

**JEE MAIN + ADVANCED**

**MATHEMATICS**

**TOPIC NAME**

**PARABOLA**

**(PRACTICE SHEET)**

# LEVEL- 1

Question based on

## Standard form of equation of parabola

**Q.1** The equation of the directrix of the parabola  $x^2 = -8y$  is

- (A)  $x = 2$  (B)  $y = 2$   
(C)  $y = -2$  (D)  $x = -2$

**Q.2** The equation to the parabola whose focus is  $(0, -3)$  and directrix is  $y = 3$  is

- (A)  $x^2 = -12y$  (B)  $x^2 = 12y$   
(C)  $y^2 = 12x$  (D)  $y^2 = -12x$

**Q.3** If  $(0, 0)$  be the vertex and  $3x - 4y + 2 = 0$  be the directrix of a parabola, then the length of its latus rectum is

- (A)  $4/5$  (B)  $2/5$   
(C)  $8/5$  (D)  $1/5$

**Q.4** If  $2x + y + \lambda = 0$  is a focal chord of the parabola  $y^2 = -8x$ , then the value of  $\lambda$  is

- (A)  $-4$  (B)  $4$   
(C)  $2$  (D)  $-2$

**Q.5** The focal distance of a point  $(x_1, y_1)$  on the parabola  $y^2 = 12x$  is

- (A)  $x_1 + 3$  (B)  $x_1 + 6$   
(C)  $y_1 + 6$  (D)  $y_1 + 3$

**Q.6** The vertex of a parabola is  $(a, b)$  and its latus rectum is  $l$ . If the axis of the parabola is along the positive direction of  $y$ -axis, then its equation is-

- (A)  $(x + a)^2 = \left(\frac{l}{4}\right)(2y - 2b)$   
(B)  $(x - a)^2 = \left(\frac{l}{2}\right)(2y - 2b)$   
(C)  $(x + a)^2 = \left(\frac{l}{2}\right)(2y - 2b)$   
(D)  $(x - a)^2 = \left(\frac{l}{8}\right)(2y - 2b)$

**Q.7** The length of latus rectum of the parabola  $x^2 = -y$  is-

- (A)  $1$  (B)  $1/4$   
(C)  $4$  (D)  $1/2$

**Q.8** The distance between the focus and the directrix of the parabola  $x^2 = -8y$ , is-

- (A)  $8$  (B)  $2$   
(C)  $4$  (D)  $6$

**Q.9** If focus of the parabola is  $(3, 0)$  and length of latus rectum is  $8$ , then its vertex is-

- (A)  $(2, 0)$  (B)  $(1, 0)$   
(C)  $(0, 0)$  (D)  $(-1, 0)$

**Q.10** For any parabola focus is  $(2, 1)$  and directrix is  $2x - 3y + 1 = 0$ , then equation of the latus rectum is-

- (A)  $3x + 2y + 8 = 0$  (B)  $2x - 3y - 1 = 0$   
(C)  $2x - 3y + 1 = 0$  (D)  $3x - 2y + 4 = 0$

**Q.11** If  $(a, b)$  is the mid point of a chord passing through the vertex of the parabola  $y^2 = 4x$ , then-

- (A)  $a = 2b$  (B)  $2a = b$   
(C)  $a^2 = 2b$  (D)  $2a = b^2$

**Q.12** The area of the triangle formed by the lines joining the vertex of the parabola  $x^2 = 12y$  to the ends of its latus rectum is-

- (A)  $16$  sq. units (B)  $12$  sq. units  
(C)  $18$  sq. units (D)  $24$  sq. units

Question based on

## Reduction to standard equation & general equation of a parabola

**Q.13** Vertex of the parabola  $9x^2 - 6x + 36y + 9 = 0$  is-

- (A)  $(1/3, -2/9)$  (B)  $(-1/3, 1/2)$   
(C)  $(-1/3, -1/2)$  (D)  $(1/3, 1/2)$

**Q.14** The equation of the latus rectum of the parabola  $x^2 + 4x + 2y = 0$  is-

- (A)  $3y = 2$  (B)  $2y + 3 = 0$   
(C)  $2y = 3$  (D)  $3y + 2 = 0$

- Q.15** The focus of the parabola  $y^2 - x - 2y + 2 = 0$  is-  
 (A) (1,2) (B) (1/4,0)  
 (C) (3/4, 1) (D) (5/4, 1)
- Q.16** Vertex of the parabola  $y^2 + 2y + x = 0$  lies in the quadrant  
 (A) Second (B) First  
 (C) Third (D) Fourth
- Q.17** The equation of the axis of the parabola  $x^2 - 4x - 3y + 10 = 0$  is  
 (A)  $y + 2 = 0$  (B)  $x + 2 = 0$   
 (C)  $x - 2 = 0$  (D)  $y - 2 = 0$
- Q.18** The vertex of the parabola  $x^2 + 4x + 2y - 7 = 0$  is-  
 (A) (-2, 2) (B) (2, 11)  
 (C) (-2, 11) (D) (-2, 11/2)
- Q.19** The latus rectum of the parabola  $y^2 - 4y - 2x - 8 = 0$  is-  
 (A) 3 (B) 2  
 (C) 1 (D) 4
- Q.20** The point of intersection of the latus rectum and axes of the parabola  $y^2 + 4x + 2y - 8 = 0$  is  
 (A) (5/4, -1) (B) (7/5, 5/2)  
 (C) (9/4, -1) (D) None of these
- Q.21** The length of the latus rectum of the parabola  $x = ay^2 + by + c$  is-  
 (A)  $\frac{a}{4}$  (B)  $\frac{a}{3}$   
 (C)  $\frac{1}{a}$  (D)  $\frac{1}{4a}$
- Q.22** Which of the following is not the equation of parabola  
 (A)  $4x^2 + 9y^2 - 12xy + x + 1 = 0$   
 (B)  $4x^2 - 12xy + 9y^2 + 3x + 5 = 0$   
 (C)  $2x^2 + y^2 - 4xy = 8$   
 (D)  $4x^2 + 9y^2 - 12xy + x + 1 = 0$

- Q.23** Which one is the equation of the parabola  
 (A)  $(x - y)^2 = 2$  (B)  $\frac{x}{y} + \frac{24}{x} = 0$   
 (C)  $\frac{x}{y} - \frac{y}{x} = 0$  (D)  $2x^2 + 5y^2 = 7$
- Q.24** The equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents a parabola, if-  
 (A)  $a = b = 1, h = 0$  (B)  $a = b, h = 0$   
 (C)  $h^2 = ab$  (D) None of these
- Q.25** Given the ends of latus rectum, the number of parabolas that can be drawn is-  
 (A) 0 (B) 1  
 (C) 2 (D) 3

