

JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

TRIGONOMETRIC

EQUATION

(PRACTICE SHEET)

LEVEL - 1

Question
based on

General solution of standard Trigonometrical Equation

- Q.1** The general solution of $\tan\left(\frac{2}{3}\theta\right) = \sqrt{3}$ is –
 (A) $\frac{3n\pi}{2} + \frac{\pi}{2}; n \in I$ (B) $\frac{n\pi}{2}; \pm \frac{\pi}{2}n \in I$
 (C) $n\pi \pm \frac{\pi}{2}; n \in I$ (D) None of these
- Q.2** If $\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$ then general value of θ is –
 (A) $n\pi; n \in I$ (B) $n\pi \pm \frac{\pi}{3}; n \in I$
 (C) $\frac{n\pi}{3} + \frac{\pi}{12}; n \in I$ (D) none of these
- Q.3** Find the general value of θ , when $\sec\theta = \frac{2}{\sqrt{3}}$
 (A) $n\pi + \frac{\pi}{6}$ (B) $n\pi - \frac{\pi}{6}$
 (C) $2n\pi \pm \frac{\pi}{6}$ (D) $n\pi + (-1)^n \frac{\pi}{6}$
- Q.4** Find the general value of θ , when $\cos\left(\frac{-\theta}{2}\right) = 0$
 (A) $(n+1)\pi; n \in I$ (B) $n\pi; n \in I$
 (C) $(2n+1)\pi; n \in I$ (D) $2n\pi; n \in I$
- Q.5** If $\tan a\theta - \tan b\theta = 0$, then the values of θ for a series in –
 (A) A.P. (B) G.P.
 (C) H.P. (D) None of these
- Q.6** Find the general solution of $2 \sin x + \tan x = 0$
 (A) $n\pi, (3k \pm 1) \frac{2\pi}{3}; k \in I$
 (B) $2n\pi, (3k+1) \frac{2\pi}{3}; k \in I$
 (C) $2n\pi, (3k \pm 1) \frac{2\pi}{3}; k \in I$
 (D) None of these
- Q.7** The solution set of $(2 \cos x - 1)(3 + 2 \cos x) = 0$ in the interval $0 \leq x \leq 2\pi$ is –
 (A) $\{\pi/3\}$
 (B) $\{\pi/3, 5\pi/3\}$
 (C) $\{\pi/3, 5\pi/3, \cos^{-1}(-3/2)\}$
 (D) None of these
- Q.8** The general solution of the equation $\tan^2\theta + 2\sqrt{3} \tan\theta = 1$ is given by –
 (A) $\theta = \frac{\pi}{2}$ (B) $\left(n + \frac{1}{2}\right)\pi$
 (C) $(6n+1) \frac{\pi}{12}$ (D) $\frac{n\pi}{12}$
- Q.9** If $\cos\theta + \cos 7\theta + \cos 3\theta + \cos 5\theta = 0$, then $\theta =$
 (A) $\frac{n\pi}{4}; n \in I$ (B) $\frac{n\pi}{2}; n \in I$
 (C) $\frac{n\pi}{8}; n \in I; n \neq 8k$ (D) $\frac{n\pi}{3}; n \in I$
- Q.10** The value of θ satisfying $\sin 7\theta = \sin 4\theta - \sin\theta$ and $0 < \theta < \pi/2$ are –
 (A) $\frac{\pi}{9}, \frac{\pi}{4}$ (B) $\frac{\pi}{3}, \frac{\pi}{9}$
 (C) $\frac{\pi}{6}, \frac{\pi}{9}$ (D) $\frac{\pi}{3}, \frac{\pi}{4}$
- Q.11** The general solution of equation $\sin^2\theta \sec\theta + \sqrt{3} \tan\theta = 0$ is –
 (A) $\theta = n\pi + (-1)^{n+1} \frac{\pi}{3}$ (B) $\theta = n\pi$
 (C) $\theta = n\pi + (-1)^{n+1} \frac{\pi}{6}$ (D) $\theta = \frac{n\pi}{2}$
- Q.12** If $(1 + \tan\theta)(1 + \tan\phi) = 2$, then $\theta + \phi =$
 (A) 30° (B) 45°
 (C) 60° (D) 75°
- Q.13** If α is a root of $25 \cos^2\theta + 5 \cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to –
 (A) $\frac{24}{25}$ (B) $-\frac{24}{25}$ (C) $\frac{13}{18}$ (D) $-\frac{13}{18}$

