JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

TRIGONOMETRIC RATIO

(PRACTICE SHEET)

LEVEL - 01

Question based on Trigonometrical ratios or function

Q.1 If 5 tanθ = 4, then $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} =$

(A) 5/9 (B) 14/5
(C) 9/5 (D) 5/14

Q.2 $\tan^2\theta \sec^2\theta(\cot^2\theta - \cos^2\theta) =$

(A) 0 (B) – 1
(C) 1 (D) 2

Q.3 If sin θ + cosec θ = 2 then the value of $\sin^8\theta + \text{cosec}^8\theta$ is equal to-

(A) 2 (B) 2^8
(C) 2^4 (D) None of these

Q.4 If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x =$ ....

(A) 0 (B) – 1
(C) 2 (D) 1

Q.5 If x = r sin θ.cos ϕ; y = r sin θ.sin ϕ and z = r cos θ then the value of $x^2 + y^2 + z^2$ is independent of-

(A) θ, ϕ (B) r, θ
(C) r, ϕ (D) r

Q.6 If $\alpha\cos^2 3\theta + \beta\cos^4\theta = 16\cos^6\theta + 9\cos^2\theta$ is an identity then-

(A) α = 1, β = 18 (B) α = 1, β = 24
(C) α = 3, β = 24 (D) α = 4, β = 2

Question based on Sign & Allied angle of Trigo. Ratios

Q.7 $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \ldots\ldots\ldots \sin^2 85^\circ + \sin^2 90^\circ =$

(A) 7 (B) 8
(C) $9\frac{1}{2}$ (D) 10

Q.8 cosA + sin(270º+A) – sin(270º–A)+cos(180º + A) =

(A) – 1 (B) 0
(C) 1 (D) None of these

Q.9 If $\sin\theta = -\frac{1}{\sqrt{2}}$ and tan θ = 1, then θ lies in which quadrant-

(A) First (B) Second
(C) Third (D) Fourth

Q.10 If A lies in the second quadrant & 3 tan A + 4 = 0, the value 2 cot A – 5 cos A + sin A is equals to-
(A) 23/11 (B) 22/10
(C) 23/10 (D) None of these

Q.11 $\cos 24^\circ + \cos 5^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ =$

(A) 1/2 (B) – 1/2 (C) $\frac{\sqrt{3}}{2}$ (D) 1

Q.12 $x = y \cos \frac{2\pi}{3} = z \cos \frac{4\pi}{3}$, then

$xy + yz + zx =$
(A) – 1 (B) 0
(C) 1 (D) 2

Question based on Sum and Difference formulae

Q.13 $\tan 20^\circ + 2 \tan 50^\circ =$
(A) tan 70º (B) cot 70º
(C) sin 70º (D) tan 30º

Q.14 If $A - B = \frac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B) =$

(A) 1 (B) 2 (C) –1 (D) –2

Q.15 tan 5x tan 3x tan 2x = ...
(A) tan 5x – tan 3x – tan2x
(B) $\frac{\sin 5x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$
(C) 0
(D) None of these

Q.16 If $0 < \beta < \alpha < \frac{\pi}{4}$, $\cos(\alpha + \beta) = \frac{3}{5}$ and $\cos(\alpha - \beta) = \frac{4}{5}$, then $\sin 2\alpha$ is equal to –
(A) 1 (B) 0
(C) 2 (D) none

Question based on Formulae for product into sum or difference conversion

Q.17 $2 \sin\left(\frac{5\pi}{12}\right) \sin\left(\frac{\pi}{12}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Q.18 $\cos^2 48° - \sin^2 12° =$

(A) $\frac{\sqrt{5}-1}{4}$ (B) $\frac{\sqrt{5}+1}{8}$

(C) $\frac{\sqrt{3}-1}{4}$ (D) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Q.19 $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) =$

(A) $\sin 2\alpha$ (B) $\cos 2\beta$

(C) $\cos 2\alpha$ (D) $\sin 2\beta$

Question based on **Formulae for sum or difference into Product conversion**

Q.20 $\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} =$

(A) $\tan 6\theta$ (B) $\tan 3\theta$

(C) $\cot 2\theta$ (D) $\cot 6\theta$

Q.21 If $\cos A = m \cos B$, then-

(A) $\cot \frac{A+B}{2} = \frac{m+1}{m-1} \tan \frac{B-A}{2}$

(B) $\tan \frac{A+B}{2} = \frac{m+1}{m-1} \cot \frac{B-A}{2}$

(C) $\cot \frac{A+B}{2} = \frac{m+1}{m-1} \tan \frac{A-B}{2}$

(D) None of these

Q.22 If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then $\cos 2\alpha + \cos 2\beta$ is equal to -

(A) $-2\sin(\alpha+\beta)$ (B) $-2\cos(\alpha+\beta)$

(C) $2\sin(\alpha+\beta)$ (D) $2\cos(\alpha+\beta)$

Q.23 If $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$, then-

(A) tan A, tan B, tan C are in A.P.

(B) tan A, tan B, tan C are in G.P.

(C) tan A, tan B, tan C are in H.P.

