JEE MAIN + ADVANCED

MATHEMATICS

TOPIC NAME

DEFINITE INTEGRATION

(PRACTICE SHEET)

LEVEL- 1

Question based on **Definition of Definite Integration**

Q.1 $\int_0^{\pi/4} \tan^2 x\, dx$ equals-

(A) π/4 (B) 1 + (π/4)

(C) 1 – (π/4) (D) 1 – (π/2)

Q.2 The value of $\int_0^{2a} \frac{dx}{\sqrt{2ax - x^2}}$ is-

(A) π (B) π/2 (C) π/4 (D) 2π

Q.3 The value of $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$ is-

(A) log (9/8) (B) log (4/3)

(C) log (3/4) (D) None of these

Q.4 $\int_0^{\infty} \frac{e^{\tan^{-1} x}}{1+x^2} dx$ equals-

(A) 1 (B) $e^{\pi/2} + 1$

(C) $e^{\pi/2} - 1$ (D) None of these

Q.5 $\int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$ equals -

(A) $\log_e \frac{2}{3}$ (B) $\log_e 3$

(C) $\frac{1}{2}\log_e \frac{4}{3}$ (D) $\log_e \frac{4}{3}$

Q.6 $\int_0^{\pi/2} \sin\theta \sin 2\theta\, d\theta$ equals-

(A) π/3 (B) 2π/3

(C) 2/3 (D) 4π/3

Q.7 $\int_0^{\pi/2} \sqrt{1+\sin 2x}\, dx$ equals-

(A) 1/2 (B) 1

(C) 2 (D) None of these

Q.8 $\int_0^{\pi/4} \frac{dx}{1+\cos 2x}$ equals -

(A) –1 (B) 1 (C) 1/2 (D) – 1/2

Q.9 $\int_0^{a} x^2 \sin x^3 dx$ equals -

(A) $-\frac{1}{3}(1-\cos a^3)$ (B) $3(1-\cos a^3)$

(C) $(1-\cos a^3)$ (D) $\frac{1}{3}(1-\cos a^3)$

Q.10 $\int_0^{\infty} x e^{-x^2} dx$ equals-

(A) 1 (B) 2

(C) 1/2 (D) None of these

Q.11 $\int_1^{2} \frac{1}{x\sqrt{x^2-1}} dx$ equals-

(A) π/2 (B) π/3 (C) π/4 (D) π

Q.12 The value of $\int_0^{1} (x^3 + 3e^x + 4)(x^2 + e^x)\, dx$ is-

(A) (3e + 2)/6 (B) (3e – 2)/6

(C) $(3e - 2)^2/36$ (D) None of these

Q.13 $\int_2^{3} \frac{dx}{\sqrt{5x - 6 - x^2}}$ equals-

(A) –π/2 (B) π/2 (C) –π (D) π

Q.14 $\int_0^{\infty} \frac{x \tan^{-1} x}{(1+x^2)^2} dx$ equals-

(A) π/2 (B) π/6 (C) π/4 (D) π/8

Q.15 $\int_0^{\pi/4} \sec x \log(\sec x + \tan x)\, dx$ =

(A) $\frac{1}{2}[\log(1+\sqrt{2})]^2$ (B) $[\log(1+\sqrt{2})]^2$

(C) $\frac{1}{2}[\log(\sqrt{2}-1)]^2$ (D) $[\log(\sqrt{2}-1)]^2$

Q.16 $\int_1^2 \frac{1-x}{1+x}\,dx$ equals-

(A) (1/2) log (3/2) –1 (B) 2 log (3/2) –1
(C) log (3/2) – 1 (D) None of these

Q.17 $\int_1^2 \frac{dx}{\sqrt{x^2-4x+5}}$ equals-

(A) log ($\sqrt{2}$ – 1) (B) log ($\sqrt{2}$ +1)
(C) – log (2 $\sqrt{2}$ –1) (D) – log (2 $\sqrt{2}$ +1)

Q.18 $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ equals-

(A) $e\left(\frac{e}{2}-1\right)$ (B) 1
(C) e (e – 1) (D) None of these

Q.19 $\int_{ka}^{kb} f(x)\,dx$ equals-

(A) $k^2\int_a^b f(x)\,dx$ (B) $k\int_a^b f(x)\,dx$
(C) $k\int_a^b f(kx)\,dx$ (D) $k^3\int_a^b f(kx)\,dx$

Q.20 $\int_{a-c}^{b-c} f(x+c)\,dx$ equals-

(A) $\int_a^b f(x+c)\,dx$ (B) $\int_a^b f(x)\,dx$
(C) $\int_{a-2c}^{b-2c} f(x)\,dx$ (D) $\int_a^b f(x+2c)\,dx$

Q.21 If $\frac{d}{dx}f(x) = g(x)$, then the value of $\int_a^b f(x)g(x)\,dx$ is-

(A) f(b) – f(a)
(B) g(b) – g(a)
(C) $\frac{1}{2}[\{g(b)\}^2 - \{g(a)\}^2]$
(D) $\frac{1}{2}[\{f(b)\}^2 - \{f(a)\}^2]$

Q.22 $\int_0^{\pi/2} (a\cos^2 x + b\sin^2 x)\,dx$ is equal to-

(A) (a + b) π/4 (B) (a + b) π/2
(C) (a + b) π/3 (D) None of these

Q.23 $\int_{\pi/4}^{\pi/2} e^x(\log\sin x + \cot x)\,dx$ =

(A) $e^{\pi/4}\log 2$ (B) $-e^{\pi/4}\log 2$
(C) $\frac{1}{2}e^{\pi/4}\log 2$ (D) $-\frac{1}{2}e^{\pi/4}\log 2$