Q.14 The general solution of the equation $\cos x + \sec x = 2$ is given by-

- (A) $2n\pi$; $n \in I$ (B) $n\pi$; $n \in I$
 (C) $\frac{n\pi}{4}$; $n \in I$ (D) $\frac{n\pi}{2}$; $n \in I$

Q.15 The general solution of $\sin x + 3 \sin 2x + \sin 3x = \cos x + 3 \cos 2x + \cos 3x$ then in the interval $0 \leq x \leq 2\pi$, $x =$

- (A) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}$
 (B) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
 (C) $\frac{4\pi}{3}, \frac{9\pi}{3}, \frac{2\pi}{3}, \frac{13\pi}{8}$
 (D) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{3}, \frac{4\pi}{3}$

Q.16 The solution set of the equation

$4\sin \theta \cos \theta - 2 \cos \theta - 2\sqrt{3} \sin \theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is-

- (A) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$
 (C) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (D) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

Q.17 The general value of θ satisfying the equation

$$\sin^2 \theta - 2 \cos \theta + \frac{1}{4} = 0$$

- (A) $2n\pi \pm \frac{\pi}{3}$ (B) $2n\pi \pm \frac{\pi}{4}$
 (C) $2n\pi \pm \frac{\pi}{6}$ (D) None of these

Q.18 If $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \tan 4\theta \tan 7\theta$, then $\theta =$

- (A) $\frac{n\pi}{4}$ (B) $\frac{n\pi}{7}$
 (C) $\frac{n\pi}{12}$; $n \neq 6(2k+1)$ (D) $n\pi$

Question based on

General solution of square of Trigonometric Equation

Q.19 If $2 \tan^2 \theta = \sec^2 \theta$, then the general value of θ is

- (A) $n\pi + \frac{\pi}{4}$; $n \in I$ (B) $n\pi - \frac{\pi}{4}$; $n \in I$
 (C) $n\pi \pm \frac{\pi}{4}$; $n \in I$ (D) $2n\pi \pm \frac{\pi}{4}$; $n \in I$

Q.20 If $3(\sec^2 \theta + \tan^2 \theta) = 5$, then the general value of θ is -

- (A) $2n\pi + \frac{\pi}{6}$ (B) $2n\pi \pm \frac{\pi}{6}$
 (C) $n\pi \pm \frac{\pi}{6}$ (D) $n\pi \pm \frac{\pi}{3}$

Q.21 If $2 \cot^2 \theta = \operatorname{cosec}^2 \theta$, then the general value of θ is-

- (A) $n\pi \pm \frac{\pi}{4}$ (B) $2n\pi \pm \frac{\pi}{4}$
 (C) $n\pi + (-1)^n \frac{\pi}{4}$ (D) $2n\pi \pm \frac{\pi}{2}$

Question based on

General solution of Trigonometric Equation $a \cos \theta + b \sin \theta = c$

Q.22 The equation $a \sin x + b \cos x = c$, where

$|c| > \sqrt{a^2 + b^2}$ has -

- (A) A unique solution
 (B) Infinite no. of solutions
 (C) No solution
 (D) None of these