(D) None of these

Q.24 The value of $\sin 78° - \sin 66° - \sin 42° + \sin 6°$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) none

Question based on Trigonometrical ratio of multiple angle

Q.25 $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} =$

(A) $\dfrac{1}{2}\tan\theta$ (B) $\dfrac{1}{2}\cot\theta$

(C) $\tan\theta$ (D) $\cot\theta$

Q.26 $1 - 2\sin^2\left(\dfrac{\pi}{4} + \theta\right) =$

(A) $\cos 2\theta$ (B) $-\cos 2\theta$

(C) $\sin 2\theta$ (D) $-\sin 2\theta$

Q.27 $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}}$; $\forall\ 0 < \theta < \pi/4$ is

(A) $\cos\theta$ (B) $\sin\theta$

(C) $2\cos\theta$ (D) $2\sin\theta$

Q.28 $\cos^2 A\,(3 - 4\cos^2 A)^2 + \sin^2 A(3 - 4\sin^2 A)^2$ is equal to-

(A) $\cos 4A$ (B) $\sin 4A$

(C) 1 (D) None of these

Q.29 $\tan\theta.\ \tan\left(\dfrac{\pi}{3} + \theta\right)\ \tan\left(\dfrac{\pi}{3} - \theta\right)$ is equal to -

(A) $\tan 2\theta$ (B) $\tan 3\theta$

(C) $\tan^3\theta$ (D) none

Question based on The greatest and least value of expression

Q.30 The minimum value of $\cos 2\theta + \cos\theta$ for real values of θ is -

(A) $-\dfrac{9}{8}$ (B) 0

(C) -2 (D) none of these

Q.31 The maximum value of $\sin(\theta + \pi/6) + \cos(\theta + \pi/6)$ is attained at

(A) $\dfrac{\pi}{12}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{2}$

Q.32 The maximum value of $12\sin\theta - 9\sin^2\theta$ is-

(A) 3 (B) 4

(C) 5 (D) None of these

Q.33 Minimum value of $5\sin^2\theta + 4\cos^2\theta$ is -

(A) 1 (B) 2 (C) 3 (D) 4

Question based on Miscellaneous point & series

Q.34 The value of cos12º. cos24º. cos36º. cos48º. cos72º. cos 84º is

(A) $\frac{1}{64}$ (B) $\frac{1}{32}$

(C) $\frac{1}{16}$ (D) $\frac{1}{128}$

Q.35 $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5} =$

(A) $\frac{1}{16}$ (B) 0 (C) $-\frac{1}{8}$ (D) $-\frac{1}{16}$

Q.36 The value of tan 6º tan 42º tan 66º tan 78º is-

(A) 1 (B) 1/2 (C) 1/4 (D) 1/8

Q.37 A quadratic equation whose roots are $\text{cosec}^2\theta$ and $\sec^2\theta$, can be -

(A) $x^2 - 2x + 2 = 0$ (B) $x^2 - 3x + 3 = 0$

(C) $x^2 - 5x + 5 = 0$ (D) $x^2 + 4x - 4 = 0$

Q.38 Find the number of solution of the equation

$30\,|\sin x| = x$ in $0 \le x \le 2\pi$

(A) 4 (B) 2 (C) 8 (D) 6

Q.39 Total number of solution of the equation

$3x + 2\tan x = \frac{5\pi}{2}$ in $x \in [0, 2\pi] =$

(A) 1 (B) 2 (C) 3 (D) 4

Q.40 The number of solutions of

$|\cos x| = \sin x$, $0 \le x \le 4\pi$ is -

(A) 8 (B) 4

(C) 2 (D) None of these

Q.41 The value of $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \ldots\ldots$ to n terms is equal to -

(A) 1 (B) 0

(C) n/2 (D) none of these

LEVEL-2

Q.1 Which of the following is correct-

(A) sin 1º > sin 1 (B) sin1º< sin 1

(C) sin 1º = sin 1 (D) sin 1º = $\frac{\pi}{180}$ sin 1

Q.2 If cos A=$\frac{3}{4}$ then the value of sin $\frac{A}{2}$ sin $\frac{5A}{2}$ is-

(A) $\frac{1}{32}$ (B) $\frac{11}{8}$

(C) $\frac{11}{32}$ (D) $\frac{11}{16}$

Q.3 If $\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha} = \lambda$ then $\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$ is equal to-

(A) $\frac{1}{\lambda}$ (B) λ

(C) 1 – λ (D) 1 + λ

Q.4 The least value of

$\cos^2\theta - 6\sin\theta.\cos\theta + 3\sin^2\theta + 2$ is-

(A) $4 + \sqrt{10}$ (B) $4 - \sqrt{10}$

(C) 0 (D) 4

Q.5 If sin α + sin β = a and cos α – cos β = b, then $\tan\left(\frac{\alpha-\beta}{2}\right)$ is equal to-

(A) $-\frac{a}{b}$ (B) $-\frac{b}{a}$

(C) $\sqrt{a^2+b^2}$ (D) None of these

Q.6 If ABCD is a cyclic quadrilateral, then the value of cos A – cos B + cos C – cos D-

(A) 0

(B) 1

(C) 2 (cos B – cos D)

(D) 2 (cos A – cos C)

Q.7 The value of $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{7\pi}{7}$ is-

(A) 1 (B) – 1

(C) 1/2 (D) 0

Q.8 If $\tan\left(\frac{\alpha}{2}\right)$ and $\tan\left(\frac{\beta}{2}\right)$ are the roots of the equation $8x^2 - 26x + 15 = 0$ then cos (α + β) is equal to-

