JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

BINOMIAL THEOREM

(PRACTICE SHEET)

LEVEL- 1

Question based on **Binomial Theorem for positive integral Index**

Q.1 Fourth term in the expansion of $\left(\frac{a}{3}+9b\right)^{10}$ is-

(A) $40\,a^7\,b^3$ (B) $40a^3b^7$

(C) $1890\,a^6b^4$ (D) $1890a^4b^6$

Q.2 Second term in the expansion of $(2x + 3y)^5$ will be -

(A) $46\,x^2y^3$ (B) $30\,x^3y^2$

(C) $240\,x^4\,y$ (D) $810\,xy^4$

Q.3 The 5^{th} term of the expansion of $(x - 2)^8$ is -

(A) ${}^8C_5x^3(-2)^5$ (B) ${}^8C_5x^3\,2^5$

(C) ${}^8C_4x^4\,(-2)^4$ (D) ${}^8C_6x^2\,(-2)^6$

Q.4 The number of terms in expansion of $(x - 3x^2 + 3x^3)^{20}$ is-

(A) 60 (B) 61

(C) 40 (D) 41

Q.5 The term with coefficient ${}^6C_2$ in the expansion of $(1+ x)^6$ is-

(A) T_1 and T_3 (B) T_2 and T_4

(C) T_3 and T_5 (D) None of these

Q.6 If n is a positive integer, then r^{th} term in the expansion of $(1-x)^n$ is-

(A) ${}^nC_r\,(-x)^r$ (B) ${}^nC_rx^r$

(C) ${}^nC_{r-1}(-x)^{\,r-1}$ (D) ${}^nC_{r-1}x^{r-1}$

Q.7 If the 4^{th} term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then the values of a and n are-

(A) 1/2, 6 (B) 1, 3

(C) 1/2, 3 (D) can not be found

Q.8 The coefficient of $(3r)^{th}$ term and coefficient of $(r + 2)^{th}$ term in the expansion of $(1 + x)^{2n}$ are equal then (where r > 1, n > 2), positive integer)-

(A) r = n/2 (B) r = n/3

(C) $r = \frac{n+1}{2}$ (D) $r = \frac{n-1}{2}$

Q.9 The coefficient of a^2b^3 in $(a + b)^5$ is-

(A) 10 (B) 20

(C) 30 (D) 40

Q.10 The coefficient of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then n is equal to-

(A) 35 (B) 45

(C) 55 (D) None of these

Q.11 The coefficient of x^5 in the expansion of $(2 + 3x)^{12}$ is-

(A) ${}^{12}C_5 2^5,\ 3^7$ (B) ${}^{12}C_6 2^6.3^6$

(C) ${}^{12}C_5 2^7.3^5$ (D) None of these

Q.12 If the expansion of $\left(x^2-\frac{1}{4}\right)^n$, the coefficient of third term is 31, then the value of n is-

(A) 30 (B) 31

(C) 29 (D) 32

Q.13 If A and B are coefficients of x^r and x^{n-r} respectively in the expansion of $(1+ x)^n$, then-

(A) A = B

(B) $A \neq B$

(C) $A = \lambda$, B for some λ

(D) None of these

Q.14 If $(1 + by)^n = (1 + 8y + 24y^2 + \ldots..)$ then the value of b and n are respectively-

(A) 4, 2 (B) 2, –4

(C) 2, 4 (D) –2, 4

Q.15 The number of terms in the expansion of $(1 + 5\sqrt{2}\,x)^9 + (1 - 5\sqrt{2}\,x)^9$ is-

(A) 5 (B) 7 (C) 9 (D) 10

Q.16 The number of non zero terms in the expansion of $[(1 + 3\sqrt{2}\,x)^9 - (1 - 3\sqrt{2}\,x)^9]$ is -

(A) 9 (B) 10

(C) 5 (D) 15

Q.17 After simplification, the total number of terms in the expansion of $(x + \sqrt{2})^4 + (x - \sqrt{2})^4$ is-

(A) 10 (B) 5

(C) 4 (D) 3

Q.18 The number of terms in the expansion of $[(x - 3y)^2 (x + 3y)^2]^3$ is-

(A) 6 (B) 7

(C) 8 (D) None of these

Q.19 The number of terms in $(x + a)^{100} + (x - a)^{100}$ after solving the expansion is -

(A) 202 (B) 51

(C) 101 (D) None of these

Q.20 The coefficient of x^5 in the expansion of $(1+ x^2)^5 (1+ x)^4$ is -

(A) 30 (B) 60

(C) 40 (D) None of these

Q. 21 The coefficient of x^5 in the expansion of $(1 + x)^3.(1 - x)^6$ is -

(A) 6 (B) 22 (C) – 6 (D) 8

Q.22 The coefficient of x^4 in the expansion of $(1+ x + x^2 + x^3)^{11}$ is-

(A) 990 (B) 495

(C) 330 (D) None of these

Question based on Particular term in the expansion

Q.23 The coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is -

(A) $\frac{405}{256}$ (B) $\frac{504}{259}$

(C) $\frac{450}{263}$ (D) None of these

Q.24 The coefficient of x^{-26} in the expansion of $\left(x^2-\frac{2}{x^4}\right)^{11}$ is

(A) 330×2^6 (B) -330×2^6

(C) 330×2^7 (D) -330×2^7

Q.25 The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ will be -

(A) 3/2 (B) 5/4

(C) 5/2 (D) None of these

Q.26 The term independent of y in the binomial expansion of $\left(\frac{1}{2}y^{1/3}+y^{-1/5}\right)^8$ is -

(A) sixth (B) seventh

(C) fifth (D) None of these

Q.27 If x^4 occurs in the r^{th} term in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$, then r equals-

(A) 7 (B) 8 (C) 9 (D) 10

Q.28 The term containing x in the expansion of $\left(x^2+\frac{1}{x}\right)^5$ is -

(A) 2^{nd} (B) 3^{rd} (C) 4^{th} (D) 5^{th}

Q.29 The term independent of x in $\left(2x+\frac{1}{3x}\right)^6$ is -

(A) 160/9 (B) 80/9

(C) 160/27 (D) 80/3

Q.30 The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\sqrt{\frac{3}{2x^2}}\right)^{10}$ is-

