JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

PROGRESSION

(PRACTICE SHEET)

LEVEL- 1

Question based on **Arithmetic Progression (A.P.)**

Q.1 10^{th} term of the progression – 4 – 1+ 2 + 5 +.... is-
(A) – 23 (B) 23 (C) – 32 (D) 32

Q.2 If 4^{th} term of an AP is 64 and its 54^{th} term is – 61, then its common difference is –
(A) 5/2 (B) – 5/2 (C) 3/50 (D) – 3/50

Q.3 Which term of the series 3 + 8 + 13 + 18 + ... is 498-
(A) 95^{th} (B) 100^{th} (C) 102^{th} (D) 101^{th}

Q.4 The number of terms in the series 101 + 99 + 97 + ...+ 47 is-
(A) 25 (B) 28 (C) 30 (D) 20

Q.5 If $(m + 2)^{th}$ term of an A.P. is $(m + 2)^2 – m^2$, then its common difference is-
(A) 4 (B) – 4 (C) 2 (D) – 2

Q.6 If m^{th} terms of the series 63 + 65 + 67 + 69 + ... and 3 + 10 + 17 + 24 + ... be equal, then m =
(A) 11 (B) 12 (C) 13 (D) 15

Q.7 If the 9^{th} term of an A.P. be zero, then the ratio of its 29^{th} and 19^{th} term is-
(A) 1 : 2 (B) 2 : 1 (C) 1 : 3 (D) 3 : 1

Q.8 If fourth term of an A.P. is thrice its first term and seventh term – 2 (third term) = 1, then its common difference is-
(A) 1 (B) 2 (C) – 2 (D) 3

Q.9 If p^{th}, q^{th} and r^{th} terms of an A.P. are a, b and c respectively, then a(q – r) + b (r – p) + c (p – q) is equal to -
(A) 0 (B) 1
(C) a + b + c (D) p + q + r

Q.10 The 19^{th} term from the end of the series 2 + 6 + 10 + ...+ 86 is –
(A) 6 (B) 18 (C) 14 (D) 10

Q.11 In the following two A.P.'s how many terms are identical?
2, 5, 8, 11.... to 60 terms, 3, 5, 7, ... 50 terms
(A) 15 (B) 16 (C) 17 (D) 18

Q.12 The first term of an A.P. is 2 and common difference is 4. The sum of its 40 terms will be –
(A) 3200 (B) 1600
(C) 200 (D) 2800

Q.13 If n^{th} term of an AP is 1/3 (2n + 1), then the sum of its 19 terms is-
(A) 131 (B) 132 (C) 133 (D) 134

Q.14 The sum of numbers lying between 10 and 200 which are divisible by 7 will be-
(A) 2800 (B) 2835
(C) 2870 (D) 2849

Q.15 If the sum of n terms of an AP is $2n^2 + 5n$, then its nth term is-
(A) 4n-3 (B) 4n + 3
(C) 3n + 4 (D) 3n – 4

Q.16 If the ratio of sum of n terms of two A.P's is (3n + 8) : (7n + 15), then the ratio of 12^{th} terms is-
(A) 16 : 7 (B) 7 :16
(C) 7 : 12 (D) 12 : 5

Q.17 If the ratio of the sum of n terms of two AP's is 2n : (n + 1), then ratio of their 8^{th} terms is-
(A) 15 : 8 (B) 8 : 13
(C) n : (n– 1) (D) 5 : 17

Q.18 The sum of three consecutive terms of an increasing A.P. is 51. If the product of the first and third of these terms be 273, then third term is-
(A) 13 (B) 17 (C) 21 (D) 9

Q.19 If we divide 20 into four parts which are in A.P. such that product of the first and the fourth is to the product of the second and third is the same as 2 : 3, then the smallest part is-
(A) 1 (B) 2 (C) 3 (D) 4

Q.20 Three numbers are in A.P. The product of the extremes is 5 times the mean, also the sum of the two largest is 8 times the least, the numbers are-

(A) 3, 9, 15 (B) 6, 18, 30

(C) 3, 8, 13 (D) 6, 16, 26

Q.21 If the angles of a quadrilateral are in A.P. whose common difference is 10º, then the angles of the quadrilateral are-

(A) 65º, 85º, 95º, 105º

(B) 75º, 85º, 95º, 105º

(C) 65º, 75º, 85º, 95º

(D) 65º, 95º, 105º, 115º

Q.22 Three numbers are in A.P., If their sum is 33 and their product is 792, then the smallest of these numbers is –

(A) 14 (B) 11 (C) 8 (D) 4

Q.23 The sum of first four terms of an A.P. is 56 and the sum of its last four terms is 112. If its first term is 11, then number of its terms is-

(A) 10 (B) 11

(C) 12 (D) None of these

Q.24 If the numbers a, b, c, d, e form an A.P., then the value of $a - 4b + 6c - 4d + e$ is-

(A) 1 (B) 2

(C) 0 (D) None of these

Q.25 If $a^2(b + c), b^2(c + a), c^2(a + b)$ are in A.P., then a, b, c, are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.26 If a, b, c are in A.P., then

$\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.27 If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P. then a, b, c are also-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.28 If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then a, b, c will be in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.29 If $\frac{1}{p+q}, \frac{1}{r+p}, \frac{1}{q+r}$ are in A.P. then-

(A) p^2, q^2, r^2 are in A.P.

(B) q^2, p^2, r^2 are in A.P.

(C) q^2, r^2, p^2 are in A.P.

(D) p, q, r are in A.P.

Q.30 The middle term of the progression 4, 9, 14,....104 is-

(A) 44 (B) 49 (C) 59 (D) 54

Question based on Arithmetic Mean (A.M.)

Q.31 If x, y, z are in A.P. and A.M. of x and y is a and that to y and z is b, then A.M. of a and b is -

(A) x (B) y

(C) z (D) 1/2(x + y)

Q.32 If A_1, A_2 be two arithmetic means between 1/3 and 1/24, then their values are-

(A) 7/72, 5/36 (B) 17/72, 5/36

(C) 7/36, 5/72 (D) 5/72, 17/72

Q.33 The AM of 1, 3, 5, ..., (2n – 1) is –

(A) n + 1 (B) n + 2

(C) n^2 (D) n

Q.34 Given two numbers a and b, let A denotes the single A.M. and S denote the sum of n A.M.'s between a and b, then S/A depends on-

(A) n, a, b (B) n , b

(C) n, a (D) n

Question based on Geometrical Progression (G.P.)

Q.35 If the first term of a G.P. be 5 and common ratio be – 5, then which term is 3125 –

(A) 6th (B) 5th (C) 7th (D) 8th

Q.36 The fifth term of a GP is 81 and its 8th term is 2187, then its third term is-

(A) 3 (B) 9

(C) 27 (D) None of these

Q.37 In any G.P. the first term is 2 and last term is 512 and common ratio is 2, then 5th term from end is-
(A) 16 (B) 32
(C) 64 (D) None of these

Q.38 Which term of the progression

18, –12, 8, ... is $\frac{512}{729}$?
(A) 9th (B) 10th
(C) 8th (D) None of these

Q.39 If third term of a G.P is 4, then product of first 5 term is-
(A) 4^3 (B) 4^4
(C) 4^5 (D) None of these

Q.40 If third and seventh terms of a GP are 15 and 135 respectively, then its fifth term will be-
(A) 5 (B) 9 (C) 45 (D) 90

Q.41 For which values of x do the numbers 1, x^2, $6 - x^2$ taken in that order form a geometric progression-
(A) $x = \pm 2$ (B) $x = \pm\sqrt{2}$
(C) $x = \pm 3$ (D) $x = \pm\sqrt{3}$

Q.42 Three numbers a, b, 12 are in G.P. and a, b, 9 are in A.P., then a and b are –
(A) 3, 6 (B) – 3, 6
(C) 3, – 6 (D) – 3, – 6

Q.43 The second; third and sixth terms of an A.P. are consecutive terms of a G.P. The common ratio of the G.P. is-
(A) 1 (B) 3 (C) – 1 (D) – 3

Q.44 Total number of terms in the progression 96 + 48 + 24 + 12 + ...+ 3/16 is-
(A) 9 (B) 10 (C) 15 (D) 20

