JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

INVERSE TRIGONOMETRIC FUNCTIONS

(PRACTICE SHEET)

LEVEL- 1

Question based on **Properties of Inverse Trigonometric Function**

Q.1 $\sin^{-1}x + \sin^{-1}\frac{1}{x} + \cos^{-1}x + \cos^{-1}\frac{1}{x} =$

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{2}$ (D) None of these

Q.2 If $x > 0$, $\sin^{-1}(2\pi + x) + \cos^{-1}(2\pi + x)$

(A) $2\pi + \frac{\pi}{2}$ (B) $\frac{\pi}{2}$

(C) $x + \frac{\pi}{2}$ (D) None of these

Q.3 $\sin^{-1}\sin 15 + \cos^{-1}\cos 20 + \tan^{-1}\tan 25 =$

(A) $19\pi - 60$ (B) $30 - 9\pi$

(C) $19 - 60\pi$ (D) $60\pi - 19$

Q.4 $\cos^{-1}\left(\frac{\pi}{3} + \sec^{-1}(-2)\right) =$

(A) -1 (B) 1

(C) 0 (D) None of these

Q.5 $\sin^{-1}\sin\frac{23\pi}{7} + \cos^{-1}\cos\frac{39\pi}{7}$

(A) $\frac{\pi}{7}$ (B) $\frac{2\pi}{7}$

(C) $\frac{3\pi}{7}$ (D) $\frac{4\pi}{7}$

Q.6 $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$ is equal to-

(A) $-\frac{17\pi}{15}$ (B) $\frac{17\pi}{15}$

(C) $\frac{2\pi}{15}$ (D) $\frac{13\pi}{15}$

Q.7 $\sin\left[\frac{\pi}{6} + \sin^{-1}\left(-\frac{1}{2}\right)\right] =$

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) 1

Q.8 $\tan\left(90° - \cot^{-1}\frac{1}{3}\right) =$

(A) 3 (B) 2/3

(C) 1/3 (D) $\frac{1}{\sqrt{10}}$

Q.9 $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] =$

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Q.10 $\sin\left(\cos^{-1}\frac{12}{13}\right) =$

(A) $\frac{5}{13}$ (B) $\frac{12}{13}$

(C) $\frac{13}{5}$ (D) $\frac{5}{12}$

Q.11 If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then

$\cos^{-1}x + \cos^{-1}y =$

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) π

Q.12 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is equal to -

(A) 0 (B) 3

(C) – 3 (D) 9

Q.13 If $\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1}x$, then x =

(A) 0 (B) 1

(C) 1/2 (D) 1/4

Q.14 $\cos[\cos^{-1}(\sqrt{3}/2) + \sin^{-1}(1/\sqrt{2})]$ is equal to-

(A) $(\sqrt{3}+1)/2\sqrt{2}$ (B) $(\sqrt{3}-1)/2\sqrt{2}$

(C) $(-\sqrt{3}+1)/2\sqrt{2}$ (D) $(-\sqrt{3}-1)/2\sqrt{2}$

Q.15 The value of $\sin^{-1}(\sin 10)$ is -

(A) 10 (B) $10 - 3\pi$

(C) $3\pi - 10$ (D) None of these

Q.16 $\cos^{-1}\sqrt{\frac{1+\cos x}{2}}$; $\forall\ 0 < x < \pi$ is

(A) x (B) $\frac{x}{2}$

(C) 2x (D) None of these

Q.17 $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) =$

(A) 5 (B) 13

(C) 15 (D) 6

Q.18 $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] =$

(A) $\frac{3-\sqrt{5}}{2}$ (B) $\frac{3+\sqrt{5}}{2}$

(C) $\frac{2}{3-\sqrt{5}}$ (D) $\frac{2}{3+\sqrt{5}}$

Q.19 $\sin\cot^{-1}\tan\cos^{-1}x$ is equal to

(A) x (B) $\sqrt{1-x^2}$

(C) $\frac{1}{x}$ (D) none of these

Q.20 Let $f(x) = \sin^{-1}x + \cos^{-1}x$. Then $\frac{\pi}{2}$ is equal to

(A) f(–2) (B) $f(k^2 - 2k + 3), k \in R$

(C) $f\left(\frac{1}{1+k^2}\right), k \in R$ (D) none

Question based on Formulae for sum & difference of Inverse Trigonometric Function