Question based on

**Equation of parabola when its vertex & focus are given**

- Q.26** If (0, a) be the vertex and (0, 0) be the focus of a parabola then its equation will be  
 (A)  $y^2 = 4a(a + x)$  (B)  $x^2 = 4a(a - y)$   
 (C)  $x^2 = 4a(a + y)$  (D)  $y^2 = 4a(a - x)$
- Q.27** If vertex and focus of a parabola are on x- axis and at distances p and q respectively from the origin, then its equation is-  
 (A)  $y^2 = -4(p - q)(x + p)$   
 (B)  $y^2 = 4(p - q)(x - p)$   
 (C)  $y^2 = -4(p - q)(x - p)$   
 (D) None of these
- Q.28** Find the equation of the parabola having the vertex at (0,1) and the focus at (0,0)  
 (A)  $x^2 + 4y - 4 = 0$  (B)  $x^2 + 4y + 4 = 0$   
 (C)  $x^2 - 4y + 4 = 0$  (D)  $x^2 - 4y - 4 = 0$
- Q.29** If (2, 0) and (5, 0), are the vertex and focus of a parabola respectively then its equation is  
 (A)  $y^2 = -12x - 24$  (B)  $y^2 = 12x - 24$   
 (C)  $y^2 = 12x + 24$  (D)  $y^2 = -12x + 24$

Question based on

### Parametric equation of parabola

- Q.30**  $x - 2 = t^2$ ,  $y = 2t$  are the parametric equations of the parabola-  
 (A)  $y^2 = -4x$  (B)  $y^2 = 4x$   
 (C)  $x^2 = -4y$  (D)  $y^2 = 4(x - 2)$
- Q.31** The equations  $x = \frac{t}{4}$ ,  $y = \frac{t^2}{4}$  represents-  
 (A) An ellipse (B) A parabola  
 (C) A circle (D) A hyperbola
- Q.32** The parametric equation of a parabola is  $x = t^2 + 1$ ,  $y = 2t + 1$ . The Cartesian equation of its directrix is-  
 (A)  $x = 0$  (B)  $y = 0$   
 (C)  $x + 1 = 0$  (D) None of these
- Q.33** Any point on the parabola whose focus is  $(0, 1)$  and the directrix is  $x + 2 = 0$  is given by-  
 (A)  $(t^2 + 1, 2t + 1)$  (B)  $(t^2 + 1, 2t - 1)$   
 (C)  $(t^2, 2t)$  (D)  $(t^2 - 1, 2t + 1)$
- Q.34** Which of the following are not parametric coordinates of any point on the parabola  $y^2 = 4ax$   
 (A)  $(a/m^2, 2/m)$  (B)  $(a, 2a)$   
 (C)  $(at^2, 2at)$  (D)  $(am^2, -2am)$
- Q.35** The parametric equations of the parabola  $y^2 - 12x - 2y - 11 = 0$  are-  
 (A)  $x = 3t^2 - 1$ ,  $y = 6t + 1$   
 (B)  $x = 3t^2 + 1$ ,  $y = 6t - 1$   
 (C)  $x = 6t + 1$ ,  $y = 3t^2 - 1$   
 (D) None of these

Question based on

### Chords

- Q.36** In the parabola  $y^2 = 6x$ , the equation of the chord through vertex and negative end of latus rectum, is -  
 (A)  $x = 2y$  (B)  $y + 2x = 0$   
 (C)  $y = 2x$  (D)  $x + 2y = 0$
- Q.37** If  $(2, -8)$  is at an end of a focal chord of the parabola  $y^2 = 32x$  then the other end of the chord is-  
 (A)  $(32, 32)$  (B)  $(-2, 8)$   
 (C)  $(32, -32)$  (D) None of these

- Q.38** The length of the chord of the parabola  $y^2 = 4x$  which passes through the vertex and makes  $30^\circ$  angle with x-axis is-  
 (A)  $\sqrt{3}/2$  (B)  $3/2$   
 (C)  $8\sqrt{3}$  (D)  $\sqrt{3}$
- Q.39** Lines  $y = x$  and  $y = -x$  intersect the parabola  $y^2 = 4x$  at A and B other than the origin. The length AB is  
 (A) 12 (B) 8  
 (C) 4 (D) 16
- Q.40** The length of the chord passing through the vertex of the parabola  $y^2 = 4ax$  and making an angle  $\theta$  with x-axis is  
 (A)  $4a \sin \theta \cos^2 \theta$  (B)  $4a \cos \theta \operatorname{cosec}^2 \theta$   
 (C)  $4a \sin \theta \sec^2 \theta$  (D)  $4a \cos \theta \sin^2 \theta$
- Q.41** The length of the intercept made by the parabola  $x^2 - 7x + 4y + 12 = 0$  on x-axis is  
 (A) 4 (B) 3  
 (C) 1 (D) 2
- Q.42** The length of the intercept made by the parabola  $2y^2 + 6y = 8 - 5x$  on y-axis is  
 (A) 7 (B) 5  
 (C) 3 (D) 1
- Q.43** Length of the chord intercepted by the parabola  $y = x^2 + 3x$  on the line  $x + y = 5$  is  
 (A)  $6\sqrt{2}$  (B)  $\sqrt{2}$   
 (C)  $6\sqrt{3}$  (D) None of these
- Q.44** If PSQ is the focal chord of the parabola  $y^2 = 8x$  such that  $SP = 6$ . Then the length SQ is-  
 (A) 4 (B) 6  
 (C) 3 (D) None of these
- Q.45** If length of the two segments of focal chord to the parabola  $y^2 = 8ax$  are 2 and 4, then the value of a is-  
 (A)  $1/3$  (B)  $2/3$   
 (C)  $4/3$  (D) 4

Question based on

### Condition of tangent & Equation of Tangent

- Q.46** If the line  $x + y - 1 = 0$  touches the parabola  $y^2 = kx$ , then the value of k is-  
 (A) 2 (B) -4  
 (C) 4 (D) -2

