JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

ELLIPSE

(PRACTICE SHEET)

LEVEL- 1

Question based on **Equation and properties of the ellipse**

Q.1 The equation to the ellipse (referred to its axes as the axes of x and y respectively) whose foci are (± 2, 0) and eccentricity 1/2, is-

(A) $\frac{x^2}{12}+\frac{y^2}{16}=1$ (B) $\frac{x^2}{16}+\frac{y^2}{12}=1$

(C) $\frac{x^2}{16}+\frac{y^2}{8}=1$ (D) None of these

Q.2 The eccentricity of the ellipse $9x^2 + 5y^2 - 30y = 0$ is-

(A) 1/3 (B) 2/3

(C) 3/4 (D) None of these

Q.3 If the latus rectum of an ellipse be equal to half of its minor axis, then its eccentricity is-

(A) 3/2 (B) $\sqrt{3}/2$ (C) 2/3 (D) $\sqrt{2}/3$

Q.4 If distance between the directrices be thrice the distance between the foci, then eccentricity of ellipse is-

(A) 1/2 (B) 2/3 (C) $1/\sqrt{3}$ (D) 4/5

Q.5 The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if-

(A) $\Delta = 0, h^2 < ab$ (B) $\Delta \neq 0, h^2 < ab$

(C) $\Delta \neq 0, h^2 > ab$ (D) $\Delta \neq 0, h^2 = ab$

Q.6 Equation of the ellipse whose focus is (6, 7) directrix is $x + y + 2 = 0$ and $e = 1/\sqrt{3}$ is-

(A) $5x^2 + 2xy + 5y^2 - 76x - 88y + 506 = 0$

(B) $5x^2 - 2xy + 5y^2 - 76x - 88y + 506 = 0$

(C) $5x^2 - 2xy + 5y^2 + 76x + 88y - 506 = 0$

(D) None of these

Q.7 The eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose latus rectum is half of its major axis is-

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{\frac{2}{3}}$

(C) $\frac{\sqrt{3}}{2}$ (D) None of these

Q.8 The equation of the ellipse whose centre is at origin and which passes through the points (– 3,1) and (2,–2) is-

(A) $5x^2 + 3y^2 = 32$ (B) $3x^2 + 5y^2 = 32$

(C) $5x^2 - 3y^2 = 32$ (D) $3x^2 + 5y^2 + 32 = 0$

Q.9 The equation of the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point (– 3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$, is-

(A) $3x^2 + 6y^2 = 33$ (B) $5x^2 + 3y^2 = 48$

(C) $3x^2 + 5y^2 - 32 = 0$ (D) None of these

Q.10 Latus rectum of ellipse $4x^2 + 9y^2 - 8x - 36y + 4 = 0$ is-

(A) 8/3 (B) 4/3

(C) $\frac{\sqrt{5}}{3}$ (D) 16/3

Q.11 The latus rectum of an ellipse is 10 and the minor axis is equal to the distance between the foci. The equation of the ellipse is-

(A) $x^2 + 2y^2 = 100$ (B) $x^2 + \sqrt{2}y^2 = 10$

(C) $x^2 - 2y^2 = 100$ (D) None of these

Q.12 If the distance between the foci of an ellipse be equal to its minor axis, then its eccentricity is-

(A) 1/2 (B) $1/\sqrt{2}$

(C) 1/3 (D) $1/\sqrt{3}$

Q.13 The equation $2x^2 + 3y^2 = 30$ represents-

(A) A circle (B) An ellipse

(C) A hyperbola (D) A parabola

Q.14 The equation of the ellipse whose centre is (2, –3), one of the foci is (3, –3) and the corresponding vertex is (4, – 3) is-

(A) $\frac{(x-2)^2}{3}+\frac{(y+3)^2}{4}=1$

(B) $\frac{(x-2)^2}{4}+\frac{(y+3)^2}{3}=1$

(C) $\frac{x^2}{3}+\frac{y^2}{4}=1$

(D) None of these

Q.15 Eccentricity of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$ is-

(A) $1/\sqrt{3}$ (B) $\sqrt{3}/2$

(C) 1/2 (D) None of these

Q.16 The equation of ellipse whose distance between the foci is equal to 8 and distance between the directrix is 18, is-