Q.23 General solution of

$$\sin^3 x + \cos^3 x + \frac{3}{2} \sin 2x = 1$$

- (A) $x = n\pi$ when n is even integer
 (B) $x = n\pi + \frac{\pi}{2}$ when n is odd integer
 (C) $x = 2n\pi$ when n is odd integer
 (D) $x = n\pi - \frac{\pi}{2}$ when n is even integer

Question based on

Most General Value

Q.24 The most general value of θ which satisfies both

the equations $\tan \theta = \sqrt{3}$ and $\operatorname{cosec} \theta = -\frac{2}{\sqrt{3}}$ is

- (A) $n\pi + \frac{4\pi}{3}$; $n \in I$ (B) $n\pi + \frac{2\pi}{3}$; $n \in I$
 (C) $2n\pi + \frac{4\pi}{3}$; $n \in I$ (D) $2n\pi + \frac{2\pi}{3}$; $n \in I$

LEVEL- 2

- Q.1** The general solutions of the equation $\sec^2 x = \sqrt{2} (1 - \tan^2 x)$ are given by-
- (A) $n\pi + \frac{\pi}{8}$ (B) $n\pi \pm \frac{\pi}{4}$
 (C) $n\pi \pm \frac{\pi}{8}$ (D) None of these
- Q.2** The general solution of the equation $7 \cos^2 x + \sin x \cos x - 3 = 0$ is given by-
- (A) $n\pi + \frac{\pi}{2}$ ($n \in I$)
 (B) $n\pi - \frac{\pi}{4}$ ($n \in I$)
 (C) $n\pi + \tan^{-1} \frac{4}{3}$ ($n \in I$)
 (D) $n\pi - \frac{\pi}{4}$, $k\pi + \tan^{-1} \frac{4}{3}$ ($n, k \in I$)
- Q.3** Find the general solution of x, $\cos^2 2x + \cos^2 3x = 1$
- (A) $(2k + 1) \frac{\pi}{10}$, $k \in I$
 (B) $(\pi k + 1) \frac{\pi}{10}$; $k \in I$
 (C) $(2k - 1) \frac{\pi}{10}$, $k \in I$
 (D) Both (A) and (C)
- Q.4** The set of values of x for which $\sin x \cdot \cos^3 x > \cos x \cdot \sin^3 x$, $0 \leq x \leq 2\pi$, is-
- (A) $(0, \pi)$ (B) $\left(0, \frac{\pi}{4}\right)$
 (C) $\left(\frac{\pi}{4}, \pi\right)$ (D) None of these
- Q.5** The general solution of the equation $(\sqrt{3} - 1) \sin \theta + (\sqrt{3} + 1) \cos \theta = 2$ is -
- (A) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$
 (B) $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{12}$
 (C) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$
 (D) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}$
- Q.6** If $0 \leq x \leq 2\pi$, $0 \leq y \leq 2\pi$ and $\sin x + \sin y = 2$ then the value of $x + y$ is-
- (A) π (B) $\pi/2$
 (C) 3π (D) None of these
- Q.7** If $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$, the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is-
- (A) π (B) 2π
 (C) $5\pi/2$ (D) None of these
- Q.8** If $\cos x = \sqrt{1 - \sin 2x}$, $0 < x < \pi$, then a value of x is-
- (A) $\tan^{-1} 2$ (B) 0
 (C) π (D) None of these
- Q.9** The number of values of x in $[0, 5\pi]$ satisfying the equation $3\cos 2x - 10\cos x + 7 = 0$ are-
- (A) 5 (B) 6 (C) 8 (D) 10
- Q.10** Total number of solution of $16^{\cos^2 x} + 16^{\sin^2 x} = 10$ in $x \in [0, 3\pi]$ is equal to-
- (A) 4 (B) 8
 (C) 12 (D) 16

