JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

HYPERBOLA

(PRACTICE SHEET)

LEVEL- 1

Question based on **Equation & Properties of Hyperabola**

Q.1 The vertices of a hyperbola are at (0, 0) and (10, 0) and one of its foci is at (18, 0). The equation of the hyperbola is -

(A) $\frac{x^2}{25}-\frac{y^2}{144}=1$

(B) $\frac{(x-5)^2}{25}-\frac{y^2}{144}=1$

(C) $\frac{x^2}{25}-\frac{(y-5)^2}{144}=1$

(D) $\frac{(x-5)^2}{25}-\frac{(y-5)^2}{144}=1$

Q.2 If the latus rectum of an hyperbola be 8 and eccentricity be $\frac{3}{\sqrt{5}}$, then the equation of the hyperbola is-

(A) $4x^2 - 5y^2 = 100$ (B) $5x^2 - 4y^2 = 100$

(C) $4x^2 + 5y^2 = 100$ (D) $5x^2 + 4y^2 = 100$

Q.3 The foci of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ are-

(A) (2, 3), (5, 7) (B) (4, 1), (–6, 1)

(C) (0, 0), (5, 3) (D) None of these

Q.4 The foci of the hyperbola $4x^2 - 9y^2 - 36 = 0$ are-

(A) $[\pm\sqrt{11}, 0]$ (B) $[\pm\sqrt{12}, 0]$

(C) $[\pm\sqrt{13}, 0]$ (D) $[0, \pm\sqrt{12}]$

Q.5 Foci of the hyperbola $\frac{x^2}{16}-\frac{(y-2)^2}{9}=1$ are

(A) (5, 2); (–5, 2) (B) (5, 2); (5, –2)

(C) (5, 2); (–5, –2) (D) None of these

Q.6 The eccentricity of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passing through the points (3, 0), $(3\sqrt{2}, 2)$ will be-

(A) $\sqrt{13}$ (B) $\frac{\sqrt{13}}{3}$

(C) $\frac{\sqrt{13}}{4}$ (D) $\frac{\sqrt{13}}{2}$

Q.7 Equation of the hyperbola with eccentricity 3/2 and foci at (± 2, 0) is-

(A) $\frac{x^2}{4}-\frac{y^2}{5}=\frac{4}{9}$ (B) $\frac{x^2}{9}-\frac{y^2}{9}=\frac{4}{9}$

(C) $\frac{x^2}{4}-\frac{y^2}{9}=1$ (D) None of these

Q.8 If the centre, vertex and focus of a hyperbola be (0, 0), (4, 0) and (6, 0) respectively, then the equation of the hyperbola is-

(A) $4x^2 - 5y^2 = 8$ (B) $4x^2 - 5y^2 = 80$

(C) $5x^2 - 4y^2 = 80$ (D) $5x^2 - 4y^2 = 8$

Q.9 The eccentricity of the hyperbola can never be equal to-

(A) $\sqrt{\frac{9}{5}}$ (B) $2\sqrt{\frac{1}{9}}$

(C) $3\sqrt{\frac{1}{8}}$ (D) $\sqrt{2}$

Q.10 The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is-

(A) $\frac{4}{3}$ (B) $\frac{4}{\sqrt{3}}$

(C) $\frac{2}{\sqrt{3}}$ (D) None of these

Q.11 If the length of the transverse and conjugate axes of a hyperbola be 8 and 6 respectively, then the difference of focal distances of any point of the hyperbola will be-

(A) 8 (B) 6

(C) 14 (D) 2

Q.12 If m is a variable, the locus of the point of intersection of the lines $\frac{x}{3}-\frac{y}{2}=m$ and $\frac{x}{3}+\frac{y}{2}=\frac{1}{m}$ is a/ an-

(A) parabola (B) ellipse

(C) hyperbola (D) None of these

Q.13 The equation of the hyperbola whose foci are (6, 5) , (– 4, 5) and eccentricity 5/4 is-