(A) $5x^2 - 9y^2 = 180$ (B) $9x^2 + 5y^2 = 180$

(C) $x^2 + 9y^2 = 180$ (D) $5x^2 + 9y^2 = 180$

Q.17 In an ellipse the distance between its foci is 6 and its minor axis is 8. Then its eccentricity is-

(A) $\frac{4}{5}$ (B) $\frac{1}{\sqrt{52}}$

(C) $\frac{3}{5}$ (D) $\frac{1}{2}$

Q.18 The eccentricity of an ellipse is 2/3, latus rectum is 5 and centre is (0, 0). The equation of the ellipse is -

(A) $\frac{x^2}{81}+\frac{y^2}{45}=1$ (B) $\frac{4x^2}{81}+\frac{4y^2}{45}=1$

(C) $\frac{x^2}{9}+\frac{y^2}{5}=1$ (D) $\frac{x^2}{81}+\frac{y^2}{45}=5$

Q.19 The length of the latus rectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{49} = 1$ is -

(A) 98/6 (B) 72/7

(C) 72/14 (D) 98/12

Q.20 For the ellipse $\frac{x^2}{64} + \frac{y^2}{28} = 1$, the eccentricity is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$

(C) $\frac{2}{\sqrt{7}}$ (D) $\frac{1}{3}$

Q.21 The equation of the ellipse whose one of the vertices is (0, 7) and the corresponding directrix is y = 12, is-

(A) $95x^2 + 144y^2 = 4655$

(B) $144x^2 + 95y^2 = 4655$

(C) $95x^2 + 144y^2 = 13680$

(D) None of these

Q.22 The foci of the ellipse, $25(x + 1)^2 + 9(y + 2)^2 = 225$, are at-

(A) (–1, 2) and (–1, –6)

(B) (–2, 1) and (– 2, 6)

(C) (– 1, – 2) and (–2, – 1)

(D) (–1, –2) and (–1, – 6)

Q.23 The eccentricity of the ellipse represented by the equation $25x^2 + 16y^2 - 150x - 175 = 0$ is -

(A) 2/5 (B) 3/5

(C) 4/5 (D) None of these

Q.24 The equation of the ellipse whose foci are (± 5, 0) and one of its directrix is 5x = 36, is -

(A) $\frac{x^2}{36}+\frac{y^2}{11}=1$ (B) $\frac{x^2}{6}+\frac{y^2}{\sqrt{11}}=1$

(C) $\frac{x^2}{6}+\frac{y^2}{11}=1$ (D) None of these

Q.25 If S and S′ are two foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a < b) and P (x_1, y_1) a point on it, then SP + S′ P is equal to-

(A) 2a (B) 2b

(C) $a + ex_1$ (D) $b + ey_1$

Q.26 Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with foci S and S′. If A be the area of triangle PSS′, then maximum value of A is–

(A) 12 sq. units (B) 24 sq. units

(C) 36 sq. units (D) 48 sq. units

Question based on Paramatric equation

Q.27 The parametric representation of a point on the ellipse whose foci are (– 1, 0) and (7, 0) and eccentricity 1/2 is-

(A) $(3 + 8\cos\theta, 4\sqrt{3}\sin\theta)$

(B) $(8\cos\theta, 4\sqrt{3}\sin\theta)$

(C) $(3 + 4\sqrt{3}\cos\theta, 8\sin\theta)$

(D) None of these

Question based on **Ellipse and a point, Ellipse and a line**

Q.28 The position of the point (4,– 3) with respect to the ellipse $2x^2 + 5y^2 = 20$ is-
(A) outside the ellipse
(B) on the ellipse
(C) on the major axis
(D) None of these

Q.29 If $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then its eccentric angle θ is equal to-
(A) 0 (B) 90º
(C) 45º (D) 60º