(A) $^{10}C_1$ (B) 5/12

(C) 1 (D) None of these

Q.31 If 9^{th} term in the expansion of $(x^{1/3} + x^{-1/3})^n$ does not depend on x, then n is equal to-

(A) 10 (B) 13 (C) 16 (D) 18

Q.32 The constant term in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is-

(A) $\frac{2n!}{n!}$ (B) $\frac{n!}{2n!}$ (C) $\frac{2n!}{2!n!}$ (D) $\frac{2n!}{n!n!}$

Q.33 The coefficient of the term independent of y in the expansion of $\left(y-\frac{1}{y^2}\right)^{3n}$ is -

(A) ${}^{3n}C_{n-1}\,(-1)^{n-1}$ (B) ${}^{3n}\,C_n$

(C) ${}^{3n}\,C_n(-1)^n$ (D) None of these

Q.34 The number of integral terms in the expansion of $(5^{1/2} + 7^{1/6})^{642}$ is -

(A) 106 (B) 108

(C) 103 (D) 109

Question based on Middle Term

Q.35 Middle term in the expansion of $(x^2 - 2x)^{10}$ will be -

(A) ${}^{10}C_4 x^{17}\, 2^4$ (B) $-{}^{10}C_5\ 2^5 x^{15}$

(C) $-{}^{10}C_4\, 2^4 x^{17}$ (D) ${}^{10}C_5\ 2^4 x^{15}$

Q.36 The middle term in the expansion of $\left(\frac{3}{x^2}-\frac{x^3}{6}\right)^9$ is -

(A) $\frac{189}{8}x^2, \frac{21}{16}x^7$ (B) $\frac{189}{8}x^2, -\frac{21}{16}x^7$

(C) $-\frac{189}{8}x^2, -\frac{21}{16}x^7$ (D) None of these

Q.37 The middle term of the expansion $\left(\frac{x}{a}-\frac{a}{x}\right)^8$ is-

(A) $56a^2/x^2$ (B) $-56a^2/x^2$

(C) 70 (D) –70

Q.38 The middle term in the expansion of $\left(x^3-\frac{1}{x^3}\right)^{10}$ is-

(A) 252 (B) – 252

(C) 210 (D) – 210

Q.39 If the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924\, x^6$, then n =

(A) 10 (B) 12

(C) 14 (D) None of these

Q.40 The greatest coefficient in the expansion of $(1+x)^{10}$ is-

(A) $\frac{10!}{5!\,6!}$ (B) $\frac{10!}{(5!)^2}$

(C) $\frac{10!}{5!\,7!}$ (D) None of these

Q.41 The middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^6$ is -

(A) ${}^{18}C_{10}x^{10}$ (B) ${}^{18}C_9(-x)^9$

(C) ${}^{18}C_9\,x^9$ (D) $-\,{}^{18}C_{10}\,x^{10}$

Question based on Term from end

Q.42 The 5th term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^3}\right)^9$ is -

(A) $63x^3$ (B) $-\frac{252}{x^3}$

(C) $\frac{672}{x^{18}}$ (D) None of these

Q.43 If in the expansion of $\left(2^{1/3}+\frac{1}{3^{1/3}}\right)^n$, the ratio of 6th terms from beginning and from the end is 1/6, then the value of n is -

(A) 5 (B) 7

(C) 9 (D) None of these

Question based on Binomial Coefficient

Q.44 If $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + ...+ C_nx^n$, then the value of $C_1+ C_2+ C_3 + ...+ C_n$ is-

(A) 2^{n+1} (B) 2^{n-1}

(C) $2^n + 1$ (D) $2^n - 1$

Q.45 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then the value of $C_0 + 2C_1 + 3C_2 + ...+(n+1)C_n$ is -

(A) $2^n (n + 1)$ (B) $2^{n-1} (n + 1)$
(C) $2^{n-1} (n + 2)$ (D) $2^n (n + 2)$

Q.46 If $C_0, C_1, C_2, ..., C_{15}$ are coefficients of different terms in the expansion of $(1 + x)^{15}$, then $C_0 + C_2 + C_4 + ...+ C_{14}$ is equal to-

(A) 2^{15} (B) 2^{14} (C) 2^7 (D) 2^8

Q.47 If $(1+x)^n = 1 + C_1x + C_2x^2 + ...+ C_nx^n$, then $C_1 + C_3 + C_5 + ...$ is equal to-

(A) 2^n (B) $2^n - 1$ (C) $2^n + 1$ (D) 2^{n-1}

Q.48 $n!\left(\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+......+\frac{1}{n!}\right)$ is equal to -

(A) 2^n (B) 2^{n-1} (C) 2^{n+1} (D) 2^{-n+1}

Q.49 $\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+.......=$

(A) $\frac{2^n}{n!}$ (B) $\frac{2^{n-1}}{n!}$
(C) 0 (D) None of these

Q.50 The value of ${}^8C_0 + {}^8C_2 + {}^8C_4 + {}^8C_6 + {}^8C_8$ is-

(A) 32 (B) 64 (C) 128 (D) 256

Q.51 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then $C_0C_1 + C_1C_2 + C_2C_3 + ...+ C_{n-1}C_n$ is equal to-

(A) $\frac{2n!}{n!\,n!}$ (B) $\frac{2n!}{n!(n+1)!}$
(C) $\frac{2n!}{(n-1)!\,(n+1)!}$ (D) $\frac{2n!}{(n-1)!\,n\,!}$

Q.52 ${}^nC_0 - \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 - ......+ (-1)^n \frac{{}^nC_n}{n+1} =$

(A) n (B) 1/n
(C) $\frac{1}{n+1}$ (D) $\frac{1}{n-1}$

Q.53 In the expansion of $(1 + x)^n \left(1+\frac{1}{x}\right)^n$, the term independent of x is-

(A) $C_0^2 + 2\,C_1^2 + ...+ (n+1)\,C_n^2$
(B) $(C_0 + C_1 + ...+ C_n)^2$
(C) $C_0^2 + C_1^2 + ...+ C_n^2$
(D) None of these