LEVEL- 3

- Q.1** The solution of the equation $\log_2(\sin x + \cos x) - \log_2(\cos x) + 1 = 0$:
- (A) $\tan^{-1}\left(-\frac{1}{2}\right)$ (B) 0
 (C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) None of these
- Q.2** The set of solution satisfying inequality $|\sin x| < \frac{1}{2}$ is-
- (A) $\left(n\pi, n\pi + \frac{\pi}{6}\right) (n \in I)$
 (B) $\left(2n\pi, 2n\pi + \frac{\pi}{6}\right)$
 (C) $\left(n\pi + \frac{\pi}{6}, n\pi + \frac{5\pi}{6}\right)$
 (D) None of these
- Q.3** The solution of equation $13 - 4 \cos^2 x = 12 \sin x$ is -
- (A) $n\pi + (-1)^n \sin^{-1}\left(\frac{3}{2}\right)$
 (B) $n\pi + (-1)^n \sin^{-1}\left(-\frac{3}{2}\right)$
 (C) $n\pi$
 (D) No solution
- Q.4** The solution set of equation $\cos^5 x = 1 + \sin^4 x$ is-
- (A) $n\pi (n \in I)$ (B) $2n\pi (n \in I)$
 (C) $4n\pi (n \in I)$ (D) None of these
- Q.5** The number of ordered pairs (x, y) satisfying $y = 2 \sin x$ and $y = 5x^2 + 2x + 3$ is -
- (A) 0 (B) 1 (C) 2 (D) ∞
- Q.6** If $0 \leq x \leq 3\pi, 0 \leq y \leq 3\pi$ and $\cos x \cdot \sin y = 1$ then the possible number of values of the ordered pair (x, y) is -
- (A) 6 (B) 12 (C) 8 (D) 15
- Q.7** The most general values of x for which $\sin x + \cos x = \min_{a \in \mathbb{R}} \{1, a^2 - 4a + 6\}$ are given by -
- (A) $2n\pi$
 (B) $2n\pi + \frac{\pi}{2}$
 (C) $n\pi + (-1)^n \cdot \frac{\pi}{4} - \frac{\pi}{4}$
 (D) None of these
- Q.8** The number of distinct solutions of $\sin 5\theta \cdot \cos 3\theta = \sin 9\theta \cdot \cos 7\theta$ in $[0, \pi/2]$ is-
- (A) 4 (B) 5 (C) 8 (D) 9
- Q.9** The values of $x \in [-2\pi, 2\pi]$ such that $\frac{\sin x + i \cos x}{1+i}, i = \sqrt{-1}$, is purely imaginary, are given by -
- (A) $n\pi - \frac{\pi}{4}$ (B) $n\pi + \frac{\pi}{4}$
 (C) $n\pi$ (D) None of these
- Q.10** The general solution of the equation $\tan 2\theta \cdot \tan \theta = 1$ for $n \in I$ is, θ is equal to-
- (A) $(2n+1)\frac{\pi}{4}$ (B) $(2n+1)\frac{\pi}{6}$
 (C) $(2n+1)\frac{\pi}{2}$ (D) $(2n+1)\frac{\pi}{3}$
- Q.11** Number of ordered pairs (a, x) satisfying the equation $\sec^2(a+2)x + a^2 - 1 = 0; -\pi < x < \pi$ is-
- (A) 2 (B) 1
 (C) 3 (D) Infinite
- Q.12** The general solution of the equation $\sin^{50} x - \cos^{50} x = 1$ is-
- (A) $2n\pi + \frac{\pi}{2}$ (B) $2n\pi + \frac{\pi}{3}$
 (C) $n\pi + \frac{\pi}{2}$ (D) $n\pi + \frac{\pi}{3}$

Q.13 For any real value of $\theta \neq \pi$, the value of the

expression $y = \frac{\cos^2 \theta - 1}{\cos^2 \theta + \cos \theta}$ is-

- (A) $1 \leq y \leq 2$ (B) $y < 0$ and $y > 2$
 (C) $-1 \leq y \leq 1$ (D) $y \geq 1$

Q.14 Total number of integral values of 'n' so that $\sin x (\sin x + \cos x) = n$ has at least one solution is-