- Q.47** The straight line  $x + y = k$  touches the parabola  $y = x - x^2$ , if  $k =$   
 (A) 0 (B) -1  
 (C) 1 (D) None of these
- Q.48** The line  $y = mx + c$  may touch the parabola  $y^2 = 4a(x + a)$ , if-  
 (A)  $c = am - (a/m)$  (B)  $c = a/m$   
 (C)  $c = -a/m$  (D)  $c = am + (a/m)$
- Q.49** The straight line  $2x + y - 1 = 0$  meets the parabola  $y^2 = 4x$  in  
 (A) Two real and different points  
 (B) Two imaginary points  
 (C) Two coincident points  
 (D) One real point and one point at infinity
- Q.50** The line  $\ell x + my + n = 0$  will touch the parabola  $y^2 = 4ax$ , if  
 (A)  $\ell m = an^2$  (B)  $mn = a\ell^2$   
 (C)  $\ell n = am^2$  (D)  $mn = a\ell$
- Q.51** For what value of  $k$ , the line  $2y - x + k = 0$  touches the parabola  $x^2 + 4y = 0$ -  
 (A) 2 (B) -1/2  
 (C) -2 (D) 1/2
- Q.52** At which point the line  $x = my + \frac{a}{m}$  touches the parabola  $x^2 = 4ay$   
 (A)  $(2am, am^2)$  (B)  $(am^2, 2am)$   
 (C)  $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$  (D)  $\left(\frac{2a}{m}, \frac{a}{m^2}\right)$
- Q.53** If a tangent to the parabola  $4y^2 = x$  makes an angle of  $60^\circ$  with the  $x$ -axis, then its point of contact is-  
 (A)  $\left(\frac{1}{48}, \frac{1}{8\sqrt{3}}\right)$  (B)  $\left(\frac{3}{16}, \frac{\sqrt{3}}{8}\right)$   
 (C)  $\left(\frac{1}{48}, -\frac{1}{8\sqrt{3}}\right)$  (D)  $\left(\frac{3}{16}, -\frac{\sqrt{3}}{8}\right)$
- Q.54** The equation of the tangent to the parabola  $y^2 = 4x$  at the point  $(1, 2)$  is-  
 (A)  $x - y + 1 = 0$  (B)  $x + y - 1 = 0$   
 (C)  $x + y + 1 = 0$  (D)  $x - y - 1 = 0$
- Q.55** The point where the line  $x + y = 1$  touches the parabola  $y = x - x^2$ , is-  
 (A)  $\left(\frac{1}{2}, \frac{1}{2}\right)$  (B)  $(1, 0)$   
 (C)  $(0, 1)$  (D)  $(-1, -2)$
- Q.56** The equation of the tangent at vertex to the parabola  $4y^2 + 6x = 8y + 7$  is  
 (A)  $x = 11/6$  (B)  $y = 2$   
 (C)  $x = -11/6$  (D)  $y = -2$
- Q.57** The equation of the tangent to the parabola  $y = 2 + 4x - 4x^2$  with slope  $-4$  is  
 (A)  $4x + y - 6 = 0$  (B)  $4x + y + 6 = 0$   
 (C)  $4x - y - 6 = 0$  (D) None of these
- Q.58** The point on the curve  $y^2 = x$  the tangent at which makes an angle of  $45^\circ$  with  $x$ -axis will be given by  
 (A)  $(1/2, 1/2)$  (B)  $(1/2, 1/4)$   
 (C)  $(2, 4)$  (D)  $(1/4, 1/2)$
- Q.59** The point of contact of the line  $2x - y + 2 = 0$  with the parabola  $y^2 = 16x$  is-  
 (A)  $(3, 4)$  (B)  $(2, 4)$   
 (C)  $(1, 4)$  (D)  $(-2, 1)$
- Q.60** The equation of tangent to the parabola  $x^2 = y$  at one extremity of latus rectum in the first quadrant is  
 (A)  $y = 4x + 1$  (B)  $x = 4y + 1$   
 (C)  $4x + 4y = 1$  (D)  $4x - 4y = 1$
- Q.61** The equation of the common tangents to the parabolas  $y^2 = 4x$  and  $x^2 = 32y$  is-  
 (A)  $x + 2y = 4$  (B)  $x = 2y + 4$   
 (C)  $x = 2y - 4$  (D)  $x + 2y + 4 = 0$
- Q.62** Area of triangle formed by the tangents at three points  $t_1, t_2$  and  $t_3$  of the parabola  $y^2 = 4ax$  is-  
 (A)  $\frac{a}{2} (t_1 - t_2) (t_2 - t_3) (t_3 - t_1)$   
 (B)  $a^2 (t_1 - t_2) (t_2 - t_3) (t_3 - t_1)$   
 (C)  $\frac{a^2}{2} (t_1 + t_2) (t_2 + t_3) (t_3 + t_1)$   
 (D)  $\frac{a^2}{2} (t_1 - t_2) (t_2 - t_3) (t_3 - t_1)$
- Q.63** The abscissa of the point of intersection of two tangents to the parabola is the .....between the abscissa of the points contact.  
 (A) H.M. (B) G.M.  
 (C) A.M. (D) None of these

## LEVEL- 2

- Q.1** A tangent to the parabola  $y^2 = 4ax$  at P (p, q) is perpendicular to the tangent at the other point Q, then coordinates of Q are-  
 (A)  $(a^2/p, -4a^2/q)$  (B)  $(-a^2/p, -4a^2/q)$   
 (C)  $(-a^2/p, 4a^2/q)$  (D)  $(a^2/p, 4a^2/q)$
- Q.2** The coordinates of a point on the parabola  $y^2 = 8x$  whose focal distance is 4 is-  
 (A) (2,4) (B) (4,2)  
 (C) (2,-9) (D) (4,-2)
- Q.3** PQ is a double ordinate of  $y^2 = 4ax$ . The locus of its point of trisection is  
 (A)  $y^2 = 2ax$  (B)  $3y^2 = 4ax$   
 (C)  $9y^2 = 4ax$  (D)  $9y^2 = 2ax$
- Q.4** Which one of the following represented parametrically, represents equation of a parabola-  
 (A)  $x = 3 \cos t; y = 4 \sin t$   
 (B)  $x^2 - 2 = -2 \cos t; y = 4 \cos^2 \frac{t}{2}$   
 (C)  $\sqrt{x} = \tan t; \sqrt{y} = \sec t$   
 (D)  $x = \sqrt{1 - \sin t}; y = \sin \frac{t}{2} + \cos \frac{t}{2}$
- Q.5** If  $(x_1, y_1)$  and  $(x_2, y_2)$  are ends of a focal chord of the parabola  $y^2 = 4ax$ , then square of G.M. of  $x_1$  and  $x_2$  is-  
 (A)  $-4a^2$  (B)  $4a^2$  (C)  $a^2$  (D)  $-a^2$
- Q.6** The equation of the common tangent to the parabolas  $y^2 = 4ax$  and  $x^2 = 4by$  is  
 (A)  $xa^{1/3} + yb^{1/3} + (ab)^{2/3} = 0$   
 (B)  $xa^{2/3} + yb^{2/3} + (ab)^{2/3} = 0$   
 (C)  $xa^{1/3} + yb^{1/3} = (ab)^{2/3}$   
 (D)  $xa^{2/3} + yb^{2/3} = (ab)^{2/3}$
- Q.7** If the chord  $y = mx + c$  subtends a right angle at the vertex of the parabola  $y^2 = 4ax$ , then the value of c is-  
 (A)  $-4am$  (B)  $4am$  (C)  $-2am$  (D)  $2am$
- Q.8** The length of L.R. of the parabola  $y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$  is-  
 (A)  $\frac{2u^2 \cos^2 \alpha}{g}$  (B)  $\frac{u^2 \sin^2 2\alpha}{g}$   
 (C)  $\frac{u^2 \cos^2 2\alpha}{g}$  (D) None of these
- Q.9** If a tangent line at a point P on a parabola makes angle  $\alpha$  with its focal distance, then angle between the tangent and axis of the parabola is-  
 (A)  $\alpha$  (B)  $\alpha/2$   
 (C)  $2\alpha$  (D)  $90^\circ$
- Q.10** If a focal chord of parabola  $y^2 = 4ax$  makes an angle  $\theta$  with its axis, then the length of perpendicular from vertex to this chord is-  
 (A)  $a \tan \theta$  (B)  $a \cos \theta$   
 (C)  $a \sin \theta$  (D)  $a \sec \theta$
- Q.11** An equilateral triangle is inscribed in the parabola  $y^2 = 4x$  whose vertex is at the vertex of the parabola. The area of this triangle is-  
 (A)  $48\sqrt{3}$  (B)  $16\sqrt{3}$   
 (C)  $64\sqrt{3}$  (D)  $8\sqrt{3}$
- Q.12** The equation of common tangent to the circle  $x^2 + y^2 = 2a^2$  and parabola  $y^2 = 8ax$  is-  
 (A)  $y = x + a$  (B)  $y = \pm x \pm 2a$   
 (C)  $y = -x + a$  (D)  $y = -x + 2a$
- Q.13** The equation of a tangent to the parabola  $y^2 = 12x$  is  $y = x + 3$ . The point on this line from which other tangent to the parabola is perpendicular to the given tangent is-  
 (A) (0, 4) (B) (-3, 3)  
 (C) (-2, 3) (D) None of these