(A) $-\frac{627}{725}$ (B) $\frac{627}{725}$

(C) $-\frac{725}{627}$ (D) – 1

Q.9 If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

(A) 3 (B) 2

(C) 1 (D) 0

Q.10 If sin A, cos A and tan A are in G.P., then $\cos^3A + \cos^2A$ is equal to-

(A) 1 (B) 2

(C) 4 (D) None of these

Q.11 If $x + \frac{1}{x} = 2\cos\theta$, then $x^3 + \frac{1}{x^3} =$

(A) cos 3θ (B) 2 cos 3θ

(C) $\frac{1}{2}\cos 3\theta$ (D) $\frac{1}{3}\cos 3\theta$

Q.12 Exact value of tan 200º (cot 10º – tan 10º) is-

(A) 1 (B) 2

(C) 0 (D) None of these

Q.13 $\frac{\cos 20^\circ + 8\sin 70^\circ \sin 50^\circ \sin 10^\circ}{\sin^2 80^\circ}$ is equal to -

(A) 1 (B) 2

(C) $\frac{3}{4}$ (D) None of these

Q.14 The sign of the product sin 2 sin 3 sin 5 is -

(A) Negative (B) Positive

(C) 0 (D) None of these

Q.15 $\frac{3\cos\theta + \cos 3\theta}{3\sin\theta - \sin 3\theta}$ is equal to-

(A) $1 + \cot^2\theta$ (B) $\cot^4\theta$

(C) $\cot^3\theta$ (D) $2 \cot \theta$

Q.16 If $3 \sin \theta + \cos \theta = 2$ then $3 \cos \theta - \sin \theta$ is equal to-

(A) $-\sqrt{6}$ (B) $\sqrt{6}$

(C) $\sqrt{5}$ (D) $-\sqrt{5}$

Q.17 No. of solution in the equation $x = 4 \sin x$ when $x \in [0, 2\pi]$-

(A) 1 (B) 2

(C) 3 (D) 4

Q.18 The numerical value of

$\tan\frac{\pi}{3} + 2\tan\frac{2\pi}{3} + 4\tan\frac{4\pi}{3} + 8\tan\frac{8\pi}{3}$ is equal to-

(A) $-5\sqrt{3}$ (B) $\frac{-5}{\sqrt{3}}$

(C) $5\sqrt{3}$ (D) $\frac{5}{\sqrt{3}}$

Q.19 The value of $\sum_{r=0}^{10} \cos^3\left(\frac{\pi r}{3}\right)$ is equal to-

(A) $\frac{-9}{2}$ (B) $\frac{-7}{2}$

(C) $\frac{-9}{8}$ (D) $\frac{-1}{8}$

Q.20 If $|\tan A| < 1$, and $|A|$ is acute then $\frac{\sqrt{1+\sin 2A} + \sqrt{1-\sin 2A}}{\sqrt{1+\sin 2A} - \sqrt{1-\sin 2A}}$ is equal to -

(A) $\tan A$ (B) $-\tan A$

(C) $\cot A$ (D) $-\cot A$

Q.21 If $\sec A + \tan A = 3$ then $\cos A$ is :

(A) $\frac{5}{3}$ (B) $\pm\frac{3}{5}$

(C) $\frac{3}{5}$ (D) none

LEVEL-3

Q.1 The value of sin 10° + sin 20° + sin 30° + ... + sin 360° is equal to-

(A) 0 (B) 1 (C) $\sqrt{3}$ (D) 2

Q.2 The value of the expression

($\sqrt{3}$ sin 75º – cos 75º) is -

(A) 2 sin 15° (B) $1+\sqrt{3}$

(C) 2 sin 105° (D) $\sqrt{2}$

Q.3 cos 52° + cos 68° + cos 172° =

(A) 0 (B) 1

(C) 2 (D) None of these

Q.4 If cosec A + cot A = $\frac{11}{2}$, then tan A is -

(A) $\frac{21}{22}$ (B) $\frac{15}{16}$ (C) $\frac{44}{117}$ (D) $\frac{117}{43}$

Q.5 If triangle ABC, $\angle C = \frac{2\pi}{3}$, then the value of $\cos^2A + \cos^2B - \cos A \cdot \cos B$ is equal to-

(A) $\frac{3}{4}$ (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.6 If $f(\theta) = 5\cos\theta + 3\cos\left(\theta+\frac{\pi}{3}\right) + 3$, then range of f(θ) is-

(A) [–5, 11] (B) [–3, 9]

(C) [–2, 10] (D) [–4, 10]

Q.7 $\cos^2\frac{3\pi}{5}+\cos^2\frac{4\pi}{5}$ is equal to -

(A) 4/5 (B) 5/2 (C) 5/4 (D) 3/4

Q.8 If θ_1 and θ_2 are two values lying in [0, 2π] for which tan θ = λ, then $\tan\frac{\theta_1}{2} \cdot \tan\frac{\theta_2}{2}$ is -

(A) Zero (B) –1 (C) 2 (D) 1

Q.9 If A+B +C = π, then $\Sigma \tan\frac{A}{2}\tan\frac{B}{2}$ is equal to -

(A) 0 (B) –1 (C) 1/2 (D) 1

Q.10 $\sin^2\theta = \frac{(x+y)^2}{4xy}$, where x ∈ R, y ∈ R, gives real θ if and only if -