Q.54 If $(1+x)^n = C_0 + C_1x + C_2x^2 +...+ C_nx^n$, then $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ...+ C_{n-r}C_n$ is equal to-

(A) $\frac{2n!}{(n-r)!(n+r)!}$ (B) $\frac{2n!}{n!(n+r)!}$
(C) $\frac{2n!}{n!(n-r)!}$ (D) $\frac{2n!}{(n-1)!(n+1)!}$

Q.55 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...C_n.\, x^n$ then the value of $C_0 + 3C_1 + 5C_2 + ...+ (2n + 1)\, C_n$ is-

(A) $n.2^n$ (B) $(n-1).\,2^n$
(C) $(n + 2).2^{n-1}$ (D) $(n + 1).2^n$

Q.56 If $C_0, C_1, C_2.......C_n$ are binomial coefficients in the expansion of $(1 + x)^n$, $n \in N$, then $\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+.....+\frac{C_n}{n+1}$ is equal to-

(A) $\frac{2^{n+1}-1}{n+1}$ (B) $(n + 1)\,.\,2^{n+1}$
(C) $\frac{2^n-1}{n+1}$ (D) None of these

Q.57 If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then for n odd, $C_1^2 + C_3^2 + C_5^2 + ...+ C_n^2$ is equal to

(A) 2^{2n-2} (B) 2^n (C) $\frac{(2n)!}{2(n!)^2}$ (D) $\frac{(2n)!}{(n!)^2}$

Q.58 If $(1+ x + x^2)^{2n} = a_0 + a_1x + a_2x^2 +....$then the value of $a_0 - a_1 + a_2 - a_3 + ...$ is-

(A) 2^n (B) 3^n (C) 1 (D) 0

Q.59 The sum of the coefficients in the expansion of $(a + 2b + c)^{10}$ is -

(A) 4^{10} (B) 3^{10} (C) 2^{10} (D) 10^4

Q.60 The sum of coefficients of even powers of x in the expansion of $(1+ x + x^2 + x^3)^5$ is -

(A) 512 (B) – 512
(C) 215 (D) None of these

LEVEL- 2

Q.1 If $(1 + x - 2x^2)^6 = 1 + C_1x + C_2x^2 + C_3x^3 + ...+ C_{12}\, x^{12}$, then the value of $C_2 + C_4 + C_6 + ...+ C_{12}$ is -

(A) 30 (B) 32
(C) 31 (D) None of these

Q.2 If $C_0, C_1, C_2, ...C_n$ are binomial coefficients of different terms in the expansion of $(1+ x)^n$ then $C_0 - 2.C_1 + 3.C_2 - 4.C_3 + ...+ (-1)^n .(n + 1)C_n$ equals-

(A) $-n.2^{n-1}$ (B) 0
(C) $2^{n-1}. (2 - n)$ (D) None of these

Q.3 If C_r stands for ${}^nC_r$, then the sum of first (n+1) terms of the series $aC_0 - (a + d) C_1 + (a + 2d)C_2 - (a + 3d)C_3 + ...$ is-

(A) $a/2^n$ (B) na
(C) 0 (D) None of these

Q.4 The value of $(\sqrt{5} + 1)^5 - (\sqrt{5} - 1)^5$ is -

(A) 252 (B) 252 (C) 452 (D) 532

Q.5 If $(2x - 3x^2)^6 = a_0 + a_1x + a_2x^2 + ...+a_{12}x^{12}$, then values of a_0 and a_6 are -

(A) 0, 6 (B) 0, 2^6 (C) 1, 6 (D) 0

Q.6 If the $(r+1)^{th}$ term in the expansion of $\left(\sqrt[3]{\frac{a}{\sqrt{b}}} + \sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{21}$ contains same powers of a and b, then the value of **r** is -

(A) 9 (B) 10 (C) 8 (D) 6

Q.7 If n is odd, then

$C_0^2 - C_1^2 + C_2^2 - C_3^2 + ...+ (-1)^n C_n^2 =$

(A) 0 (B) 1
(C) ∞ (D) $\frac{n!}{(n/2)^2!}$

Q.8 In the expansion of $(1+ x)^n$, $\frac{T_{r+1}}{T_r}$ is equal to

(A) $\frac{n+1}{r}x$ (B) $\frac{n+r+1}{r}x$
(C) $\frac{n-r+1}{r}x$ (D) $\frac{n+r}{r+1}x$

Q.9 If $(1+ x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then the value of $1^2 C_1 + 2^2 C_2 + 3^2 C_3 + ...+ n^2 C_n$ is

(A) $n(n + 1)\, 2^{n-2}$ (B) $n(n + 1)\, 2^{n-1}$
(C) $n(n + 1)\, 2^n$ (D) None of these

Q.10 The sum of coefficients of odd powers of x in the expansion of $(1+ x)^n$ is -

(A) $2^n + 1$ (B) $2^n - 1$
(C) 2^n (D) 2^{n-1}

Q.11 The sum of $C_0^2 - C_1^2 + C_2^2 - ....+ (-1)^n C_n^2$ where n is an even integer, is-

(A) ${}^{2n}C_n$ (B) $(-1)^n\ {}^{2n}C_n$
(C) ${}^{2n}C_{n-1}$ (D) $(-1)^{n/2}.\ {}^nC_{n/2}$

Q.12 If $C_0, C_1, C_2,.....C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then the value of $\sum_{r=0}^{n}(r+1)C_r$ is -

(A) $n\, 2^n$ (B) $(n+1)\, 2^{n-1}$
(C) $(n+2)\, 2^{n-1}$ (D) $(n + 2)\, 2^{n-2}$

Q.13 If $1 \le r \le n-1$ then ${}^{n-1}C_r + {}^{n-2}C_r + ...+ {}^rC_r$ equals-

(A) ${}^nC_r$ (B) ${}^nC_{r+1}$
(C) ${}^{n+1}C_r$ (D) None of these

Q.14 The coefficient of 1/x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is -

(A) $\frac{n!}{(n-1)!(n+1)!}$ (B) $\frac{(2n)!}{(n-1)!(n+1)!}$
(C) $\frac{(2n)!}{(2n-1)!(2n+1)!}$ (D) None of these