Q.21 $\tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc} =$

(A) $\tan^{-1}a - \tan^{-1}b$ (B) $\tan^{-1}a - \tan^{-1}c$

(C) $\tan^{-1}b - \tan^{-1}c$ (D) $\tan^{-1}c - \tan^{-1}a$

Q.22 If $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3} = \sin^{-1}x$, then x is equal to -

(A) 0 (B) $\frac{\sqrt{5}-4\sqrt{2}}{9}$

(C) $\frac{\sqrt{5}+4\sqrt{2}}{9}$ (D) $\frac{\pi}{2}$

Q.23 If $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ then x =

(A) –1 (B) $\frac{1}{6}$

(C) –1, $\frac{1}{6}$ (D) None of these

Q.24 $\cot^{-1}\frac{xy+1}{x-y}+\cot^{-1}\frac{yz+1}{y-z}+\cot^{-1}\frac{zx+1}{z-x}=$

(A) 0

(B) 1

(C) $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z$

(D) None of these

Q.25 $\cot[\tan^{-1}(1/7) + \tan^{-1}(1/13)]$ is equal to -

(A) 2/9 (B) 9/2

(C) 7/9 (D) 9/7

Q.26 The value of $\tan\left\{\cos^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{2}{\sqrt{13}}\right)\right\}$ is

(A) $\frac{7}{16}$ (B) $\frac{17}{6}$

(C) $\frac{6}{17}$ (D) none

LEVEL- 2

Q.1 The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is -

(A) $\frac{3\pi}{5}$ (B) $\frac{7\pi}{5}$ (C) $\frac{\pi}{10}$ (D) $-\frac{\pi}{10}$

Q.2 If $\theta = \cot^{-1}\sqrt{\cos x} - \tan^{-1}\sqrt{\cos x}$, then $\sin\theta$ =

(A) $\tan\frac{1}{2}x$ (B) $\tan^2(x/2)$

(C) $\frac{1}{2}\tan^{-1}(x/2)$ (D) None of these

Q.3 If a, b, c be positive real numbers and the value of

$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} +$

$\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ then $\tan\theta$ is equal to -

(A) 0 (B) 1

(C) $\frac{a+b+c}{abc}$ (D) None of these

Q.4 The value of

$\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$ is equal to -

(A) $\pi/4$ (B) $5\pi/12$

(C) $3\pi/4$ (D) $13\pi/12$

Q.5 If $x^2 + y^2 + z^2 = r^2$, then

$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right) =$

(A) π (B) $\pi/2$

(C) 0 (D) None of these

Q.6 If $xy + yz + zx = 1$, then,

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z =$

(A) π (B) $\pi/2$

(C) 1 (D) None of these

Q.7 The principal value of

$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is-

(A) π (B) $\pi/2$ (C) $\pi/3$ (D) $4\pi/3$

Q.8 If $3\cos^{-1}(x^2 - 7x + 25/2) = \pi$, then x =

(A) only 3 (B) only 4

(C) 3 or 4 (D) None of these

Q.9 If $\tan(x + y) = 33$ and $x = \tan^{-1}3$, then y will be

(A) 0.3 (B) $\tan^{-1}(1.3)$

(C) $\tan^{-1}(0.3)$ (D) $\tan^{-1}\left(\frac{1}{18}\right)$

Q.10 $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x\right)$, $(x \neq 0)$

is equal to -

(A) x (B) 2x (C) 2/x (D) None

Q.11 The value of

$3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$ is-

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) None

Q.12 The value of $\sin^2\left(\cos^{-1}\frac{1}{2}\right) + \cos^2\left(\sin^{-1}\frac{1}{3}\right)$ is-