(A) x + y = 0 (B) x = y

(C) |x| = |y| ≠ 0 (D) none of these

Q.11 If a = sin 170° + cos 170°, then -

(A) a > 0 (B) a < 0

(C) a = 0 (D) a = 1

Q.12 $\sin^2A + \sin^2(A-B) + 2\sin A\cos B\sin(B-A)$ is equal to -

(A) $\sin^2A$ (B) $\sin^2B$

(C) $\cos^2A$ (D) $\cos^2B$

Q.13 If $U_n = 2\cos n\theta$, then $U_1U_n - U_{n-1}$ is equal to -

(A) U_{n+2} (B) U_{n+1}

(C) U_2U_{n+1} (D) None of these

Q.14 The number of real solutions of the equation $\sin(e^x) = 2^x + 2^{-x}$ is -

(A) 1 (B) 0

(C) 2 (D) Infinite

Q.15 If $\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta$ then c is equal to -

(A) –5 (B) 1

(C) 5 (D) None of these

Q.16 If $f(x) = \frac{\sin 3x}{\sin x}$, x ≠ nπ, then the range of values of f(x) for real values of x is -

(A) [–1, 3] (B) (–∞, –1]

(C) (3, +∞) (D) [–1, 3)

Q.17 If A = tan 6° tan 42° and B = cot 66° cot 78°, then -

(A) A = 2B (B) $A = \frac{1}{3}B$

(C) A = B (D) 3A = 2B

Q.18 If $\sin\theta = n\sin(\theta + 2\alpha)$, then $\tan(\theta + \alpha)$ is equal to -

(A) $\frac{1+n}{2-n}\tan\alpha$ (B) $\frac{1-n}{1+n}\tan\alpha$

(C) $\tan\alpha$ (D) $\frac{1+n}{1-n}\tan\alpha$

Statement type Questions

Each of the questions given below consist of Statement -I and Statement- II. Use the following key to choose the appropriate answer.

(A) If both Statement-I Statement-II are true, and Statement-II is the correct explanation of Statement-I.

(B) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I

(C) If Statement-I is true but Statement-II is false

(D) If Statement-I is false but Statement-II is true.

Q.19 **Statement- I :** $\cos 36^\circ > \tan 36^\circ$
Statement- II : $\cos 36^\circ > \sin 36^\circ$

Q.20 **Statement- I :** If A, B, C are the angles of a triangle such that angle A is obtuse, then $\tan B \tan C > 1$.
Statement- II : In any triangle,
$\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$.

Q.21 **Statement-I :** The number of roots of the equation $\sin px = x^2 - x + \frac{5}{4}$ is 2.
Statement-II : In $[0, 2\pi]$, $\sin x = \frac{1}{2}$ has exactly two solutions.

Q.22 **Statement-I :** sin1 cos2 tan 3 having positive sign.
Statement-II : $1^C = 57°17'45"$

Q.23 **Statement-I :** $\log_\pi \tan\left(-\frac{3\pi}{4}\right) = 0$
Statement-II : $\tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{\pi}{4}$

Q.24 **Statement-I :** $\sec^2\theta.\ \text{cosec}^2\theta = \sec^2\theta + \text{cosec}^2\theta$
Statement-II : $1 + \tan^2\theta = \sec^2\theta$ & $1 + \cot^2\theta = \text{cosec}^2\theta$

Passage based Questions

Passage-1

Measurement of an angle by three ways. First is degree and others are grade and radian. The relation between them

$$\frac{D}{90} = \frac{G}{100} = \frac{2C}{\pi}$$

Q.25 45° is equal to -

(A) 50^g (B) 45^g
(C) 40^g (D) 39^g

Q.26 $\frac{23\pi^c}{4}$ is equal to -

(A) 675° (B) 1080°
(C) 745° (D) 1035°

Q.27 200^g is equal to -

(A) 180° and $\frac{3\pi^c}{2}$ (B) 180° and π^c
(C) 200° and π^c (D) None

Passage-2

Increasing product series
$\cos\alpha.\cos2\alpha.\cos2^2\alpha$.................... $\cos(2^{n-1}\alpha)$

$= \frac{\sin 2^n\alpha}{2^n\sin\alpha}$ if $\alpha \ne n\pi$
$= 1$ if $\alpha = 2k\pi$
$= -1$ if $\alpha = (2k+1)\pi$
$k \in I$

where kI (Integer)

Q.28 The value of $\cos\frac{2\pi}{14}.\cos\frac{4\pi}{14}.\cos\frac{8\pi}{14}$ is -

(A) $-\frac{1}{8}$ (B) $\frac{1}{8}$
(C) $\frac{1}{16}$ (D) $-\frac{1}{16}$

Q.29 The value of $8\sin\frac{\pi}{48}.\cos\frac{\pi}{48}.\cos\frac{\pi}{24}.\cos\frac{\pi}{12}$ is equal to -

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

LEVEL -4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 If cos x + cos y + cos α = 0 and

sin x + sin y + sin α = 0, then $\cot\left(\frac{x+y}{2}\right)=$

[AIEEE-2002]

(A) sin α (B) cos α (C) cot α (D) 2 sin α

Q.2 cos 1°. cos 2°. cos 3°.... cos 179° =

[AIEEE-2002]

(A) 0 (B) 1 (C) 2 (D) 3

Q.3 Let α, β be such that π < α – β < 3π.