Q.15 If 6^{th} term in the expansion of $\left(\frac{1}{x^{8/3}}+x^2\log_{10}x\right)^8$ is 5600, then x is equal to-

(A) 8 (B) 10

(C) 9 (D) None of these

Q.16 If the coefficient of third term in the expansion of $\left(x^{2/3}+\frac{1}{x^{1/3}}\right)^n$ is 27 more than the coefficient of second term, then the value of n is -

(A) 8 (B) 9

(C) 10 (D) None of these

Q.17 The sum of the terms of the series $[3.{}^nC_0 - 8.{}^nC_1 + 13.{}^nC_2 - 18.\ {}^nC_3 + ...+ (n+1)]$ is-

(A) $3.\ 2^n - 5n.2^{n-1}$ (B) 0

(C) $3.2^n + 5n.2^{n-1}$ (D) None of these

Q.18 If sum of all the coefficients in the expansion of $(x^{3/2} + x^{-1/3})^n$ is 128, then the coefficient of x^5 is -

(A) 35 (B) 45

(C) 7 (D) None of these

Q.19 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_n.x^n$ then the value of $C_0C_n + C_1C_{n-1} + C_2C_{n-2} + ...+ C_nC_0$ is -

(A) 1

(B) $\frac{(2n)!}{(n)!\ (n)!}$

(C) $\left(\frac{(2n)!}{n!}\right)^2$

(D) $(2^n)^2$

Q.20 Find the value

$$\frac{(18^3+7^3+3.18.7.25)}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$$

(A) 1 (B) 5

(C) 25 (D)100

Q.21 If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$ is equal to-

(A) $\frac{a_2}{a_2+a_3}$ (B) $\frac{1}{2}\frac{a_2}{a_2+a_3}$

(C) $\frac{2a_2}{a_2+a_3}$ (D) $\frac{2a_3}{a_2+a_3}$

Q.22 If the coefficients of four consecutive terms in the expansion of $(1+x)^n$ are a_1,a_2,a_3 and a_4, then $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}, \frac{a_3}{a_3+a_4}$ are in -

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.23 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then $C_0C_2 + C_1C_3 + C_2C_4 + ...+ C_{n-2}\,C_n =$

(A) $\frac{2n!}{(n+1)!(n+2)!}$ (B) $\frac{2n!}{n!(n+2)!}$

(C) $\frac{2n!}{(n-2)!\,(n+2)!}$ (D) $\frac{2n!}{(n-1)!\,(n+2)!}$

Q.24 The greatest coefficient in the expansion of $(1+x)^{2n+2}$ is -

(A) $\frac{2n!}{(n!)^2}$ (B) $\frac{(2n+2)!}{[(n+1)!]^2}$

(C) $\frac{(2n+2)!}{n!\,(n+1)!}$ (D) $\frac{(2n)!}{n!(n+1)!}$

Q.25 The middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$ is-

(A) $\frac{1.3.5.....(2n-3)}{n!}$ (B) $\frac{1.3.5.....(2n-1)}{n!}$

(C) $\frac{1.3.5.....(2n+1)}{n!}$ (D) None of these

Q.26 The term independent of x in the expansion of $\left(x-\frac{1}{x}\right)^4\left(x+\frac{1}{x}\right)^3$ is-

(A) –3 (B) 0

(C) 1 (D) 3

Q.27 If the sum of the coefficients in the expansion of $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$ vanishes, then the value of α is -

(A) 2　　(B) –1
(C) 1　　(D) –2

Q.28 If the sum of coefficients in the binomial expansion of $(x + y)^n$ is 4096 then greatest coefficient in the expansion is -

(A) 922　　(B) 942
(C) 787　　(D) 924

Q.29 The value of $({}^nC_2 - 2.\,{}^nC_3 + 3.{}^nC_4 - 4.{}^nC_5 + ...)$ is equal to -

(A) 1　　(B) 0
(C) –1　　(D) None of these

Q.30 The sum of 12 terms of the series

${}^{12}C_1 . \frac{1}{3} + {}^{12}C_2 . \frac{1}{9} + {}^{12}C_3 . \frac{1}{27} + ...$ is -

(A) $\left(\frac{4}{3}\right)^{12} - 1$　　(B) $\left(\frac{3}{4}\right)^{12} - 1$

(C) $\left(\frac{3}{4}\right)^{12} + 1$　　(D) None of these

Q.31 If n = 10 then $(C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^n (C_n^2))$ equals-

(A) $(-1)^5 . {}^{10}C_5$　　(B) 0
(C) ${}^{10}C_5$　　(D) $(-1)^6\ {}^{10}C_6$

Q.32 If n = 11 then $(C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^n (C_n^2)$ equals-

(A) $(-1)^5 .\ {}^{10}C_5$　　(B) 0
(C) ${}^{10}C_6$　　(D) None of these

Q.33 The sum of (n +1) terms of the series

$C_0^2 + 3C_1^2 + 5C_2^2 + ...$ is -

(A) ${}^{2n-1}C_{n-1}$　　(B) ${}^{2n-1}C_n$
(C) $2(n+1)^{2n-1}C_n$　　(D) None of these

Q.34 If $(1 + x)^n = \sum_{r=0}^{n} {}^nC_r . (x)^r$

then $\left(1 + \frac{{}^nC_1}{{}^nC_0}\right) . \left(1 + \frac{{}^nC_2}{{}^nC_1}\right) ..... \left(1 + \frac{{}^nC_n}{{}^nC_{n-1}}\right) =$

(A) $\frac{n^{n-1}}{(n-1)!}$　　(B) $\frac{(n+1)^{n-1}}{(n-1)!}$

(C) $\frac{(n+1)^n}{n!}$　　(D) $\frac{(n+1)^{n+1}}{n!}$

Q.35 The term independent of x in the expansion of

$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is -

(A) $T_4 = 180$　　(B) $T_5 = -210$
(C) $T_4 = -180$　　(D) $T_5 = 210$

LEVEL- 3

Q.1 If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in A.P., then r -

(A) 6 (B) 7 (C) 8 (D) 9

Q.2 The value of x , for which the 6^{th} term in the expansion of $\left(2^{\log_2\sqrt{(9^{x-1}+7)}}+\frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right)^7$ is 84 is equal to -