Q.24 $\int_0^{\pi/2} \frac{\sin x\cos x}{1+\sin^4 x}\,dx$ =

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$

Q.25 $\int_1^2 \frac{dx}{\sqrt{(x-1)(2-x)}}$ equals-

(A) 1 (B) π/2 (C) π (D) 2π

Q.26 $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}\,dx$ =

(A) – log 2 (B) log 2
(C) π/2 (D) 0

Q.27 $\int_1^e \frac{e^x}{x}(1 + x\log x)\,dx$ =

(A) e^e (B) $e^e - e$
(C) $e^e + e$ (D) None of these

Q.28 $\int_0^{\pi/4} \sec^7\theta\sin^3\theta\,d\theta$ =

(A) 1/12 (B) 3/12
(C) 5/12 (D) None of these

Q.29 $\int_0^{\pi/2}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta$ =

(A) π log 2 (B) $\frac{\pi}{\log 2}$
(C) π (D) None of these

Q.30 $\int_0^{\pi/4} \tan^4 x\, dx$ equals -

(A) $\frac{\pi}{4}+\frac{2}{3}$ (B) $\frac{\pi}{4}-\frac{2}{3}$

(C) $\frac{\pi}{4}+\frac{1}{3}$ (D) $\frac{\pi}{4}-\frac{1}{3}$

Q.31 $\int_1^3 \left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$, equals-

(A) π (B) 2π

(C) 3π (D) None of these

Q.32 $\int_{-\pi/4}^{\pi/2} e^{-x}\sin x\, dx =$

(A) $-\frac{1}{2}e^{-\pi/2}$ (B) $-\frac{\sqrt{2}}{2}e^{-\pi/4}$

(C) $-\sqrt{2}\,(e^{-\pi/4}+e^{-\pi/2})$ (D) 0

Q.33 $\int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx$ equals-

(A) 1 (B) 2

(C) 0 (D) 4

Q.34 $\int_{\pi/6}^{\pi/4} \frac{\tan x+\cot x}{\tan^{-1}x+\cot^{-1}x}\, dx$ equals-

(A) 0 (B) $(\sqrt{3}+1)/\sqrt{3}$

(C) $(\log 3)/\pi$ (D) None of these

Q.35 $\int_0^3 \sqrt{\frac{x^3}{3-x}}\, dx$ equals-

(A) $3\pi/16$ (B) $27\pi/8$

(C) $3\pi/32$ (D) $9\pi/8$

Q.36 $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ equals-

(A) π/ab (B) $2\pi/ab$

(C) ab/π (D) $\pi/2\,ab$

Q.37 $\int_0^{\pi/4} \frac{\sin x+\cos x}{3+\sin 2x}\, dx$ is equal to

(A) log 2 (B) log 3

(C) $\frac{1}{4}\log 3$ (D) $\frac{1}{8}\log 3$

Q.38 $\int_0^1 e^{2\ell nx}\, dx$ is equal to-

(A) 0 (B) 1/4 (C) 1/3 (D) 1/2

Q.39 $\int_0^{\infty} \frac{x^2}{(x^2+4)(x^2+9)}\, dx$ is equal to-

(A) $\pi/20$ (B) $\pi/40$ (C) $\pi/10$ (D) $\pi/80$

Q.40 $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$ is equal to-

(A) $\frac{2\sqrt{2}}{3}$ (B) $\frac{4\sqrt{2}}{3}$

(C) $\frac{8\sqrt{2}}{2}$ (D) None of these

Q.41 $\int_0^{-\pi/4} \frac{1+\tan x}{1-\tan x}\, dx$ is equal to-

(A) $-\frac{1}{2}\log 2$ (B) $\frac{1}{4}\log 2$

(C) $\frac{1}{3}\log 2$ (D) None of these

Q.42 $\int_0^{\pi/2} e^{\sin^2 x}\sin 2x\, dx$ equals-

(A) e (B) e + 1

(C) e – 1 (D) 2 e

Q.43 If $\int_0^{\pi/3} \frac{\cos x}{3+4\sin x}\, dx = k\log\left(\frac{3+2\sqrt{3}}{3}\right)$, then k is equal to-

(A) 1/2 (B) 1/3

(C) 1/4 (D) 1/8

Question based on Property (P-3) of Definite Integration

Q.44 If $f(x) = \begin{cases} x^2, & \text{when } 0 \le x < 1 \\ \sqrt{x}, & \text{when } 1 \le x < 2 \end{cases}$, then $\int_0^2 f(x)\,dx$ equals-

(A) $\frac{1}{3}(4\sqrt{2} - 1)$ (B) $\frac{1}{3}(4\sqrt{2} + 1)$

(C) 0 (D) does not exist

Q.45 $\int_0^1 |3x - 1|\,dx$ equals-

(A) 5/6 (B) 5/3 (C) 10/3 (D) 5

Q.46 $\int_0^\pi |\cos x|\,dx$ equals -

(A) 1 (B) 2 (C) 0 (D) –1

Q.47 $\int_{1/e}^{e} |\log x|\,dx =$

(A) $e^{-1} - 1$ (B) 2 (1–1/e)