- Q.14** The locus of the point of intersection of perpendicular tangent to the parabola  $x^2 - 8x + 2y + 2 = 0$  is-  
 (A)  $2y - 15 = 0$  (B)  $2y + 15 = 0$   
 (C)  $2x + 9 = 0$  (D) None of these
- Q.15** The slope of tangent lines drawn from  $(3, 8)$  to the parabola  $y^2 = -12x$  are-  
 (A)  $3, 1/3$  (B)  $-3, -1/3$   
 (C)  $3, -1/3$  (D)  $-3, 1/3$
- Q.16** The equation of the common tangent of the parabolas  $x^2 = 108y$  and  $y^2 = 32x$ , is-  
 (A)  $2x + 3y = 36$  (B)  $2x + 3y + 36 = 0$   
 (C)  $3x + 2y = 36$  (D)  $3x + 2y + 36 = 0$
- Q.17** On the parabola  $y = x^2$ , the point least distance from the straight line  $y = 2x - 4$  is-  
 (A)  $(1, 1)$  (B)  $(1, 0)$   
 (C)  $(1, -1)$  (D)  $(0, 0)$
- Q.18** AB, AC are tangents to a parabola  $y^2 = 4ax$ . If  $l_1, l_2, l_3$  are the lengths of perpendiculars from A, B, C on any tangent to the parabola, then-  
 (A)  $l_1, l_2, l_3$  are in GP  
 (B)  $l_2, l_3, l_1$  are in GP  
 (C)  $l_3, l_1, l_2$  are in GP  
 (D) None of these
- Q.19** A circle with centre at the focus of the parabola  $y^2 = 4px$  touches the directrix. Then a point of intersection of the circle and the parabola is-  
 (A)  $(-p, 2p)$  (B)  $(p, -2p)$   
 (C)  $(p, \pm 2p)$  (D)  $(-p, -2p)$
- Q.20** An equilateral triangle is inscribed in the parabola  $y^2 = 4ax$  with one vertex at the origin. The radius of the circum circle of that triangle is-  
 (A)  $2a$  (B)  $4a$  (C)  $6a$  (D)  $8a$
- Q.21**  $L(2, 4)$  and  $L'(2, -4)$  are the ends of the latus-rectum of a parabola. P is a point on the directrix. Then the area of  $\Delta PLL' =$   
 (A) 16 (B) 8  
 (C) 4 (D) 1
- Q.22** The equation of the locus of a point which moves so as to be at equal distances from the point  $(a, 0)$  and the y-axis is  
 (A)  $y^2 - 2ax + a^2 = 0$  (B)  $y^2 + 2ax + a^2 = 0$   
 (C)  $x^2 - 2ay + a^2 = 0$  (D)  $x^2 + 2ay + a^2 = 0$
- Q.23** If the line  $\ell x + my + n = 0$  is a tangent to the parabola  $y^2 = 4ax$ , then locus of its point of contact is  
 (A) a straight line (B) a circle  
 (C) a parabola (D) two straight lines
- Q.24** M is the foot of the  $\perp$  from a point P on the parabola  $y^2 = 8(x - 3)$  to its directrix and S is the focus of the parabola and SPM is an equilateral triangle, then the length of each side of the triangle is  
 (A) 2 (B) 3 (C) 4 (D) 8
- Q.25** Equation of the directrix of the parabola whose focus is  $(0, 0)$  and the tangent at the vertex is  $x - y + 1 = 0$  is  
 (A)  $x - y = 0$  (B)  $x - y - 1 = 0$   
 (C)  $x - y + 2 = 0$  (D)  $x + y - 1 = 0$
- Q.26** The point on  $y^2 = 4ax$  nearest to the focus has its abscissae equal to  
 (A)  $-a$  (B)  $a$   
 (C)  $a/2$  (D)  $0$
- Q.27** The tangents at the points  $(at_1^2, 2at_1), (at_2^2, 2at_2)$  on the parabola  $y^2 = 4ax$  are at right angles if  
 (A)  $t_1 t_2 = -1$  (B)  $t_1 t_2 = 1$   
 (C)  $t_1 t_2 = 2$  (D)  $t_1 t_2 = -2$

## LEVEL- 3

- Q.1** The angle subtended by the double ordinate of length  $2a$  of the parabola  $y^2 = ax$ , at the vertex is equals to-
- (A)  $\frac{\pi}{4}$  (B)  $\frac{\pi}{3}$   
(C)  $\frac{\pi}{2}$  (D) none of these
- Q.2** The portion of a tangent to a parabola  $y^2 = 4ax$  cut off between the directrix and the curve subtends an angle  $\theta$  at the focus, then  $\theta$  is equal to -
- (A)  $\frac{\pi}{4}$  (B)  $\frac{\pi}{3}$   
(C)  $\frac{\pi}{2}$  (D) none of these
- Q.3** If three points E, F, G are taken on the parabola  $y^2 = 4ax$  so that their ordinates are in G.P., then the tangents at E and G intersect on the-
- (A) directrix (B) axis  
(C) abscissa of F (D) ordinate of F
- Q.4** A circle has its centre at the vertex of the parabola  $x^2 = 4y$  and the circle cuts the parabola at the ends of its latus rectum. The equation of the circle is-
- (A)  $x^2 + y^2 = 5$  (B)  $x^2 + y^2 = 4$   
(C)  $x^2 + y^2 = 1$  (D) none of these
- Q.5** A triangle ABC of area  $5a^2$  is inscribed in the parabola  $y^2 = 4ax$  such that vertex A lies at the vertex of parabola and BC is a focal chord. Then the length of focal chord is-
- (A)  $5a$  (B)  $\frac{25a}{4}$  (C)  $\frac{5a}{4}$  (D)  $25a$
- Q.6** The equation of directrix of a parabola is  $3x + 4y + 15 = 0$  and equation of tangent at vertex is  $3x + 4y - 5 = 0$ . Then the length of latus rectum is equal to
- (A) 15 (B) 14 (C) 13 (D) 16
- Q.7** For the parabola  $y^2 + 8x - 12y + 20 = 0$ , which of the following is not correct-
- (A) vertex (2, 6)  
(B) focus (0, 6)  
(C) length of the latus rectum = 4  
(D) axis is  $y = 6$
- Q.8** Length of the tangent drawn from an end of the latus rectum of the parabola  $y^2 = 4ax$  to the circle of radius  $a$  touching externally the parabola at vertex is equal to-
- (A)  $\sqrt{3} a$  (B)  $2a$   
(C)  $\sqrt{7} a$  (D)  $3a$
- Q.9** The vertex of parabola is (2, 2) and the co-ordinates of extremities of latus rectum are (-2, 0) & (6, 0). The equation of parabola is equal to-
- (A)  $y^2 - 4y + 8x - 12 = 0$   
(B)  $x^2 + 4x - 8y - 12 = 0$   
(C)  $x^2 - 4x + 8y - 12 = 0$   
(D)  $x^2 - 8y - 4x + 20 = 0$
- Q.10** If M is the foot of perpendicular from a point P of a parabola  $y^2 = 4ax$  to its directrix and SPM is an equilateral triangle, where S is the focus. Then SP equal to-
- (A)  $a$  (B)  $2a$   
(C)  $3a$  (D)  $4a$
- Q.11** If the point P(2, -2) is the one end of the focal chord PQ of the parabola  $y^2 = 2x$  then slope of the tangent at Q is-
- (A) -2 (B) 2  
(C) 1/2 (D) -1/2



**Assertion-Reason type Question:**

**Note :** Question Number 12 to 19 are assertion-reason type question. Each question has two statements. (Statement-1 and Statement-2) Each question has four options A, B, C and D. Out of which only one is correct.