(A) 4, 3 (B) 0, 3

(C) 0, 2 (D) 1, 2

Q.3 The coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+ \ldots\ldots\ldots..+(1+x)^{30}$ is -

(A) ${}^{51}C_5$ (B) ${}^{9}C_5$

(C) ${}^{31}C_6 - {}^{21}C_6$ (D) ${}^{30}C_5 + {}^{20}C_5$

Q.4 The coefficient of x^5 in the expansion of $(x^2 - x - 2)^5$ is-

(A) – 83 (B) – 82

(C) – 81 (D) 0

Q.5 In the expansion of $(1+3x+2x^2)^6$ the coefficient of x^{11} is -

(A) 144 (B) 288

(C) 216 (D) 576

Q.6 Coefficients of x^r $[0 < r < (n-1)]$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots\ldots.. + (x+2)^{n-1}$ -

(A) ${}^nC_r(3^r - 2^n)$ (B) ${}^nC_r(3^{n-r} - 2^{n-r})$

(C) ${}^nC_r(3^r + 2^{n-r})$ (D) None of these

Q.7 If x + y = 1, then $\sum_{r=0}^{n} r^2\, {}^nC_r x^r y^{n-r}$ equals -

(A) nxy (B) nx (x+ yn)

(C) nx (nx+y) (D) None of these.

Q.8 $(1+x)^n - nx - 1$ is divisible by (where $n \in N$)–

(A) $2x^3$ (B) 2x

(C) x^2 (D) All of these

Q.9 In the expansion of $(5^{1/2} + 7^{1/8})^{1024}$, the number of integral terms is -

(A) 128 (B) 129 (C) 130 (D) 131

Q.10 Let $R = (5\sqrt{5} + 11)^{2n+1}$ and $f = R - [R]$, where [.] denotes the greatest integer function. The value of $R.f$ is -

(A) 4^{2n+1} (B) 4^{2n}

(C) 4^{2n-1} (D) 4^{-2n}

Q.11 The greatest integer less than or equal to $(\sqrt{2}+1)^6$ is -

(A) 196 (B) 197

(C) 198 (D) 199

Q.12 The number of integral terms in the expansion of $(5^{1/2} + 7^{1/6})^{642}$ is -

(A) 106 (B) 108

(C) 103 (D) 109

Q.13 The remainder when 5^{99} is divides by 13 is -

(A) 6 (B) 8

(C) 9 (D) 10

Q.14 When 2^{301} is divided by 5, the least positive remainder is

(A) 4 (B) 8 (C) 2 (D) 6

Q.15 If the sum of the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ is a and if the sum of the coefficients in the expansion of $(1 + x^2)^n$ is b, then -

(A) $a = 3b$ (B) $a = b^3$

(C) $b = a^3$ (D) none of these

Q.16 $\sum_{k=0}^{10} {}^{20}C_k =$

(A) $2^{19} + \frac{1}{2}\,{}^{20}C_{10}$ (B) 2^{19}

(C) ${}^{20}C_{10}$ (D) none of these

Q.17 If $n \in N$ such that $(7 + 4\sqrt{3})^n = I + F$, where $I \in N$ and $0 < F < I$, Then the value of $(I + F)(1 - F)$ is -

(A) 0 (B) 1

(C) 7^{2n} (D) 2^{2n}

Q.18 The value of ${}^{95}C_4 + \sum_{j=1}^{5} {}^{100-j}C_3$ is -

(A) ${}^{95}C_5$ (B) ${}^{100}C_4$

(C) ${}^{99}C_4$ (D) ${}^{100}C_5$

Q.19 If $(5 + 2\sqrt{6})^n = I + f$, where $I \in N$, $n \in N$ and $0 < f < 1$, then I equals -

(A) $\frac{1}{-f} - f$ (B) $\frac{1}{1+f} - f$

(C) $\frac{1}{1-f} - f$ (D) $\frac{1}{1-f} + f$

Q.20 If $({}^{15}C_r + {}^{15}C_{r-1})({}^{15}C_{15-r} + {}^{15}C_{16-r}) = ({}^{16}C_{13})^2$, then the value of r is -

(A) r = 3 (B) r = 2

(C) r = 4 (D) none of these

Passage Based Questions (Q. 21 - 23)

The numerically greatest term in the expansion of $(x + a)^n$ is given by $\frac{n+1}{\left|\frac{x}{a}\right| + 1} = k$ (say)

(a) If k is an integer then T_k and T_{k+1} are the numerically greatest term

(b) If k is not an integer. Let m is its integral part then T_{m+1} is the numerically greatest term.

Q.21 The numerically greatest term in the expansion of $(3 - 5x)^{15}$ when $x = \frac{1}{5}$ is –

(A) T_4 (B) T_5 & T_6

(C) T_4 & T_5 (D) T_6

Q.22 The value of numerically greatest term in the expansion of $(3 + 5x)^{11}$ when $x = \frac{1}{5}$

(A) 55×3^{10} (B) 110×3^9

(C) 55×3^8 (D) 55×3^9

Q.23 The value of numerically greatest term in the expansion of $(3x + 2)^9$ when x = 3/2

(A) $\frac{7 \times 3^{13}}{2}$ (B) 7×3^{13}

(C) 7×3^{14} (D) None of these

Question based on Statements (Q. 24 - 26)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement- I and Statement- II are true, and Statement - II is the correct explanation of Statement– I.

(B) If both Statement - I and Statement - II are true but Statement - II is not the correct explanation of Statement – I.

(C) If Statement - I is true but Statement - II is false

(D) If Statement - I is false but Statement- II is true.

Q.24 **Statement I** : The number of terms in $(1 + x + x^2 + ...+ x^{10})^5$ is 51.

Statement II : The sum of the products of ${}^nC_0, {}^nC_1, {}^nC_2 ..... {}^nC_n$ (taken two together) is equal to $2^{2n-1} - \frac{(2n)!}{2.(n!)^2}$

Q.25 **Statement I** : The coefficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$ is 120.

Statement II : The sum of the coefficients in the expansion of $(1 + 2x - 3y + 5z)^3$ is 125.