(C) 1 – 1/e (D) None of these

Q.48 $\int_0^1 |\sin 2\pi x|\,dx$ is equal to-

(A) 0 (B) $-1/\pi$ (C) $1/\pi$ (D) $2/\pi$

Q.49 $\int_{-1}^{1} |1 - x|\,dx$ is equal to-

(A) –2 (B) 0 (C) 2 (D) 4

Q.50 $\int_{-3}^{3} |x|\,dx$ equals-

(A) 0 (B) 9/2 (C) 6 (D) 9

Question based on Property (P-4) of Definite Integration

Q.51 The value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}$ is-

(A) $\pi/2$ (B) $\pi/4$

(C) π (D) 2π

Q.52 $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x}$ equals-

(A) 0 (B) $\pi/4$

(C) $\pi^2/4$ (D) $\pi^2/2$

Q.53 $\int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x}\,dx$ equals-

(A) $\pi^2/8$ (B) $\pi^2/16$

(C) $\pi^2/4$ (D) 0

Q.54 $\int_0^1 f(x)\,dx$ equals-

(A) $\int_0^1 f(1-x)\,dx$ (B) $\int_0^1 f(-x)\,dx$

(C) $2\int_0^{1/2} f(x)\,dx$ (D) None of these

Q.55 Which of the following is correct?

(A) $\int_0^a f(x)\,dx = -\int_0^a f(a-x)\,dx$

(B) $\int_0^a f(x)\,dx = 2\int_0^{a/2} f(x)\,dx$

(C) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(D) $\int_0^a f(x)\,dx = -\int_0^a f(a+x)\,dx$

Q.56 $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx$ equals-

(A) $\pi/2$ (B) $\pi/4$

(C) π (D) 2π

Q.57 $\int_0^{\pi/2} \frac{\tan x}{1 + \tan x}\,dx$ equals-

(A) π (B) $\pi/3$

(C) $\pi/4$ (D) $\pi/2$

Q.58 The value of $\int_0^\pi \frac{x}{1 + \sin x}\,dx$ is-

(A) π (B) $\pi/2$

(C) $\pi/4$ (D) 2π

Q.59 $\int_0^a f(x)\,dx$ is equal to-

(A) $\int_0^a f(a+x)\,dx$ (B) $\int_0^a f(2a+x)\,dx$

(C) $\int_0^a f(x+a)\,dx$ (D) $\int_0^a f(a-x)\,dx$

Q.60 $\int_0^\infty \frac{\log x}{1+x^2}\,dx$ equals-

(A) π (B) 0

(C) log 2 (D) π log 2

Q.61 $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$ equals

(A) $\pi/2$ (B) $\pi/3$

(C) $\pi/4$ (D) π

Q.62 $\int_0^{\pi/4} \log\ (1+\tan\theta)d\theta =$

(A) $\frac{\pi}{4}\log 2$ (B) $\frac{\pi}{4}\log\frac{1}{2}$

(C) $\frac{\pi}{8}\log 2$ (D) $\frac{\pi}{4}\log\frac{1}{2}$

Q.63 $\int_0^{\pi/2} \frac{\cos^2 x}{2+\sin x+\cos x}\,dx$ is equal to-

(A) $\frac{1}{\sqrt{2}}(\tan^{-1}\sqrt{2}+\cot^{-1}\sqrt{2})$

(B) $\frac{1}{\sqrt{2}}(\tan^{-1}\sqrt{2}-\cot^{-1}\sqrt{2})$

(C) $\frac{1}{2}(\tan^{-1}\sqrt{2}-\cot^{-1}\sqrt{2})$

(D) None of these

Question based on Property (P-5) of Definite Integration

Q.64 $\int_{-\pi/2}^{\pi/2} \sqrt{\frac{1}{2}(1-\cos 2x)}\,dx$ equals-

(A) 0 (B) 2

(C) 1/2 (D) None of these

Q.65 $\int_{-3}^{3} \frac{x^2\sin x}{1+x^6}\,dx$ equals-

(A) 4 (B) 2

(C) 0 (D) None of these

Q.66 $\int_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx$ equals-

(A) 1 (B) 0

(C) $\sqrt{2}$ (D) 2

Q.67 The value of the integral $\int_{-2}^{2}|1-x^2|\,dx$ is-

(A) 0 (B) 4

(C) 2 (D) None of these

Q.68 If $f : R \to R$ and $g : R \to R$ are two continuous functions, then the value of the integral $\int_{-\pi/2}^{\pi/2}[f(x)+f(-x)]\,[g(x)-g(-x)]\,dx$ is-

(A) π (B) 1

(C) –1 (D) 0

Q.69 $\int_{-a}^{a} f(x)\,dx = 0$, if-

(A) $f(-x) = f(x)$ (B) $f(a-x) = -f(x)$

(C) $f(-x) = -f(x)$ (D) $f(a+x) = -f(x)$

Q.70 $\int_{-\pi/2}^{\pi/2}(\sin^3 x+\cos^3 x)\,dx$ equals-

(A) 0 (B) 1/3

(C) 4/3 (D) 2/3

Q.71 $\int_{-\pi/2}^{\pi/2}\frac{dx}{1+\cos x}$ equals-

(A) 0 (B) 2

(C) 1 (D) 3

Q.72 $\int_{-\pi/2}^{\pi/2}\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta$ equals-