Q.45 The sum of the first 10 terms of a certain G.P. is equal to 244 times the sum of the first 5 terms. Then the common ratio is-
(A) 3 (B) 4 (C) 5 (D) None

Q.46 The sum of the infinite terms of
$1 - 1/3 + 1/3^2 - 1/3^3 + ...$ is-
(A) 3/4 (B) 4/3 (C) – 3/4 (D) – 4/3

Q.47 The sum $1 + \frac{2}{x} + \frac{4}{x^2} + \frac{8}{x^3} + ...$ (upto ∞) is finite if –
(A) $x < 2$ (B) $x > 2$
(C) $x < 1$ (D) $x < 1/2$

Q.48 If the sum to n terms of a series be $3(2^n - 1)$, then it is-
(A) A.P. (B) G.P.
(C) H.P. (D) None of these

Q.49 The value of $9^{1/3} . 9^{1/9} . 9^{1/27} ...$ upto ∞, is-
(A) 1 (B) 3
(C) 9 (D) None of these

Q.50 If $3 + 3\alpha + 3\alpha^2 + ... \infty = \frac{45}{8} (\alpha > 0)$; then α equals-
(A) 15/23 (B) 15/7
(C) 7/15 (D) 23/15

Q.51 If the sum of an infinite GP be 3 and the sum of the squares of its term is also 3, then its first term and common ratio are –
(A) 3/2, 1/2 (B) 1/2, 3/2
(C) 1, 1/2 (D) None of these

Q.52 Every term of an infinite GP is thrice the sum of all the successive terms. If the sum of first two terms is 15, then the sum of the GP is-
(A) 20 (B) 16 (C) 28 (D) 30

Q.53 A geometric progression consists of an even number of terms. The sum of all the terms is three times that of the odd terms, the common ratio of the progression will be-
(A) 1/2 (B) 2 (C) 3 (D) 1/3

Q.54 If first term of a decreasing infinite G.P. is 1 and sum is S, then sum of squares of its terms is-
(A) S^2 (B) $1/S^2$
(C) $S^2/(2S - 1)$ (D) $S^2/(2S + 1)$

Q.55 If sum of three numbers of a G.P. is 19 and their product is 216, then its c.r. is-
(A) 1/2 (B) 1/3 (C) 3/2 (D) 3/4

Q.56 If the product of three numbers in GP is 3375 and their sum is 65, then the smallest of these numbers is -
(A) 3 (B) 5 (C) 4 (D) 6

Q.57 If the product of three terms of G.P. is 512. If 8 added to first and 6 added to second term, so that number may be in A.P., then the numbers are-

(A) 2, 4, 8 (B) 4, 8, 16

(C) 3, 6, 12 (D) None of these

Q.58 In the four numbers first three are in G.P. and last three are in A.P. whose common difference is 6. If the first and last numbers are same, then first will be-

(A) 2 (B) 4 (C) 6 (D) 8

Q.59 Break the numbers 155 into three parts so that the obtained numbers form a G.P., the first term being less than the third one by 120-

(A) 5, 65, 125 (B) 10, 65, 120

(C) 5, 25, 125 (D) None of these

Q.60 Find three numbers in G.P. such that their sum is 14 and the sum of their squares is 84 -

(A) 3, 6, 12 (B) 2, 6, 18

(C) 1, 3, 9 (D) 2, 4, 8

Q.61 Determine the first term and the common ratio of the geometric progression, the sum of whose first and third terms is 40 and the second and fourth term is 80 -

(A) 8, 3 (B) 8, 2

(C) 7, 3 (D) 7, 2

Q.62 The sum of three positive numbers constituting an arithmetic progression is 15. If we add 1,4,19 to those numbers respectively. We get a geometric progression, then the numbers are-

(A) 2, 5, 8 (B) 8, 5, 2

(C) 5, 8, 2 (D) All of these

Q.63 The fractional value of $0.\dot{1}\dot{2}\dot{5}$ is-

(A) 125/999 (B) 23/990

(C) 61/550 (D) None of these

Q.64 If x, y, z are in G.P. then $x^2 + y^2$, $xy + yz$, $y^2 + z^2$ are in -

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.65 If a, b, c, d are in G.P. then a + b, b + c, c + d are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.66 If a, b, c are in G.P. then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in -

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Question based on Geometrical Mean (G.M.)

Q.67 If three geometric means be inserted between 2 and 32, then the third geometric mean will be-

(A) 8 (B) 4 (C) 16 (D) 12

Q.68 The product of three geometric means between 4 and 1/4 will be -

(A) 4 (B) 2 (C) – 1 (D) 1

Q.69 The ratio between the GM's of the roots of the equations $ax^2 + bx + c = 0$ and $\ell x^2 + mx + n = 0$ is-

(A) $\sqrt{\frac{b\ell}{an}}$ (B) $\sqrt{\frac{c\ell}{an}}$

(C) $\sqrt{\frac{an}{c\ell}}$ (D) $\sqrt{\frac{cn}{a\ell}}$

Q.70 If G be the geometric mean of x and y, then $\frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} =$

(A) G^2 (B) $\frac{1}{G^2}$ (C) $\frac{2}{G^2}$ (D) $3G^2$

Q.71 The A.M. of two numbers is 34 and GM is 16, the numbers are-

(A) 2 and 64 (B) 64 and 3

(C) 64 and 4 (D) None of these

Q.72 Two numbers are in the ratio 4 : 1. If their AM exceeds their GM by 2, then the numbers are-(A) 4, 1 (B) 16, 4

(C) 12, 3 (D) None of these

Q.73 a, b, c are in A.P. If x is the GM between a and b and y is the GM between b and c, then the A.M. between x^2 and y^2 will be-

(A) a^2 (B) b^2

(C) c^2 (D) None of these

Question based on Arithmetic-Geometrical Progression (A.G.P.)

Q.74 Sum to infinite of the series

$1 + \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + ...$ is-

(A) 5/4 (B) 6/5

(C) 25/16 (D) 16/9

Q.75 The sum of infinite terms of the progression $1 + 3x + 5x^2 + 7x^3 + ...(x < 1)$ is-

(A) $\frac{1+x}{1-x}$ (B) $\left(\frac{1+x}{1-x}\right)^2$

(C) $\frac{1+x}{(1-x)^2}$ (D) None of these

Q.76 $1+2(1+1/n) + 3(1+1/n)^2 + ... \infty$ terms, equals-

(A) $n(1+1/n)$ (B) n^2

(C) $n(1+1/n)^2$ (D) None of these

Not in AIEEE syllabus

Question based on Harmonic Progression (H.P.)

Q.77 If fourth term of an HP is 3/5 and its 8^{th} term is 1/3, then its first term is–

(A) 2/3 (B) 3/2

(C) 1/4 (D) None of these

Q.78 The fifth term of the H.P. $2, 2\frac{1}{2}, 3\frac{1}{3}$,..... will be-

(A) $5\frac{1}{5}$ (B) $3\frac{1}{5}$

(C) $\frac{1}{10}$ (D) 10

Q.79 If first and second terms of a HP are a and b, then its n^{th} term will be-

(A) $\frac{ab}{a+(n-1)ab}$ (B) $\frac{ab}{b+(n-1)(a+b)}$

(C) $\frac{ab}{b+(n-1)(a-b)}$ (D) None of these

Q.80 If the m^{th} term of a H.P. be n and n^{th} term be m, then the r^{th} term will be-

(A) $\frac{r}{mn}$ (B) $\frac{mn}{r+1}$

(C) $\frac{mn}{r}$ (D) $\frac{mn}{r-1}$

Q.81 If b + c, c + a, a + b are in H.P., then a^2, b^2, c^2 will be in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.82 If a, b, c be in H.P. then $a-\frac{b}{2}, \frac{b}{2}, c-\frac{b}{2}$ will be in -

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.83 If a, b, c are in A.P., then

$\frac{bc}{ca+ab}, \frac{ca}{bc+ab}, \frac{ab}{bc+ca}$ are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Question based on Harmonic Mean (H.M.)