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ & $\cos\alpha + \cos\beta = -\frac{27}{65}$,

then the value of $\cos\frac{\alpha-\beta}{2}$ is- **[AIEEE-2004]**

(A) $-\frac{3}{\sqrt{130}}$ (B) $\frac{3}{\sqrt{130}}$

(C) $\frac{6}{65}$ (D) $\frac{-6}{65}$

Q.4 Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$,

where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then tan 2α =

[AIEEE-2010]

(A) $\frac{25}{16}$ (B) $\frac{56}{33}$ (C) $\frac{19}{12}$ (D) $\frac{20}{7}$

Q.5 If $A = \cos^2\theta + \sin^4\theta$, then for all value of θ-

[AIEEE-2011]

(A) $1 \le A \le 2$ (B) $\frac{3}{4} \le A \le 1$

(C) $\frac{13}{16} \le A \le 1$ (D) None of these

Q.6 In a ΔPQR, if 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to : **[AIEEE-2012]**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{6}$

Q.7 The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as – **[JEE Main- 2013]**

(A) tan A + cot A (B) sec A + cosec A

(C) sin A cos A + 1 (D) sec A cosec A + 1

SECTION-B

Q.1 For 0 < φ < π/2, if $x = \sum_{n=0}^{\infty}\cos^{2n}\phi$,

$y = \sum_{n=0}^{\infty}\sin^{2n}\phi$, $z = \sum_{n=0}^{\infty}\cos^{2n}\phi\,\sin^{2n}\phi$, then

[IIT-1993]

(A) xyz = xz + y (B) xyz = xy + z

(C) xyz = yz + x (D) None of these

Q.2 If K = sin (π/18) sin (5π/18) sin (7π/18), then the numerical value of K is- **[IIT-1993]**

(A) 1/8 (B) 1/16

(C) 1/2 (D) None of these

Q.3 If A > 0, B > 0 and A + B = π/3, then the maximum value of tan A tan B is-

[IIT-1993]

(A) 1 (B) 1/3 (C) $\sqrt{3}$ (D) $1/\sqrt{3}$

Q.4 The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi-\alpha)\right]$

$-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$ is equal to

[IIT-1994]

(A) 0 (B) 1

(C) 3 (D) sin 4α + cos 6α

Q.5 $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2$

$+ 4(\sin^6 x + \cos^6 x) =$ **[IIT-1995]**

(A) 11 (B) 12 (C) 13 (D) 14

Q.6 $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true, if and only if-

[IIT-1996]

(A) x + y ≠ 0 (B) x = y, x ≠ 0

(C) x = y (D) x ≠ 0, y ≠ 0

Q.7 The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}\,x)$ attains its maximum is- **[IIT-1998]**

(A) 0 (B) 1
(C) 2 (D) Infinite

Q.8 Which of the following number(s) is rational- **[IIT-1998]**

(A) sin 15º (B) cos 15º
(C) sin 15º cos 15º (D) sin 15º cos 75º

Q.9 Let n be an odd integer.

If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ for every value of θ, then- **[IIT-1998]**

(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n$
(C) $b_0 = -1, b_1 = n$ (D) $b_0 = 0, b_1 = n^2 + 3n + 3$

Q.10 The function $f(x) = \sin^4 x + \cos^4 x$ increases if- **[IIT-1999]**

(A) $0 < x < \frac{\pi}{8}$ (B) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
(C) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (D) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Q.11 In a triangle PQR, $R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$, then- **[IIT-1999]**

(A) a + b = c (B) b + c = a
(C) a + c = b (D) b = c

Q.12 For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta)$. Then- **[IIT-1999]**

(A) $f_2\left(\frac{\pi}{16}\right) = 2$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$
(C) $f_4\left(\frac{\pi}{64}\right) = 0$ (D) None of these

Q.13 Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ **[IIT-2000]**

(A) ≥ 0 only when θ ≥ 0
(B) ≤ 0 for all real θ
(C) ≥ 0 for all real θ
(D) ≤ 0 only when θ ≤ 0

Q.14 If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then tan α equals- **[IIT-2001]**

(A) 2(tan β + tan γ) (B) tan β + tan γ
(C) tan β + 2 tan γ (D) 2 tan β + tan γ

Q.15 The maximum value of $(\cos\alpha_1).\cos(\alpha_2)...(\cos\alpha_n)$, under the restrictions $0 \le \alpha_1.\alpha_2....\alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2).(\cot\alpha_3)....(\cot\alpha_n) = 1$ is- **[IIT-2001]**

(A) $\frac{1}{2^{n/2}}$ (B) $\frac{1}{2^n}$ (C) $\frac{1}{2n}$ (D) 1

Q.16 If θ & φ are acute angles such that $\sin\theta = \frac{1}{2}$ and $\cos\phi = \frac{1}{3}$ then θ + φ lies in- **[IIT Scr-2004]**

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left(\frac{\pi}{6}, \pi\right]$

Q.17 $\cos(\alpha + \beta) = \frac{1}{e}$, $\cos(\alpha - \beta) = 1$ find no. of ordered pair of (α, β), $-\pi \le \alpha, \beta \le \pi$ **[IIT Scr-2005]**

(A) 0 (B) 1 (C) 2 (D) 4

Q.18 If $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$, $t_4 = (\cot\theta)^{\cot\theta}$ and let $\theta \in (0, \frac{\pi}{4})$ then- **[IIT -2006]**