(A) 0 (B) 1

(C) 2 (D) None of these

Q.73 $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is -

(A) $\frac{\pi^2}{4}$ (B) zero

(C) π^2 (D) $\frac{\pi}{2}$

Q.74 $\int_{-1}^{1} \frac{e^x+1}{e^x-1} dx$ equals-

(A) log $(e^x + 1)$ (B) log $(e^x - 1)$

(C) 1 (D) 0

Q.75 $\int_{-1}^{1} x^{17}\cos^4 x\ dx$ is equal to -

(A) –2 (B) 2 (C) 0 (D) 1

Q.76 $\int_{-1}^{1} \sin^{-1}\left(\frac{x}{1+x^2}\right) dx$ is equal to -

(A) $\pi/4$ (B) $\pi/2$ (C) π (D) 0

Q.77 $\int_{-\pi/2}^{\pi/2} \cos^3\theta\ (1 + \sin\theta)^2\ d\theta$ is equal to -

(A) 8/5 (B) 5/8 (C) –8/5 (D) –5/8

Q.78 $\int_{-1}^{1} \log\left(\frac{1+x}{1-x}\right) dx$ is equal to -

(A) π (B) 1 (C) 0 (D) 2

Q.79 $\int_{-a}^{a} \sin x\ f(\cos x)\ dx$ is equal to-

(A) f(a) (B) – f(a) (C) 2 f(a) (D) 0

Q.80 $\int_{-1}^{1} \sin^{11} x dx$ is equal to-

(A) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}$

(B) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}\cdot\frac{\pi}{2}$

(C) 1

(D) 0

Q.81 $\int_{-\pi}^{\pi} (1-x^2)\sin x \cos^2 x\ dx$ is equal to-

(A) 0 (B) $\pi - \frac{\pi^3}{3}$

(C) $2\pi - \pi^3$ (D) $\frac{7}{4} - 2\pi^3$

Question based on Property (P-6, P-7) of Definite Integration

Q.82 $\int_{0}^{2\pi} \cos^4 x\, dx$ equals-

(A) $3\pi/8$ (B) $3\pi/4$

(C) $3\pi/2$ (D) 3π

Q.83 $\int_{0}^{2\pi} \frac{\sin 2\theta}{a - b\cos\theta} d\theta$ equals-

(A) 1 (B) 2 (C) $\pi/4$ (D) 0

Q.84 $\int_{0}^{400\pi} \sqrt{1-\cos 2x}\ dx$ is equal to-

(A) $400\sqrt{2}$ (B) $800\sqrt{2}$

(C) 0 (D) None of these

Q.85 Which of the following is correct?

(A) $\int_0^a f(x)dx = \int_0^a f(a+x)dx$

(B) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx$

(C) $\int_0^a f(x)dx = \int_0^{-a} f(-x)dx$

(D) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Question based on Property (P-8) of Definite Integration

Q.86 $\int_{1}^{2} \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} dx$ is equal to-

(A) 2/1 (B) 3/4

(C) 1/2 (D) None of these

Q.87 $\int_0^{\pi/2} \frac{\phi(x)}{\phi(x)+\phi(\pi/2-x)} dx$ is equal to-

(A) $\pi/4$ (B) $\pi/2$
(C) π (D) None of these

Q.88 If f(x) = f(a + b – x), then $\int_a^b x f(x)\,dx$ is equal to

(A) $(a + b) \int_a^b f(x)\,dx$ (B) $\frac{1}{2}(a + b) \int_a^b f(x)\,dx$

(C) $(b - a) \int_a^b f(x)\,dx$ (D) $\frac{1}{2}(b - a) \int_a^b f(x)\,dx$

Q.89 $\int_0^{2\pi} |\sin^3\theta|\,d\theta$, equals-

(A) 0 (B) 3/8
(C) 8/3 (D) π

Q.90 $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx$ equals-

(A) b – a (B) a + b

(C) $\frac{1}{2}(b - a)$ (D) $\frac{1}{2}(a + b)$

Question based on Some important Formulae

Q.91 $\int_0^{\pi/2} \log\cos x\; dx$ equals-

(A) $(\pi/2)\log(1/2)$ (B) $\pi\log 2$
(C) $-\pi\log 2$ (D) $2\pi\log 2$

Q.92 $\int_0^{\pi/2} \sin^2\theta\cos^6\theta\,d\theta$ equals-

(A) $-\pi/16$ (B) $\pi/16$
(C) $5\pi/256$ (D) $-5\pi/256$

Q.93 $\int_0^{\pi/2} \sin^5 x\,dx$ equals-

(A) 8/15 (B) 4/15

(C) $\frac{8\sqrt{\pi}}{15}$ (D) $\frac{8\pi}{15}$

Q.94 $\int_0^{\pi/2} \log\sin 2x\,dx$ equals-

(A) $(\pi/2)\log 2$ (B) $-(\pi/2)\log 2$
(C) $(\pi/4)\log 2$ (D) $-(\pi/4)\log 2$

Q.95 $\int_0^{\pi/4} \log\sin 2x\,dx$ equals to-

(A) $(\pi/4)\log 2$ (B) $(\pi/2)\log 2$
(C) $-(\pi/4)\log 2$ (D) $-(\pi/2)\log 2$