(A) $\frac{17}{36}$ (B) $\frac{59}{36}$ (C) $\frac{36}{59}$ (D) None

Q.13 Solution of equation

$\tan(\cos^{-1}x) = \sin(\cot^{-1}1/2)$ is-

(A) $x = \frac{\sqrt{7}}{3}$ (B) $x = \frac{\sqrt{5}}{3}$

(C) $x = \frac{3\sqrt{5}}{2}$ (D) None of these

Q.14 $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$ is equal to-

(A) $\sqrt{\frac{x^2+2}{x^2+3}}$ (B) $\sqrt{\frac{x^2+2}{x^2+1}}$

(C) $\sqrt{\frac{x^2+1}{x^2+2}}$ (D) None of these

Q.15 If $a \le \tan^{-1}x + \cot^{-1}x + \sin^{-1}x \le b$. Then-

(A) a = 0, b = π (B) b = $\frac{\pi}{2}$

(C) a = $\frac{\pi}{4}$ (D) None of these

Q.16 If $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \frac{3\pi}{2}$. Then

$\alpha\beta + \beta\gamma + \gamma\alpha$ is -

(A) 1 (B) 0 (C) 3 (D) –3

Q.17 If $\cos^{-1}x > \sin^{-1}x$, then-

(A) $x < 0$ (B) $-1 < x < 0$

(C) $0 \le x < \frac{1}{\sqrt{2}}$ (D) $-1 \le x < \frac{1}{\sqrt{2}}$

Q.18 The principal value of

$\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right\}$ is-

(A) $\frac{3\pi}{20}$ (B) $\frac{7\pi}{20}$ (C) $\frac{7\pi}{10}$ (D) none

LEVEL- 3

Q.1 If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then x equals-

(A) –1 (B) 1

(C) 0 (D) None of these

Q.2 $\sum_{r=1}^{n} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$ is equal to -

(A) $\tan^{-1}(2^n)$ (B) $\tan^{-1}(2^n) - \frac{\pi}{4}$

(C) $\tan^{-1}(2^{n+1})$ (D) $\tan^{-1}(2^{n+1}) - \frac{\pi}{4}$

Q.3 If $\tan^{-1}\frac{1}{a-1} = \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{a^2-x+1}$, then x is-

(A) $\frac{a}{2}$ (B) a^3

(C) $a^2 - a + 1$ (D) $a^2 + a - 1$

Q.4 $\tan^{-1}n + \cot^{-1}(n+1)$ is equal to-

(A) $\cot^{-1}(n^2 + n + 1)$

(B) $\cot^{-1}(n^2 - n + 1)$

(C) $\tan^{-1}(n^2 + n + 1)$

(D) None of these

Q.5 The value of $\sin\left[\cot^{-1}\left(\cot\frac{17\pi}{3}\right)\right]$ is-

(A) $-\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) None of these

Q.6 $\sec(\text{cosec}^{-1}x)$ is equal to-

(A) $\text{cosec}(\sec^{-1}x)$ (B) cot x

(C) π (D) None of these

Q.7 If $\sum_{i=1}^{20}\sin^{-1}x_i = 10\pi$ then $\sum_{i=1}^{20}x_i$ is equal to-

(A) 20 (B) 10

(C) 0 (D) None of these

Q.8 The value of $\cot^{-1}3 + \sec^{-1}\frac{\sqrt{5}}{2}$ is-

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) None of these

Q.9 $-\frac{2\pi}{5}$ is the principal value of -

(A) $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)$ (B) $\sin^{-1}\left(\sin\frac{7\pi}{5}\right)$

(C) $\sec^{-1}\left(\sec\frac{7\pi}{5}\right)$ (D) None of these

Q.10 If $\theta = \sin^{-1}(\sin(-600°))$, then one of the possible value of θ is-

