 (B) 4
(C) 1 (D) none of these

**Q.9** The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is **[IIT-1998]**

(A) 0 (B) 5
(C) 6 (D) 10

**Q.10** If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then - **[IIT-1999]**

(A) $a < 2$ (B) $2 \le a \le 3$
(C) $3 < a \le 4$ (D) $a > 4$

**Q.11** For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the roots is square of the other, then p is equal to - **[IIT Sc.-2000]**

(A) 1/3 (B) 1
(C) 3 (D) 2/3

**Q.12** If α and β (α < β), are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then **[IIT Sc. - 2000]**

(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$
(C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

**Q.13** If b > a, then the equation $(x - a)(x - b) - 1 = 0$, has - **[IIT Sc.-2000]**

(A) both roots in [a, b]
(B) both roots in $(-\infty, a)$
(C) both roots in $(b, +\infty)$
(D) one root in $(-\infty, a)$ and other in $(b, +\infty)$

**Q.14** The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is- **[IIT Sc.-2002]**

(A) $(-\infty, -2) \cup (2, \infty)$
(B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(C) $(-\infty, -1) \cup (1, \infty)$
(D) $(\sqrt{2}, \infty)$

**Q.15** If one root of the equation $x^2 + px + q = 0$ is square of the other then for any p & q, it will satisfy the relation- **[IIT Sc.-2004]**

(A) $p^3 - q(3p - 1) + q^2 = 0$
(B) $p^3 - q(3p + 1) + q^2 = 0$
(C) $p^3 + q(3p - 1) + q^2 = 0$
(D) $p^3 + q(3p + 1) + q^2 = 0$

**Q.16** Let $x^2 + 2ax + 10 - 3a > 0$ for every real value of x, then- **[IIT Sc.-2004]**

(A) $a > 5$ (B) $a < -5$
(C) $-5 < a < 2$ (D) $2 < a < 5$

**Q.17** α, β are roots of equation $ax^2 + bx + c = 0$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in G.P., $\Delta = b^2 - 4ac$, then **[IIT Sc.-2005]**

(A) $\Delta b = 0$ (B) $bc \ne 0$
(C) $\Delta \ne 0$ (D) $\Delta = 0$

**Q.18** Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is **[IIT -2007]**

(A) $\frac{2}{9}(p - q)(2q - p)$

(B) $\frac{2}{9}(q - p)(2p - q)$

(C) $\frac{2}{9}(q - 2p)(2q - p)$

(D) $\frac{2}{9}(2p - q)(2q - p)$

**Q.19** The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is : **[IIT -2009]**

(A) 2 (B) 3 (C) 5 (D) 6

**Q.20** Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is - **[IIT -2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

**Q.21** Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[IIT -2011]**

(A) 1 (B) 2 (C) 3 (D) 4

**Q.22** Let $a$, $b$ and $c$ be three real numbers satisfying

$$[a \ b \ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \ 0 \ 0] \quad \text{.......(E)}$$

Let $b = 6$, with $a$ and $c$ satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

**[IIT -2011]**

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

**Q.23** A value of b for which the equation

$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$,

have one root in common is - **[IIT -2011]**

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$
(C) $i\sqrt{5}$ (D) $\sqrt{2}$

**Q.24** The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **[IIT -2011]**

(A) 0 (B) 1
(C) 2 (D) 4

**Q.25** Let α (a) and β(a) be the roots of the equation $\left(\sqrt[3]{1+a} - 1\right)x^2 + \left(\sqrt{1+a} - 1\right)x + \left(\sqrt[6]{1+a} - 1\right) = 0$ where $a > -1$. Then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$ are

**[IIT -2012]**

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and $-1$
(C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

# ANSWER KEY

## LEVEL- 1

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | B | D | B | B | B | B | D | D | A | D | B | A | A | C | C | D | A | A | C | C |
| Ques. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| Ans. | C | C | A | D | C | B | C | B | B | A | D | C | A | C | B | C | C | A | C | B |
| Ques. | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| Ans. | D | B | B | C | A | A | A | D | C | A | B | D | B | A | B | C | C | B | B | B |
| Ques. | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| Ans. | B | C | C | C | B | B | D | C | D | A | D | D | C | C | B | D | B | C | C | B |
| Ques. | 81 | 82 | 83 | 84 | 85 | | | | | | | | | | | | | | | |
| Ans. | C | C | A | C | A | | | | | | | | | | | | | | | |