(A) $t_4 < t_2 < t_1 < t_3$ (B) $t_4 < t_1 < t_3 < t_2$
(C) $t_4 < t_3 < t_2 < t_1$ (D) $t_2 < t_1 < t_3 < t_4$

Q.19 If $\frac{\sin^4 x}{2}+\frac{\cos^4}{3}=\frac{1}{5}$ then **[IIT -2009]**

(A) $\tan^2 x = \frac{2}{3}$

(B) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$

(D) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

Numeric Response Question:

Q.20 The maximum value of the expression

$\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is……

[IIT 2010]

Q.21 Let θ, φ ∈ [0, 2π] be such that $2\cos\theta(1-\sin\varphi)$ $=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1$, $\tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$. Then φ cannot satisfy

[IIT 2012]

(A) $0<\varphi<\frac{\pi}{2}$ (B) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$

(C) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$ (D) $\frac{3\pi}{2}<\varphi<2\pi$

Q.22 Let $f:(-1,1)\to \mathbb{R}$ be such that $f(\cos 4\theta)$ $=\frac{2}{2-\sec^2\theta}$ for $\theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are) **[IIT 2012]**

(A) $1-\sqrt{\frac{3}{2}}$ (B) $1+\sqrt{\frac{3}{2}}$

(C) $1-\sqrt{\frac{2}{3}}$ (D) $1+\sqrt{\frac{2}{3}}$

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	C	A	D	A	B	C	B	C	C	A	B	A	B	A	A	B	B	C	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	A	B	B	B	C	D	C	C	B	A	A	B	D	A	**D**	**A**	**C**	**A**	**C**	**B**
Q.No.	**41**																			
Ans.	B																			

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Ans.	B	C	B	B	B	A	B	A	D	A	B	B	B	A	C	B	B	A	D	C	C

LEVEL- 3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	D	A	C	A	D	D	B	D	C	B	B	B	B	C	D	C	D	B	D
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**											
Ans.	D	A	C	A	A	D	B	A	A											

LEVEL- 4

SECTION-A

Q.No.	1	2	3	4	5	6	7
Ans.	C	A	A	B	B	A	D

SECTION-B

1.[B] $x = 1 + \cos^2\phi + \cos^2\phi + \cos^6\phi + \ldots\ldots + \infty$

$= \dfrac{1}{1-\cos^2\phi} = \text{cosec}^2\phi$

$y = 1 + \sin^2\phi + \sin^4\phi + \sin^6\phi + \ldots\ldots\infty$

$= \dfrac{1}{1-\sin^2\phi} = \sec^2\phi$

$z = 1 + \cos^2\phi + \cos^2\phi\sin^2\phi + \cos^4\phi\sin^4\phi \ldots\ldots\infty$

$= \dfrac{1}{1-\cos^2\phi\sin^2\phi} = \dfrac{1}{1-\frac{1}{y}, \frac{1}{x}}$

$z = \dfrac{xy}{xy-1}$

$xyz - z = xy$

$xyz = xy + z$

2.[A] $k = \sin\dfrac{\pi}{18}\sin\dfrac{5\pi}{18}\sin\dfrac{7\pi}{18}$

$= \dfrac{1}{2}\left[\left(2\sin\dfrac{\pi}{18}\sin\dfrac{7\pi}{18}\right)\sin\dfrac{5\pi}{18}\right]$

$= \dfrac{1}{2}\left[\left(\cos\dfrac{\pi}{3} - \cos\dfrac{8\pi}{18}\right)\sin\dfrac{5\pi}{18}\right]$

$= \dfrac{1}{2}\left[\dfrac{1}{2}\sin\dfrac{5\pi}{18} - \dfrac{1}{2}\times 2\cos\dfrac{8\pi}{18}\sin\dfrac{5\pi}{18}\right]$

$= \dfrac{1}{2}\left[\dfrac{1}{2}\sin\dfrac{5\pi}{18} - \dfrac{1}{2}\left(\sin\dfrac{13\pi}{18} - \sin\dfrac{\pi}{6}\right)\right]$

$= \dfrac{1}{2}\left[\dfrac{1}{2}\sin\dfrac{5\pi}{18} - \dfrac{1}{2}\sin\left(\pi - \dfrac{5\pi}{18}\right) + \dfrac{1}{4}\right]$

$= \dfrac{1}{8}$

3.[B] A + B = π/3

Let y = tan A tan B

$= \tan A \tan(\pi/3 - A)$

$= \tan A \left[\dfrac{\sqrt{3} - \tan A}{1 + \sqrt{B}\tan A}\right]$

$= \dfrac{\sqrt{3}t - t^2}{1 + \sqrt{3}t}$ where t = tan A $0 < t < \sqrt{3}$

$t^2 + (\sqrt{3}y - \sqrt{3}) + y = 0$

$D \geq 0$

$3(y-1)^2 - 4y \geq 0$

$(3y-1)(y-3) \geq 0$

$y \leq \dfrac{1}{3}$ or $y \geq 3$

$0 < \tan A < \sqrt{3}$ and $0 < \tan B < \sqrt{3}$

$0 < \tan A \tan B < 3$

$y \leq 1/3$

4.[B] $3[\cos^4\alpha + \sin^4\alpha] - 2[\cos^6\alpha + \sin^6\alpha]$

$= 3[(\cos^2\alpha + \sin^2\alpha) - 2\sin^2\alpha\cos^2\alpha]$

$- 2[(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha(\sin^2\alpha + \cos^2\alpha)]$