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$ (D) $-\frac{2\pi}{3}$

Q.11 $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$ is equal to -

(A) $\frac{6}{25}$ (B) $\frac{24}{25}$

(C) $\frac{4}{5}$ (D) $-\frac{24}{25}$

Q.12 If $\sin^{-1}\sin x = \cos^{-1}\cos x$; $\forall\ 0 < x < \pi$ then x =

(A) $\left[0, \frac{\pi}{4}\right]$ (B) $\left(0, \frac{\pi}{2}\right]$

(C) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (D) $\left[0, \frac{\pi}{2}\right)$

Q.13 If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$, then x is-

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $-\frac{1}{2}$ (D) None of these

Q.14 The principal value of $\cos^{-1}\left(-\sin\frac{7\pi}{6}\right)$ is-

(A) $\frac{5\pi}{3}$ (B) $\frac{7\pi}{6}$

(C) $\frac{\pi}{3}$ (D) None of these

Q.15 The number of positive integral solutions of the equation

$\tan^{-1}x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$ is-

(A) one (B) two

(C) zero (D) None of these

Q.16 The value of

$$\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)}\right.\right.$$

$$\left.\left.+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}\sqrt{2}\right)\right]$$ is -

(A) 0 (B) $\pi/4$

(C) $\pi/6$ (D) $\pi/2$

Q.17 The value of $\tan\left\{\left(\cos^{-1}\left(-\frac{2}{7}\right)-\pi/2\right)\right\}$ is-

(A) $\frac{2}{3\sqrt{5}}$ (B) $\frac{2}{3}$

(C) $\frac{1}{\sqrt{5}}$ (D) $\frac{4}{\sqrt{5}}$

Q.18 If $\cos^{-1}(a) + \cos^{-1}(b) + \cos^{-1}(c) = 3\pi$ and $f(1) = 2$, $f(x + y) = f(x)\, f(y)$ for all x, y; then

$$a^{2f(1)} + b^{2f(2)} + c^{2f(3)} + \frac{(a+b+c)}{a^{2f(1)} + b^{2f(2)} + c^{2f(3)}}$$

is equal to -

(A) 0 (B) 1 (C) 2 (D) 3

Q.19 $\tan^{-1}\tan\left(\frac{5\pi}{7}\right)$ is equal to-

(A) $\frac{2\pi}{7}$ (B) $\frac{5\pi}{7}$ (C) $-\frac{2\pi}{7}$ (D) $\frac{\pi}{7}$

Q.20 The principal value of

$\sin^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}(1) + \cos^{-1}\cos\left(-\frac{\pi}{2}\right)$ is -

(A) $\frac{5\pi}{12}$ (B) $-\frac{5\pi}{12}$

(C) $\frac{\pi}{12}$ (D) $\frac{7\pi}{12}$

Q.21 If $\sin^{-1}x + \tan^{-1}x = y$ $(-1 < x < 1)$, then which is not possible -

(A) $y = \frac{3\pi}{2}$ (B) $y = 0$

(C) $y = \frac{\pi}{2}$ (D) $y = -\frac{\pi}{2}$

Q.22 The number of positive integral solutions of $\cos^{-1}\left(4x^2 - 8x + \frac{7}{2}\right) = \frac{\pi}{3}$ is -

(A) one (B) two

(C) three (D) None of these

Statement type Questions

Each of the questions given below consists of Statement–I and Statement–II. Use the following Key to choose the appropriate answer.

(A) If both Statement-I and Statement-II are true, and Statement-II is the correct explanation of Statement– I.

(B) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement – I.

(C) If Statement-I is true but Statement-II is false.

(D) If Statement- I is false but Statement-II is true.

Q.23 **Statement I :** The equation

$\sec^{-1}x + \cot^{-1}x < \frac{-\pi}{2}$ has no solution.

Statement II : sec x is not defined at $\frac{\pi}{2}$.

Q.24 **Statement I :** The equation $\sin^{-1}x = \cos^{-1}x$ has one and only one solution.

Statement II : The equation $\tan^{-1}x = 1$ has only one solution.