- (A) 2 (B) 1
 (C) 3 (D) Zero

Q.15 If $r > 0$, $-\pi \leq \theta \leq \pi$ and r, θ satisfy $r \sin \theta = 3$ and $r = 4(1 + \sin \theta)$, then the number of possible solutions of the pair (r, θ) is-

- (A) 2 (B) 4
 (C) 0 (D) Infinite

Q.16 The value of θ satisfying

$3\cos^2 \theta - 2\sqrt{3} \sin \theta \cos \theta - 3 \sin^2 \theta = 0$ are-

- (A) $n\pi - \frac{2\pi}{3}, n\pi + \frac{\pi}{6}$ (B) $n\pi - \frac{\pi}{3}, n\pi + \frac{\pi}{6}$
 (C) $2n\pi - \frac{\pi}{3}, n\pi$ (D) $2n\pi + \frac{\pi}{3}, n\pi$

Statement type Questions

Each of the questions given below consists of **Statement – I** and **Statement – II**. Use the following **Key** to choose the appropriate answer.

- (A) If both **Statement- I** and **Statement- II** are true, and **Statement-II** is the correct explanation of **Statement– I**.
 (B) If both **Statement - I** and **Statement - II** are true but **Statement - II** is not the correct explanation of **Statement – I**.
 (C) If **Statement - I** is true but **Statement - II** is false.
 (D) If **Statement - I** is false but **Statement - II** is true.

Q.17 **Statement I** : If $\cos \theta = -\frac{1}{2}$ then general solution for θ is $2n\pi \pm \frac{\pi}{6}$.

Statement II : If $\cos \theta = \cos \alpha$, then $\theta = 2n\pi \pm \alpha$

Q.18 **Statement I**: If $\tan m\theta + \cot n\theta = 0$ then general value of $\theta = \frac{(2r+1)\pi}{2(m-n)}$ where $r \in I$.

Statement II : If $\tan \theta = \cot \alpha \therefore \theta = r\pi + \alpha$

Q.19 **Statement I**: $\cot \theta - \tan \theta = 2$, then

$\theta = (4n + 1) \frac{\pi}{8}$

Statement II : $\sin 2x + \cos 2x + \sin x + \cos x + 1 = 0$ has no solution in the 1st quadrant.

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 Find the no. of roots of the equation $\tan x + \sec x = 2 \cos x$ in the interval $[0, 2\pi]$ -

[AIEEE-2002]

- (A) 1 (B) 2
(C) 3 (D) 4

Q.2 General solution of $\tan 5\theta = \cot 2\theta$ is-

[AIEEE-2002]

- (A) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$ (B) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$
(C) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$ (D) None of these

Q.3 The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2 \sin^2 x + 5 \sin x - 3 = 0$ is –

[AIEEE-2006]

- (A) 6 (B) 1
(C) 2 (D) 4

Q.4 If $0 < x < \pi$, and $\cos x + \sin x = \frac{1}{2}$,

then $\tan x$ is – [AIEEE-2006]

- (A) $(4 - \sqrt{7})/3$ (B) $-(4 + \sqrt{7})/3$
(C) $(1 + \sqrt{7})/4$ (D) $(1 - \sqrt{7})/4$

Q.5 The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are –

[AIEEE-2011]

- (A) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$
(B) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$
(C) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$
(D) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

SECTION-B

Q.1 The number of solutions of the equation $\tan x + \sec x = 2 \cos x$ lying in the interval $[0, 2\pi]$ is [IIT-1993]

- (A) 0 (B) 1 (C) 2 (D) 3

Q.2 Let $2 \sin^2 x + 3 \sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval- [IIT-1994]

- (A) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (B) $\left(-1, \frac{5\pi}{6}\right)$
(C) $(-1, 2)$ (D) $\left(\frac{\pi}{6}, 2\right)$