Q.84 The HM between 1/21 and – 1/5 is -

(A) $\frac{1}{8}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$

Q.85 If H is H.M. between two numbers a and b, then $\frac{1}{H-a} + \frac{1}{H-b}$ equals -

(A) a – b (B) a + b

(C) $\frac{1}{a} - \frac{1}{b}$ (D) $\frac{1}{a} + \frac{1}{b}$

Q.86 The HM between $\frac{a}{b}$ and $\frac{b}{a}$ is-

(A) $\frac{2ab}{a+b}$ (B) $\frac{2a^2b^2}{a^2+b^2}$

(C) $\frac{2ab}{a^2+b^2}$ (D) $\frac{2a^2b^2}{a+b}$

Q.87 If 4 HM's be inserted between 2/3 and 2/13, then the second HM is-

(A) 2/5 (B) 2/7

(C) 2/11 (D) 2/17

Question based on **Relation between A.M., G.M. & H.M.**

Q.88 If A,G & 4 are A.M, G.M & H.M of two numbers respectively and $2A + G^2 = 27$, then the numbers are-

(A) 8, 2 (B) 8, 6 (C) 6, 3 (D) 6, 4

Q.89 If x, y, z are AM, GM and HM of two positive numbers respectively, then correct statement is -

(A) $x < y < z$ (B) $y < x < z$

(C) $z < y < x$ (D) $z < x < y$

Q.90 If sum of A.M. and H.M. between two positive numbers is 25 and their GM is 12, then sum of numbers is-

(A) 9 (B) 18

(C) 32 (D) 18 or 32

Q.91 The A.M. between two positive numbers exceeds the GM by 5, and the GM exceeds the H.M. by 4. Then the numbers are-

(A) 10, 40 (B) 10, 20

(C) 20, 40 (D) 10, 50

Question based on **Special Series**

Q.92 Sum of the series 1+ 3+ 7 + 15 + 31+ .... to n terms is-

(A) $2^n - 2 - n$ (B) $2^{n+1}+2 + n$

(C) $2^{n+1}-2-n$ (D) None of these

Q.93 The number of terms in the sequence 1, 3, 6, 10, 15, 21, ... , 5050 is-

(A) 50 (B) 100

(C) 101 (D) 105

Q.94 Sum of n terms of $1 + (1 + x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + ...$ is-

(A) $\frac{1-x^n}{1-x}$

(B) $\frac{x(1-x^n)}{1-x}$

(C) $\frac{n(1-x)-x(1-x^n)}{(1-x)^2}$

(D) None of these

Q.95 $\sum_{k=1}^{n} k^3$ is equal to-

(A) $2\sum_{k=1}^{n} k^2$ (B) $\left(\sum_{k=1}^{n} k\right)^2$

(C) $\left(\sum_{k=1}^{n} k\right)^3$ (D) $3\sum_{k=1}^{n} k^2$

LEVEL- 2

Q.1 Find the sum of all the even positive integers less than 200 which are not divisible by 6-

(A) 6535 (B) 6539
(C) 6534 (D) 6532

Q.2 The sum of n terms of the series

$\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots$ is-

(A) $n \log \left(\frac{a}{b}\right)$

(B) n log (ab)

(C) $\frac{n^2}{2} \log \frac{a}{b} + \frac{n}{2} \log (ab)$

(D) $\frac{n^2}{2} \log \frac{a}{b} - \frac{n}{2} \log (ab)$

Q.3 The sum of 40 terms of the series

1+ 2 + 3 + 4 + 5 + 8 + 7+ 16 + 9 + ... is-

(A) $398 + 2^{20}$ (B) $398 + 2^{21}$
(C) $398 + 2^{19}$ (D) None of these

Q.4 If first and $(2n - 1)^{th}$ terms of an A.P., G.P. and H.P. are equal and their n^{th} terms are respectively a, b, c, then -

(A) a = b = c (B) a + c = b
(C) $ac - b^2 = 0$ (D) None of these

Q.5 Certain numbers appear in both the arithmetic progressions 17, 21, 25.... and 16, 21, 26.... find the sum of the first two hundred terms appearing in both-

(A) 4022 (B) 402200
(C) 201100 (D) 398000

Q.6 If S denotes the sum to infinity and S_n the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$, such that $S - S_n < \frac{1}{1000}$, then the least value of n is-

(A) 11 (B) 9
(C) 10 (D) 8

Q.7 The sum of 10 terms of the series

$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \ldots$ is -

(A) $\left(\frac{x^{20} - 1}{x^2 - 1}\right)\left(\frac{x^{22} + 1}{x^{20}}\right) + 20$

(B) $\left(\frac{x^{18} - 1}{x^2 - 1}\right)\left(\frac{x^{11} + 1}{x^9}\right) + 20$

(C) $\left(\frac{x^{18} - 1}{x^2 - 1}\right)\left(\frac{x^{11} - 1}{x^9}\right) + 20$

(D) None of these

Q.8 If 0 < x, y, a, b < 1, then the sum of the infinite terms of the series

$\sqrt{x}\,(\sqrt{a} + \sqrt{x}) + \sqrt{x}\,(\sqrt{ab} + \sqrt{xy}) + \sqrt{x}\,(b\sqrt{a} + y\sqrt{x}) + \ldots$ is-

(A) $\frac{\sqrt{ax}}{1 + \sqrt{b}} + \frac{x}{1 + \sqrt{y}}$ (B) $\frac{\sqrt{x}}{1 + \sqrt{b}} + \frac{x}{1 + \sqrt{y}}$

(C) $\frac{\sqrt{x}}{1 - \sqrt{b}} + \frac{\sqrt{x}}{1 - \sqrt{y}}$ (D) $\frac{\sqrt{ax}}{1 - \sqrt{b}} + \frac{x}{1 - \sqrt{y}}$

Q.9 If sum of 3 terms of a G.P. is S. product is P, and sum of reciprocal of its terms is R, then $P^2 R^3$ equals to -

(A) S (B) S^3
(C) $2S^2$ (D) S^2/R

Q.10 If A and G are respectively A.M. and G.M. of roots of a quadratic equation, then it is-

(A) $x^2 + 2Ax + G^2 = 0$
(B) $x^2 - 2Ax + G^2 = 0$
(C) $x^2 - Ax + G = 0$
(D) None of these

Q.11 If t_n be the n^{th} term of an A.P. and if $t_7 = 9$, then the value of the c.d. that would make $t_1t_2t_7$ least is-

(A) 33/40 (B) 33/20
(C) 33/10 (D) None of these

Q.12 If m^{th} terms of the series 63 + 65 + 67 + 69 + ... and 3 + 10 + 17 + 24 + ... be equal, then m =

(A) 11 (B) 12 (C) 13 (D) 15

Q.13 A ball falls from a height of 100 mts. on a floor. If in each rebound it describes 4/5 height of the previous falling height, then the total distance travelled by the ball before coming to rest is-

(A) ∞ (B) 500 mts

(C) 1000 mts (D) 900 mts

Q.14 If A,G and H are respectively A.M., G.M., and H.M. of three positive numbers a, b and c, then the equation whose roots are a, b and c is given by-

(A) $x^3 - 3Ax^2 + 3G^3 x + G^3 = 0$

(B) $x^3 - 3Ax^2 + 3(G^3/H) x - G^3 = 0$

(C) $x^3 + 3Ax^2 + 3(G^3/H) x - G^3 = 0$

(D) $x^3 - 3Ax^2 - 3(G^3/H) x + G^3 = 0$

Q.15 The G.M. of roots of the equation $x^2 - 2ax + b^2 = 0$ is equal to which type of mean of roots of $x^2 - 2bx + a^2 = 0$?

(A) A.M. (B) G.M.

(C) H.M. (D) None of these

Q.16 If a and ℓ be the first and last term of an A.P. and S be the sum of its all terms; then its common difference is-

(A) $\frac{\ell^2 + a^2}{2S - \ell - a}$ (B) $\frac{\ell^2 - a^2}{2S - \ell - a}$

(C) $\frac{\ell^2 - a^2}{2S + \ell + a}$ (D) None of these

Q.17 If x,y,z are in A.P. , then magnitude of its common difference is-

(A) $\sqrt{(x^2 - yz)}$ (B) $\sqrt{(y^2 - zx)}$

(C) $\sqrt{(z^2 - xy)}$ (D) None of these

Q.18 If sum of infinite G.P. is x and sum of square of its terms is y, then common ratio is-

(A) $\frac{x^2 + y}{x^2 - y}$ (B) $\frac{x^2 - y}{x^2 + y}$

(C) $\frac{x^2 + y^2}{x^2 - y^2}$ (D) $\frac{x^2 - y^2}{x^2 + y^2}$

Q.19 In the following two A.P.'s how many terms are identical ?