Q.25 **Statement I :** $\sin^{-1}\sin\ x \neq \sin\ \sin^{-1}x$, if $-1 \le x \le 1$

Statement II : $\sin\theta$ and $\sin^{-1}\theta$ are different functions

Q.26 **Statement I :** Equation $2\sin^{-1}x + 3\sin^{-1}y = \frac{5\pi}{2}$ and $y = px - 5$ hold simultaneously when p is equal to 6.

Statement II :

The range of $\sin^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Q.27 **Statement I :** The maximum value or $\sin^{-1}x + \text{cosec}^{-1}x + \cos^{-1}x + \sec^{-1}x + \tan^{-1}x$ is $\frac{3\pi}{2}$

Statement II :

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ and $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$

Passage based Questions

Passage

Every bijection $f : A \to B$ there exists a bijection $g : B \to A$ defined by $g(y) = x$ if and only if $f(x) = y$. The function $g : B \to A$ is called the inverse of function $f : A \to B$ and is denoted by f^{-1}.

Q.28 The value of $\cos[\tan^{-1}\tan 2]$ is -

(A) $\frac{1}{\sqrt{5}}$ (B) $-\frac{1}{\sqrt{5}}$

(C) cos 2 (D) – cos2

Q.29 If $\pi \le x \le 2\pi$ then $\cos^{-1}\cos x$ is equal to -

(A) x (B) – x

(C) $2\pi + x$ (D) $2\pi - x$

Q.30 If $x + \frac{1}{x} = 2$, the principal value of $\sin^{-1}x$ is -

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) π (D) $\frac{3\pi}{2}$

Passage

The inverse of a function $f : A \to B$ exists iff f is one-one onto i.e. a bijection and is given by $f(x) = y\ \ f^{-1}(y) = x$

Q.31 The trigonometric equation $\sin^{-1}x = 2\sin^{-1}$ a has a solution for -

(A) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ (B) All real values of a

(C) $|a| < \frac{1}{2}$ (D) $|a| \le \frac{1}{\sqrt{2}}$

Q.32 The value of $\sin\left[\frac{\pi}{6} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to -

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$

(C) $-\frac{\sqrt{3}}{2}$ (D) 0

Q.33 If $\sin^{-1}(\sin x) = \pi - x$ then x belongs to –

(A) R (B) $[0, \pi]$

(C) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (D) $[\pi, 2\pi]$

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 The value of $\cos^{-1}(-1) - \sin^{-1}(1)$ is-

[AIEEE - 2002]

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) $-\frac{3\pi}{2}$

Q.2 If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to - **[AIEEE - 2005]**

(A) $2\sin 2\alpha$ (B) 4

(C) $4\sin^2\alpha$ (D) $-4\sin^2\alpha$

Q.3 If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is- **[AIEEE - 2007]**

(A) 1 (B) 3 (C) 4 (D) 5

Q.4 The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is -

[AIEEE-2008]

(A) $\frac{3}{17}$ (B) $\frac{2}{17}$ (C) $\frac{5}{17}$ (D) $\frac{6}{17}$

SECTION –B

Q.1 If $\sin^{-1}x = \frac{\pi}{5}$, $x \in (-1, 1)$, then $\cos^{-1}x =$

[IIT Scr. 1992]

(A) $\frac{3\pi}{10}$ (B) $\frac{5\pi}{10}$ (C) $-\frac{3\pi}{10}$ (D) $\frac{9\pi}{10}$

Q.2 $\tan(\cos^{-1}x)$ is equal to- **[IIT Scr. 1993]**

(A) $\frac{\sqrt{1-x^2}}{x}$ (B) $\frac{x}{1+x^2}$

(C) $\frac{\sqrt{1+x^2}}{x}$ (D) $\sqrt{1-x^2}$

Q.3 If we consider only the principal values of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is-

[IIT - 1994]

(A) $\frac{\sqrt{29}}{3}$ (B) $\frac{29}{3}$ (C) $\frac{\sqrt{3}}{29}$ (D) $\frac{3}{29}$

Q.4 The number of real solution of

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is-

[IIT - 1999]

(A) zero (B) one (C) two (D) Infinite

Q.5 If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - .....\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - .....\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$, then x equals **[IIT - 2001]**

(A) $\frac{1}{2}$ (B) 1 (C) $-\frac{1}{2}$ (D) – 1

Q.6 For which value of x,

$\sin(\cot^{-1}(x+1)) = \cos(\tan^{-1}x)$ **[IIT Scr. 2004]**

(A) 1/2 (B) 0 (C) 1 (D) –1/2

Q.7 Let (x, y) be such that

$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$.