Q.96 $\int_0^{\pi} \log\sin^2 x\,dx$ is equal to-

(A) $2\pi\log(1/2)$ (B) $\pi\log 2$
(C) $\pi/2\log(1/2)$ (D) None of these

Q.97 $\int_0^{\pi/2} \log\sec x\,dx$ equals-

(A) $\pi\log 2$ (B) $(\pi/2)\log 2$
(C) $-\pi\log 2$ (D) $-(\pi/2)\log 2$

Q.98 $\int_0^{1} \log\sin\left(\frac{\pi}{2}x\right) dx$ equals -

(A) $\pi\log 2$ (B) $-\pi\log 2$
(C) log 2 (D) – log 2

Q.99 $\int_0^{\pi/2} \sin^2 x\cos^5 x\,dx$ equals -

(A) 16/105 (B) 8/105
(C) $(16/105)\pi$ (D) $(8/105)\pi$

Q.100 $\int_0^{\pi/2} \sin^3 x\,dx$ equals-

(A) 2/3 (B) $4\pi/3$ (C) 3/2 (D) $2\pi/3$

Q.101 $\int_{-\pi/2}^{\pi/2} \cos^3 x\,dx$ equals-

(A) 0 (B) $\pi/2$ (C) $3\pi/2$ (D) 4/3

Q.102 $\int_0^{1} \frac{x^3}{\sqrt{1-x^2}} dx$ equals-

(A) $2\pi/3$ (B) 2/3
(C) 3/2 (D) None of these

Q.103 $\int_0^1 \sqrt{x(1-x)}\,dx$ equals-

(A) $\pi/4$ (B) $\pi/8$ (C) $\pi/2$ (D) $\pi/3$

Q.104 $\int_0^\infty \frac{\log(1+x^2)}{1+x^2}\,dx$ equals-

(A) $\pi \log 2$ (B) $-\pi \log 2$
(C) $\pi/2 \log 2$ (D) $-\pi/2 \log 2$

Q.105 $\int_0^1 x^2(1-x^2)^{3/2}\,dx$ equals-

(A) $\pi/32$ (B) $\pi/16$ (C) $\pi/8$ (D) $\pi/4$

Q.106 $\int_0^{\pi/4} \sin^4 2x\,dx$ equals

(A) $2\pi/32$ (B) $3\pi/32$
(C) $\pi/32$ (D) $3\pi/16$

Q.107 If $f(x) = \int_{x^2}^{x^3} \log t\,dt\ (x > 0)$, then $f'(x)$ is equal to-

(A) $(4x^2 - 9x) \log x$ (B) $(9x^2 + 4x) \log x$
(C) $(9x^2 - 4x) \log x$ (D) $(x^2 + x) \log x$

Q.108 The derivative of $F(x) = \int_{x^2}^{x^3} \frac{1}{\log t}\,dt\ (x > 0)$ is-

(A) $\frac{1}{3\log x} - \frac{1}{2\log x}$ (B) $\frac{1}{3\log x}$

(C) $\frac{3x^2}{3\log x}$ (D) $(\log x)^{-1}.\ x(x-1)$

Question based on Summation of series by Integration

Q.109 $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{3n}\right)$ equals-

(A) log 2 (B) log 4
(C) 0 (D) $\log_e 3$

Q.110 $\lim_{n\to\infty}\left[\frac{1^2}{1^3+n^3}+\frac{2^2}{2^3+n^3}+.....+\frac{r^2}{r^3+n^3}+.....+\frac{1}{2n}\right]$ equals-

(A) (1/2) log 3 (B) (1/3) log 2
(C) 3 log 2 (D) (1/2) log 2

Q.111 $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{6n}\right]$ is equal to-

(A) log 4 (B) log 6
(C) log 8 (D) log 2

Q.112 $\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+......+\frac{1}{n}\right]$ is equal to-

(A) $\frac{\pi}{4}\log 2$ (B) $\frac{\pi}{4} + \log\sqrt{2}$

(C) $\frac{\pi}{2} + \log\sqrt{2}$ (D) None of these

Q.113 $\lim_{n\to\infty}\frac{1^{99}+2^{99}+......+n^{99}}{n^{100}}$ is equal to

(A) $\frac{99}{100}$ (B) $\frac{1}{100}$

(C) $\frac{1}{99}$ (D) $\frac{1}{101}$

Q.114 $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}$ equal to-

(A) $-1+\sqrt{2}$ (B) $-1+\sqrt{5}$

(C) $1+\sqrt{5}$ (D) $1+\sqrt{2}$

Q.115 $\lim_{n\to\infty}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+.....+\frac{1}{nb}\right]$ is equal to

(A) log (b/a) (B) log (a/b)
(C) log a (D) log b

LEVEL- 2

Q.1 $\int_0^{2\pi} \sin^4 x \cos^6 x \, dx$ equals-

(A) $3\pi/256$ (B) $3\pi/128$
(C) $3\pi/64$ (D) None of these

Q.2 If $\int_0^{\pi} \log \sin x \, dx = k$, then the value of

$\int_0^{\pi/4} \log (1 + \tan x) dx$ is -

(A) $-\frac{k}{4}$ (B) $\frac{k}{4}$
(C) $-\frac{k}{8}$ (D) $\frac{k}{8}$

Q.3 $\int_0^{\pi} \log \sin x \, dx$ equals-

(A) $(-\pi/2) \log 2$ (B) $(\pi/2) \log 2$
(C) $-\pi\log 2$ (D) $\pi\log 2$

Q.4 $\int_0^{a} [f(x) + f(a-x)] dx$ equals-

(A) $\int_0^a f(x)dx$ (B) $-\int_0^a f(x)dx$
(C) $2\int_0^a f(x)dx$ (D) None of these

Q.5 $\int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx$ equals-

(A) $-\pi \log (1/2)$ (B) $\pi \log (1/2)$
(C) $\frac{\pi}{2} \log (1/2)$ (D) $-\frac{\pi}{2} \log (1/2)$