Q.30 Find the equation of the tangent to the ellipse $x^2 + 2y^2 = 4$ at the points where ordinate is 1.
(A) $x + \sqrt{2}\, y - 2\sqrt{2} = 0$ & $x - \sqrt{2}\, y + 2\sqrt{2} = 0$
(B) $x - \sqrt{2}\, y - 2\sqrt{2} = 0$ & $x - \sqrt{2}\, y + 2\sqrt{2} = 0$
(C) $x + \sqrt{2}\, y + 2\sqrt{2} = 0$ & $x + \sqrt{2}\, y + 2\sqrt{2} = 0$
(D) None of these

Q.31 Find the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which make equal intercepts on the axes.
(A) $y = x \pm \sqrt{a^2 + b^2}$ & $y = -x \pm \sqrt{a^2 + b^2}$
(B) $y = x + \sqrt{a^2 + b^2}$ & $y = -x \pm \sqrt{a^2 + b^2}$
(C) $y = x + \sqrt{a^2 + b^2}$ & $y = x \pm \sqrt{a^2 + b^2}$
(D) None of these

Q.32 Find the equations of tangents to the ellipse $9x^2 + 16y^2 = 144$ which pass through the point (2,3).
(A) y = 3 and y = –x + 5
(B) y = 5 and y = –x + 3
(C) y = 3 and y = x – 5
(D) None of these

Q.33 If any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intercepts lengths h and k on the axes, then-
(A) $\frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$ (B) $\frac{h^2}{a^2} + \frac{k^2}{b^2} = 2$
(C) $\frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$ (D) $\frac{a^2}{h^2} + \frac{b^2}{k^2} = 2$

Q.34 The equation of the tangent at the point (1/4, 1/4) of the ellipse $\frac{x^2}{4} + \frac{y^2}{12} = 1$, is-
(A) 3x + y = 48 (B) 3x + y = 3
(C) 3x + y = 16 (D) None of these

Q.35 The line $x \cos\alpha + y \sin\alpha = p$ will be a tangent to the conic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if-
(A) $p^2 = a^2 \sin^2\alpha + b^2 \cos^2\alpha$
(B) $p^2 = a^2 + b^2$
(C) $p^2 = b^2 \sin^2\alpha + a^2 \cos^2\alpha$
(D) None of these

Q.36 If y = mx + c is tangent on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, then the value of c is-
(A) 0 (B) 3/m
(C) $\pm\sqrt{9m^2 + 4}$ (D) $\pm 3\sqrt{1 + m^2}$

Q.37 The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line y = mx + c intersect in real points only if-
(A) $a^2m^2 < c^2 - b^2$
(B) $a^2m^2 > c^2 - b^2$
(C) $a^2m^2 \geq c^2 - b^2$
(D) $c \geq b$

Q.38 If the straight line y = 4x + c is a tangent to the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$, then c will be equal to-
(A) ± 4 (B) ± 6
(C) ± 1 (D) None of these

Q.39 The equation of the tangents to the ellipse $4x^2 + 3y^2 = 5$ which are parallel to the line y = 3x + 7 are
(A) $y = 3x \pm \sqrt{\frac{155}{3}}$ (B) $y = 3x \pm \sqrt{\frac{155}{12}}$
(C) $y = 3x \pm \sqrt{\frac{95}{12}}$ (D) None of these

Q.40 The equation of tangent to the ellipse $x^2 + 3y^2 = 3$ which is $\perp^r$ to line $4y = x - 5$ is-

(A) $4x + y + 7 = 0$ (B) $4x + y - 7 = 0$

(C) $4x + y - 3 = 0$ (D) None of these

LEVEL- 2

Q.1 The area of quadrilateral formed by tangents at the ends of latus-rectum of the ellipse $x^2 + 2y^2 = 2$ is-

(A) $\frac{8}{\sqrt{2}}$ (B) $8\sqrt{2}$

(C) 8 (D) None of these

Q.2 The equation $\frac{x^2}{10-a} + \frac{y^2}{4-a} = 1$ represents an ellipse if -

(A) $a < 4$ (B) $a > 4$

(C) $4 < a < 10$ (D) $a > 10$

Q.3 If the focal distance of an end of the minor axis of an ellipse (referred to its axes as the axes of x and y respectively) is k and the distance between its foci is 2h, then its equation is-