Match the statement in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. **[IIT - 2007]**

Column I	Column II
(A) If a = 1 and b = 0, then (x, y)	(P) lies on the circle $x^2 + y^2 = 1$
(B) If a = 1 and b = 1, then (x, y)	(Q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If a = 1 and b = 2, then (x, y)	(R) lies on y = x
(D) If a = 2 and b = 2, then (x, y)	(S) lies on $(4x^2 - 1)(y^2 - 1) = 0$

Q.8 If $0 < x < 1$, then

$\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$ is equal to- **[IIT - 2008]**

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Q.9 The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is –

[JEE - Advance 2013]

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

Q.10 Match List-I with List-II and select the correct answer using the code given below the lists :

[JEE - Advance 2013]

List – I **List - II**

(P) $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right)^{1/2}$ takes value (1) $\frac{1}{2}\sqrt{\frac{5}{3}}$

(Q) If $\cos x+\cos y+\cos z=0$ $=\sin x+\sin y+\sin z$ then possible value of $\cos\frac{x-y}{2}$ is (2) $\sqrt{2}$

(R) If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x$ $=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of sec x is (3) $\frac{1}{2}$

(S) If $\cot(\sin^{-1}\sqrt{1-x^2})$ $=\sin(\tan^{-1}(x\sqrt{6}))$, $x\neq 0$, then possible value of x is (4) 1

Codes :

	P	Q	R	S
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

ANSWER KEY

LEVEL-1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	D	B	D	A	D	A	C	C	A	B	A	B	B	C
Q.No.	16	17	18	19	20	21	22	23	24	25	26				
Ans.	B	C	A, D	A	C	B	C	B	A	B	B				

LEVEL-2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	D	B	A	C	B	B	A	C	C	C	D	B	B	C	A	C	D	D

LEVEL-3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	C	B	A	A	A	B	A	D	B	B	C	B	A	A	C	C	D
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33							
Ans.	A	D	B	B	D	A	D	D	D	B	D	A	C							

LEVEL-4

SECTION-A

Q.No.	1	2	3	4
Ans.	B	C	B	D

SECTION-B

1.[A] $\sin^{-1}x = \pi/5$

$\pi/2 - \cos^{-1}x = \pi/5$

$\cos^{-1}x = \pi/2 - \pi/5$

$\cos^{-1}x = 3\pi/10$

2.[A] $\tan(\cos^{-1}x)$

$= \tan^{-1}\tan^{-1}\dfrac{\sqrt{1-x^2}}{x} = \dfrac{\sqrt{1-x^2}}{x}$

3.[D] $\tan\left(\cot^{-1}\dfrac{1}{5\sqrt{2}} - \sin^{-1}\dfrac{4}{\sqrt{17}}\right)$

$= \tan\left(\cos^{-1}\dfrac{1}{5\sqrt{2}} - \cos^{-1}\dfrac{1}{\sqrt{17}}\right)$

$= \tan\cos^{-1}\left(\dfrac{1}{5\sqrt{2}}\dfrac{1}{\sqrt{17}} + \dfrac{7}{5\sqrt{2}}\times\dfrac{4}{\sqrt{17}}\right)$

$= \tan\cos^{-1}\left(\dfrac{+29}{5\sqrt{34}}\right) = \tan\cos^{-1}\dfrac{29}{5\sqrt{34}}$