Q.3 The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$ for all x is-

[IIT-1994]

- (A) 0 (B) 1
(C) 2 (D) infinite

Q.4 The smallest positive root of the equation $\tan x - x = 0$ lies on- [IIT-1994]

- (A) $\left(0, \frac{\pi}{2}\right)$
(B) $\left(\frac{\pi}{2}, \pi\right)$
(C) $\left(\pi, \frac{3\pi}{2}\right)$
(D) $\left(\frac{3\pi}{2}, 2\pi\right)$

Q.5 General value of θ satisfying equation $\tan^2 \theta + \sec 2\theta = 1$ is- [IIT-1996]

- (A) $n\pi$ (B) $n\pi + \frac{\pi}{3}$
(C) $n\pi - \frac{\pi}{3}$ (D) all of these

Q.6 The solution set of the system of equations:

$$x + y = \frac{2\pi}{3}, \cos x + \cos y = \frac{3}{2}, \text{ where } x \text{ \& } y$$

are real in: **[IIT-1998]**

- (A) a finite non-empty set
- (B) null set
- (C) ∞
- (D) none of these

Q.7 The number of values of x in the interval $[0, 5\pi]$ satisfying the equation

$$3 \sin^2 x - 7 \sin x + 2 = 0 \text{ is- } \quad \text{[IIT-1998]}$$

- (A) 0
- (B) 5
- (C) 6
- (D) 10

Q.8 The number of integral values of k for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution is- **[IIT-2002]**

- (A) 4
- (B) 8
- (C) 10
- (D) 12

Q.9 For which interval for θ , the inequation $(2 \sin^2 \theta - 5 \sin \theta + 2) > 0$. When $0 < \theta < 2\pi$

[IIT-2006]

- (A) $\left(\frac{13\pi}{48}, 2\pi\right)$
- (B) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$
- (C) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
- (D) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

Q.10 The number of solutions of the pair of equations $2\sin^2 \theta - \cos 2\theta = 0, 2\cos^2 \theta - 3 \sin \theta = 0$ in the interval $[0, 2\pi]$ is- **[IIT-2007]**

- (A) zero
- (B) one
- (C) two
- (D) four

Numerical Response Question:

Q.11 The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and

$$\tan \theta = \cot 5\theta \text{ as well as } \sin 2\theta = \cos 4\theta \text{ is.....}$$

[IIT-2010]

Q.12 The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is.}$$

[IIT-2011]

ANSWER KEY

LEVEL-1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	A	D	C	C	A	A	B	C	C	A	B	B	B	A
Q.No.	15	16	17	18	19	20	21	22	23	24				
Ans.	B	D	A	C	C	C	A	C	A	C				

LEVEL-2

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	C	D	D	B	A	A	B	A	C	C

LEVEL-3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	D	D	B	A	A	C	D	A	B	C	C	B	A	A
Q.No.	16	17	18	19											
Ans.	B	D	C	B											

LEVEL-4

SECTION-A

Q.No.	1	2	3	4	5
Ans.	B	D	D	B	D

SECTION-B

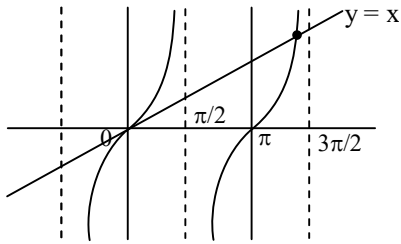
1.[C] $\tan x + \sec x = 2 \cos x$
 $\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \cos x$
 $\sin x + 1 = 2 \cos^2 x$
 $\sin x + 1 = 2 - 2 \sin^2 x$
 $2 \sin^2 x + \sin x - 1 = 0$
 $2 \sin^2 x + 2 \sin x - \sin x - 1 = 0$
 $2 \sin x (\sin x + 1) - 1 (\sin x + 1) = 0$
 $(2 \sin x - 1) (\sin x + 1) = 0$
 $\sin x = 1/2$ or $\sin x = -1$ (not possible)
 $x = \pi/6, 5\pi/6$