(A) $\frac{x^2}{k^2} + \frac{y^2}{h^2} = 1$

(B) $\frac{x^2}{k^2} + \frac{y^2}{k^2 - h^2} = 1$

(C) $\frac{x^2}{k^2} + \frac{y^2}{h^2 - k^2} = 1$

(D) $\frac{x^2}{k^2} + \frac{y^2}{k^2 + h^2} = 1$

Q.4 The locus of the mid-points of the portion of the tangents to the ellipse intercepted between the axes is -

(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$ (B) $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$

(C) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 4$ (D) None of these

Q.5 If S and T are foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B is an end of the minor axis. If STB is an equilateral triangle the eccentricity of ellipse is-

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{\sqrt{3}}{2}$

Q.6 The sum of the squares of the perpendicular on any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from two points on the minor axis each distance $\sqrt{a^2 - b^2}$ from the centre is -

(A) a^2 (B) b^2

(C) $2a^2$ (D) $2b^2$

Q.7 If (5, 12) and (24, 7) are the focii of an ellipse passing through origin, then the eccentricity of ellipse is -

(A) $\frac{\sqrt{386}}{38}$ (B) $\frac{\sqrt{386}}{12}$

(C) $\frac{\sqrt{386}}{13}$ (D) $\frac{\sqrt{386}}{25}$

Q.8 The common tangent of $x^2 + y^2 = 4$ and $2x^2 + y^2 = 2$ is-

(A) $x + y + 4 = 0$ (B) $x - y + 7 = 0$

(C) $2x + 3y + 8 = 0$ (D) None

Q.9 The eccentric angles of the extremities of latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are given by-

(A) $\tan^{-1}\left(\pm\frac{ae}{b}\right)$ (B) $\tan^{-1}\left(\pm\frac{be}{a}\right)$

(C) $\tan^{-1}\left(\pm\frac{b}{ae}\right)$ (D) $\tan^{-1}\left(\pm\frac{a}{be}\right)$

Q.10 A point, ratio of whose distance from a fixed point and line x = 9/2 is always 2 : 3. Then locus of the point will be -

(A) Hyperbola (B) Ellipse
(C) Parabola (D) Circle

Q.11 If the minor axis of an ellipse subtends an angle 60° at each focus then the eccentricity of the ellipse is -

(A) $\sqrt{3}/2$ (B) $1/\sqrt{2}$
(C) $2/\sqrt{3}$ (D) None

Q.12 LL′ is the latus rectum of an ellipse and ΔSLL′ is an equilateral triangle. The eccentricity of the ellipse is -

(A) $1/\sqrt{5}$ (B) $1/\sqrt{3}$
(C) $1/\sqrt{2}$ (D) $\sqrt{2}/\sqrt{3}$

Q.13 If the latus rectum of the ellipse $x^2\tan^2\alpha + y^2\sec^2\alpha = 1$ is 1/2 then α =

(A) π/12 (B) π/6
(C) 5π/12 (D) None

Q.14 If P is a point on the ellipse of eccentricity e and A, A′ are the vertices and S, S′ are the focii then ΔSPS′ : Δ APA′ =

(A) e^3 (B) e^2
(C) e (D) 1/e

Q.15 The tangent at P on the ellipse meets the minor axis in Q, and PR is drawn perpendicular to the minor axis and C is the centre. Then CQ . CR =

(A) b^2 (B) $2b^2$
(C) a^2 (D) $2a^2$

Q.16 The circle on SS′ as diameter touches the ellipse then the eccentricity of the ellipse is
(where S and S′ are the focus of the ellipse)

(A) $2/\sqrt{3}$ (B) $\sqrt{3}/2$
(C) $1/\sqrt{2}$ (D) None of these

Q.17 The tangent at any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the tangents at the vertices A, A′ in L and L′. Then AL. A′L′ =

(A) a + b (B) $a^2 + b^2$
(C) a^2 (D) b^2

Q.18 The tangent at any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to meets the major and minor axes in P and Q respectively, then $\frac{a^2}{CP^2}+\frac{b^2}{CQ^2}=$

(A) 4 (B) 3
(C) 2 (D) 1

Q.19 The locus of extremities of the latus rectum of the family of ellipses $b^2x^2 + a^2y^2 = a^2b^2$ is