(A) $\frac{(x-1)^2}{16}-\frac{(y-5)^2}{9}=1$

(B) $\frac{x^2}{16}-\frac{y^2}{9}=1$

(C) $\frac{(x-1)^2}{9}-\frac{(y-5)^2}{16}=1$

(D) None of these

Q.14 The equation $\frac{x^2}{12-\lambda}+\frac{y^2}{8-\lambda}=1$ represents

(A) a hyperbola if $\lambda < 8$

(B) an ellipse if $\lambda > 8$

(C) a hyperbola if $8 < \lambda < 12$

(D) None of these

Q.15 The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a rectangular hyperbola if-

(A) $\Delta \neq 0, h^2 > ab, a + b = 0$

(B) $\Delta \neq 0, h^2 < ab, a + b = 0$

(C) $\Delta \neq 0, h^2 = ab, a + b = 0$

(D) None of these

Q.16 The equation $\frac{x^2}{1-k}-\frac{y^2}{1+k}=1$, $k > 1$ represents-

(A) circle (B) ellipse

(C) hyperbola (D) None of these

Q.17 If e and e′ be the eccentricities of two conics S and S′ such that $e^2 + e'^2 = 3$, then both S and S′ are-

(A) ellipse (B) parabolas

(C) hyperbolas (D) None of these

Q.18 A point moves in a plane so that its distances PA and PB from two fixed points A and B in the plane satisfy the relation $|PA - PB| = k$ ($k \neq 0$), then the locus of P is-

(A) a parabola

(B) an ellipse

(C) a hyperbola

(D) a branch of a hyperbola

Q.19 The equation of the conic with focus at (1, –1), directrix along $x - y + 1 = 0$ and with eccentricity $\sqrt{2}$ is-

(A) $x^2 - y^2 = 1$

(B) $xy = 1$

(C) $2xy - 4x + 4y + 1 = 0$

(D) $2xy + 4x - 4y - 1 = 0$

Q.20 The length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is-

(A) $\frac{2a^2}{b}$ (B) $\frac{2b^2}{a}$

(C) $\frac{b^2}{a}$ (D) $\frac{a^2}{b}$

Q.21 The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola-

(A) the length of whose transverse axis is $4\sqrt{3}$

(B) the length of whose conjugate axis is 4

(C) whose centre is (–1, 2)

(D) whose eccentricity is $\sqrt{\frac{19}{3}}$

Q.22 The length of the transverse axis of a hyperbola is 7 and it passes through the point (5, –2). The equation of the hyperbola is-

(A) $\frac{4}{49}x^2-\frac{196}{51}y^2=1$

(B) $\frac{49}{4}x^2-\frac{51}{196}y^2=1$

(C) $\frac{4}{49}x^2-\frac{51}{196}y^2=1$

(D) none of these

Q.23 The latus rectum of a hyperbola $\frac{x^2}{16}-\frac{y^2}{p}=1$ is $4\frac{1}{2}$. Its eccentricity e =

(A) 4/5 (B) 5/4 (C) 3/4 (D) 4/3

Q.24 Consider the set of hyperbola $xy = k$, $k \in R$. Let e_1 be the eccentricity when $k = 4$ and e_2 be the eccentricity when $k = 9$. Then $e_1^2 + e_2^2 =$

(A) 2 (B) 3 (C) 4 (D) 1

Q.25 The eccentricity of the hyperbola $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by -

(A) $e=+\sqrt{\frac{a^2+b^2}{a^2}}$ (B) $e=+\sqrt{\frac{a^2-b^2}{a^2}}$

(C) $e=+\sqrt{\frac{b^2-a^2}{a^2}}$ (D) $e=+\sqrt{\frac{a^2+b^2}{b^2}}$

Q.26 If e and e' be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2}+\frac{1}{e'^2}=$

(A) 0 (B) 1

(C) 2 (D) None of these

Question based on Line and Hyperbola

Q.27 The equation of a tangent parallel to y = x drawn to $\frac{x^2}{3}-\frac{y^2}{2}=1$ is-