## LEVEL- 2

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | A | D | A | C | D | A | D | C | C | B | B | A | C | A | D | C | C | A | A | A |
| Ques. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| Ans. | B | A | B | B | C | A | A | A | B | D | C | C | D | B | A | B | C | A | B | B |
| Ques. | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | | | | | | | |
| Ans. | D | A | B | A | B | B | A | C | B | A | C | B | D | | | | | | | |

## LEVEL- 3

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | A,D | B | A | C | A | B,C | A | A | C | C | A | A | B | A | A | B | D | A | B | B,D |
| Ques. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | | | | | | | |
| Ans. | D | A | C | A,B | A | C | D | B | A | C | B | B | D | | | | | | | |

## LEVEL- 4

### SECTION-A

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | D | C | A | D | B | C | C | A | A | D | B | C | A | B | A | A | C | A | C | C | D | C |

### SECTION-B

**1.[A]** $e^{\frac{\sin^2 x}{1-\sin^2 x}\log 2} = e^{\tan^2 x \ln 2} = e^{\ln 2^{\tan^2 x}}$

$= 2^{\tan^2 x}$ is root of given equation

$x^2 - 9x + 8 = 0$

$\Rightarrow (x - 8)(x - 1) = 0 \;\therefore\; x = 1, 8$

$\therefore\; 2^{\tan^2 x} = 1 \Rightarrow \tan^2 x = 0$ (rejected)

& $2^{\tan^2 x} = 8 \Rightarrow \tan^2 x = 3$

$\therefore\; \tan x = \sqrt{3}$

$\therefore\; x = \dfrac{\pi}{3}$

Hence $\dfrac{\cos x}{\cos x + \sin x} = \dfrac{\frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \dfrac{1}{1+\sqrt{3}}$

**2.[B]** $3x^2 + 2x + p(p - 1) = 0; \quad \because \alpha\beta < 0$

$\dfrac{p(p-1)}{3} < 0;\; p(p - 1) < 0;\; \therefore\; \boxed{0 < p < 1}$

**3. [C]** $\because D = q^2 - 4p \geq 0$

$\therefore p \leq \frac{q^2}{4}$

(i) $q = 1 \rightarrow p \leq \frac{1}{4}$ (wrong)

(ii) $q = 2 \rightarrow p \leq 1 \therefore p = 1$ …(i)

(iii) $q = 3 \rightarrow p \leq \frac{9}{4} \therefore p = 1, 2$ …(ii)

(iv) $q = 4 \rightarrow p \leq 4 \therefore p = 1, 2, 3, 4$ …(iii)

Hence 7 solutions are possible.

**4.[D]** $x^2 + x + 1 = 0$

Here $\alpha = \omega$ ; $\beta = \omega^2$

Now to find equation whose roots are $\alpha^{19} = \omega^{19} = \omega$

& $\beta^7 = \omega^{14} = \omega^2$

$\because$ roots are same

$\therefore$ equation is same : $x^2 + x + 1 = 0$

**5.[C]** p, q are roots of $x^2 + px + q = 0$

Here

$\alpha\beta \equiv pq = q$

$\therefore (p - 1) q = 0$

$\therefore \boxed{p = 1}$ or $q = 0$

$\alpha + \beta \equiv p + q = -p ; \Rightarrow q = -2$

$\alpha + \beta \equiv p + q = -p ; \Rightarrow 2p = 0; \therefore \boxed{p = 0}$

Hence p = 1 or 0

**6.[C]** Let $p = a$ ; $q = a + d$ ; $r = a + 2d$; $s = a + 3d$ are in AP

Here $p + q = 2a + d = 2$ (As per equation ..(1))

& $r + s = 2a + 5d = 18$ (As per equation ..(2))

$\therefore \boxed{d = 4}$ & hence $a = -1$

$\therefore p = -1; q = 3; r = 7; s = 11$

Now $A = pq = -3$

& $B = rs = 77$

**7.[A]** $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

Squaring : $(x + 1) + (x - 1) - 2\sqrt{x^2 - 1} = 4x - 1$

$\Rightarrow -2\sqrt{x^2 - 1} = 2x - 1$

Squaring :

$\Rightarrow 4x^2 - 4 = 4x^2 + 1 - 4x$

$\therefore x = \frac{5}{4}$

but it doesn't satisfy given equation

Hence no solution.