$= \tan\tan^{-1}\dfrac{3}{29} = \dfrac{3}{29}$

4.[C] $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \pi/2$

$\cos^{-1}\dfrac{1}{\sqrt{x^2+x+1}} + \sin^{-1}\sin^{-1}\sqrt{x^2+x+1} = \pi/2$

is true if $\dfrac{1}{\sqrt{x^2+x+1}} = \sqrt{x^2+x+1}$

$1 = x^2 + x^1 + 1$

$\Rightarrow x(x+1) = 0 \qquad \Rightarrow x = 0, -1$

5.[B] $\sin^{-1}\left(x - \dfrac{x^2}{2} + \dfrac{x^3}{4} - .....\right) +$

$\cos^{-1}\left(x^2 - \dfrac{x^4}{2} + \dfrac{x^6}{4} .....\right) = \pi/2$

is hold then

$x - \dfrac{x^2}{2} + \dfrac{x^3}{4} - .... = x^2 - \dfrac{x^4}{2} + \dfrac{x^6}{4} .....$

$\Rightarrow \dfrac{x}{1+x/2} = \dfrac{x}{1+x^2/2}$

$\Rightarrow \dfrac{2x}{2+x} = \dfrac{2x^2}{2+x^2}$

$\Rightarrow \dfrac{2x^2}{2+x^2} - \dfrac{2x}{2+x} = 0$

$\Rightarrow 2x\left(\dfrac{x}{2+x^2} - \dfrac{1}{2+x}\right) = 0$

$x \neq 0$

$2x + x^2 - 2 - x^2 = 0$

$x = 1$

6.[D] $\sin\cot^{-1}(x+1) = \cos\tan^{-1}x$

$\Rightarrow \sin\tan^{-1}\dfrac{1}{x+1} = \cos\tan^{-1}x$

$\Rightarrow \sin\sin^{-1}\dfrac{\dfrac{1}{x+1}}{\sqrt{1+\dfrac{1}{(x+1)^2}}} = \cos\cos^{-1}\dfrac{1}{\sqrt{1+x^2}}$

$\Rightarrow \dfrac{1}{\sqrt{(x+1)^2+1}} = \dfrac{1}{\sqrt{1+x^2}}$

$\Rightarrow x + x^2 = x + (x+1)^2 \Rightarrow x^2 = x^2 + 2x + 1$

$\Rightarrow x = -1/2$

7. [A → P ; B → Q ; C → P ; D → S]

(A) If b = 0 and a = 1 then

$\sin^{-1}x + \cos^{-1}y = \pi/2$

$\Rightarrow \sin^{-1}x = \pi/2 - \cos^{-1}y$

$\Rightarrow \sin^{-1}x = \sin^{-1}y$

$\Rightarrow x = y$

(B) If a = 1 and b = 1 then

$\sin^{-1}x + \cos^{-1}y + \cos^{-1}xy = \pi/2$

$\Rightarrow \cos^{-1}(y.xy - \sqrt{1-y^2}\ \sqrt{1-x^2y^2}\ )$

$= \pi/2 - \sin^{-1}x$

$xy^2 - \sqrt{1-y^2}\ \sqrt{1-x^2y^2} = x$

$x^2(y^2-1)^2 = (1-y^2)(1-x^2y^2)$

$\Rightarrow (x^2-1)(y^2-1) = 0$

(C) If a = 1and b = 2 then

$\sin^{-1}x + \cos^{-1}y + \cos^{-1}2xy = \pi/2$

$\cos^{-1}\left[2xy^2 - \sqrt{1-y^2}\sqrt{1-4x^2y^2}\right] = \cos^{-1}x$

$x^2(2y^2-1)^2 = (1-y^2)(1-4x^2y^2)$

$\Rightarrow x^2 + y^2 = 1$

(D) If a = 2 and b = 2 then

$2xy^2 - \sqrt{1-y^2}\sqrt{1-4x^2y^2} = 2x$

$\Rightarrow 4x^2(y^2-1)^2 = (1-y^2)(1-4x^2y^2)$

$\Rightarrow (y^2-1)[4x^2(y^2-1) + (1-4x^2y^2)] = 0$

$(4x^2-1)(y^2-1)$

8.[C]