2, 5, 8, 11.... to 60 terms, 3, 5, 7, ... 50 terms

(A) 15 (B) 16 (C) 17 (D) 18

Q.20 If $1 + r + r^2 + ... + r^n = (1+ r) (1+ r^2) (1+ r^4) (1+ r^8)$, then the value of n is-

(A) 13 (B) 14 (C) 15 (D) 16

Q.21 In an A.P. of which a is the first term, if the sum of the first p terms is zero, then the sum of the next q term is-

(A) $\frac{a(p-q)q}{p-1}$ (B) $-\frac{a(p+q)q}{p-1}$

(C) $\frac{a(p+q)p}{p-1}$ (D) None of these

Q.22 Find sum of the series

$1.3^2 + 2.5^2 + 3.7^2 + ...$ to 20 terms-

(A) 188090 (B) 94045

(C) 325178 (D) 812715

Q.23 If a, b, c are in A.P. and $x = 1 + a + a^2 + ...$, $y = 1 + b + b^2 + ...$ and $z = 1 + c + c^2 + ...$, (where a, b, c < 1)), then x, y, z are in-

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.24 The sum of 10 terms of the series.

0.7 + .77 + .777 + ... is-

(A) $\frac{7}{9}\left(89 + \frac{1}{10^{10}}\right)$ (B) $\frac{7}{81}\left(89 + \frac{1}{10^{10}}\right)$

(C) $\frac{7}{81}\left(89 + \frac{1}{10^{9}}\right)$ (D) None of these

Q.25 The value of ${}_a\log_b{}^x$ where a = 0.2, $b = \sqrt{5}$, $x = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\infty\right)$, is-

(A) 1 (B) 2 (C) 1/2 (D) 4

Q.26 Find the sum of the series up to n term

1.3.5 + 3.5.7 + 5.7.9 + ..

(A) $8n^3 + 12n^2 - 2n - 3$

(B) $n (8n^3 + 11n^2 - n - 3)$

(C) $n(2n^3 + 8n^2 + 7n - 2)$

(D) None of these

Q.27 If A.M. between p and q ($p \geq q$) is two times the GM, then p : q is-

(A) 1 : 1

(B) 2 : 1

(C) $(2 + \sqrt{3}) : (2 - \sqrt{3})$

(D) 3 :1

Q.28 The sum of the first ten terms of the geometric progression is S_1 and the sum of the next ten terms (11th through 20th) is S_2. then the common ratio will be-

(A) $\pm\sqrt[10]{\frac{S_1}{S_2}}$ (B) $\pm\sqrt{\frac{S_2}{S_1}}$

(C) $\pm\sqrt[10]{\frac{S_2}{S_1}}$ (D) $\sqrt{\frac{S_1}{S_2}}$

Q.29 If $x = a + (a/r) + (a/r^2) + ...$,

$y = b - (b/r) + (b/r^2) - ....$ and

$z = c + (c/r^2) + (c/r^4) + ...$, then (xy/z) is-

(A) $\frac{ab}{c}$ (B) $\frac{bc}{a}$ (C) $\frac{ca}{b}$ (D) abc

Q.30 The series of natural numbers is divided into groups as follows ; (1), (2, 3), (4, 5, 6), (7, 8, 9, 10) and so on. Find the sum of the numbers in the nth group is-

(A) $\frac{1}{2}[n(n^2+1)]$ (B) $\frac{n(n^2+1)}{4}$

(C) $\frac{2n(n+1)}{3}$ (D) $\frac{n^2(n+1)}{2}$

Q.31 The sum to infinity of the following series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ...$ shall be-

(A) ∞ (B) 1

(C) 0 (D) None of these

Q.32 The number of terms in the sequence 1, 3, 6, 10, 15, 21,...., 5050 is-

(A) 50 (B) 100 (C) 101 (D) 105

Q.33 The sum of the infinite series $1^2 + 2^2 x + 3^2 x^2 + ...$ is-

(A) $(1+x)/(1-x)^3$ (B) $(1+x)/(1-x)$

(C) $x/(1-x)^3$ (D) $1/(1-x)^3$

Q.34 If the sum of four numbers in A.P. be 48 and that the product of the extremes is to the product of the means is 27 to 35 then the numbers are-

(A) 3, 9, 15, 21 (B) 9, 5, 7, 3

(C) 6, 10, 14, 18 (D) None of these

Q.35 The sum of infinite series $1 - \frac{3}{2} + \frac{5}{4} - \frac{7}{8} + ...$ is-

(A) $\frac{2}{9}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{9}$ (D) $\frac{9}{2}$

Q.36 If a, b, c are in G.P. and A.M. between a, b and b, c are respectively p and q, then $(a/p) + (c/q)$ is equal to-

(A) 0 (B) 1 (C) 2 (D) 1/2

Q.37 The solution of the equation $(8)^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+.....)} = 4^3$ in the interval $(-\pi, \pi)$ are-

(A) $\pm\frac{\pi}{3}, \pm\frac{\pi}{6}$ (B) $\pm\frac{\pi}{3}, \pm\pi$

(C) $\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$ (D) None of these

Q.38 If a, b, c, d are in G.P., then the value of $(a - c)^2 + (b - c)^2 + (b - d)^2 - (a - d)^2$ is-

(A) 0 (B) 1

(C) a + d (D) a – d

Q.39 The third term of an A.P. is 9 and the difference of the seventh and the second term is 20. If the number 2001 is the nth term of the sequence then n is-

(A) equal to 499

(B) is equal to 500

(C) equal to 501

(D) can have no value

Q.40 Given the geometric progression 3, 6,12, 24,..... the term 12288 would occur as the-

(A) 11th term (B) 12th term (C) 13th term (D) 14th term

LEVEL- 3

Q.1 The maximum sum of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + \ldots\ldots.$ is -

(A) 310 (B) 300

(C) 320 (D) None of these

Q.2 Let a, b be the roots of $x^2 - 3x + p = 0$ and let c, d be the roots of $x^2 - 12x + q = 0$, where a, b, c, d form an increasing G.P. Then the ratio of q + p : q – p is equal to -

(A) 8 : 7 (B) 11 : 10

(C) 17 : 15 (D) None of these

Q.3 If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ $(x \neq 0)$, then a, b, c, d are in -

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.4 The sum of the first n terms of the series $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots\ldots\ldots..$ is -

(A) $\frac{6n}{n+1}$ (B) $\frac{9n}{n+1}$

(C) $\frac{12n}{n+1}$ (D) $\frac{15n}{n+1}$

Q.5 If $\sum_{r=1}^{n} t_r = 2(3^n - 1)\ \forall n \geq 1$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{t_r} =$

(A) 3 (B) $\frac{3}{2}$

(C) $\frac{3}{4}$ (D) $\frac{3}{8}$

Q.6 Let the sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ form an A.P., then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots + a_{2n-1}^2 - a_{2n}^2$ is equal to -

(A) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (B) $\frac{2n}{n-1}(a_{2n}^2 - a_1^2)$

(C) $\frac{n}{n+1}(a_1^2 + a_{2n}^2)$ (D) None of these

Q.7 If 1, $\log_9(3^{1-x} + 2)$ and $\log_3(4.3^x - 1)$ are in A.P., then x is equal to -

(A) $\log_4 3$ (B) $\log_3 4$

(C) $1 - \log_3 4$ (D) $\log_3 0.25$

Q.8 If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, their cubes respectively, then $\frac{S_3(1+8S_1)}{S_2^{\,2}}$ is equal to -

(A) 1 (B) 3 (C) 9 (D) 10

Q.9 The sum of three consecutive terms in a geometric progression is 14. If 1 is added to the first and the second terms and 1 is subtracted from the third, the resulting new terms are in arithmetic progression. Then the lowest of the original terms is -

(A) 1 (B) 2 (C) 4 (D) 8

Q.10 If S_n denotes the sum of n terms of an A.P., then $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is equal to -

(A) 0 (B) 1 (C) 1/2 (D) 2

Q.11 If $a_1, a_2, a_3, \ldots\ldots, a_{24}$ are in A.P. and $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + \ldots\ldots + a_{23} + a_{24}$ is equal to-

(A) 909 (B) 75 (C) 750 (D) 900

Q.12 The value of x + y + z is 15 if a, x, y, z, b are in A.P. while the value of $\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$ is $\frac{5}{3}$ if a, X, Y, Z, b are in H.P., then a and b are-

(A) 1, 9 (B) 3, 7

(C) 7, 3 (D) 9, 1

Q.13 If $I_n = \int_0^{\pi/4} \tan^n x \sec^2 x \, dx$, then $I_1, I_2, I_3, \ldots$ are in -

(A) A. P. (B) G.P.