(A) x – y + 1 = 0 (B) x – y + 2 = 0

(C) x + y – 1 = 0 (D) x – y + 2 = 0

Q.28 The line lx + my + n = 0 will be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, if -

(A) $a^2l^2 + b^2m^2 = n^2$ (B) $a^2l^2 – b^2m^2 = n^2$

(C) $am^2 – b^2n^2 = a^2l^2$ (D) None of these

Q.29 The equation of tangents to the hyperbola $x^2 – 4y^2 = 36$ which are perpendicular to the line x–y+4=0

(A) $y = – x + 3\sqrt{3}$ (B) $y = x – 3\sqrt{3}$

(C) $y = – x \pm 2$ (D) None of these

Q.30 The line y = x + 2 touches the hyperbola $5x^2 – 9y^2 = 45$ at the point-

(A) (0, 2) (B) (3, 1)

(C) (–9/2, –5/2) (D) None of these

Q.31 Equation of tangent to the hyperbola $2x^2 – 3y^2 = 6$ which is parallel to the line y = 3x + 4 is-

(A) y = 3x + 5

(B) y = 3x – 5

(C) y = 3x + 5 and y = 3x – 5

(D) none of these

Q.32 If the straight line $x \cos\alpha + y \sin\alpha = p$ be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then-

(A) $a^2 \cos^2\alpha + b^2 \sin^2\alpha = p^2$

(B) $a^2 \cos^2\alpha – b^2 \sin^2\alpha = p^2$

(C) $a^2 \sin^2\alpha + b^2 \cos^2\alpha = p^2$

(D) $a^2 \sin^2\alpha – b^2 \cos^2\alpha = p^2$

Q.33 The value of m for which y = mx + 6 is a tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{49}=1$ is-

(A) $\sqrt{\frac{17}{20}}$ (B) $\sqrt{\frac{20}{17}}$

(C) $\sqrt{\frac{3}{20}}$ (D) $\sqrt{\frac{20}{3}}$

Q.34 Equation of one of common tangent to parabola $y^2 = 8x$ and hyperbola $3x^2 – y^2 = 3$ is-

(A) 2x – y – 1 = 0 (B) 2x – y + 1 = 0

(C) y + 2x + 1 = 0 (D) y – 2x + 1 = 0

LEVEL- 2

Q.1 The latus rectum subtends a right angle at other focus of a hyperbola then its eccentricity is-

(A) $\sqrt{3}+1$ (B) $\sqrt{2}+1$

(C) $-\sqrt{3}+\sqrt{2}$ (D) 2

Q.2 The equation of the hyperbola whose foci are the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ and the eccentricity is 2, is -

(A) $\frac{x^2}{4}+\frac{y^2}{12}=1$ (B) $\frac{x^2}{4}-\frac{y^2}{12}=1$

(C) $\frac{x^2}{12}+\frac{y^2}{4}=1$ (D) $\frac{x^2}{12}-\frac{y^2}{4}=1$

Q.3 A tangent to a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ intercepts a length of unity from each of the coordinate axes, then the point (a, b) lies on the-

(A) $x^2-y^2=2$ (B) $x^2-y^2=1$

(C) $x^2-y^2=-1$ (D) none of these

Q.4 A common tangent to $9x^2-16y^2=144$ and $x^2+y^2=9$ is -

(A) $y=\frac{3}{\sqrt{7}}x+\frac{15}{\sqrt{7}}$ (B) $y=3\sqrt{\frac{2}{7}}x+\frac{15}{\sqrt{7}}$

(C) $y=2\sqrt{\frac{3}{7}}x+15\sqrt{7}$ (D) none of these

Q.5 The product of the lengths of the perpendiculars drawn from foci on any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is -

(A) a^2 (B) b^2

(C) a^2b^2 (D) a^2/b^2

Q.6 The area of quadrilateral formed by focii of hyperbola $\frac{x^2}{4}-\frac{y^2}{3}=1$ and its conjugate hyperbola is-