Q.26 **Statement I** : $\sum_{K=1}^{n} K \cdot ({}^nC_K)^2 = n \cdot {}^{2n-1}C_{n-1}$

Statement II : If 2^{2003} is divided by 15 the remainder is 1.

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 If the coefficient of $(r + 2)^{th}$ and $(3r)^{th}$ term in the exp. of $(1+ x)^{2n}$ are equal then **[AIEEE 2002]**

(A) n = 2r + 1 (B) n = 2r – 1

(C) n = 2r (D) None of these

Q.2 If $(1+ x)^n = C_0 + C_1x + C_2x^2 +....+ C_nx^n$, then $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} +.......+ \frac{nC_n}{C_{n-1}} =$ **[AIEEE-2002]**

(A) $\frac{n}{2}$ (B) n (n + 1)

(C) $\frac{n(n+1)}{12}$ (D) $\frac{n(n+1)}{2}$

Q.3 The coefficient of x^{39} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is- **[AIEEE-2002]**

(A) – 455 (B) – 105

(C) + 455 (D) + 105

Q.4 The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is - **[AIEEE- 2003]**

(A) 35 (B) 32

(C) 33 (D) 34

Q.5 The coefficient of the middle term in the binomial expansion in powers of x of $(1+ \alpha x)^4$ and of $(1- \alpha x)^6$ is the same if α equals- **[AIEEE 2004]**

(A) $-\frac{5}{3}$ (B) $\frac{10}{3}$

(C) $-\frac{3}{10}$ (D) $\frac{3}{5}$

Q.6 The coefficient of x^n in expansion of $(1+ x) (1- x)^n$ is- **[AIEEE 2004]**

(A) (n – 1) (B) $(-1)^n (1- n)$

(C) $(-1)^{n-1} (n- 1)^2$ (D) $(-1)^{n-1} n$

Q.7 If $S_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{^nC_r}$, then $\frac{t_n}{s_n}$ is equal to- **[AIEEE 2004]**

(A) $\frac{1}{2}n$ (B) $\frac{1}{2}n - 1$ (C) n –1 (D) $\frac{2n-1}{2}$

Q.8 If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation - **[AIEEE-2005]**

(A) $m^2 - m (4r - 1) + 4r^2 - 2 = 0$

(B) $m^2 - m (4r + 1) + 4r^2 + 2 = 0$

(C) $m^2 - m (4r + 1) + 4r^2 - 2 = 0$

(D) $m^2 - m (4r - 1) + 4r^2 + 2 = 0$

Q.9 If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation - **[AIEEE-2005]**

(A) a – b = 1 (B) a + b = 1

(C) $\frac{a}{b} = 1$ (D) ab = 1

Q.10 For natural numbers m, n if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ...$, and $a_1 = a_2 = 10$, then (m, n) is– **[AIEEE 2006]**

(A) (35, 20) (B) (45, 35)

(C) (35, 45) (D) (20, 45)

Q.11 In the binomial expansion of $(a - b)^n$, n ≥ 5, the sum of 5^{th} and 6^{th} terms is zero, then $\frac{a}{b}$ equals- **[AIEEE 2007]**

(A) $\frac{5}{n-4}$ (B) $\frac{6}{n-5}$

(C) $\frac{n-5}{6}$ (D) $\frac{n-4}{5}$

Q.12 The sum of the series $^{20}C_0 - {^{20}C_1} + {^{20}C_2} - {^{20}C_3} + \ldots\ldots - \ldots.. + {^{20}C_{10}}$ is- **[AIEEE 2007]**

(A) $-{^{20}C_{10}}$ (B) $\frac{1}{2}\,{^{20}C_{10}}$

(C) 0 (D) $^{20}C_{10}$

Q.13 **Statement- 1:**

$$\sum_{r=0}^{n}(r+1)\,{^nC_r} = (n+2)\,2^{n-1}$$

Statement -2:

$$\sum_{r=0}^{n}(r+1)\,{^nC_r}\,x^r = (1+x)^n + nx\,(1+x)^{n-1}$$

[AIEEE-2008]

(A) Statement-1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is true, Statement -2 is true; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is true, Statement -2 is false

(D) Statement-1 is false, Statement-2 is true

Q.14 The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is- **[AIEEE-2009]**

(A) 0 (B) 2 (C) 7 (D) 8

Q.15 Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{^{10}C_j}, S_2 = \sum_{j=1}^{10} j\,{^{10}C_j}$ and

$$S_3 = \sum_{j=1}^{10} j^2\,{^{10}C_j}.$$

Statement – 1 : $S_3 = 55 \times 2^9$.

Statement – 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

[AIEEE-2010]

(A) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(B) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(C) Statement -1 is true, Statement -2 is false.

(D) Statement -1 is false, Statement -2 is ture.

Q.16 The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is : **[AIEEE-2011]**

(A) 144 (B) –132 (C) –144 (D) 132

Q.17 The term independent of x in expansion of

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10} \text{ is –}$$

[JEE Main- 2013]

(A) 210 (B) 310 (C) 4 (D) 120

SECTION - B

Q.1 If n is an integer between 0 and 21; then the minimum value of n! (21 – n)! is - **[IIT-1990]**

(A) 9 ! 12 ! (B) 10 ! 11 !

(C) 20 ! (D) 2 !