Q.6 $\int_{-1}^{3/2} | x \sin \pi x | dx$ equals-

(A) $(3/\pi) + 1/\pi^2$ (B) $(3/\pi) - 1/\pi^2$
(C) $1/\pi (\pi + 1)$ (D) None of these

Q.7 If $I = \int_0^{\pi/4} \sin^2 x \, dx$ and $J = \int_0^{\pi/4} \cos^2 x \, dx$ then I is equal to-

(A) $\pi/4 - J$ (B) 2 J
(C) J (D) J/2

Q.8 $\int_0^{\pi} \sin mx \sin nx \, dx$ equals $(m, n \in Z, m \neq n)$

(A) m – n (B) 0 (C) m + n (D) 1

Q.9 If $f(x + 1) + f(x + 7) = 0, \forall x \in R$ then possible value of 't' for which $\int_a^{a+t} f(x) dx$ is independent of a, is

(A) 13 (B) 6
(C) 12 (D) None of these

Q.10 $\int_0^{\pi/4} \cos^{3/2} 2\theta \cos \theta \, d\theta$ equals-

(A) $\frac{3\pi}{16}$ (B) $\frac{3\pi}{16\sqrt{2}}$
(C) $\frac{3}{8\sqrt{2}}$ (D) None of these

Q.11 If $f(x) = |x| + |x - 1|$, then $\int_0^2 f(x) dx$ equals-

(A) 3 (B) 2 (C) 0 (D) –1

Q.12 $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx$ is equal to

(A) $3 + 2\pi$ (B) $4 - \pi$
(C) $2 + \pi$ (D) None of these

Q.13 $\int_0^1 e^{x^2} (x - \alpha) dx = 0$, then-

(A) $1 < \alpha < 2$ (B) $\alpha < 0$
(C) $0 < \alpha < 1$ (D) $\alpha = 0$

Q.14 If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then-

(A) $I_1 = I_2$ (B) $I_1 < I_2$
(C) $I_1 > I_2$ (D) None of these

Q.15 $\int_0^{\pi} \log (1 - \cos x) \, dx$ equals-

(A) $\pi \log 2$ (B) $-\pi \log 2$
(C) $(\pi/2) \log 2$ (D) $-(\pi/2) \log 2$

Q.16 $\int_0^{\pi} x \sin x \cos^4 x \, dx$ is equal to-

(A) $3\pi/5$ (B) $2\pi/5$

(C) $\pi/5$ (D) None of these

Q.17 $\int_0^{\pi} \frac{dx}{a + b\cos x}$ is equal to-

(A) $\pi/\sqrt{a^2 - b^2}$ (B) $\pi/\sqrt{a^2 + b^2}$

(C) π/ab (D) $(a + b)\pi$

Q.18 $\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx$ is equal to

(A) 0 (B) 1/2

(C) –1/2 (D) None of these

Q.19 If $f(a - x) = f(x)$ and $\int_0^{a/2} f(x)\,dx = p$, then $\int_0^{a} f(x)\,dx$ is equal to-

(A) 2p (B) 0

(C) p (D) None of these

Q.20 $\int_0^{2\pi} |\sin x|\, dx =$

(A) 2 (B) 1

(C) 0 (D) 4

Q.21 $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2 \, dx$, where p and q are integers, is equal to-

(A) $-\pi$ (B) 0 (C) 2π (D) π

Q.22 $\int_1^5 (|x-3| + |1-x|)\, dx$ is equal to-

(A) 21 (B) 5/6

(C) 10 (D) 12

Q.23 The value of α which satisfy

$\int_{\pi/2}^{\alpha} \sin x \, dx = \sin 2\alpha$, $(\alpha \in [0, 2\pi])$ are equal to-

(A) $7\pi/6$ (B) $3\pi/2$

(C) $\pi/2$ (D) all of these

Q.24 If $I_n = \int_0^{\infty} e^{-x} x^{n-1}\, dx$ then $\int_0^{\infty} e^{-\lambda x} x^{n-1} dx =$

(A) λI_n (B) $\frac{1}{\lambda} I_n$ (C) $\frac{I_n}{\lambda^n}$ (D) $\lambda^n I_n$

Q.25 $\int_1^{e^{37}} \frac{\pi \sin(\pi \ell n x)}{x} dx$ is equal to-

(A) 1 (B) 2 (C) e (D) 37

Q.26 The value of integral $\int_0^1 e^{x^2} dx$ lies in the interval-

(A) (0, 1) (B) (–1, 0)

(C) (1, e) (D) None of these

Q.27 $\lim_{n\to\infty} \left[\frac{1}{2n} + \frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-4}} + \ldots + \frac{1}{\sqrt{3n^2+2n-1}}\right]$

is equal to-

(A) $\pi/4$ (B) $\pi/3$

(C) $\pi/2$ (D) $\pi/6$

Q.28 $\lim_{n\to\infty} \left(1+\frac{1}{n^2}\right)^{2/n^2} \cdot \left(1+\frac{2^2}{n^2}\right)^{4/n^2} \left(1+\frac{3^2}{n^2}\right)^{6/n^2}$

$\ldots\ldots\ldots \left(1+\frac{n^2}{n^2}\right)^{2n/n^2}$ is equal to

(A) 4/e (B) 3/e

(C) 2/e (D) None of these

Q.29 Let $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $I_2 = \int_1^2 \frac{dx}{x}$, then-