(A) 14 (B) 24

(C) 12 (D) None of these

Q.7 The equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ are-

(A) $y=\pm x\pm\sqrt{b^2-a^2}$

(B) $y=\pm x\pm\sqrt{a^2-b^2}$

(C) $y=\pm x\pm(a^2-b^2)$

(D) $y=\pm x\pm\sqrt{a^2+b^2}$

Q.8 A hyperbola has axes along coordinate axes. Its transverse axis is 2a and it passes through (h,k) then its eccentricity is-

(A) $\sqrt{\frac{h^2+k^2+a^2}{h^2-a^2}}$ (B) $\sqrt{\frac{h^2-a^2}{h^2+k^2+a^2}}$

(C) $\sqrt{\frac{h^2+k^2-a^2}{h^2-a^2}}$ (D) $\sqrt{\frac{h^2-a^2}{h^2+k^2-a^2}}$

Q.9 If the focii of the ellipse $\frac{x^2}{k^2a^2}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ coincides then value of k =

(A) $\pm\sqrt{3}$ (B) $\pm\sqrt{2}$

(C) $\sqrt{3}$ (D) $\sqrt{2}$

Q.10 The locus of the point of intersection of the lines $bxt-ayt=ab$ and $bx+ay=abt$ is -

(A) a parabola (B) an ellipse

(C) a hyperbola (D) None of these

Q.11 Let LL′ be the latus rectum through the focus S of a hyperbola and A′ be the farther vertex of the conic. If $\Delta A'LL'$ is equilateral then its eccentricity e =

(A) $\sqrt{3}$ (B) $\sqrt{3}+1$

(C) $(\sqrt{3}+1)/\sqrt{2}$ (D) $(\sqrt{3}+1)/\sqrt{3}$

Q.12 If the latus rectum subtends a right angle at the centre of the hyperbola then its eccentricity is

(A) $e = (\sqrt{13})/2$ (B) $e = (\sqrt{5} - 1)/2$

(C) $e = (\sqrt{5} + 1)/2$ (D) $e = (\sqrt{3} + 1)/2$

Q.13 The equation $x = \frac{e^t + e^{-t}}{2}$; $y = \frac{e^t - e^{-t}}{2}$; $t \in R$ represents

(A) an ellipse (B) a parabola

(C) a hyperbola (D) a circle

Q.14 If the tangent at the point (2 sec θ, 3 tan θ) of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is parallel to $3x - y + 4 = 0$, then the value of θ is-

(A) 45° (B) 60° (C) 30° (D) 75°

Q.15 The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ have in common-

(A) centre only

(B) centre, foci and directrices

(C) centre, foci and vertices

(D) centre and vertices only

Q.16 The tangents to the hyperbola $x^2 - y^2 = 3$ are parallel to the straight line $2x + y + 8 = 0$ at the following points-

(A) (2, 1) (B) (2, –1)

(C) (–2, 1) (D) (–2, –1)

Q.17 P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then OT.ON is equal to -

(A) e^2 (B) a^2 (C) b^2 (D) $\frac{b^2}{a^2}$

Q.18 If e_1, e_2 are the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively, then the relation between e_1 and e_2 is -

(A) $3e_1^2 + e_2^2 = 2$ (B) $e_1^2 + 2e_2^2 = 3$

(C) $2e_1^2 + e_2^2 = 3$ (D) $e_1^2 + 3e_2^2 = 2$

Q.19 The value of m for which line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of the equation $x^2 - (a + b)x - 4 = 0$, then $(a + b)$ is equal to-

(A) 2 (B) 4

(C) 0 (D) none of these

Q.20 The area of triangle formed by lines $x^2 - y^2 = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is-

(A) $2a^2$ (B) $4a^2$

(C) a^2 (D) None of these

Q.21 If the distances between the foci and the distance between the directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3: 2 then a : b is-

(A) $\sqrt{2} : 1$ (B) $\sqrt{3} : \sqrt{2}$

(C) 1 : 2 (D) 2 : 1

Q.22 If the eccentricity of the hyperbola $x^2 - y^2 \sec^2 \alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2 \alpha + y^2 = 25$, then a value of α is-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Q.23 If the tangent at the point P(a sec α, b tan α) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets its transverse axis at T, then ST (S focus) must be equal to

(A) $a(e - \cos\alpha)$ (B) $b(e + \cos\alpha)$

(C) $a(e + \cos\alpha)$ (D) $\sqrt{a^2e^2 + b^2\cot^2\alpha}$

Questions based on Statements (Q. 24-27)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement- I and Statement- II are true, and Statement - II is the correct explanation of Statement– I.