Q.2 The expansion $[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$ is a polynomial of degree - **[IIT- 1992]**

(A) 5 (B) 6

(C) 7 (D) 8

Q.3 If the r^{th} term in the expansion of $(x/3 - 2/x^2)^{10}$ contains x^4, then r is equal to - **[IIT-(Scr.)- 1992]**

(A) 2 (B) 3

(C) 4 (D) 5

Q.4 The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} {^{100}C_m}(x-3)^{100-m}\,2^m$ is - **[IIT (Scr.)-1992]**

(A) $^{100}C_{47}$ (B) $^{100}C_{53}$

(C) $-{^{100}C_{53}}$ (D) $-{^{100}C_{100}}$

Q.5 The value of $C_0 + 3C_1 + 5C_2 + 7C_3 + \ldots\ldots.. + (2n+1)\,C_n$ is equal to - **[IIT (Scr.)-1993]**

(A) 2^n (B) $2^n + n.2^{n-1}$

(C) $2^n.(n+1)$ (D) None of these

Q.6 The largest term in the expansion of $(3 + 2x)^{50}$ where $x = 1/5$ is - **[IIT (Scr.)-1993]**

(A) 5^{th} (B) 51^{th}

(C) 6^{th} and 7^{th} (D) 8^{th}

Q.7 The sum of the rational terms in the expansion of $(\sqrt{2} + 3^{1/5})^{10}$ is - **[IIT-Re-ex- 1997]**

(A) 41 (B) 42

(C) 40 (D) 43

Q.8 If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals – **[IIT - 1998]**

(A) $(n-1)\, a_n$ (B) na_n

(C) $1/2\, na_n$ (D) None of these

Q.9 If n is an odd natural number, then $\sum_{r=0}^{n} \frac{(-1)^r}{{}^nC_r}$ equal **[IIT- 1998]**

(A) 0 (B) 1/n

(C) n/2n (D) none of these

Q.10 If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and –6 respectively, then m is - **[IIT - 1999]**

(A) 6 (B) 9

(C) 12 (D) 24

Q.11 For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} = \ldots\ldots\ldots$ **[IIT-Sc- 2000]**

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$

(C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

Q.12 In the binomial expansion of $(a - b)^n$, $n \ge 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then $\frac{a}{b}$ equals- **[IIT-Sc- 2001]**

(A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$

(C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

Q.13 Find coefficient of t^{24} in the expansion of $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is- **[IIT-Sc- 2003]**

(A) ${}^{12}C_6 + 2$ (B) ${}^{12}C_6 + 1$

(C) ${}^{12}C_6 + 3$ (D) ${}^{12}C_6$

Q.14 If ${}^{n-1}C_r = (k^2 - 3)\, {}^nC_{r+1}$, then k lies between **[IIT-Sc- 2004]**

(A) $(-\infty, -2)$ (B) $(2, \infty)$

(C) $[-\sqrt{3}, \sqrt{3}]$ (D) $(\sqrt{3}, 2]$

Q.15 $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \ldots\ldots\ldots + \binom{30}{20}\binom{30}{30} =$ **[IIT-Sc- 2005]**

(A) $\binom{30}{10}$ (B) $\binom{60}{20}$

(C) $\binom{31}{10}$ (D) $\binom{31}{11}$

Q.16 For r = 0, 1, …, 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1+ x)^{30}$. Then $\sum_{r=1}^{10} A_r (B_{10}B_r - C_{10}A_r)$ is equal to **[IIT - 2010]**

(A) $B_{10} - C_{10}$ (B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$

(C) 0 (D) $C_{10} - B_{10}$

Q.17 The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n = **[JEE - Advance 2013]**

ANSWER KEY

LEVEL- 1

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	C	D	C	C	A	A	A	C	C	D	A	C	A	C	D	B	B	B
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	C	A	A	C	B	A	C	C	C	D	C	D	C	B	B	B	C	B	B	B
Ques.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	B	B	B	D	C	B	D	B	B	C	C	C	C	A	D	C	C	C	A	A

LEVEL- 2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	C	B	B	A	A	C	A	D	D	C	B	B	B	B	B	A	B	A
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
Ans.	C	A	C	B	B	B	C	D	A	A	A	B	C	C	D					

LEVEL- 3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D	D	C	C	D	B	C	C	B	A	B	B	B	C	B	A	B	B	C	A
21	22	23	24	25	26														
C	D	A	D	D	C														

LEVEL- 4

SECTION-A

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Ans.	C	D	A	C	C	B	A	C	D	C	D	B	A	B	C	C	A

SECTION-B

1.[B] Min n! (21–n)! = ?

$^{21}C_n$ is maximum when n = $\frac{21+1}{2} = 11$

$\therefore$ Max $(^{21}C_n) = {}^{21}C_{11}$

$\therefore [n!\,(21-n)!]_{min} = \frac{21!}{21!C_n/\max} = \frac{21!}{^{21}C_{11}}$

= 10 ! 11 !

2.[C] $(x+\sqrt{x^3-1})^5 + (x-\sqrt{x^3-1})^5 = 2(T_1+T_3+T_5)$

Here $T_1 = {}^5C_0.x^5$

$T_3 = {}^5C_2.x^3(\sqrt{x^3-1})^3 = {}^5C_2.x^3(x^3-1)$

$T_5 = {}^5C_4.x(\sqrt{x^3-1}) = {}^5C_4.x.(x^3-1)^2$

Clearly highest power of x is 7 & that occurs is 5th term

3.[B] Here $T_{r+1} = {}^{10}C_r.\left(\frac{x}{3}\right)^{10-r}.\left(\frac{-2}{x^2}\right)^r$