(C) H.P. (D) None of these

Q.14 A G.P. consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of the terms at the even places is S_2, then S_2/S_1 is -

(A) Dependent on a
(B) Independent of r
(C) Independent of a and r
(D) Dependent on r

Q.15 If $x^{18} = y^{21} = z^{28}$, then 3, $3 \log_y x$, $3 \log_z y$, $7 \log_x z$ are in -

(A) A.P. (B) G.P. (C) H.P. (D) None

Q.16 The sum of integers from 1 to 100 that are divisible by 2 or 3, is -

(A) 3300 (B) 3330
(C) 3000 (D) None of these

Q.17 The sum of an infinitely decreasing G.P. is equal to 4 and the sum of the cubes of its terms is equal to $\frac{64}{7}$. Then 5th term of the progression is -

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$
(C) $\frac{1}{16}$ (D) $\frac{1}{32}$

Q.18 If the sum of the first 2n terms of the A.P. 2, 5, 8, ... is equal to the sum of first n terms of the A.P. 57, 59, 61, ..., then n equals -

(A) 10 (B) 11
(C) 12 (D) 13

Q.19 Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \ldots\ldots + \frac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots\ldots n^3}$; n = 1, 2, 3, ... Then S_n is not greater than-

(A) 1/2 (B) 1
(C) 2 (D) 4

Q.20 The number of common terms to the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ... 466 is -

(A) 21 (B) 19
(C) 20 (D) 91

Passage Based Questions (Q. 21 - 23)

Suppose a series of n terms is given by $S_n = t_1 + t_2 + t_3 + \ldots\ldots + t_n$,
Then $S_{n-1} = t_1 + t_2 + t_3 + \ldots\ldots + t_{n-1}$, n > 1 (n ∈ N) subtracting, we get $S_n - S_{n-1} = t_n$, n > 1 Further if we put n = 1 in the first sum then $S_1 = t_1$ .Thus we can write $t_n = S_n - S_{n-1}$ and $t_1 = S_1$. The above result can be used to find the terms of any kind of series, independent of its nature, provided the sum to first n terms is given.

Q.21 If sum of n terms of a series is of the form $an^2 + bn$, where a and b are constants, then the fourth term of the series is–

(A) a + b (B) 7a + b
(C) 9 a + 3 b (D) 16a + 4b

Q.22 The sum of n terms of a series is $a.2^n - b$, where a and b are constants then the series is –

(A) A.P.
(B) G.P.
(C) A.G.P.
(D) G.P. from second term onwards

Q.23 If the sum to n terms of a series is given by $\frac{n(n+1)(n+2)}{6}$ then the n^{th} term of the series is –

(A) Σn^2 (B) $(\Sigma n)^2$
(C) Σn (D) $\Sigma n + n$

Questions based on Statements (Q. 24-28)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement- I and Statement- II are true, and Statement - II is the correct explanation of Statement– I.
(B) If both Statement - I and Statement - II are true but Statement - II is not the correct explanation of Statement – I.
(C) If Statement - I is true but Statement - II is false.

(D) If Statement - I is false but Statement - II is true.

Q.24 **Statement I :** If A and G be the A.M and G.M. between two positive real numbers a and b then a, b are given by $A \pm \sqrt{(A+G)(A-G)}$.

Statement II : Using $x^2 - (a + b)x + ab = 0$; where $a + b = 2A$, $ab = G^2$, we calculate x.

Q.25 **Statement I :** The sum of all numbers of the form n^3 which lie between 100 and 10,000 is 53261.

Statement II : If $= \frac{a-b}{b-c} = \frac{a}{c}$ then a, b, c are in G.P.

Q.26 **Statement I :** The number of terms of the A.P. 3, 7, 11, 15, ... to be taken so that the sum is 465 is 15.

Statement II : The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is 2632.

Q.27 **Statement I :** If a, b, c, x are all real numbers and $(a^2 + b^2)\, x^2 - 2b\,(a + c)x + (b^2 + c^2) = 0$ then a, b, c are in G.P. and x is their common ratio.

Statement II : If the ratio of the sum of m terms and n terms of an A.P. is $m^2 : n^2$ then the ratio of its m^{th} and n^{th} terms will be $(2m - 1) : (2n-1)$

Q.28 **Statement I :** 1 + 3 + 7 + 13 + ... up to n terms $= \frac{n(n^2+2)}{3}$.

Statement II : $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is Harmonic mean of a and b if $n = -\frac{1}{2}$.

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 The sum of the series $1^3 - 2^3 + 3^3 - ......+ 9^3 =$ **[AIEEE- 2002]**

(A) 300 (B) 125 (C) 425 (D) 0

Q.2 If the sum of an infinite GP is 20 and sum of their square is 100 then common ratio will be = **[AIEEE- 2002]**

(A) 1/2 (B) 1/4 (C) 3/5 (D) 1

Q.3 If the third term of an A.P. is 7 and its 7^{th} term is 2 more than three times of its 3^{rd} term, then sum of its first 20 terms is- **[AIEEE- 2002]**

(A) 228 (B) 74 (C) 740 (D) 1090

Q.4 Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation- **[AIEEE- 2004]**

(A) $x^2 + 18x + 16 = 0$ (B) $x^2 - 18x + 16 = 0$
(C) $x^2 + 18x - 16 = 0$ (D) $x^2 - 18x - 16 = 0$

Q.5 Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals- **[AIEEE- 2004]**

(A) 0 (B) 1 (C) 1/mn (D) $\frac{1}{m} + \frac{1}{n}$

Q.6 The sum of the first n terms of the series $1^2 + 2.\ 2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is- **[AIEEE- 2004]**

(A) $\frac{3n(n+1)}{2}$ (B) $\frac{n^2(n+1)}{2}$

(C) $\frac{n(n+1)^2}{4}$ (D) $\left[\frac{n(n+1)}{2}\right]^2$

Q.7 If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in - **[AIEEE- 2005]**

(A) GP
(B) AP
(C) Arithmetic - Geometric Progression
(D) HP

Q.8 Let $a_1, a_2, a_3, ...$ be terms of an A.P. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, $p \neq q$ then $\frac{a_6}{a_{21}}$ equals – **[AIEEE- 2006]**

(A) $\frac{7}{2}$ (B) $\frac{2}{7}$ (C) $\frac{11}{41}$ (D) $\frac{41}{11}$

Q.9 If $a_1, a_2, ... a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to – **[AIEEE- 2006]**

(A) $(n-1)(a_1 - a_n)$ (B) na_1a_n
(C) $(n-1)a_1a_n$ (D) $n(a_1 - a_n)$

Q.10 In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- **[AIEEE- 2007]**

(A) $\frac{1}{2}(1 - \sqrt{5})$ (B) $\frac{1}{2}\sqrt{5}$

(C) $\frac{1}{2}\sqrt{5}$ (D) $\frac{1}{2}(\sqrt{5} - 1)$

Q.11 The sum to infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ **[AIEEE- 2009]**

(A) 2 (B) 3 (C) 4 (D) 6

Q.12 A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11},...$ are in an AP with common difference – 2, then the time taken by him to count all notes is - **[AIEEE- 2010]**

(A) 24 minutes (B) 34 minutes
(C) 125 minutes (D) 135 minutes

Q.13 A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after : **[AIEEE- 2011]**

(A) 18 months (B) 19 months
(C) 20 months (D) 21 months

Q.14 The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is – **[JEE Main - 2013]**