2.[D] $2 \sin^2 x + 3 \sin x - 2 > 0$
 $2 \sin^2 x + 4 \sin x - \sin x - 2 > 0$
 $2 \sin x (\sin x + 2) - 1 (\sin x + 2) > 0$
 $(2 \sin x - 1) (\sin x + 2) > 0$
 $\sin x > \frac{1}{2}$
 $x \in \left(\frac{\pi}{6}, \frac{5\pi}{6} \right)$ (i)

$x^2 - 2x + x - 2 < 0$
 $x(x-2) + 1(x-2) < 0$
 $(x+1)(x-2) < 0$
 $-1 < x < 2$ (ii)
 From (i) and (ii) we get
 $\pi/6 < x < 2$
 $x \in (\pi/6, 2)$

3.[D] $a_1 + a_2 \cos 2x + \frac{1}{2} a_3 (1 - \cos 2x) = 0$
 $2a_1 + 2a_2 \cos 2x + a_3 - a_3 \cos 2x = 0$
 $\cos 2x = \left(\frac{2a_1 + a_3}{a_3 - 2a_2} \right)$
 $\cos 2x = \frac{2a_1 + a_3}{a_3 - 2a_2}$
 since $-1 \leq \cos 2x \leq 1$
 then $-1 \leq \frac{2a_1 + a_3}{a_3 - 2a_2} \leq 1$
 Hence, Infinite triplet possible.

4.[C] By the graphs of $y = x$ and $y = \tan x$



We see the smallest positive roots of $\tan x = x$ lies in $(\pi, 3\pi/2)$

5.[B] $\tan^2\theta + \frac{1 + \tan^2\theta}{1 - \tan^2\theta} = 1$

$$\frac{\tan^2\theta(1 - \tan^2\theta) + 1 + \tan^2\theta}{1 - \tan^2\theta} = 1$$

$$2 \tan^2\theta - \tan^4\theta + 1 = 1 - \tan^2\theta$$

$$\tan^4\theta - 3 \tan^2\theta = 0$$

$$\tan^2\theta(\tan^2\theta - 3) = 0$$

$$\tan^2\theta = 0$$

$$\theta = n\pi$$

$$\tan^2\theta = \tan^2\pi/3$$

$$\theta = n\pi \pm \pi/3$$

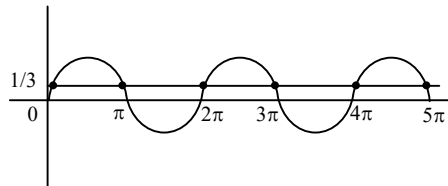
6.[B] $\cos x + \cos y = 3/2$

$$2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} = 3/2$$

$$2 \cos \frac{\pi}{3} \cos \frac{x-y}{2} = 3/2$$

$$\cos \frac{x-y}{2} = 3/2 \text{ which is not possible.}$$

7.[C] By graph



$$3 \sin^2 x - 6 \sin x + 2 = 0$$

$$3 \sin x (\sin x - 2) - 1 (\sin x - 2) = 0$$

$$(\sin x - 2)(3 \sin x - 1) = 0$$

$$\text{If } \sin x - 2 = 0$$

$$\sin x \neq 2$$

$$\sin x = 1/3$$

$$\text{Hence no of value lies } [0, 5\pi] = 6$$

8. [B] $\frac{7}{\sqrt{74}} \cos x + \frac{5}{\sqrt{74}} \sin x = \frac{2k+1}{\sqrt{74}}$

$$\cos(x - \alpha) = \frac{2k+1}{\sqrt{74}} \text{ where } \alpha = \tan^{-1} \frac{5}{7}$$