**A** → If both statements are correct and Statement (2) is correct explanation of statement (1).

**B** → If both statements are correct but Statement (2) is not correct explanation of statement (1).

**C** → If statement (1) is correct and statement (2) is wrong.

**D** → If statement (1) is wrong and statement (2) is correct.

**Q.12** **Statement-(1):** The curve  $9y^2 - 16x - 12y - 57 = 0$  is symmetric about line  $3y = 2$ .

**Statement-(2):** A parabola is symmetric about its axis.

**Q.13** **Statement-(1):** Two perpendicular tangents of parabola  $y^2 = 16x$  always meet on  $x + 4 = 0$ .

**Statement-(2):** Two perpendicular tangents of a parabola, always meet on axis.

**Q.14** **Statement-(1) :** If 4 & 3 are length of two focal segments of focal chord of parabola  $y^2 = 4ax$  than latus rectum of parabola will be  $48/7$  units

**Statement-(2) :** If  $l_1$  &  $l_2$  are length of focal segments of focal chord than its latus rectum is

$$\frac{2l_1l_2}{l_1+l_2}.$$

**Q.15** **Statement-(1) :** Let  $(x_1, y_1)$  and  $(x_2, y_2)$  are the ends of a focal chord of  $y^2 = 4x$  then  $4x_1x_2 + y_1y_2 = 0$

**Statement-(2) :** PSQ is the focal of a parabola with focus S and latus rectum  $\lambda$  then  $SP + SQ = 2\lambda$ .

**Q.16** **Statement-(1) :** Straight line  $y + x = k$  touch the parabola  $y = x - x^2$  if  $k = 1$

**Statement-(2) :** Discriminant of  $(x - 1)^2 = x - x^2$  is zero.

**Q.17** **Statement-(1) :** PQ is a focal chord of a parabola. Then the tangent at P to the parabola is parallel to the normal at Q.

**Statement-(2):** If  $P(t_1)$  and  $Q(t_2)$  are the ends of a focal chord of the parabola  $y^2 = 4ax$ , then  $t_1t_2 = -1$ .

**Q.18** **Statement-(1):** The tangents drawn to the parabola  $y^2 = 4ax$  at the ends of any focal chord intersect on the directrix.

**Statement-(2) :** The point of intersection of the tangents at drawn at  $P(t_1)$  and  $Q(t_2)$  are the parabola  $y^2 = 4ax$  is  $\{at_1t_2, a(t_1 + t_2)\}$

**Q.19** **Statement-(1) :** The lines from the vertex to the two extremities of a focal chord of the parabola  $y^2 = 4ax$  are at an angle of  $\frac{\pi}{2}$ .

**Statement-(2) :** If extremities of focal chord of a parabola are  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$ , then  $t_1t_2 = -1$ .

**Passage-1 (Q. No. 20 to Q. No. 22)**

A equilateral triangle is inscribed in the parabola  $y^2 = 8x$ . If one vertex of the triangle is at the vertex of parabola then-

**Q.20** Length of side of the triangle is-

(A)  $8\sqrt{3}$  (B)  $16\sqrt{3}$

(C)  $4\sqrt{3}$  (D) 8

**Q.21** Area of the triangle is-

(A)  $64\sqrt{3}$  (B)  $48\sqrt{3}$

(C)  $192\sqrt{3}$  (D) None of these

**Q.22** Radius of the circum-circle of the triangle is-

(A) 4 (B) 8 (C) 16 (D) 32

**Passage-2 (Q. No. 23 to Q. No. 25)**

Conic possesses enormous properties which can be probed by taking their standard forms. Unlike circle these properties rarely follow by geometrical considerations. Most of the properties of conics are proved analytically. For example, the properties of a parabola can be proved by taking its standard equation  $y^2 = 4ax$  and a point  $(at^2, 2at)$  on it.

**Q.23** If the tangent and normal at any point P on the parabola whose focus is S, meets its axis in T and G respectively, then

- (A)  $PG = PT$
- (B) S is mid- point of T and G
- (C)  $ST = 2SG$
- (D) None of these

**Q.24** The angle between the tangents drawn at the extremities of a focal chord must be

- (A)  $30^\circ$
- (B)  $60^\circ$
- (C)  $90^\circ$
- (D)  $120^\circ$

**Q.25** If the tangent at any point P meets the directrix at K, then  $\angle KSP$  must be

- (A)  $30^\circ$
- (B)  $60^\circ$
- (C)  $90^\circ$
- (D) None of these

### COLUMN MATCHING QUESTIONS

**Q.26** Match the following :

- | <b>Column-I</b>                                              | <b>Column-II</b> |
|--------------------------------------------------------------|------------------|
| (A) The focus of the parabola $(y - 4)^2 = 12(x - 2)$        | (P) (1, 2)       |
| (B) The vertex of the parabola $y^2 - 5x - 4y + 9 = 0$       | (Q) (-3, 1)      |
| (C) The foot of the directrix of the parabola $x^2 + 8y = 0$ | (R) (3, 0)       |
| (D) The vertex of the parabola $x^2 + 6x - 2y + 11 = 0$      | (S) (5, 4)       |
|                                                              | (T) (0, 2)       |

## LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

### SECTION-A

**Q.1** The length of the latus-rectum of the parabola  $x^2 - 4x - 8y + 12 = 0$  is- [AIEEE-2002]

- (A) 4 (B) 6 (C) 8 (D) 10

**Q.2** The equation of tangents to the parabola  $y^2 = 4ax$  at the ends of its latus rectum is-

[AIEEE-2002]

- (A)  $x - y + a = 0$  (B)  $x + y + a = 0$   
(C)  $x + y - a = 0$  (D) both (A) and (B)

**Q.3** If  $a \neq 0$  and the line  $2bx + 3cy + 4d = 0$  passes through the points of intersection of the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ , then-

[AIEEE 2004]

- (A)  $d^2 + (2b + 3c)^2 = 0$   
(B)  $d^2 + (3b + 2c)^2 = 0$   
(C)  $d^2 + (2b - 3c)^2 = 0$   
(D)  $d^2 + (3b - 2c)^2 = 0$

**Q.4** The locus of the vertices of the family of parabolas  $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$  is-

[AIEEE 2006]

- (A)  $xy = \frac{3}{4}$  (B)  $xy = \frac{35}{16}$   
(C)  $xy = \frac{64}{105}$  (D)  $xy = \frac{105}{64}$

**Q.5** The equation of a tangent to the parabola  $y^2 = 8x$  is  $y = x + 2$ . The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is-

[AIEEE 2007]