(A) $\frac{7}{81}(179 + 10^{-20})$ (B) $\frac{7}{9}(99 + 10^{-20})$
(C) $\frac{7}{81}(179 - 10^{-20})$ (D) $\frac{7}{9}(99 - 10^{-20})$

Q.15 If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then – **[JEE Main - 2013]**

(A) 6x = 3y = 2z (B) 6x = 4y = 3z
(C) x = y = z (D) 2x = 3y = 6z

SECTION - B

Q.1 Let a_n be n^{th} term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$ then the common ratio is- **[IIT-1992]**

(A) α/β (B) β/α
(C) $\sqrt{(\alpha/\beta)}$ (D) $\sqrt{(\beta/\alpha)}$

Q.2 If the sum of first n natural numbers is 1/5 times the sum of their squares, then the value of n is- **[IIT-1992]**

(A) 5 (B) 6 (C) 7 (D) 8

Q.3 If ratio of H.M. and G.M. between two positive numbers a and b (a > b) is 4: 5, then a : b is- **[IIT-1992]**

(A) 1: 1 (B) 2 : 1
(C) 4 : 1 (D) 3 : 1

Q.4 $\log_3 2$, $\log_6 2$ and $\log_{12} 2$ are in - **[IIT-1993]**

(A) A.P. (B) G.P.
(C) H.P. (D) None of these

Q.5 For $0 < \phi < \pi/2$ if $x = \sum_{n=0}^{\infty}\cos^{2n}\phi$, $y = \sum_{n=0}^{\infty}\sin^{2n}\phi$; $z = \sum_{n=0}^{\infty}\cos^{2n}\phi\,\sin^{2n}\phi$, then- **[IIT-1993]**

(A) xyz = xz + y (B) xyz = xy + z
(C) xyz = yz + x (D) None of these

Q.6 If $\ell n(a + c)$, $\ell n(c - a)$, $\ell n(a - 2b + c)$ are in A.P., then- **[IIT-1994]**

(A) a, b, c are in A.P. (B) a^2, b^2, c^2 are in A.P.
(C) a, b, c are in G.P. (D) a, b, c are in H.P.

Q.7 If the sum of n terms of an A.P. is $nP + \frac{1}{2}n(n-1)Q$ then its common difference is - **[IIT-1994]**

(A) P + Q (B) 2P + 3Q
(C) 2Q (D) Q

Q.8 If p, q, r in A.P. and are positive, the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for- **[IIT-1995]**

(A) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (B) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$
(C) all p and r (D) No. p and r

Q.9 If cos (x–y), cos x and cos (x+ y) are in H.P., then cos x sec (y/2) equals- **[IIT-1997]**

(A) 1 (B) 2
(C) $\sqrt{2}$ (D) None of these

Q.10 If x be the AM and y, z be two GM's between two positive numbers, then $\frac{y^3 + z^3}{xyz}$ is equal to- **[IIT-1997]**

(A) 1 (B) 2 (C) 3 (D) 4

Q.11 Let T_r be the r^{th} term of an A.P., for r = 1, 2, 3, ...if for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals- **[IIT-1998]**

(A) $\frac{1}{mn}$ (B) $\frac{1}{m} + \frac{1}{n}$
(C) 1 (D) 0

Q.12 If x > 1, y > 1, z > 1 are in G.P., then $\frac{1}{1+\ell nx}$, $\frac{1}{1+\ell ny}$, $\frac{1}{1+\ell nz}$ are in& **[IIT-1998]**

(A) A.P. (B) H.P.
(C) G.P. (D) None of these

Q.13 Let $a_1, a_2, ... a_{10}$ be in A.P. and $h_1, h_2, ...h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is- **[IIT-1999]**

(A) 2 (B) 3 (C) 5 (D) 6

Q.14 The harmonic mean of the root of the equation

$(5+\sqrt{2})x^2-(4+\sqrt{5})x+8+2\sqrt{5}=0$ is **[IIT-1999]**

(A) 2 (B) 4 (C) 6 (D) 8

Q.15 The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is- **[IIT-2000]**

(A) 2489 (B) 4735
(C) 2317 (D) 2632

Q.16 Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then- **[IIT Sc.-2000]**

(A) $a=\frac{7}{4}, r=\frac{3}{7}$ (B) $a=2, r=\frac{3}{8}$

(C) $a=\frac{3}{2}, r=\frac{1}{2}$ (D) $a=3, r=\frac{1}{4}$

Q.17 Let α, β be the roots of $x^2-x+p=0$ and γ, δ be the roots of $x^2-4x+q=0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are- **[IIT Sc.-2001]**

(A) – 2, – 32 (B) – 2, 3
(C) – 6, 3 (D) – 6, – 32

Q.18 Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are- **[IIT Sc.-2001]**

(A) Not in A.P./G.P./H.P.
(B) in A.P.
(C) in G.P.
(D) in H.P.

Q.19 If the sum of the first 2n terms of the A.P. 2, 5, 8,.... is equal to the sum of the first n terms of the A.P.57, 59, 61,.... then n equals- **[IIT Sc.-2001]**

(A) 10 (B) 12 (C) 11 (D) 13

Q.20 If $a_1, a_2, ..., a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1+a_2+.........+a_{n-1}+a_n$ is - **[IIT Sc.-2002]**

(A) $n(c)^{1/n}$ (B) $(n+1)c^{1/n}$
(C) $2nc^{1/n}$ (D) $(n+1)(2c)^{1/n}$

Q.21 An infinite G.P., with first term x & sum of the series is 5 then - **[IIT Sc.-2004]**

(A) $x \geq 10$ (B) $0 < x < 10$
(C) $x < -10$ (D) $-10 < x < 0$

Q.22 Suppose for distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$. **[IIT-2008]**

Statement-1 : The number b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

Statement-2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.
(B) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1
(C) Statement–1 is True, Statement–2 is False
(D) Statement–1 is False, Statement–2 is True

Q.23 If the sum of n terms of an A.P. is cn^2, then the sum of squares of these n terms are : **[IIT- 2009]**

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Q.24 Let S_k, k = 1, 2, ……, 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is – **[IIT- 2010]**

(A) 0 (B) 4 (C) 6 (D) 8

Q.25 Let $a_1, a_2, a_3, ….., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[IIT 2011]**

(A) 5 (B) 6 (C) 8 (D) 9

Q.26 The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is **[IIT 2011]**

(A) 8 (B) 6 (C) 5 (D) 3

Q.27 Let $a_1, a_2, a_3,$ .... be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[IIT 2012]**

(A) 22 (B) 23 (C) 24 (D) 25

Q.28 Let $S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(s) **[JEE – Advance 2013]**

(A) 1056 (B) 1088 (C) 1120 (D) 1332

Q.29 A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 = **[JEE - Advance 2013]**

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	B	B	A	C	B	B	A	C	C	A	C	B	B	B	A	C	B	A
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	B	D	B	C	A	A	A	A	A	D	B	B	D	D	B	B	B	A	C	C
Q.No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	B	A	B	B	A	A	B	B	B	C	A	B	B	C	C	B	B	D	C	D
Q.No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	B	A	A	B	B	B	C	D	B	B	C	B	B	C	C	D	B	D	C	C
Q.No.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95					
Ans.	A	B	C	A	D	C	B	C	C	C	A	C	B	C	B					

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	B	C	B	A	A	D	B	B	B	C	D	B	A	B	B	B	C	C
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	B	A	C	B	D	C	C	C	A	A	B	B	A	C	A	C	C	A	C	C

LEVEL- 3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	B	A	D	A	C	C	B	A	D	A, D	C	D	A	D	B	B	C	C
Q.No.	21	22	23	24	25	26	27	28												
Ans.	B	D	C	A	C	B	B	C												

LEVEL- 4

SECTION-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	C	B	A	B	D	C	C	D	B	B	D	A	C

SECTION-B

1.[A] $\alpha = a_2 + a_4 \ldots\ldots\ldots + a_{200}$

$\beta = a_1 + a_3 \ldots\ldots.. + a_{199}$

$$\frac{\alpha}{\beta} = \frac{ra_1 + ra_3 + \ldots\ldots. + ra_{199}}{a_1 + a_3 \ldots\ldots\ldots + a_{199}} = r$$