(A) $I_1 = I_2$ (B) $I_2 > I_1$

(C) $I_1 > I_2$ (D) $I_1 > 2 I_2$

Q.30 If [ ] denotes the greatest integer function, then $\int_0^{3/2} [x^2]\, dx$ is equal to-

(A) $2 - \sqrt{2}$ (B) $2 + \sqrt{2}$

(C) $1 - \sqrt{2}$ (D) $1 + \sqrt{2}$

Q.31 $\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{1/n}$ is equal to

(A) e (B) 1/e

(C) $\sqrt{e}$ (D) None of these

Q.32 If [x] denotes the greatest integer less than or equal to x, then $\int_0^2 x^2[x]\,dx$ is equal to

(A) 5/3 (B) 7/3
(C) 8/3 (D) 4/3

Q.33 If f(x) is a function of x, then $\int_{-\pi/2}^{\pi/2} f(\cos x)dx$ is equal to

(A) 0 (B) $\int_0^{\pi/2} f(\cos x)dx$
(C) $4\int_0^{\pi/2} f(\cos x)\,dx$ (D) $2\int_0^{\pi/2} f(\sin x)dx$

Q.34 If $\int_{-1}^{4} f(x)\,dx = 4$ and $\int_{2}^{4}[3-f(x)]\,dx = 7$, then $\int_{2}^{-1} f(x)\,dx$ is equal to-

(A) 2 (B) –2
(C) –5 (D) None of these

Q.35 $\int_{-\pi}^{\pi}(x^3 + x\cos x + \tan^5 x + 2)\,dx$ is equal to

(A) π (B) 2π (C) 4π (D) 0

Q.36 $\int_0^{\infty}\frac{dx}{(x+\sqrt{x^2+1})^3} =$

(A) 3/8 (B) 1/8
(C) –3/8 (D) None of these

Q.37 $\int_0^1 (1+e^{-x^2})\,dx$ is equal to

(A) –1 (B) 2
(C) $1 + e^{-1}$ (D) None of these

Q.38 If h(a) = h(b) then value of integral $\int_a^b [f(g(h(x))]^{-1} f'(g(h(x)).g'(h(x))\ h'(x)dx$ is equal to

(A) 0 (B) f(a) – f(b)
(C) f(g(a)) – f(g(b)) (D) None of these

Q.39 $\int_0^{\pi/2}|\sin x-\cos x|\,dx$ equals

(A) 0 (B) $\sqrt{2}-1$
(C) $2(\sqrt{2}-1)$ (D) $2(\sqrt{2}+1)$

Q.40 $\int_0^a x^4\sqrt{a^2-x^2}\,dx =$

(A) π/32 (B) $\frac{\pi}{32}a^6$
(C) $\frac{\pi}{16}a^6$ (D) $\frac{\pi}{8}a^6$

Q.41 $\int_0^{2\pi}\sqrt{1+\sin(x/2)}\,dx$ equals

(A) 0 (B) 2
(C) 8 (D) 4

Q.42 $\int_{-1}^{1} e^{|x|}dx$ equals

(A) 2(e – 1) (B) 2(e + 1)
(C) 0 (D) None of these

Q.43 $\int_0^1 \frac{x^3dx}{(x^2+1)^{3/2}} =$

(A) $(\sqrt{2}-1)^2$ (B) $\frac{(\sqrt{2}-1)^2}{\sqrt{2}}$
(C) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (D) None of these

Q.44 $\int_0^{\pi}\frac{x\tan x}{\sec x+\cos x}\,dx$ equals-

(A) $\pi^2/4$ (B) $\pi^2/2$ (C) $3\pi^2/2$ (D) $\pi^2/3$

Q.45 $\int_0^1 |3x-1|\,dx$ equals-

(A) 5/6 (B) 5/3
(C) 10/3 (D) 5

Q.46 If $\int_0^{\pi/2}\sin^4 x\cos^2 x\,dx = \frac{\pi}{32}$ then $\int_0^{\pi/2}\cos^4 x\sin^2 x\,dx$ equals

(A) π/32 (B) 3π/32
(C) π/2 (D) None of these

Q.47 $\int_a^b \frac{|x|}{x} dx$, a < b is equal to

(A) b – a (B) a – b
(C) b + a (D) |b| – |a|

Q.48 $\int_0^{\pi/2} \frac{\cos x}{1+\cos x+\sin x} dx =$

(A) $\frac{\pi}{4}+\frac{1}{2}\log 2$ (B) $\frac{\pi}{4}$ + log 2
(C) $\frac{\pi}{4}-\frac{1}{2}$ log 2 (D) $\frac{\pi}{4}$ – log 2

Q.49 Let f(x) = maximum {x + |x|, x –[x]} where [x] represents greatest integer ≤ x then $\int_{-2}^{2} f(x)\,dx$ is equal to

(A) 3 (B) 2
(C) 1 (D) None of these

Q.50 Let $I_1 = \int_0^1 \frac{\tan^{-1} x}{x} dx$, $I_2 = \frac{1}{2}\int_0^{\pi/2} \frac{t}{\sin t} dt$ then

(A) $I_1 = I_2$ (B) $I_1 < I_2$
(C) $I_1 > I_2$ (D) None of these

Q.51 If [x] represent G.I.F. then $\int_0^{\infty}\left[\frac{2}{e^x}\right]dx$ is equal to:

(A) $\log_e 2$ (B) e^2
(C) 0 (D) 2/e

Q.52 The value of $\sum_{n=1}^{1000}\int_{n-1}^{n} e^{x-[x]}dx$ is

([x] is the greatest integer function)