$= 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[1 - 3\sin^2\alpha\cos^2\alpha]$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha$

$= 1$

5.[C] $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2$

$+ 4(\sin^6 x + \cos^6 x)$

$= 3(1 - 2\sin x\cos x)^2 + 6(1 + 2\sin x\cos x) + 4$

$[(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)]$

$= 3(1 + 4\sin^2 x\cos^2 x - 4\sin x\cos x)$

$+ 6(1 + 2\sin x\cos x) + 4[1 - 3\sin^2 x\cos^2 x]$

$= 13$

6.[B] $\sec^2\theta \geq 1$

$\dfrac{4xy}{(x+y)^2} \geq 1$

$\dfrac{4xy - (x+y)^2}{(x+y)^2} \geq 0$

$\dfrac{(x-y)^2}{(x+y)^2} \leq 0$

Which is holds if and only if x = y

However if x = y = 0 the expression is in defined

so $\sec^2\theta = \dfrac{4xy}{(x+y)^2}$ if and only if $x = y \neq 0$

7.[C] $f(x) = \cos x + \cos\sqrt{2}x$

when x = 0 then

f(x) = 2

8.[C] (A) sin (45 – 30) = sin 45 cos 30 – cos 45 sin 30

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ which is irrational

(B) cos (45 – 30) = cos 45 cos 30 – sin 45 sin 30

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ Which is irrational

(C) $\dfrac{1}{2} \times 2\sin 15\cos 15$

$= \dfrac{1}{2}\sin 30$

$= \dfrac{1}{4}$ which is rational

9.[B] $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r\theta$

Put θ = 0 we get $b_0 = 0$

$\dfrac{\sin n\theta}{\sin\theta} = \sum_{r=1}^{n} b_r \sin^{r-1}\theta$

$\lim_{\theta\to 0}\dfrac{\sin n\theta}{\sin\theta} = \sum_{r=1}^{n} b_r \lim_{\theta\to 0}\sin^{r-1}\theta$

$n = b_1$

10.[B] $f(x) = \sin^4 x + \cos^4 x = 1 - 2\sin^2 x\cos^2 x$

$= 1 - \dfrac{1}{2}\sin^2 2x = 1 - \dfrac{1}{4}(1 - \cos 4x)$

$= \dfrac{3}{4} + \dfrac{1}{4}\cos 4x$

$f'(x) = -\sin 4x$

$(2\pi + 1)\pi < 4x < (2\pi + 2)\pi$

$\dfrac{(2\pi+1)\pi}{4} < x < (2\pi + 2)\pi/4$

$x \in (\pi/4, \pi/2) \cup \left(\dfrac{3\pi}{4}, \pi\right) \cup \ldots\ldots$

Thus f(x) increases in (π/4, 3π/8)

11.[A] $\tan P/2 + \tan \theta/2 = -b/a$

$\tan P/2 \tan \theta/2 = c/a$

$P + Q + R = \pi$

$P + Q = \pi/2$

$P/2 + Q/2 = \pi/4$

$\tan (P/2 + Q/2) = \tan \pi/4$

$$\frac{\tan P/2 + \tan Q/2}{1 - \tan P/2 \tan Q/2} = 1$$

$$\frac{-b/a}{1 - c/a} = 1$$

$$\frac{-b}{a - c} = 1$$

$a - c = -b$

$a + b = c$

12.[B] $f_n(\theta) = \tan \theta/2 \left(\frac{1+\cos\theta}{\cos\theta}\right)\left(\frac{1+\cos 2\theta}{\cos 2\theta}\right)\ldots\ldots \left(\frac{1+\cos 2^n\theta}{\cos 2^n\theta}\right)$

$$= \frac{\sin\theta/2}{\cos\theta/2} \; \frac{2\cos\theta/2}{\cos\theta} \times \frac{2\cos^2\theta}{\cos 2\theta} \times \ldots\ldots \times \frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}$$

$= (2 \sin\theta/2 \cos \theta/2)\, (2 \cos \theta)\, (2 \cos 2\theta)\ldots\ldots \dfrac{2\cos 2^{n-1}\theta}{\cos 2^n\theta}$

$= (2 \sin \theta \cos \theta)\, (2 \cos 2\theta)\ldots\ldots \dfrac{2\cos 2^{n-1}\theta}{\cos 2^n\theta}$

$$= \frac{\sin 2^n\theta}{\cos 2^n\theta}$$

$f_n(\theta) = \tan 2^n\theta$

$f_3(\pi/32) = \tan 8\pi/32 = 1$

13.[C] $f(\theta) = \sin \theta\, 2 \sin 2\theta \cos \theta$

$= \sin 2\theta \sin 2\theta$

$= \sin^2 2\theta \geq \theta$ for all real θ

14.[C] $\beta = \pi/2 - \alpha$

$\tan \beta \tan (\pi/2 - \alpha)$

$\tan \beta \tan \alpha = 1$

$\gamma = \alpha - \beta$

$\tan \gamma = \tan (\alpha - \beta)$

$$\tan \gamma = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\tan \gamma = \frac{\tan\alpha - \tan\beta}{2}$$

$2 \tan \gamma + \tan \beta = \tan \alpha$

15.[A] $\cot \alpha_1 + \cot \alpha_2 + \cot \alpha_3 \ldots\ldots \cot \alpha_n = 1$