2.[C] $\because \Sigma n = \frac{1}{5}\Sigma n^2$

$$\Rightarrow \frac{n(n+1)}{2} = \frac{1}{5}\frac{n(n+1)(2n+1)}{6} \quad \Rightarrow n = 7$$

3.[C] We know

If $A = \frac{a+b}{2}$ & $G = \sqrt{ab}$

$$\therefore \frac{a}{b} = \frac{A + \sqrt{A^2 - G^2}}{A - \sqrt{A^2 - G^2}} \quad (a > b)$$

Here $\frac{H}{G} = \frac{4}{5}$ but $G^2 = AH$

$$\therefore \frac{H}{G} = \frac{G}{A} = \frac{4}{5}$$

Now $$\frac{a}{b} = \frac{1 + \sqrt{1 - \left(\frac{G}{A}\right)^2}}{1 - \sqrt{1 - \left(\frac{G}{A}\right)^2}} = \frac{1 + \sqrt{1 - \frac{16}{25}}}{1 - \sqrt{1 - \frac{16}{25}}}$$

$$\therefore \frac{a}{b} = \frac{1 + \frac{3}{5}}{1 - \frac{3}{5}} = 4$$

4.[C] $\frac{\log 2}{\log 3}, \frac{\log 2}{\log 6}, \frac{\log 2}{\log 12}$: ?

$\Rightarrow \frac{1}{\log 3}, \frac{1}{\log 6}, \frac{1}{\log 12}$: ?

We know 3, 6, 12 : GP

$\therefore$ log3, log6, log12 : AP

$\therefore \frac{1}{\log 3}, \frac{1}{\log 6}, \frac{1}{\log 12}$: HP

5.[B] Clearly $x = \frac{1}{1-\cos^2\phi} = \frac{1}{\sin^2\phi} = \sin^2\phi = \frac{1}{x}$

& $y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi} \Rightarrow \cos^2\phi = \frac{1}{y}$

& $z = \frac{1}{1-\sin^2\phi\cos^2\phi} = \frac{1}{1-\frac{1}{x}.\frac{1}{y}}$

$\therefore z = \frac{xy}{xy-1}$

xyz = xy+z

6.[D] log(a + c), log(c – a), log(a – 2b + c) : AP

a + c, c – a, a – 2b + c : GP

$(c - a)^2 = (a + c)(a - 2b + c)$

$\Rightarrow c^2 + a^2 - 2ac = a^2 - 2ab + ac + ac - 2bc + c^2$

$\Rightarrow$ 2ab + abc = 4ac

$\therefore ac = \frac{ab+bc}{2}$

$\therefore$ ab, ac, bc : AP

$\frac{1}{c}, \frac{1}{b}, \frac{1}{a}$: AP

c, b, a : HP

7.[D] $S_n = nP + \frac{1}{2}n(n-1)Q$

$\therefore S_{n-1} = (n-1)P + \frac{1}{2}(n-1)(n-2)Q$

$\therefore T_n = S_n - S_{n-1}$

$T_n = P + (n-1)Q \equiv a + (n-1)d$

$\therefore$ d = Q

8.[A] $\because$ p, q, r : AP

$\therefore q = \frac{p+r}{2}$

Now $px^2 + qx + r = 0$

Roots are real if D ≥ 0

$\Rightarrow q^2 - 4pr \geq 0 \Rightarrow \frac{(p+r)^2}{4} - 4pr \geq 0$

$\Rightarrow p^2 + r^2 + 2pr - 16pr \geq 0$

$\Rightarrow p^2 + r^2 - 14pr \geq 0 \quad \Rightarrow 1 + \frac{r^2}{p^2} - \frac{14r}{p} \geq 0$

$\Rightarrow \frac{r^2}{p^2} - \frac{14r}{p} + 49 \geq 49 - 1$

$\Rightarrow \left(\frac{r}{p} - 7\right)^2 \geq 48 \Rightarrow \left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$

9.[C] $\because$ cos(x – y), cosx, cos(x + y) : HP

$\cos x = \frac{2\cos(x+y)\cos(x-y)}{\cos(x+y)+\cos(x-y)}$

$\cos x = \frac{2(\cos^2 x - \sin^2 y)}{2\cos x \cos y}$

$\cos^2 x \cos y = \cos^2 x - \sin^2 y$

$\Rightarrow \sin^2 y = (1 - \cos y)\cos^2 x$

$\Rightarrow 4\sin^2(y/2)\cos^2(y/2) = 2\sin^2(y/2)\cos^2 x$

$\Rightarrow 2\cos^2(y/2) = \cos^2 x \Rightarrow [\cos x \sec(y/2)]^2 = 2$

$\Rightarrow \cos x.\sec(y/2) = \sqrt{2}$

10.[B] Let numbers are a & b

$\therefore x = \frac{a+b}{2}$

& $\because$ a, y, z, b are in GP

$\therefore y^2 = az$ & $z^2 = by$

Now, $\frac{y^3+z^3}{xyz} = \frac{1}{x}\left(\frac{y^2}{z} + \frac{z^2}{y}\right) = \frac{1}{x}(a+b) = 2$

11.[C] $\because T_m = a + (m-1)d = \frac{1}{n}$ ….(i)

& $T_n = a + (n-1)d = \frac{1}{m}$ ….(ii)

(i) – (ii) $(m-n)d = \frac{1}{n} - \frac{1}{m}$

$\therefore \boxed{d = \frac{1}{mn}}$

Using (i); we get $\boxed{a = \frac{1}{mn}}$

Now $T_{mn} = a + (mn-1)d = \frac{1}{mn} + \frac{mn-1}{mn} = 1$

12.[B] $\because$ x, y, z : GP

$\therefore$ *l*nx, *l*ny, *l*nz : AP

$\therefore$ 1 + *l*nx, 1+ *l*ny, 1 + *l*nz : AP

Hence their reciprocals are in HP

13.[D] Given n = 8 AM's

2, $\boxed{a_2, a_3, ..., a_9}$, 3 : AP …(i)

n = 8 HM's

& 2, $\boxed{h_2, h_3, ..., h_9}$ 3 : HP …(ii)

In (i) $d = \frac{3-2}{8+1} = \frac{1}{9}$

$\therefore a_4 = 2 + 3d = 2 + \frac{3}{9} = 2 + \frac{1}{3}$

$\therefore \boxed{a_4 = \frac{7}{3}}$

In (ii) 2, $h_2, h_3, \ldots, h_9$, 3 : HP

$\frac{1}{2}, \boxed{\frac{1}{h_2}, \frac{1}{h_3}, \ldots\ldots, \frac{1}{h_9}}, \frac{1}{3}$: AP

n = 8 AM's

$\therefore d = \frac{\frac{1}{3}-\frac{1}{2}}{8+1} = \frac{-1}{54}$

$\therefore \frac{1}{h_7} = \frac{1}{2} + 6d = \frac{1}{2} - \frac{6}{54} = \frac{7}{18}$

$\therefore \boxed{h_7 = \frac{18}{7}}$

$\therefore a_4.h_7 = \frac{7}{3}.\frac{18}{7} = 6$

14. [B] $H = \frac{2\alpha\beta}{\alpha+\beta} = \frac{2c/a}{-b/a} = \frac{-2c}{b}$

$\therefore H = \frac{-2.(8+2\sqrt{5})}{-(4+\sqrt{5})} = 4$

15.[D] Sum of nos. divisible by 3 or 5

$= (3 + 6 + \ldots\ldots + 99) + (5 + 10 + \ldots\ldots + 100) - (15 + 30 \ldots\ldots + 90)$

$= 3\,(1 + 2 \ldots\ldots + 33) + 5\,(1 + 2 \ldots\ldots + 20) - 15\,(1 + 2 \ldots\ldots + 6)$

$= 3.\frac{33.34}{2} + 5.\frac{20.21}{2} - \frac{15.6.7}{2}$

$= 1683 + 1050 - 315 = 2418$

Now sum of nos. not divisible by 3 or 5

$= (1 + 2 + 3 \ldots\ldots + 100) - 2418$

$= \frac{100\times101}{2} - 2418 = 2632$

16. [D] Given $T_2 = ar = 3/4$ & $S = \frac{a}{1-r} = 4$

$\Rightarrow \frac{ar}{r-r^2} = 4 \Rightarrow \frac{3/4}{r-r^2} = 4 \Rightarrow 16r - 16r^2 = 3$