$$-1 \leq \frac{2k+1}{\sqrt{74}} \leq 1$$

$$-\sqrt{74} \leq 2k+1 \leq \sqrt{74}$$

$$\frac{-\sqrt{74}-1}{2} \leq k \leq \frac{\sqrt{74}-1}{2}$$

$$k = -4, -3, -2, -1, 0, 3, 2, 1$$

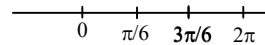
No. of integral value of $k = 8$

9.[D] $2 \sin^2\theta - 5 \sin \theta + 2 > 0$

$$2 \sin^2\theta - 4 \sin \theta - \sin \theta + 2 > 0$$

$$2 \sin \theta (\sin \theta - 2) - 1(\sin \theta - 2) > 0$$

$$(2 \sin \theta - 1)(\sin \theta - 2) > 0$$



$$\theta \in (0, \pi/6) \cup (5\pi/6, 2\pi)$$

10. [C] $2 \sin^2\theta - (1 - 2 \sin^2\theta) = 0$

$$4 \sin^2\theta - 1 = 0$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$\theta = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$$

$$2 \cos^2\theta - 3 \sin \theta = 0$$

$$2(1 - \sin^2\theta) - 3 \sin \theta = 0$$

$$2 \sin^2\theta + 3 \sin \theta - 2 = 0$$

$$2 \sin^2\theta + 4 \sin \theta - \sin \theta - 2 = 0$$

$$2 \sin \theta (\sin \theta + 2) - 1(\sin \theta + 2) = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 2) = 0$$

$$\sin \theta \neq -2$$

$$\sin \theta = \frac{1}{2} = \sin \pi/6$$

$$\theta = \pi/6, 5\pi/6$$

Hence No of pair of sol = 2

$$\begin{aligned}
11.[3] \quad & \tan \theta = \cot 5\theta \\
& \Rightarrow \cos 6\theta = 0 \\
& \Rightarrow 4 \cos^3 2\theta - 3 \cos 2\theta = 0 \\
& \Rightarrow \cos 2\theta = 0 \text{ or } \pm \frac{\sqrt{3}}{2} \\
& \sin 2\theta = \cos 4\theta \\
& \Rightarrow 2 \sin^2 2\theta + 2 \sin 2\theta - \sin 2\theta - 1 = 0 \\
& \sin 2\theta = -1 \text{ or } \sin 2\theta = \frac{1}{2} \\
& \cos 2\theta = 0 \text{ and } \sin 2\theta = -1 \\
& \Rightarrow 2\theta = -\pi/2 \Rightarrow \theta = -\pi/4 \\
& \cos \theta = 0 \text{ \& } \sin 2\theta = \frac{1}{2} \\
& 2\theta = \pi/6, 5\pi/6 \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12} \\
& \theta = \frac{-\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}
\end{aligned}$$

$$\begin{aligned}
12.[7] \quad & \text{Let } \frac{\pi}{n} = x \\
& \frac{1}{\sin x} = \frac{1}{\sin 2x} + \frac{1}{\sin 3x} \\
& \sin 2x \sin 3x = \sin x \sin 3x + \sin x \sin 2x \\
& \Rightarrow \cos x - \cos 5x = \cos 2x - \cos 4x + \cos x \\
& \quad \quad \quad - \cos 3x \\
& \cos 2x - \cos 4x + \cos 5x - \cos 3x = 0 \\
& \Rightarrow 2 \sin 3x \sin x + \sin 4x \sin (-x) = 0 \\
& 2 \sin x (\sin 3x - \sin 4x) = 0 \\
& -2.2 \sin x \cos \frac{7x}{2} \cos n \frac{x}{2} = 0 \\
& x = k\pi. \quad \frac{7x}{2} = (2k+1) \frac{\pi}{2} \\
& \frac{x}{2} = (2x+1) \frac{\pi}{2}; x \in I \\
& x = \frac{\pi}{n} \text{ only possible value of } n \text{ is '7'}.
\end{aligned}$$