- (A) (-1, 1) (B) (0, 2)  
(C) (2, 4) (D) (-2, 0)

**Q.6** A parabola has the origin as its focus and the line  $x = 2$  as the directrix. Then the vertex of the parabola is at -

[AIEEE 2008]

- (A) (1, 0) (B) (0, 1)  
(C) (2, 0) (D) (0, 2)

**Q.7** If two tangents drawn from a point P to the parabola  $y^2 = 4x$  are at right angles, then the locus of P is -

[AIEEE 2010]

- (A)  $x = 1$  (B)  $2x + 1 = 0$   
(C)  $x = -1$  (D)  $2x - 1 = 0$

**Q.8** The shortest distance between line  $y - x = 1$  and curve  $x = y^2$  is -

[AIEEE 2011]

- (A)  $\frac{\sqrt{3}}{4}$  (B)  $\frac{3\sqrt{2}}{8}$  (C)  $\frac{8}{3\sqrt{2}}$  (D)  $\frac{4}{\sqrt{3}}$

**Q.9** Given : A circle,  $2x^2 + 2y^2 = 5$  and a parabola,  $y^2 = 4\sqrt{5}x$ .

**Statement-I :** An equation of a common tangent to these curves is  $y = x + \sqrt{5}$ .

**Statement-II :** If the line,  $y = mx + \frac{\sqrt{5}}{m}$  ( $m \neq 0$ )

is their common tangent, then m satisfies  $m^4 - 3m^2 + 2 = 0$ , [JEE Main - 2013]

- (A) Statement-I is true; Statement-II is false.  
(B) Statement-I is false; Statement-II is true.  
(C) Statement-I is true; Statement-II is true; Statement-II is a correct explanation for Statement-I.  
(D) Statement-I is true; Statement-II is true; Statement-II is not a correct explanation for Statement-I.

### SECTION-B

**Q.1** The point of intersection of the tangents at the ends of the latus rectum of the parabola  $y^2 = 4x$  is ... [IIT-1994]

- (A) (-1, 0) (B) (1, 0)  
(C) (0, 1) (D) None of these

**Q.2** Consider a circle with centre lying on the focus of the parabola  $y^2 = 2px$  such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is

[IIT-1995]

- (A)  $(p/2, p)$  (B)  $(-p/2, p)$   
(C)  $(-p/2, -p)$  (D) None of these

**Q.3** The curve described parametrically by  $x = t^2 + t + 1$ ,  $y = t^2 - t + 1$  represents-

[IIT 1991, 99]

- (A) a pair of st. lines (B) an ellipse  
(C) a parabola (D) a hyperbola

**Q.4** If the line  $x - 1 = 0$  is the directrix of the parabola  $y^2 - kx + 8 = 0$ , then one of the values of k is-

[IIT-Scr. -2000]

- (A) 1/8 (B) 8 (C) 4 (D) 1/4

**Q.5** Above x-axis, the equation of the common tangents to the circle  $(x - 3)^2 + y^2 = 9$  and parabola  $y^2 = 4x$  is -

[IIT-Scr. -2001]

- (A)  $\sqrt{3}y = 3x + 1$  (B)  $\sqrt{3}y = -(x + 3)$   
(C)  $\sqrt{3}y = x + 3$  (D)  $\sqrt{3}y = -(3x + 1)$

- Q.6** The equation of the directrix of the parabola  $y^2 + 4y + 4x + 2 = 0$  is - [IIT-Scr. -2001]  
 (A)  $x = -1$  (B)  $x = 1$   
 (C)  $x = -\frac{3}{2}$  (D)  $x = \frac{3}{2}$

- Q.7** The locus of the mid-point of the line segment joining the focus to a moving point on the parabola  $y^2 = 4ax$  is another parabola with directrix - [IIT-Scr. -2002]  
 (A)  $x = -a$  (B)  $x = -a/2$   
 (C)  $x = 0$  (D)  $x = a/2$

- Q.8** If focal chord of  $y^2 = 16x$  touches  $(x - 6)^2 + y^2 = 2$  then slope of such chord is- [IIT-Scr. -2003]  
 (A) 1, -1 (B) 2,  $-\frac{1}{2}$   
 (C)  $\frac{1}{2}$ , -2 (D) 2, -2

- Q.9** The equation(s) of common tangent(s) to the parabola  $y = x^2$  and  $y = -(x-2)^2$  [IIT- 2006]  
 (A)  $y = -4(x-1)$  (B)  $y = 0$   
 (C)  $y = 4(x-1)$  (D)  $y = -30x - 50$

- Q.10** STATEMENT-1: The curve  $y = -\frac{x^2}{2} + x + 1$  is symmetric with respect to the line  $x = 1$  because  
 STATEMENT-2: A parabola is symmetric about its axis. [IIT- 2007]  
 (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.  
 (B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1  
 (C) Statement-1 is True, Statement-2 is False  
 (D) Statement-1 is False, Statement-2 is True

- Q.11** Consider the two curves [IIT-2008]  
 $C_1 : y^2 = 4x$   
 $C_2 : x^2 + y^2 - 6x + 1 = 0$   
 Then,  
 (A)  $C_1$  &  $C_2$  touch each other only at one point  
 (B)  $C_1$  &  $C_2$  touch each other exactly at two points  
 (C)  $C_1$  &  $C_2$  intersect (but do not touch) at exactly two points  
 (D)  $C_1$  and  $C_2$  neither intersect nor touch each other

- Q.12** Let A and B be two distinct points on the parabola  $y^2 = 4x$ . If the axis of the parabola touches a circle of radius  $r$  having AB as its diameter, then the slope of the line joining A and B can be - [IIT 2010]  
 (A)  $-\frac{1}{r}$  (B)  $\frac{1}{r}$  (C)  $\frac{2}{r}$  (D)  $-\frac{2}{r}$

- Q.13** Let  $(x, y)$  be any point on the parabola  $y^2 = 4x$ . Let P be the point that divides the line segment from  $(0, 0)$  to  $(x, y)$  in the ratio 1 : 3. Then the locus of P is - [IIT 2011]  
 (A)  $x^2 = y$  (B)  $y^2 = 2x$   
 (C)  $y^2 = x$  (D)  $x^2 = 2y$

- Q.14** Consider the parabola  $y^2 = 8x$ . Let  $\Delta_1$  be the area of the triangle formed by the end points of its latus rectum and the point  $P\left(\frac{1}{2}, 2\right)$  on the parabola, and  $\Delta_2$  be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then  $\frac{\Delta_1}{\Delta_2}$  is. [IIT 2011]  
 (A) 2 (B) 3 (C) 4 (D) 5

- Q.15** Let S be the focus of the parabola  $y^2 = 8x$  and let PQ be the common chord of the circle  $x^2 + y^2 - 2x - 4y = 0$  and the given parabola. The area of the triangle PQS is [IIT 2012]

**Paragraph for Questions 16 and 17**

Let PQ be a focal chord of the parabola  $y^2 = 4ax$ . The tangents to the parabola at P and Q meet at a point lying on the line  $y = 2x + a$ ,  $a > 0$ .