(B) If both Statement - I and Statement - II are true but Statement - II is not the correct explanation of Statement – I.

(C) If Statement - I is true but Statement - II is false.

(D) If Statement - I is false but Statement - II is true.

Q.24 **Statement-(1):** If $P(x_1, y_1)$ is a point on $b^2x^2 + a^2y^2 = a^2b^2$ then area $\Delta SPS' = ae\sqrt{a^2 - x_1^2}$

Statement-(2) : A tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the transverse and conjugate axis in P and Q then $\frac{a^2}{CP^2} - \frac{b^2}{CQ^2} = 1$, where C is the centre of the conic. Which of the statements is correct?

(A) both 1 and 2 (B) only 1

(C) only 2 (D) neither 1 nor 2

Q.25 **Statement-(1) :** The conic $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represent a hyperbola.

Statement- (2) : The square of the coefficient of xy is greater than the product of the coefficient of x^2 & y^2 and $\Delta \neq 0$.

Q.26 **Statement-(1):** The latus-rectum of the hyperbola $x^2 - y^2 = a^2$ is equal to the length of its transverse axis.

Statement-(2): The semi latusrectum of the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is equal to $\frac{b^2}{a}$.

Q.27 **Statement- (1) :** The equation $x^2 + 2y^2 + \lambda xy + 2x + 3y + 1 = 0$ can never represent a hyperbola.

Statement- (2): The general equation of second degree represents a hyperbola if $h^2 > ab$.

Passage : (Q.No.28 & 29)

If parametric equation of hyperbola is $x = \frac{e^t + e^{-t}}{2}$ & $y = \frac{e^t - e^{-t}}{3}$ then.

Q.28 Eccentricity of hyperbola is

(A) $\frac{\sqrt{13}}{2}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\frac{3}{2}$ (D) $\sqrt{13}$

Q.29 Eccentric angle of point $\left(2, \frac{2}{\sqrt{3}}\right)$ on hyperbola

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) None of these

LEVEL- 3

(Question asked in previous AIEEE and IIT-JEE)

SECTION -A

Q.1 The latus rectum of the hyperbola $16x^2 - 9y^2 = 144$ is- **[AIEEE-2002]**

(A) 16/3 (B) 32/3

(C) 8/3 (D) 4/3

Q.2 The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is- **[AIEEE- 2003]**

(A) 9 (B) 1

(C) 5 (D) 7

Q.3 The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is- **[AIEEE-2005]**

(A) an ellipse (B) a circle

(C) a parabola (D) a hyperbola

Q.4 The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by - **[AIEEE-2011]**

(A) $x^2 - 3y^2 = 3$ (B) $3x^2 - y^2 = 3$

(C) $-x^2 + 3y^2 = 3$ (D) $-3x^2 + y^2 = 3$

SECTION -B

Q.1 If $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ represents family of hyperbolas, where α varies then- **[IIT Scr.2003/AIEEE-07]**

(A) e remains constant

(B) abscissas of foci remain constant

(C) equation of directrices remain constant

(D) abscissa of vertices remain constant

Q.2 The point at which the line $2x + \sqrt{6}y = 2$ touches the curve $x^2 - 2y^2 = 4$, is- **[IIT Scr. 2004]**

(A) $(4, -\sqrt{6})$ (B) $(\sqrt{6}, 1)$

(C) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (D) $\left(\frac{\pi}{6}, \pi\right)$