**8.[B]** $|x - 2|^2 + |x - 2| - 2 = 0$

$\therefore (|x - 2| + 2)(|x - 2| - 1) = 0$

$\therefore |x - 2| = -2$ or $|x - 2| = 1$

Not possible

$\therefore x - 2 = \pm 1$

$\therefore x = 1, 3$

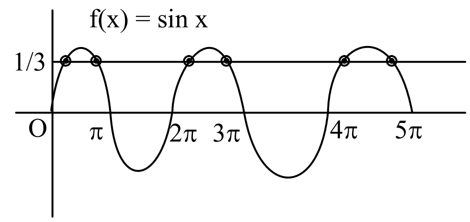
$\therefore$ sum = 4

**9. [C]** $3 \sin^2 x - 7 \sin x + 2 = 0$

$\sin^2 x - \frac{7}{3} \sin x + \frac{2}{3} = 0$

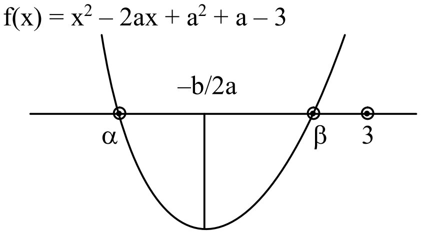
$\therefore \sin x = 2$ or $\sin x = \frac{1}{3}$

sin x = 2 is not possible.




$\therefore$ No. of solution is 6.

**10.[A]** $f(x) = x^2 - 2ax + a^2 + a - 3$




(i) $D \geq 0 \Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$

$\Rightarrow -a + 3 \geq 0$

$\therefore \boxed{a \leq 3}$ … (A)

(ii) $f(3) > 0 \Rightarrow 9 - 6a + a^2 + a - 3 > 0$

$a^2 - 5a + 6 > 0 \Rightarrow (a - 2)(a - 3) > 0$

$\therefore \boxed{a < 2 \text{ or } a > 3}$ … (B)

(iii) $\frac{-b}{2a} < 3 \Rightarrow \boxed{a < 3}$ … (C)

Hence from (A), (B), (C) : $\boxed{a < 2}$

**11.[C]** One root of $ax^2 + bx + c = 0$ is square of other root if

$ac^2 + a^2c + b^3 = 3abc$

Here $3x^2 + px + 3 = 0$

$\Rightarrow 27 + 27 + p^3 = 27p$

$\therefore p^3 - 27p + 54 = 0$

Clearly p = 3 is a root

$\therefore (p - 3)(p^2 + 3p - 18) = 0$

$\therefore p = 3$ & from $p^2 + 3p - 18 = 0$

$p = \frac{-3 \pm \sqrt{9 + 72}}{2}$

$\therefore p = -6$ or $3$

**12. [B]** $\because \alpha, \beta\ (\alpha < \beta)$ are roots of $x^2 + bx + c = 0$

$(c < 0 < b)$

$\therefore \alpha + \beta = -b \Rightarrow \alpha + \beta = -$ ive ...(i)

& $\alpha\beta = c \Rightarrow \alpha\beta = -$ive ... (ii)

$\because \alpha\beta < 0$; $\therefore$ one root is negative

& one root is positive

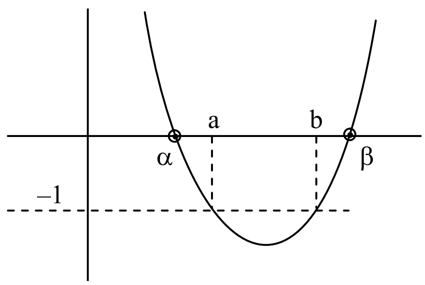
but $\alpha < \beta$ so $\boxed{\alpha < 0 \,\&\, \beta > 0}$