(A) $x^2 - ay = a^2b^2$ (B) $x^2 - ay = b^2$
(C) $x^2 + ay = a^2$ (D) $x^2 + ay = b^2$

Q.20 The length of the common chord of the ellipse $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$ and
the circle $(x-1)^2 + (y-2)^2 = 1$ is

(A) 0 (B) 1 (C) 3 (D) 8

Q.21 If any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intercepts equal lengths ℓ on the axes, then ℓ =

(A) $\sqrt{a^2+b^2}$ (B) $a^2 + b^2$
(C) $(a^2 + b^2)^2$ (D) None of these

Q.22 If C is the centre of the ellipse $9x^2 + 16y^2 = 144$ and S is one focus. The ratio of CS to major axis, is

(A) $\sqrt{7}$: 16 (B) $\sqrt{7}$: 4
(C) $\sqrt{5}$: $\sqrt{7}$ (D) None of these

Q.23 P is a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with AA′ as the major axis. Then, the maximum value of the area of the triangle APA′ is-

(A) ab (B) 2ab
(C) ab/2 (D) None of these

Q.24 If PSQ is a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$, then the harmonic mean of SP and SQ is

(A) $\frac{b^2}{a}$ (B) $\frac{a^2}{b}$ (C) $\frac{2b^2}{a}$ (D) $\frac{2a^2}{b}$

Q.25 If the eccentricity of the ellipse $\frac{x^2}{a^2+1}+\frac{y^2}{a^2+2}=1$ be $\frac{1}{\sqrt{6}}$, then latus rectum of ellipse is -

(A) $\frac{5}{\sqrt{6}}$ (B) $\frac{10}{\sqrt{6}}$

(C) $\frac{8}{\sqrt{6}}$ (D) None of these

Q.26 Locus of the point which divides double ordinate of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the ratio 1 : 2 internally, is

(A) $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$ (B) $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=\frac{1}{9}$

(C) $\frac{9x^2}{a^2}+\frac{9y^2}{b^2}=1$ (D) None of these

Q.27 A tangent having slope of –4/3 to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersect the major and minor axes at A and B respectively. If C is the centre of ellipse then area of triangle ABC is-

(A) 12 (B) 24

(C) 36 (D) 48

Q.28 If F_1 and F_2 are the feet of the perpendiculars from the foci S_1 & S_2 of an ellipse $\frac{x^2}{5}+\frac{y^2}{3}=1$ on the tangent at any point P on the ellipse, then $(S_1F_1).(S_2F_2)$ is equal to-

(A) 2 (B) 3

(C) 4 (D) 5

Q.29 Equation of one of the common tangent of $y^2 = 4x$ and $\frac{x^2}{4}+\frac{y^2}{3}=1$ is equal to-

(A) $x + 2y + 4 = 0$ (B) $x + 2y – 4 = 0$

(C) $x – 2y – 4 = 0$ (D) None of these

Q.30 The eccentricity of ellipse which meets straight line $2x – 3y = 6$ on the X axis and $4x + 5y = 20$ on the Y axis and whose principal axes lie along the co-ordinate axes is equal to-

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$

(C) $\frac{\sqrt{3}}{4}$ (D) $\frac{\sqrt{7}}{4}$

Q.31 If a circle of radius r is concentric with ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then common tangent is inclined to the major axis at an angle-

(A) $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$ (B) $\tan^{-1}\sqrt{\frac{r^2-b^2}{r^2-a^2}}$

(C) $\tan^{-1}\sqrt{\frac{a^2-r^2}{r^2-b^2}}$ (D) None of these

Q.32 If the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ meet the ellipse $\frac{x^2}{1}+\frac{y^2}{a^2}=1$ in four distinct points and $a = b^2 – 10b + 25$ then which of the following is true

(A) $b < 4$ (B) $4 < b < 6$

(C) $b > 6$ (D) $b \in R – [4, 6]$

Q.33 An ellipse and a hyperbola have the same centre "origin", the same foci. The minor-axis of the one is the same as the conjugate axis of the other. If e_1, e_2 be their eccentricities respectively, then $\frac{1}{e_1^2}+\frac{1}{e_2^2}$ is equal to

(A) 1 (B) 2

(C) 4 (D) 3

Q.34 A parabola is drawn whose focus is one of the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (where $a > b$) and whose directrix passes through the other focus and perpendicular to the major axes of the ellipse. Then the eccentricity of the ellipse for which the latus-rectum of the ellipse and the parabola are same, is

(A) $\sqrt{2}-1$ (B) $2\sqrt{2}+1$

(C) $\sqrt{2}+1$ (D) $2\sqrt{2}-1$

Questions based on Statements

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement- I and Statement- II are true, and Statement - II is the correct explanation of Statement– I.