Q.3 If a hyperbola passes through the focus of the $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with the major and minor axis of ellipse, and product of eccentricities is 1, then **[IIT-2006]**

(A) Focus of hyperbola is (5, 0)

(B) Focus of hyperbola is $(5\sqrt{3}, 0)$

(C) The equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(D) The equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Q.4 A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is- **[IIT-2007]**

(A) $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

(B) $x^2\sec^2\theta - y^2\text{cosec}^2\theta = 1$

(C) $x^2\sin^2\theta - y^2\cos^2\theta = 1$

(D) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

Q.5 An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then **[IIT-2009]**

(A) equation of ellipse is $x^2 + 2y^2 = 2$

(B) the foci of ellipse are $(\pm 1, 0)$

(C) equation of ellipse is $x^2 + 2y^2 = 4$

(D) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Q.6 The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is - **[IIT-2010]**

(A) 2 (B) 3 (C) 4 (D) $\sqrt{3}$

Passage : (Q.7 to Q.8)

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B **[IIT-2010]**

Q.7 Equation of a common tangent with positive slope to the circle as well as to the hyperbola is -

(A) $2x - \sqrt{5}\ y - 20 = 0$ (B) $2x - \sqrt{5}\ y + 4 = 0$

(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Q.8 Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$

(B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$

(D) $x^2 + y^2 - 24x - 12 = 0$

Q.9 Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then - **[IIT-2011]**

(A) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

Q.10 Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are **[IIT-2012]**

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(C) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

ANSWER KEY

LEVEL-1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	B	A	B	C	A	B	A	C	B	C	A	C	A
Q.No.	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Ans.	C	A	D	C	C	C	A	D	C	B	C	D	B
Q.No.	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**					
Ans.	A	B	A	C	C	B	A	B,C					

LEVEL-2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	B	B	B	B	B	A	B	C	A	C	D	C	C
Q.No.	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Ans.	C	D	B,C	B	C	C	C	A	B	A,C	C	A	A
Q.No.	**27**	**28**	**29**										
Ans.	D	B	C										

LEVEL-3

SECTION-A

Qus.	1	2	3	4
Ans.	B	D	D	B

SECTION-B

1.[B] $\sin^2\alpha = \cos^2\alpha . e^2 - \cos^2\alpha$

$\cos^2\alpha . e^2 = 1$

i.e. Abscissas of foci remain constant.

2.[A] Let point of contract be (x_1, y_1)

$xx_1 - 2yy_1 - 4 = 0$

$2x + \sqrt{6}\, y - 2 = 0$

$\frac{x_1}{2} = -\frac{2y_1}{\sqrt{6}} = \frac{-4}{-2}$

$x_1 = 4$

$y_1 = -\sqrt{6}$

$(4, -\sqrt{6})$

3.[A, D] $e_e = \frac{3}{5}$ $\quad e_h = \frac{5}{3}$

focii of ellipse is $(\pm 3, 0)$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

As it passes through (3, 0)

$3^2 = a^2 \Rightarrow a = \pm 3$

$e_h = \pm 5$, $b^2 = 25 - 9 = 16$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

4.[A] $\frac{x^2}{4} + \frac{y^2}{3} = 1$

i.e. $e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

Hence foci are $(\pm 1, 0)$

Also required hyperbola is $\frac{x^2}{\sin^2\theta} - \frac{y^2}{b^2} = 1$

As these two are confocal $\sqrt{b^2 + \sin^2\theta} = 1$

$\therefore\ b^2 = \cos^2\theta$

Hence required equation is

$x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$

5.[A, B] Given hyperbola is $2x^2 - 2y^2 = 1$

its foci are $(\pm 1, 0)$

As ellipse intersects it orthogonally. Hence foci of ellipse be $(\pm 1, 0)$. Also, eccentricity of hyperbola is $\sqrt{2}$ then eccentricity of ellipse is $\frac{1}{\sqrt{2}}$. Hence equation of ellipse be