Also $\alpha + \beta < 0$ $\therefore$ $\boxed{|\alpha| > \beta}$

$\therefore \alpha < 0 < \beta < |\alpha|$

**13. [D]** $f(x) = (x - a)(x - b) - 1 \quad (a < b)$

$\because f(a) = f(b) = -1$




Clearly $-\infty < \alpha < a$

& $b < \beta < +\infty$

**14.[B]** $\because f(x) = x^2 - |x + 2| + x > 0$

Also $|x + 2| = \begin{cases} -(x+2); x < -2 \\ +(x+2); x \ge -2 \end{cases}$

(i) when $\boxed{x < -2}$ : $f(x) = x^2 + x + 2 + x > 0$

$= x^2 + 2x + 2 > 0$

$= (x + 1)^2 + 1 > 0$

is always satisfied when $x < -2$

i.e. $\boxed{x \in (-\infty, -2)}$ ...(A)

(ii) When $\boxed{x \ge -2}$ $\therefore f(x) = x^2 - x - 2 + x > 0$

$\therefore x^2 > 2$

$\therefore |x| > \sqrt{2}$

$\therefore x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

but $x \ge -2$

Hence $\boxed{x \in [-2, -\sqrt{2}] \cup (\sqrt{2}, \infty)}$ ... (B)

Hence from (A) & (B)

$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

**15.[A]** $ax^2 + bx + c = 0$

if one root is square of other root then

$ac^2 + a^2c + b^3 = 3abc$

Here $q^2 + q + p^3 = 3pq$

$\Rightarrow p^3 - q(3p - 1) + q^2 = 0$

**16.[C]** $D < 0$

$4a^2 - 4(10 - 3a) < 0$

$(a^2 + 3a - 10) < 0$

$(a + 5)(a - 2) < 0$

$-5 < a < 2$

**17.[D]** $(\alpha + \beta), (\alpha^2 + \beta^2), (\alpha^3 + \beta^3)$ are in GP.

$\therefore (\alpha^2 + \beta^2)^2 = (\alpha + \beta).(\alpha^3 + \beta^3)$ ...(i)

$\because$ We know $\alpha + \beta = -\dfrac{b}{a}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dfrac{b^2}{a^2} - \dfrac{2c}{a} = \dfrac{b^2 - 2ac}{a^2}$

& $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= -\dfrac{b}{a}\left(\dfrac{b^2 - 2ac}{a^2} - \dfrac{c}{a}\right)$

$= \dfrac{-b}{a^3}(b^2 - 2ac - ac) = -\dfrac{b}{a^3}(b^2 - 3ac)$

Putting in (i),we get

$\left(\dfrac{b^2 - 2ac}{a^2}\right)^2 = \dfrac{-b}{a} \cdot \dfrac{-b}{a^3}(a^2 - 3ac)$

$(b^2 - 2ac)^2 = b^2(b^2 - 3ac)$

$\Rightarrow b^4 + 4a^2c^2 - 4acb^2 = b^4 - 3acb^2$

$\Rightarrow acb^2 - 4a^2c^2 = 0$

$\Rightarrow ac(b^2 - 4ac) = 0$

$\therefore ac\,\Delta = 0 \Rightarrow \Delta = 0$

**18.[D]** $\alpha, \beta$ are roots of $x^2 - px + r = 0$

$\therefore$ $\boxed{\alpha + \beta = p}$ ... (i)

& $\alpha\beta = r$

Also $\dfrac{\alpha}{2}, 2\beta$ are roots of $x^2 - qx + r = 0$

$\therefore$ $\boxed{\dfrac{\alpha}{2} + 2\beta = q}$ & $\alpha\beta = r$ ... (ii)

from (i) & (ii)

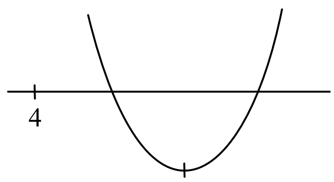
$\alpha + \beta = p$

$\alpha + 4\beta = 2q$

$\therefore \beta = \dfrac{2q - p}{3}$ & $\therefore \alpha = \dfrac{2(2p - q)}{3}$

$\therefore\ r = \alpha\beta = \frac{2}{9}\ (2p - q)\ (2q - p)$

**19. [A]**




$D > 0 \Rightarrow 64\ k^2 - 64(k^2 - k + 1) > 0$
$64\ k^2 - 64\ k^2 + 64\ k - 64 > 0$
$k > 1$
$-\frac{B}{2A} > 4 \Rightarrow 4k > 4 \Rightarrow k > 1$
$f(4) \geq 0$
$16 - 32\ k + 16k^2 - 16\ k + 16 \geq 0$
$\Rightarrow 16k^2 - 48\ k + 32 \geq 0$
$k^2 - 3k + 2 \geq 0 \Rightarrow (k - 1)\ (k - 2) \geq 0 \Rightarrow k \leq 1, k \geq 2$
so $k \in [2, \infty)$
so smallest integer value of k is 2.