$= \sqrt{1+x^2}\ [x\cos(\cos^{-1}x) + \sin(\cos^{-1}x)^2 - 1]^{1/2}$

$= \sqrt{1+x^2}\left[\left\{x\cos\cos^{-1}\dfrac{1}{\sqrt{1+\dfrac{1}{x^2}}} + \sin\sin^{-1}\dfrac{1/x}{\sqrt{1+\dfrac{1}{x^2}}}\right\}^2 - 1\right]^{1/2}$

$= \sqrt{1+x^2}\left[\left(x.\dfrac{x}{\sqrt{1+x^2}} + \dfrac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$

$= \sqrt{1+x^2}\ \left[1+x^2-1\right]^{1/2} = x\sqrt{1+x^2}$

9.[B] $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$

$= \cot\left(\sum_{n=1}^{23}\cot^{-1}(1+n(n+1))\right)$

$= \cot\left(\sum_{n=1}^{23}\cot^{-1}\left(\frac{1+n(n+1)}{(n+1)-n}\right)\right)$

$= \cot\left(\sum_{n=1}^{23}(\cot^{-1}(n+1)-\cot^{-1}(n)\right)$

$= \cot(\cot^{-1}2 - \cot^{-1}1 + \cot^{-1}3 - \cot^{-1}2 + \ldots.. + \cot^{-1}24 - \cot^{-1}23)$

$= \cot(\cot^{-1}24 - \cot^{-1}1)$

$= \frac{1+\cot(\cot^{-1}24)\cot(\cot^{-1}1)}{\cot(\cot^{-1}24)-\cot(\cot^{-1}1)}$

$= \frac{1+24\times1}{24-1} = \frac{25}{23}$

10.[B] P : $\left[\frac{1}{y^2}\left[\frac{\frac{1}{\sqrt{1+y^2}}+\frac{y.y}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}\right]^2+y^4\right]^{1/2}$

$\left[\frac{1}{y^2}\left(\frac{\sqrt{1+y^2}}{1}.y\sqrt{1-y^2}\right)^2+y^4\right]^{1/2} = [1-y^4+y^4]^{1/2} = 1$

Q : $\cos x + \cos y = -\cos z$ ……(i)

$\sin x + \sin y = -\sin z$ ……(ii)

square & add equation (i) & (ii)

$2 + 2\cos(x-y) = 1$

$\cos(x-y) = \frac{-1}{2}$

$2\cos^2\left(\frac{x-y}{2}\right) - 1 = \frac{-1}{2}$

$\cos^2\left(\frac{x-y}{2}\right) = \frac{1}{4}$

$\cos\left(\frac{x-y}{2}\right) = \pm\frac{1}{2}$

R : $\left(\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}+x\right)\right)\cos 2x$

$= \sin 2x\,[\cot x - \tan x]$

$2\sin\frac{\pi}{4}\sin x.\cos 2x = \sin 2x \times 2\cot 2x$

$\frac{\sin x}{\sqrt{2}} = 1 \Rightarrow \sin x = \sqrt{2}$ is possible

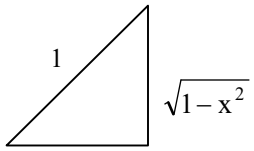
or $\cos 2x = 0$

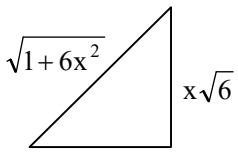
$\cos^2 x = \frac{1}{2}$

$\Rightarrow \cos x = \frac{1}{\sqrt{2}}$

$\sin x = \sqrt{2}$

S : $\cot(\sin^{-1}\sqrt{1-x^2}) = \sin\tan^{-1}(x\sqrt{6})$





$\frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{6}}{\sqrt{1+6x^2}}$

$x = 0$ not possible

$\therefore \frac{1}{\sqrt{1-x^2}} = \frac{\sqrt{6}}{\sqrt{1+6x^2}}$

$1 + 6x^2 = 6 - 6x^2$

$12x^2 = 5$

$x = \sqrt{\frac{5}{12}} = \frac{1}{2}\sqrt{\frac{5}{3}}$