$$\frac{x^2}{2} + y^2 = 1 \text{ i.e. } x^2 + 2y^2 = 2$$

6.[A] $1 = 4a^2 - b^2$ …(1)

$$\frac{2a}{e} = 1$$

$$a = \frac{e}{2} \quad \ldots(2)$$

Also $b^2 = a^2(e^2 - 1)$ ….(3)

from (1) and (3), we get

$1 = 4a^2 - a^2e^2 + a^2$

$\Rightarrow 1 = 5a^2 - a^2e^2$

$$\Rightarrow 1 = \frac{5e^2}{4} - \frac{e^4}{4}$$

$\Rightarrow e^4 - 5e^2 + 4 = 0$

$\Rightarrow (e^2 - 4)(e^2 - 1) = 0$

$\therefore e = 2$

7.[B] $y = m(x - 4) \pm 4\sqrt{1 + m^2}$

$\Rightarrow y = mx \pm \sqrt{9m^2 - 4}$

$\Rightarrow -4m \pm 4\sqrt{1 + m^2} = \pm \sqrt{9m^2 - 4}$

$\Rightarrow 16m^2 + 16 + 16m^2 \mp 32m\sqrt{1 + m^2} = 9m^2 - 4$

$\Rightarrow \mp 32m\sqrt{1 + m^2} = -23m^2 - 20$

$\Rightarrow 1024m^2 + 1024m^4 = 529m^4 + 400 + 920m^2$

$\Rightarrow 495m^4 + 104m^2 - 400 = 0$

$\Rightarrow (5m^2 - 4)(99m^2 + 100) = 0$

$$\Rightarrow m^2 = \frac{4}{5} \Rightarrow m = \pm\frac{2}{\sqrt{5}}$$

So tangent with positive slope

$$\Rightarrow y = \frac{2}{\sqrt{5}}x \pm \frac{4}{\sqrt{5}}$$

$\Rightarrow 2x - \sqrt{5}y \pm 4 = 0$

8.[A] $x^2 + y^2 - 8x = 0$

$4x^2 - 9y^2 = 36$

$$\Rightarrow x^2 + \left(\frac{4x^2 - 36}{9}\right) - 8x = 0$$

$\Rightarrow 13x^2 - 72x - 36 = 0$

$\Rightarrow (x - 6)(13x + 6) = 0$

$$\Rightarrow x = 6, -\frac{6}{13}$$

$\Rightarrow x = 6, \quad y = \pm\sqrt{12}$

equation of required circle is

$\Rightarrow (x - 6)^2 + (y - \sqrt{12})(y + \sqrt{12}) = 0$

$\Rightarrow x^2 + y^2 - 12x + 24 = 0$

9.[B, D] Let e_1 = eccentricity of hyperbola

e_2 = eccentricity of ellipse

$$\therefore e_1 = \frac{1}{e_2}$$

So eccentricity of ellipse $= \frac{\sqrt{3}}{2} = e_2$

Eccentricity of hyperbola $= \frac{2}{\sqrt{3}} = e_1$

Now focus of ellipse is $(\pm ae_2, 0) = (\pm\sqrt{3}, 0)$

Hyperbola passes through it

So $\frac{(\sqrt{3})^2}{a^2} - 0 = 1 \Rightarrow a^2 = 3$

also $b^2 = a^2(e_1^2 - 1)$

$$b^2 = 3\left(\frac{4}{3} - 1\right) = 1$$

and hyperbola

$$\frac{x^2}{3} - \frac{y^2}{1} = 1$$

also focus $(\pm ae, 0) = (\pm 2, 0)$

10. [A, B]

Equation of tangent is

$2x - y + c = 0$

$y = 2x + c$

slope $m = 2$

$\therefore \quad a^2 = 9, b^2 = 4$

$\therefore \quad c^2 = a^2m^2 - b^2 = 9 \times 4 - 4$

$c = \pm 4\sqrt{2}$

$$\therefore \quad \text{point of contact is } \left(\pm\frac{a^2m}{c}, \pm\frac{b^2}{c}\right)$$