$\Rightarrow 16r^2 - 16r + 3 = 0$

$\therefore r = \frac{1}{4}$ or $r = \frac{3}{4}$

$\therefore a = 3$ or $a = 1$

17.[A] Let $\beta = \alpha r,\ \gamma = \alpha r^2,\ \delta = \alpha r^3$

$\therefore \alpha + \beta = \alpha(1+r) = 1$ ….(i)

& $\alpha\beta = \alpha^2 r = p$ ….(ii)

& $\gamma + \delta = \alpha r^2(1+r) = 4$ ….(iii)

& $\gamma\delta = \alpha^2 r^5 = q$ ….(iv)

Now $\frac{(iii)}{(i)}$: $r^2 = 4$

$\therefore r = +2$ or $r = -2$

From (i) $\alpha = 1/3$ from (i) $= \alpha = -1$

Now $p = \alpha^2 r = \frac{2}{9}$ $\therefore p = \alpha^2 r = -2$

& $q = \frac{32}{9}$ & $q = \alpha^2 r^5 = -32$

18. [D] a, b, c, d : AP

Dividing by abcd, we get

$\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$: AP

$\therefore$ bcd, acd, abd, abc : HP

19.[C] $S_{2n} = S'_n$

$\Rightarrow \frac{2n}{2}[2.2 + (2n-1).3] = \frac{n}{2}[2.57 + (n-1).2]$

$\Rightarrow 8 + 12n - 6 = 114 + 2n - 2 \Rightarrow 10n = 110$

$\therefore n = 11$

20. [A] $\because A \ge G$

$\Rightarrow \frac{a_1 + a_2 + \ldots + a_n}{n} \ge (a_1.a_2 \ldots .a_n)^{1/n}$

$\Rightarrow \frac{a_1 + a_2 + \ldots + a_n}{n} \ge (c)^{1/n}$

$\therefore a_1 + a_2 + \ldots + a_n \ge n.c^{1/n}$

21.[B] $\because S = \frac{x}{1-r} = 5$

$\therefore 1 - r = \frac{x}{5} \Rightarrow r = 1 - \frac{x}{5}$

Now $|r| < 1 \Rightarrow \left|1-\frac{x}{5}\right| < 1 \Rightarrow |5 - x| < 5$

$\therefore -5 < x - 5 < +5$

$0 < x < 10$

22.[C] a_1, a_2, a_3, a_4 are in GP

& $b_1 = a_1$

$b_2 = b_1 + a_2 = a_1 + a_2$

$b_3 = b_2 + a_3 = a_1 + a_2 + a_3$

$b_4 = b_3 + a_4 = a_1 + a_2 + a_3 + a_4$

Clearly b_1, b_2, b_3, b_4 are not in AP, GP & HP

So statement-I is true

But statement-II is false

23.[C] $S_n = cn^2$

$\therefore T_n = c\,(2n - 1)$

$\therefore T'_n = c^2(4n^2 - 4n + 1)$

$S'_n = c^2[4\Sigma n^2 - 4\Sigma n + \Sigma 1]$

$= c^2\left[\frac{4.n(n+1)(2n+1)}{6} - \frac{4.n(n+1)}{2} + n\right]$

$= \frac{nc^2}{6}[4(n+1)(2n+1) - 12(n+1) + 6]$

$= \frac{nc^2}{6}[4(2n^2 + 3n + 1) - 12n - 12 + 6]$

$= \frac{nc^2}{6}$ $[8n^2 + 12n + 4 - 12n - 6]$

$= \frac{nc^2}{6}[8n^2 - 2] = \frac{nc^2}{3}[4n^2 - 1]$

24.[B] $S_k = \frac{k}{k!}$

$\sum_{k=1}^{100} |(k^2 - 3k + 1)s_k|$

$= 1 + 1 + \sum_{k=3}^{100} \left|\frac{(k^2 - 3k + 1)}{(k-1)!}\right|$

$= 2 + \sum_{k=3}^{100} \left|\frac{(k-1)^2 - k}{(k-1)\,!}\right|$

$= 2 + \sum \left|\frac{k-1}{(k-2)!} - \frac{k}{(k-1)\,!}\right| = 2 + 2 - \frac{100}{99\,!}$

Value $= \frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)s_k| = 4$

25.[D] $a_1 = 3$

$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[2a_1 + (5n-1)d]}{\frac{n}{2}[2a_1 + (n-1)d]}$

$\frac{S_{5n}}{S_n} = \frac{5[(6-d) + 5nd]}{(6-d) + nd}$

$\because \frac{S_{5n}}{S_n}$ is independent of n so, d = 6

$a_2 = a_1 + d = 3 + 6 = 9$

26.[A] AM ≥ 6M

$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8}$

$\geq (a^{-5}.\ a^{-4}.\ a^{-3}.\ a^{-3}.\ a^{-3}.\ 1.\ a^8.\ a^{10})^{1/8}$

$a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10} \geq 8$

Minimum value is 8

27.[D] $a_1 = 5a_{20} = 25$

$T_1 = \frac{1}{5}\ T_{20} = \frac{1}{25}$

$T_{20} = \frac{1}{5} + 19D = \frac{1}{25}$

$D = \left(\frac{1}{25} - \frac{1}{5}\right)\frac{1}{19} = -\frac{20}{5(25)(19)}$

$T_n = \frac{1}{5} - \frac{(n-1)(20)}{(125)(19)} = \frac{(25)(19) - (n-1)(20)}{(125)(19)} < 0$

$(25)(19) < (n-1)(20)$

$n - 1 > \frac{(25)(19)}{(20)} \Rightarrow n > \frac{5(19)}{4} + 1$

$\Rightarrow n > \frac{95}{4} + 1 \quad \Rightarrow \quad n > 23.75 + 1$

$\Rightarrow n > 24.75 \qquad \Rightarrow n = 25$

28.[A,D] $\therefore S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 .. + 4n^2$

$= [3^2 - 1^2 + 7^2 - 5^2$ ……….. 2n terms] + $[4^2 - 2^2 + 8^2 - 6^2$ ………….. 2n terms]

= 2[1 + 3 + 5 + 7……….. 2n terms] + 2[2 + 4 + 6 + 8…… 2n terms]

= 2[2n/2 [2 + (2n – 1)2] + 2[2n/2 (4 + (2n – 1)2]

= 2n[4n] + 2n[4n + 2]

$= 8n^2 + 8n^2 + 4n$

$= 16n^2 + 4n$

$S_n = 4n(4n + 1)$

which gives option A and D for n = 8, 9

29.[5] Sum of n cards = 1 + 2 …….. + n = $\frac{n(n+1)}{2}$

$\therefore \frac{n(n+1)}{2} - (k + k + 1) = 1224$

$\frac{n(n+1)}{2} = 1224 + 2k + 1$…(1)

$\therefore$ as $k \geq 1$ so, $\frac{n(n+1)}{2} \geq 1224 + 1 + 2$

$\Rightarrow n^2 + n \geq 2448 + 6$

$\Rightarrow \left(\frac{n+1}{2}\right)^2 \geq 2448 + 6 + \frac{1}{4}$

$\left(n + \frac{1}{2}\right)^2 \geq (49.5)^2$

$n \geq 49$ …(2)

also $k \leq n$

$\therefore \frac{n(n+1)}{2} \leq 1224 + n + n + 1$

$n^2 + n \leq 2448 + 4n + 2 \Rightarrow n^2 - 3n \leq 2450$

$\left(n - \frac{3}{2}\right)^2 \leq 2450 + \frac{9}{4} < 50^2$

$n - \frac{3}{2} < 50 \quad \Rightarrow \quad n < 51.5$….(3)

from (2) and (3)

n can be 49, 50, 51

put n = 49 in (1) we get

$49 \times 25 = 1224 + 2k + 1$

$\Rightarrow k = 0$ not possible

At n = 50 we get

25.5 = 1224 + 2k + 1

k = 25

At n = 51 we get

51.26 = 1224 + 2k + 1

102 = 2k + 1

$\therefore k \notin I$

$\therefore k = 25 \Rightarrow k - 20 = 5$