(B) If both Statement - I and Statement - II are true but Statement - II is not the correct explanation of Statement – I.

(C) If Statement - I is true but Statement - II is false.

(D) If Statement - I is false but Statement - II is true.

Q.35 **Statement- (1) :** From a point $(5, \lambda)$ perpendicular tangents are drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ then $\lambda = \pm 4$.

Statement- (2) : The locus of the point of intersection of perpendicular tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is $x^2+y^2=41$.

Passage : 1 (Q.36 to 38)

Variable tangent drawn to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ intersects major and minor axis at points A & B in first quadrant then (where, O is the centre of the ellipse)

Q.36 Area of ΔOAB is minimum when $\theta =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Q.37 Minimum value of OA. OB is

(A) 2b (B) 2ab

(C) ab (D) b

Q.38 Locus of centroid of ΔOAB is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=k^2$ then $k =$

(A) 1 (B) 2

(C) 3 (D) 4

Passage : 2 (Q.39 & 40)

A parabola P : $y^2 = 8x$, ellipse E : $\frac{x^2}{4}+\frac{y^2}{15}=1$.

Q.39 Equation of a tangent common to both the parabola P and the ellipse E is

(A) $x - 2y + 8 = 0$ (B) $2x - y + 8 = 0$

(C) $x + 2y - 8 = 0$ (D) $2x - y - 8 = 0$

Q.40 Point of contact of a common tangent to P and E on the ellipse is

(A) $\left(\frac{1}{2}, \frac{15}{4}\right)$ (B) $\left(-\frac{1}{2}, \frac{15}{4}\right)$

(C) $\left(\frac{1}{2}, -\frac{15}{2}\right)$ (D) $\left(-\frac{1}{2}, -\frac{15}{2}\right)$

COLUMN MATCHING QUESTIONS

Q.41

Column I	**Column II**
(A) eccentricity of $\frac{x^2}{64}+\frac{y^2}{39}=1$	(P) 10
(B) Length of latus-rectum of $\frac{x^2}{9}+\frac{y^2}{4}=1$	(Q) 8
(C) Length of major axis of $25x^2 + 16y^2 = 400$	(R) 5/8
(D) The length of minor axis of $16x^2 + 9y^2 = 144$	(S) 8/3
	(T) 6

LEVEL- 3

(Question asked in previous AIEEE and IIT-JEE)

SECTION -A

Q.1 If distance between the foci of an ellipse is equal to its minor axis, then eccentricity of the ellipse is- **[AIEEE-2002]**

(A) $e = \frac{1}{\sqrt{2}}$ (B) $e = \frac{1}{\sqrt{3}}$

(C) $e = \frac{1}{\sqrt{4}}$ (D) $e = \frac{1}{\sqrt{6}}$

Q.2 The equation of an ellipse, whose major axis = 8 and eccentricity = 1/2, is **[AIEEE-2002]**

(A) $3x^2 + 4y^2 = 12$ (B) $3x^2 + 4y^2 = 48$

(C) $4x^2 + 3y^2 = 48$ (D) $3x^2 + 9y^2 = 12$

Q.3 The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is x = 4, then the equation of the ellipse is- **[AIEEE- 2004]**

(A) $3x^2 + 4y^2 = 1$

(B) $3x^2 + 4y^2 = 12$

(C) $4x^2 + 3y^2 = 12$

(D) $4x^2 + 3y^2 = 1$

Q.4 In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is- **[AIEEE- 2006]**

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$

(C) $\frac{1}{\sqrt{5}}$ (D) $\frac{3}{5}$

Q.5 A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is- **[AIEEE- 2008]**

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{8}{3}$

Q.6 The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0), then the equation of the ellipse is- **[AIEEE- 2009]**