[JEE - Advance 2013]

- Q.16** If chord PQ subtends an angle  $\theta$  at the vertex of  $y^2 = 4ax$ , then  $\tan \theta =$   
 (A)  $\frac{2}{3}\sqrt{7}$  (B)  $\frac{-2}{3}\sqrt{7}$  (C)  $\frac{2}{3}\sqrt{5}$  (D)  $\frac{-2}{3}\sqrt{5}$

- Q.17** Length of chord PQ is  
 (A) 7a (B) 5a (C) 2a (D) 3a

- Q.18** A line  $L : y = mx + 3$  meets y-axis at E  $(0, 3)$  and the arc of the parabola  $y^2 = 16x$ ,  $0 \leq y \leq 6$  at the point  $F(x_0, y_0)$ . The tangent to the parabola at  $F(x_0, y_0)$  intersects the y-axis at  $G(0, y_1)$ . The slope  $m$  of the line L is chosen such that the area of the triangle EFG has a local maximum. Match List-I with List-II and select the correct answer using the code given below the lists :

[JEE - Advance 2013]

List-I	List-II
(P) $m =$	(1) $\frac{1}{2}$
(Q) Maximum area of $\Delta EFG$ is	(2) 4
(R) $y_0 =$	(3) 2
(S) $y_1 =$	(4) 1

**Codes :**

P	Q	R	S
(A) 4	1	2	3
(B) 3	4	1	2
(C) 1	3	2	4
(D) 1	3	4	2

# ANSWER KEY

## LEVEL- 1

<b>Qus.</b>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
<b>Ans.</b>	B	A	C	B	A	B	A	C	B	B	D	C	A	C	D	D	C	D	B	A
<b>Qus.</b>	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
<b>Ans.</b>	C	C	B	C	C	B	C	A	B	D	B	A	D	A	A	B	A	C	B	B
<b>Qus.</b>	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
<b>Ans.</b>	C	B	A	C	B	B	C	D	A	C	B	D	A	A	B	A	A	D	C	D
<b>Qus.</b>	61	62	63																	
<b>Ans.</b>	D	D	B																	

## LEVEL- 2

<b>Qus.</b>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
<b>Ans.</b>	A	A	C	B	C	A	A	A	A	C	A	B	D	A	C	B	A	C	C	D
<b>Qus.</b>	21	22	23	24	25	26	27													
<b>Ans.</b>	A	A	C	D	C	D	A													

## LEVEL- 3

<b>Qus.</b>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
<b>Ans.</b>	C	C	D	A	D	D	C	C	C	D	B	A	C	C	C	C	A	A	D	B
<b>Qus.</b>	21	22	23	24	25															
<b>Ans.</b>	C	C	B	C	C															

(26) A → S; B → P; C → T; D → Q

## LEVEL- 4

### SECTION-A

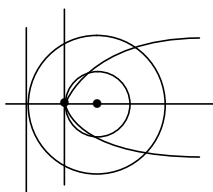
<b>Qus.</b>	1	2	3	4	5	6	7	8	9
<b>Ans.</b>	C	D	A	D	D	A	C	B	D

### SECTION-B

1.[A] It will intersect at directrix.

2.[A]  $\left(x - \frac{p}{2}\right)^2 + (y)^2 = p^2$

$\left(\frac{p}{2}, p\right)$  will satisfy this equation



3.[C] As  $x = t^2 + t + 1$   
 $y = t^2 - t + 1$

$$\frac{x-y}{2} = t \text{ and } \frac{x+y}{2} = t^2 + 1$$

$$\Rightarrow \frac{x+y}{2} = \left(\frac{x-y}{2}\right)^2 + 1$$

$$\Rightarrow 2x + 2y = x^2 + y^2 - 2xy + 4$$

$$\Rightarrow x^2 + y^2 - 2xy - 2x - 2y + 4 = 0 \text{ i.e. a parabola.}$$

4.[C]  $y^2 = kx - 8$

$$= k \left(x - \frac{8}{k}\right) \Rightarrow x - \frac{8}{k} = -\frac{y}{k}$$

$$\frac{8}{k} - \frac{k}{4} = 1 \Rightarrow 32 - k^2 = 4k$$

$$k^2 + 4k - 32 = 0$$

$$k = -8, 4$$

5.[C] Let  $(t^2, 2t)$  is any point on the parabola then equation of tangent be

$$2ty = 2(x + t^2)$$

$$\Rightarrow ty = x + t^2 \Rightarrow x - ty + t^2 = 0$$

As it is tangent to the circle  $(x-3)^2 + y^2 = 9$

$$\text{then } \left| \frac{3-0+t^2}{\sqrt{1+t^2}} \right| = 3$$

$$\text{Now } t^4 + 9 + 6t^2 = 9 + 9t^2$$

$$\Rightarrow t^4 - 3t^2 = 0 \Rightarrow t^2 = 3 \Rightarrow t = \sqrt{3}$$

[As in positive quadrant]

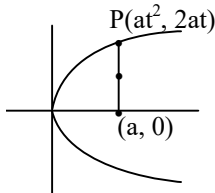
$$\text{Hence equation is } x - \sqrt{3}y + 3 = 0$$

6.[D]  $(y+2)^2 = -4x+2 = -4\left(x-\frac{1}{2}\right)$

Therefore, directrix is  $x - \frac{1}{2} = 1$

$$\Rightarrow x = \frac{3}{2}$$

7.[C]  $h = \frac{at^2+a}{2}, k = \frac{2at}{2} = at$  i.e.  $t = \frac{k}{a}$



$$\text{Now } 2h = a \left(1 + \frac{k^2}{a^2}\right)$$

$$2x = \frac{a^2 + y^2}{a} \Rightarrow y^2 = 2ax - a^2$$

$$y^2 = 4 \frac{2a}{4} \left(x - \frac{a}{2}\right)$$

$$\text{Directrix be } x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$$

8.[A] Eq<sup>n</sup> of tangent in slope form

$$y = m(x-6) \pm \sqrt{2} \sqrt{1+m^2}$$

It passes through the focus of the parabola S(4, 0)

$$y = m(x-6) \pm \sqrt{2} \sqrt{1+m^2}$$

$$(4, 0)$$

$$0 = m \pm \sqrt{2} \sqrt{1+m^2}$$

$$4m^2 = 2 + 2m^2 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$$

9.[B, C] Given curves are  $y = x^2$  and  $y = -(x-2)^2$   
taking a point  $(t, t^2)$  on  $y = x^2$ , at this point  
tangent be  $y = 2xt - t^2$

$$\text{Now, } -(x-2)^2 = 2xt - t^2$$

$$\Rightarrow x^2 + (2t-4)x + 4 - t^2 = 0$$

$$\text{As it is also tangent to } y = -(x-2)^2$$

$$\text{Hence } (2t-4)^2 - 4(4-t^2) = 0$$

$$\Rightarrow 8t(t-2) = 0 \Rightarrow t = 0, t = 2$$

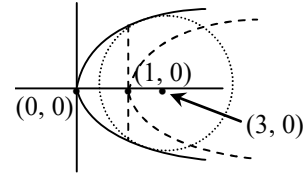
$$\text{Hence tangents are } y = 0, y = 4(x-1)$$

10.[A]  $x = \frac{(1)}{2 \times (1/2)} = 1$

Statement: 1 is true.

Statement 2 is also correct as well correct explanation.