$\therefore T_{r+1} = {}^{10}C_r.\frac{(-2)^r}{(3)^{10-r}}.x^{10-3r}$

put 10–3r = 4

$\therefore$ r = 2

$\therefore x^4$ occurs in $T_{2+1} = T_3$

4.[C] $\sum_{m=0}^{100} {}^{100}C_m.(x-3)^{100-m}\, 2^m = (x-3+2)^{100}$

$= (x-1)^{100} = (1-x)^{100}$

$\therefore$ In $(1-x)^{100}$: coefficient of $x^{53} = {}^{100}C_{53}(-1)^{53}$

5.[C] $= \sum_{r=0}^{n} (2r+1)C_r$

$= 2\sum_{r=0}^{n} rC_r + \sum_{r=0}^{n} C_r$

$= 2.n.2^{n-1} + 2^n$

$= (n+1)2^n$

6.[C] In $(x+a)^n$: let $k = \dfrac{n+1}{\left|\dfrac{x}{a}\right|+1}$

Here $k = \dfrac{50+1}{\left|\dfrac{3}{2x}\right|+1}$

put $x = 1/5$

$k = \dfrac{51}{\dfrac{15}{2}+1} = \dfrac{102}{17} = 6$

$\because$ k is integer

$\therefore$ T_k & T_{k+1} i.e. T_6 & T_7 are greatest term

7.[A] $(\sqrt{2} + 3^{1/5})^{10}$

$\because T_{r+1} = {}^{10}C_r.\,(\sqrt{2})^{10-r}\,(3^{1/5})^r$

$= {}^{10}C_r.\,2^{5-\frac{r}{2}}.\,3^{r/5}$

so r must be divisible by both 2 & 5

$\therefore$ r must be divisible by 10

$\because$ r varies from 0 to 10

$\therefore$ r = 0, 10

$\therefore T_{0+1} = {}^{10}C_0.2^5 = 32$

& $T_{10+1} = {}^{10}C_{10}.3^2 = 9$

$\therefore$ sum of rational terms = 41

8.[C] Let $b = \sum_{r=0}^{n} \dfrac{r}{{}^nC_r}$

then $b = \sum_{r=0}^{n} \dfrac{n-(n-r)}{{}^nC_r}$

$b = n\sum_{r=0}^{n} \dfrac{1}{{}^nC_r} - \sum_{r=0}^{n} \dfrac{n-r}{{}^nC_{n-r}}$

$(\because {}^nC_r = {}^nC_{n-r})$

$b = n\,a_n - b$

$2b = na_n$

$\therefore b = \dfrac{1}{2}\,na_n$

9. [A] $S = \sum_{r=0}^{n} \dfrac{(-1)^r}{{}^nC_r}$

$$S = \frac{1}{{}^nC_0} - \frac{1}{{}^nC_1} + \frac{1}{{}^nC_2} \,.......\, - \frac{1}{{}^nC_{n-2}} + \frac{1}{{}^nC_{n-1}} - \frac{1}{{}^nC_n}$$

$(\because$ n is odd$)$

$\therefore S = 0$

10.[C] $(1+x)^m(1-x)^n$

$= (1 + {}^mC_1x + {}^mC_2x^2 + ...)(1 - {}^nC_1x + {}^nC_2x^2 .....)$

coefficient of $x = -{}^nC_1 + {}^mC_1 = m - n = 3$ .... (i)

& coefficient of $x^2 = {}^nC_2 - {}^mC_1.{}^nC_1 + {}^mC_2$

$= \dfrac{n(n-1)}{2} - mn + \dfrac{m(m-1)}{2} = 6$ ...(ii)

solving (i) & (ii), we get $\left[\begin{array}{l} m = 12 \\ n = 9 \end{array}\right.$

11.[D] ${}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1}$

$= {}^{n+2}C_r$

12.[B] $\because T_5 + T_6 = 0$

$\Rightarrow {}^nC_4.a^{n-4}b^4 - {}^nC_5a^{n-5}b^5 = 0$

$\therefore {}^nC_4a^{n-4}b^4 = {}^nC_5.a^{n-5}b^5$

$\Rightarrow \dfrac{{}^nC_5}{{}^nC_4} = \dfrac{a}{b}$

$\Rightarrow \dfrac{a}{b} = \dfrac{n-5+1}{5} = \dfrac{n-4}{5}$

13.[A] $(1 + t^2)^{12}(1 + t^{12})(1 + t^{24})$

$= (1 + t^{12} + t^{24} + t^{36})\,[1 + {}^{12}C_1t^2 \ldots + {}^{12}C_6t^{12} \ldots + {}^{12}C_{12}t^{24}]$

$\therefore$ coefficient of $t^{24} = 1.{}^{12}C_{12} + 1.{}^{12}C_6 + 1$

$= 1 + {}^{12}C_6 + 1$

$= {}^{12}C_6 + 2$

14.[D] ${}^{n-1}C_r = (k^2 - 3)\,{}^nC_{r+1}$

$\Rightarrow {}^{n-1}C_r = (k^2 - 3).{}^{n-1}C_r.\ \dfrac{n}{r+1}$

$\Rightarrow k^2 = 3 + \dfrac{r+1}{n}$

Now r varies from 0 to n

$\therefore$ k^2 varies from $\left(3+\dfrac{1}{n}\right)$ to $\left(4+\dfrac{1}{n}\right)$

$\therefore k \in (\sqrt{3}, 2]$

15.[A] $({}^{30}C_0)({}^{30}C_{10}) - ({}^{30}C_1)({}^{30}C_{11}) \ldots\ldots + ({}^{30}C_{20})({}^{30}C_{30})$

$= ({}^{30}C_0)({}^{30}C_{20}) - ({}^{30}C_1)({}^{30}C_{19}) \ldots\ldots + ({}^{30}C_{20})({}^{30}C_0)$

= Coefficient of x^{20} in $(1 + x)^{30}.(1 - x)^{30}$

= Coefficient of x^{20} in $(1 - x^2)^{30}$

$= {}^{30}C_{10}(-1)^{10} = {}^{30}C_{10}$

16.[D] $A_r = {}^{10}C_r,\ B_r = {}^{20}C_r,\ C_r = {}^{30}C_r$

$$\sum_{r=1}^{10} {}^{10}C_r\left({}^{20}C_{10}\ {}^{20}C_r - {}^{30}C_{10}\ {}^{10}C_r\right)$$

$$= {}^{20}C_{10} \sum_{r=1}^{10} {}^{10}C_r\,{}^{20}C_{20-r} - {}^{30}C_{10} \sum_{r=1}^{10} {}^{10}C_r\ {}^{10}C_r$$

$= {}^{20}C_{10}\,({}^{30}C_{20} - {}^{10}C_0\,{}^{20}C_{20}) - {}^{30}C_{10}\,[{}^{20}C_{10} - ({}^{10}C_0)^2]$

$= {}^{20}C_{10}\,{}^{30}C_{20} - {}^{20}C_{10} - {}^{30}C_{10}\,{}^{20}C_{10} + {}^{30}C_{10}$

$= {}^{30}C_{10} - {}^{20}C_{10}$

$= C_{10} - B_{10}$

17.[6] $\dfrac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \dfrac{5}{10} \Rightarrow n - 3r = -3 \ldots(1)$

$\dfrac{{}^{n+5}C_{r+1}}{{}^{n+5}C_{r+2}} = \dfrac{10}{14} \Rightarrow 5n - 12r = -6 \ldots(2)$

solve (1) and (2)

r = 3

n = 6