**20.[B]** $\alpha + \beta = -p$ ...(i)
$\alpha^2 + \beta^2 = q$
$(\alpha + \beta)\ (\alpha^2 + \beta^2 - \alpha\beta) = q$
$\Rightarrow (\alpha + \beta)\ \{(\alpha + \beta)^2 - 3\alpha\beta\} = q$
$\Rightarrow\ (-p)\ \{p^2 - 3\alpha\beta\} = q$

$\alpha\beta = \frac{q + p^3}{3p}$ ...(ii)

$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

sum of roots (S) = $\frac{p^3 - 2q}{p^3 + q}$ using (i) and (ii)

product of roots = $\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

**21.[C]** $\therefore\ x^2 - 6x - 2 = 0$ has roots $\alpha, \beta$
$\alpha^2 - 2 = 6\alpha$
$\beta^2 - 2 = 6\beta$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)} = 3$$

**22.[B]** $[a\ \ b\ \ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ \ 0\ \ 0]$

$a + 8b + 7c = 0\ ;\ 9a + 2b + 3c = 0$
$7a + 7b + 7c = 0$

an solving we get $(a, b, c) = \left(-\frac{\lambda}{7}, -\frac{6\lambda}{7}, \lambda\right)$
if $b = 6$ so $\lambda = -7$
so. $(a, b, c) = (1, 6, -7)$
so the equation becomes $ax^2 + bx + c = 0$
$x^2 + 6x - 7 = 0$
$\alpha = 1, \beta = -7$

$$S = \sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{1}{1} - \frac{1}{7}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n$$

$$= 1 + \frac{6}{7} + \left(\frac{6}{7}\right)^2 + \ldots\infty = \frac{1}{1 - \frac{6}{7}} = 7$$

**23.[B]** $x^{-2} + bx - 1 = 0$ ...(i)
$x^2 + x + b = 0$ ...(ii)
(i) – (ii) we get $x = \frac{b+1}{b-1}$

$\left(\frac{b+1}{b-1}\right)^2 + b\left(\frac{b+1}{b-1}\right) - 1 = 0$

$b^2 + 3b = 0$
$b(b^2 + 3) = 0$
$b = 0$ or $b = \pm i\sqrt{3}$

**24.[C]** Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1$
Let $\alpha, \beta, \gamma, \delta$ are root of equation
$\therefore \alpha\beta\gamma\delta = -1$ so the equation has at least two real roots
$f'(x) = 4x^3 - 12x^2 + 24x + 1$
$f''(x) = 12x^2 - 24x + 24 = 12\{(x - 1)^2 + 1\}$
$\because$ so $f''(x) > 0$ so $f'(x) = 0$ has only one real roots
So $f(x) = 0$ has at most two real roots
$\therefore\ f(x) = 0$ has exactly two real roots.

**25.[B]** $(1 + a) = t^6$
$(t^2 - 1)\ x^2 + (t^3 - 1)\ x + (t - 1) = 0$

$$x = \frac{-(t^3 - 1) \pm \sqrt{(t^3 - 1)^2 - 4(t - 1)(t^2 - 1)}}{2(t^2 - 1)}$$

$$x = \frac{-(t^3 - 1) \pm (t - 1)\sqrt{(t^2 + t + 1)^2 - 4(t + 1)}}{2(t - 1)(t + 1)}$$

$$x = \frac{-(t^2 + t + 1) \pm \sqrt{(t^2 + t + 1)^2 - 4(t + 1)}}{2(t + 1)}$$

$a \to 0^+ \Rightarrow t \to 1^+$

$x = \frac{-3 \pm \sqrt{9 - 8}}{2(2)} \Rightarrow x = \frac{-3 \pm 1}{4}$

$\Rightarrow x = -1, -\frac{1}{2}$