11.[B]



$$\text{As } C_1 : y^2 = 4x$$

$$\text{and } C_2 : x^2 + y^2 - 6x + 1 = 0$$

Now

$$x^2 + 4x - 6x + 1 = 0$$

$$\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0$$

$C_1$  &  $C_2$  touch each other at exactly two points.

12.[C] Let  $A(t_1^2, 2t_1)$   $B(t_2^2, 2t_2)$

$$\text{slope} = \frac{2(t_2 - t_1)}{t_2^2 - t_1^2} = \frac{2}{t_1 + t_2}$$

equation of circle will be

$$(x - t_1^2)(x - t_2^2) + (y - 2t_1)(y - 2t_2) = 0$$

$$x^2 + y^2 - x(t_1^2 + t_2^2) - 2y(t_1 + t_2) + t_1^2 t_2^2 + 4t_1 t_2 = 0$$

As it touches x axis so

$$t_1^2 t_2^2 + 4t_1 t_2 = \frac{(t_1^2 + t_2^2)^2}{4}$$

$$4t_1^2 t_2^2 + 16t_1 t_2 = t_1^4 + t_2^4 + 2t_1^2 t_2^2$$

$$(t_1^2 - t_2^2)^2 = 16t_1 t_2 \quad \dots(1)$$

AB is diameter so.

$$(t_1^2 - t_2^2)^2 + 4(t_1 - t_2)^2 = 4r^2 \quad \dots(2)$$

from (1) and (2)

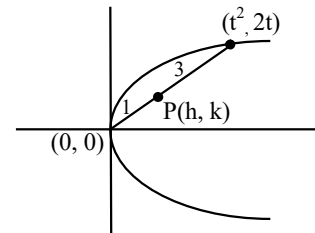
$$4t_1 t_2 + (t_1 - t_2)^2 = r^2$$

$$(t_1 + t_2)^2 = r^2$$

$$t_1 + t_2 = \pm r$$

$$\therefore \text{slope} = \pm \frac{2}{r}$$

13.[C]



$$h = \frac{t^2}{4}, k = \frac{2t}{4}$$

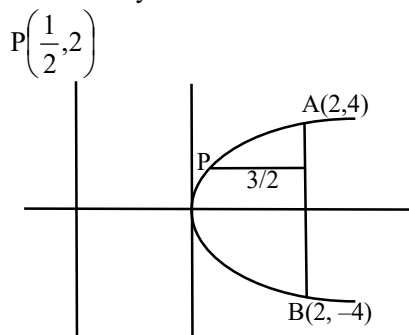
$$t^2 = 4h, t = 2k$$

$$\text{so } 4k^2 = 4h$$

$$k^2 = h$$

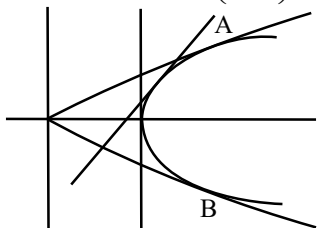
hence required locus is  $y^2 = x$

- 14.[A] It is property that area of triangle formed by joining three points lying on parabola is twice the area of triangle formed by tangent at these points  
**Alternate:**  $y^2 = 8x$



$$\Delta_1 = \frac{1}{2} |\text{Base} \times \text{height}| = \frac{1}{2} \times \frac{3}{2} \times 8 = 6$$

Equation of tangent at  $P\left(\frac{1}{2}, 2\right)$

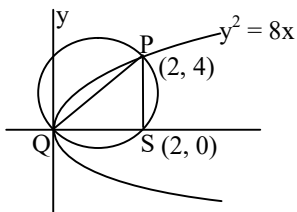


$$y(2) = 4 \left(x + \frac{1}{2}\right) \Rightarrow y = 2x + 1$$

Tangent at A :  $y = x + 2$ , tangent at B ;  $y = -x - 2$   
 Point of intersection L(-2, 0), M(1, 3), N(-1, -1)  
 $\Delta_2 = 3$

$$\therefore \frac{\Delta_1}{\Delta_2} = \frac{6}{3} = 2$$

- 15.[4]  $(x-1)^2 + (y-2)^2 = (\sqrt{5})^2$



$$x^2 + 8x - 2x - 4.2\sqrt{2x} = 0$$

$$x^2 + 6x - 8\sqrt{2x} = 0$$

$$x^{3/2} + 6x^{1/2} - 8\sqrt{2} = 0$$

$$x^{1/2} = t$$

$$t^3 + 6t - 8\sqrt{2} = 0$$

$$(t - \sqrt{2})(t^2 - \sqrt{2}t + 4) = 0$$

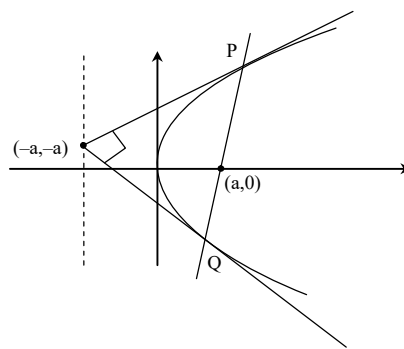
$$t = \sqrt{2} \quad x = 2$$

$$y = 4$$

$$P(2, 4) \quad Q(0, 0) \quad S(2, 0)$$

$$\text{Area } \Delta PQS = \frac{1}{2} \times 2 \times 4 = 4$$

- 16.[D] As tangents drawn at end points of focal chord intersects at directrix



So solving  $y = 2x + a$ , and  $x = -a$  we get  $(-a, -a)$

Equation of PQ T = 0

$$(-a)y - 2a(x - a) = 0 \Rightarrow 2x + y - 2a = 0$$

Solving it with parabola

$$y^2 - 4ax \left(\frac{2x + y}{2a}\right) = 0 \Rightarrow y^2 - 4x^2 - 2xy = 0$$

$$\Rightarrow m^2 - 2m - 4 = 0 \Rightarrow m_1 + m_2 = 2, m_1 m_2 = -4$$

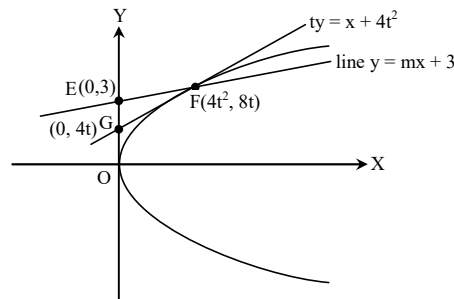
$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2} = \frac{2\sqrt{5}}{3}$$

17.[B]  $PQ = \frac{4}{m^2} \sqrt{a(a - mc)(1 + m^2)}$

$$m = -2, c = 2a$$

$$PQ = \frac{4}{4} \sqrt{a(a + 4a)(1 + 4)} = 5a$$

- 18.[A]



$$\text{Area of } \Delta EFG = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 4t^2 \\ 3 & 4t & 8t \end{vmatrix} = 24t(-2t + 1)$$

$$t = 0$$

$$t = 1/2$$

not possible local point of maxima  
 so point E(0, 3), F(1, 4), G(0, 2)

$$\text{Slope} = m = \frac{4 - 3}{1 - 0} = 1$$

$$y_0 = 4, y_1 = 2$$

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 3 & 4 & 2 \end{vmatrix}$$

$$= \frac{1}{2} [1(2) + 3(-1)] = -\frac{1}{2}$$

$$\text{Area} = \frac{1}{2} \text{ unit}^2$$