(A) $x^2 + 16y^2 = 16$ (B) $x^2 + 12y^2 = 16$

(C) $4x^2 + 48y^2 = 48$ (D) $4x^2 + 64y^2 = 48$

Q.7 Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is – **[AIEEE- 2011]**

(A) $3x^2 + 5y^2 - 32 = 0$

(B) $5x^2 + 3y^2 - 48 = 0$

(C) $3x^2 + 5y^2 - 15 = 0$

(D) $5x^2 + 3y^2 - 32 = 0$

Q.8 **Statement 1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$. **[AIEEE- 2012]**

(A) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(B) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(C) Statement 1 is true, Statement 2 is false.

(D) Statement 1 is false, Statement 2 is true.

Q.9 An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[AIEEE- 2012]**

(A) $x^2 + 4y^2 = 8$ (B) $4x^2 + y^2 = 8$

(C) $x^2 + 4y^2 = 16$ (D) $4x^2 + y^2 = 4$

Q.10 The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is – **[JEE Main - 2013]**

(A) $x^2 + y^2 - 6y - 5 = 0$ (B) $x^2 + y^2 - 6y + 5 = 0$

(C) $x^2 + y^2 - 6y - 7 = 0$ (D) $x^2 + y^2 - 6y + 7 = 0$

SECTION -B

Q.1 Let P be a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci F_1 and F_2. If A is the area of the triangle PF_1F_2, then the maximum value of A is- **[IIT-1994]**

(A) 2abe (B) abe

(C) $\frac{1}{2}$abe (D) None

Q.2 If P(x, y), $F_1 = (3,0)$, $F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2 =$ **[IIT-1996]**

(A) 8 (B) 6

(C) 10 (D) 12

Q.3 An ellipse has OB as semi - minor axis. F and F' are its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is- **[IIT- 97/AIEEE-2005]**

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Q.4 The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is **[IIT-1998]**

(A) 0 (B) 1

(C) 2 (D) infinite

Q.5 Locus of middle point of segment of tangent to ellipse $x^2 + 2y^2 = 2$ which is intercepted between the coordinate axes, is- **[IIT Scr. 2004]**

(A) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (B) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(C) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (D) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Q.6 A tangent is drawn at some point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is intersecting to the coordinate axes at points A & B then minimum area of the Δ OAB is- **[IIT Scr. 2005]** (where O is the centre of ellipse.)

(A) ab (B) $\frac{a^2 + b^2}{2}$

(C) $\frac{a^2 + b^2}{4}$ (D) $\frac{a^2 + b^2 - ab}{3}$

Q.7 The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **[IIT -2009]**

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$

(C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Passage : (Q.8 to Q.10)

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, touching the ellipse at points A and B. **[IIT 2010]**

Q.8 The coordinates of A and B are

(A) (3, 0) and (0, 2)

(B) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

(C) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5},\frac{8}{5}\right)$

Q.9 The orthocentre of the triangle PAB is

(A) $\left(5,\frac{8}{7}\right)$ (B) $\left(\frac{7}{5},\frac{25}{8}\right)$

(C) $\left(\frac{11}{5},\frac{8}{5}\right)$ (D) $\left(\frac{8}{25},\frac{7}{5}\right)$

Q.10 The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Q.11 The ellipse $E_1 : \frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is **[IIT Scr. 2012]**

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	B	C	B	B	A	B	C	A	A	B	B	B	B	D	C	B	B	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	B	A	B	A	B	A	A	A	C	A	A	A	C	D	C	C	C	D	B	A,B

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	B	B	B	C	A	D	C	B	A	B	A	C	A	C	D	D	C	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	A	D	A	A	B	A	B	B	A	D	A	A,C,D	B	A	A	C	B	C	A	B