$\Rightarrow \cos \alpha_1 \cos \alpha_2 \cos \alpha_3 \ldots\ldots \cos\alpha_n = \sin \alpha_1 \sin \alpha_2 \ldots\ldots \sin\alpha_n$

$\Rightarrow \cos^2\alpha_1 \cos^2\alpha_2 \cos^2\alpha_3 \ldots\ldots \cos^2\alpha_n \ldots \dfrac{1}{2^n}$

$(\sin 2\alpha_1)\, (\sin 2\alpha_2) \ldots\ldots (\sin 2\alpha_n) \leq \dfrac{1}{2^n}$

(equality occurs when $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n = \pi/4$)

$\therefore \cos \alpha_1 \cos \alpha_2 \ldots \cos\alpha_n \leq \dfrac{1}{2^{n/2}}$

16.[B] $\sin \theta = \dfrac{1}{2} \Rightarrow \theta = \pi/6$

$\cos \phi = \dfrac{1}{3} < \dfrac{1}{2} \Rightarrow \phi > \pi/3$

$\Rightarrow \phi + \theta > \pi/2$

Also $\phi < \pi/2 \Rightarrow \theta + \phi < 2\pi/3$

Thus $\pi/2 < \theta + \phi < 2\pi/3$

17.[D] $\cos (\alpha - \beta) = 1$

$\Rightarrow \alpha - \beta = 2n\pi$

$\alpha - \beta = -2\pi, 0$, or 2π

$\cos (\alpha + \beta) = 1/e$

$\alpha + \beta = 2n\pi \pm \phi$

where $\phi \in (0, \pi)$ and $\cos \phi = 1/e$

thus $\alpha + \beta = \pm \phi$

or $-2\pi + \phi$ or $2\pi - \phi$

As $\alpha, \beta \in (-\pi, \pi)$

$\Rightarrow \alpha - \beta = 0$

and $\alpha + \beta = -\phi, \; -2\pi + \phi, 2\pi - \phi$

$\Rightarrow \alpha = \beta = -\phi/2, \phi/2, -\pi + \phi/2, \pi - \phi/2$

Total four solution

18.[D] As $\theta \in (0, \pi/4)$ so $\tan \theta < 1$ and $\cot \theta > 1$

Clearly $(\cot \theta)^{\cot \theta} > (\cot \theta)^{\tan \theta} > 1$

and $(\cot \theta)^2 (\tan \theta)^{\cot \theta} < (\tan \theta)^{\tan \theta} < 1$

Thus $t_4 > t_3 > t_1 > t_2$

19.[A,B] $3 \sin^4x + 2 \cos^4x = 6/5$

$3 \sin^4x + 2 (1 - \sin^2x)^2 = 6/5$

$3 \sin^4x + 2 (1 + \sin^4x - 2 \sin^2x) = 6/5$

$25 \sin^4x - 20 \sin^2x + 4 = 0$

$\Rightarrow \sin^2x = 2/5$ and $\cos^2x = 3/5$

$\Rightarrow \tan^2x = 2/3$ and $\sin^8x + \cos^8x = \dfrac{1}{125}$

20.[2]
$$\frac{1}{1 + 4\cos^2\theta + \frac{3}{2}\sin 2\theta}$$

$$= \frac{1}{1 + 2(1 + \cos 2\theta) + \frac{3}{2}\sin 2\theta}$$

$$= \frac{1}{\frac{3}{2}\sin 2\theta + 2\cos 2\theta + 3}$$

Min. $= 3 - \sqrt{\dfrac{9}{4} + 4} = \dfrac{1}{2}$

Max. value = 2

21.[A,C,D]

$\tan(2\pi - \theta) > 0$

$\Rightarrow 0 < 2\pi - \theta < \frac{\pi}{2}$ or $\pi < 2\pi - \theta < \frac{3\pi}{2}$

$\Rightarrow \frac{3\pi}{2} < \theta < 2\pi$ or $\frac{\pi}{2} < \theta < \pi$...... (1)

Also $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$

$\frac{3\pi}{2} < \theta < \frac{5\pi}{3}$ .....(2)

from (1) & (2)

$\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$.....(3)

Now, $2\cos\theta(1 - \sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)$

$\cos\phi - 1$

$\Rightarrow \cos\theta + \frac{1}{2} = \sin(\theta + \varphi)$ ....(4)

Now, $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$ from (3)

so $\cos\theta \in \left(0, \frac{1}{2}\right)$

$\sin(\theta + \varphi) \in \left(\frac{1}{2}, 1\right)$

Now, check option

(A) if $0 < \varphi < \frac{\pi}{2}$

then $\theta + \varphi \in \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$ & $\sin(\theta + \varphi)$ □ $\left(\frac{1}{2}, 1\right)$

Similarly check option B, C, D.

22. [A,B] $\cos 4\theta = \frac{1}{3}$

$2\cos^2 2\theta - 1 = \frac{1}{3}$

$\cos 2\theta = \pm\sqrt{\frac{2}{3}}$

On solving

$\cos^2\theta = \frac{1 \pm \sqrt{2/3}}{2}$

$\sec^2\theta = \frac{2}{1 \pm \sqrt{2/3}}$

On putting this

$f\left(\frac{1}{3}\right) = \pm\sqrt{\frac{3}{2}} + 1$

Note : As f(1/3) is having two values, so this relation is not a function. So JEE has given zero marks to all.