41. **(A) → R ; (B) → S; (C) → P; (D) → T**

LEVEL- 3

SECTION-A

Qus.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	B	D	D	B	A,B	A	C	C

SECTION-B

1.[B] Maximum area = $\frac{1}{2}$. (2ae) b = abe

2.[C] $PF_1 + PF_2$ = Length of major axis

3.[B] $ae = b \Rightarrow e = \frac{1}{\sqrt{2}}$

4.[C] $c^2 = 4.4^2 + 1 = 65$

5.[A] $\frac{x^2}{2} + \frac{y^2}{1} = 1$

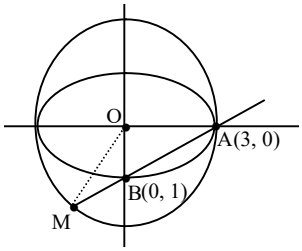
$h = \frac{\sqrt{2}}{2\cos\theta}$; $\Rightarrow \cos\theta = \frac{1}{\sqrt{2}h}$

$k = \frac{1}{2\sin\theta} \Rightarrow \sin\theta = \frac{1}{2k}$

Now required locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

6.[A] Minimum area = $\frac{1}{2}$. $\sqrt{2}$ a. $\sqrt{2}$ b = ab

7.[D]



Equation of AB is $y - 1 = \left(\frac{1-0}{0-3}\right)(x - 0)$

$\Rightarrow y - 1 = -\frac{1}{3}x$

Equation of auxiliary circle is $x^2 + y^2 = 9$

Now $x^2 + \left(1 - \frac{1}{3}x\right)^2 = 9$

$\Rightarrow x^2 + \frac{x^2 + 9 - 6x}{9} = 9$

$\Rightarrow 10x^2 - 6x - 72 = 0$

$\Rightarrow 5x^2 - 3x - 36 = 0$

$\Rightarrow (x - 3)(5x + 12) = 0$

Now, area of OAM = $\frac{1}{2}\begin{vmatrix} 3 & -\frac{12}{5} \\ 0 & \frac{9}{5} \end{vmatrix} = \frac{27}{10}$

8.[D] Equation of tangent

$y = mx \pm \sqrt{9m^2 + 4}$

It passes through (3, 4)

$4 = 3m \pm \sqrt{9m^2 + 4}$

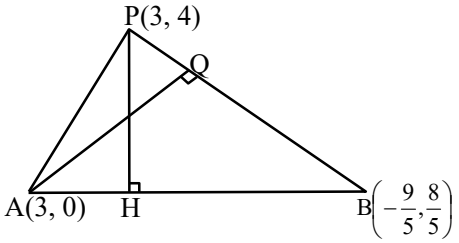
$m = \frac{1}{2}$ and undefined

So equation of the tangent will be

$x - 2y + 5 = 0$ and $x = 3$

so point of contacts are (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

9.[C]



Equation of two altitudes PH and AQ are

$3x - y - 5 = 0$ and $2x + y - 6 = 0$ respectively

So orthocenter will be $\left(\frac{11}{5}, \frac{8}{5}\right)$

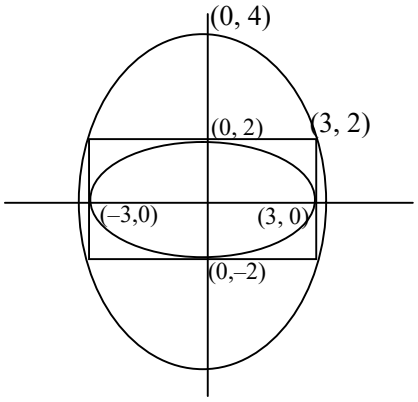
10.[A] Equation of AB is $x + 3y - 3 = 0$

So required locus will be

$(x - 3)^2 + (y - 4)^2 = \frac{(x + 3y - 3)^2}{10}$

$9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

11.[C]



Let equation of ellipse E_2 is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

it passes through (0, 4)

so $b^2 = 16$

and also passes through (3, 2)

So $\frac{9}{a^2} + \frac{4}{b^2} = 1$

$\Rightarrow \frac{9}{a^2} + \frac{1}{4} = 1$

$\Rightarrow a^2 = 12$

$\Rightarrow$ as $a < b$

so $12 = 16(1 - e^2)$

$\Rightarrow e^2 = \frac{1}{4}$

$\Rightarrow e = \frac{1}{2}$