(A) $\frac{e^{1000}-1}{1000}$ (B) $\frac{e^{1000}-1}{e-1}$
(C) $\frac{e-1}{1000}$ (D) 1000 (e –1)

Q.53 $\int_0^{1/2}$ |sinπ x| dx is equal to

(A) 0 (B) π
(C) – π (D) 1/π

Q.54 If a is such that $\int_0^a x\,dx \le a+4$ then

(A) 0 ≤ a ≤ 4 (B) –2 ≤ a ≤ 0
(C) a ≤ –2 or a ≥ 4 (D) –2 ≤ a ≤ 4

Q.55 $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ is equal to

(A) $\frac{\pi}{2}$ + log 2 (B) $\frac{\pi}{2}$ – log 2
(C) $\frac{\pi}{4}$ + log 2 (D) $\frac{\pi}{4}$ – log 2

Q.56 $\int_0^{\pi/2} \frac{\sin 8x \log(\cot x)}{\cos 2x} dx$ is equal to

(A) 0 (B) π/4
(C) π/2 (D) None of these

Q.57 Let $I_1 = \int_0^{\pi} x \log \sin x\, dx$, $I_2 = \int_0^{\pi} \log \sin x\, dx$, then

(A) $I_1 = I_2$ (B) $I_1 = \pi I_2$
(C) $2I_1 = \pi I_2$ (D) $\pi I_1 = I_2$

Q.58 Assuming a, b, c are non zero real numbers such that : $\int_0^3 (3ax^2+2bx+c)\,dx =$

$\int_1^3 (3ax^2+2bx+c)\,dx$ then

(A) a + b + c = 3 (B) a + b + c = 1
(C) a + b + c = 0 (D) a + b + c = 2

Q.59 If φ(a – x) = φ(x), then $\int_0^a x\phi(x)\,dx$ is equal to

(A) $a\int_0^a \phi(x)\,dx$ (B) $\frac{1}{2}a\int_0^a \phi(x)dx$
(C) $2a\int_0^a \phi(x)\,dx$ (D) None of these

Q.60 $\lim_{x\to 0} \frac{xe^{x^2}}{\int_0^x e^{t^2}dt} =$

(A) 0 (B) 1
(C) –1 (D) None of these

LEVEL # 3

Q.1 If $f(x) = \int_{1/x^2}^{x^2} \cos\sqrt{t}\, dt$ then $f'(1) =$

(A) cos 1 (B) 2 cos 1
(C) 4 cos 1 (D) None of these

Q.2 If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, $n \in N$, then $I_{n+2} + I_n$ equals-

(A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$
(C) $\frac{1}{n+1}$ (D) $\frac{1}{n+2}$

Q.3 If $I_1 = \int_x^1 \frac{1}{1+t^2}\, dt$ and $I_2 = \int_1^{1/x} \frac{1}{1+t^2}\, dt$ for $x > 0$, then-

(A) $I_1 = I_2$ (B) $I_1 > I_2$
(C) $I_2 > I_1$ (D) None of these

Q.4 The expression $\frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$, where [x] and {x} are integral and fractional parts of x and is equal to-

(A) $\frac{1}{n-1}$ (B) $\frac{1}{n}$
(C) n (D) n – 1

Q.5 The value of $\int_0^{[x]} \frac{2^x}{2^{[x]}}\, dx$ is

(A) [x] log 2 (B) $\frac{[x]}{\log 2}$
(C) $\frac{1}{2}\frac{[x]}{\log 2}$ (D) none of these

Q.6 The value of $\int_0^{16\pi/3} |\sin x|\, dx$ is

(A) 21 (B) 21/2 (C) 10 (D) 11

Q.7 If $\int_0^{n\pi} f(\cos^2 x)\, dx = k \int_0^{\pi} f(\cos^2 x)\, dx$, then the value of k is-

(A) 1 (B) n
(C) n/2 (D) none of these

Q.8 The value of the integral $\int_0^{100} \sin(x - [x])\, \pi dx$ is-

(A) 100/π (B) 200/π
(C) 100π (D) 200π

Q.9 The value of the integral $\int_0^1 x(1-x)^n\, dx$ is-

(A) $\frac{1}{n+1} + \frac{1}{n+2}$ (B) $\frac{1}{(n+1)(n+2)}$
(C) $\frac{1}{n+2} - \frac{1}{n+1}$ (D) (B) and (C)

Q.10 The greater value of $F(x) = \int_1^x |t|\, dt$ on the interval [–1/2, 1/2] is-

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$
(C) $-\frac{3}{8}$ (D) $-\frac{1}{2}$

Q.11 If x satisfies the equation

$$x^2\left(\int_0^{\pi/2}(2\sin t + 3\cos t)dt\right) - x\left(\int_{-3}^{3}\frac{t^2 \sin 2t}{t^2+1}dt\right) - 2 = 0,$$

Then the value of x is -

(A) –1 (B) 1
(C) $\sqrt{2/5}$ (D) none

Q.12 Let f be an odd function then

$\int_{-1}^{1} (|x| + f(x).\cos x)\, dx$ is equal to–

(A) 0 (B) 1
(C) 2 (D) None of these

Q.13 The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is-

(A) 2 (B) 3/4
(C) 0 (D) None of these

Q.14 Consider the integrals

$I_1 = \int_0^1 e^{-x}\cos^2 x\,dx$, $I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx$

$I_3 = \int_0^1 e^{-x^2}dx$, $I_4 = \int_0^1 e^{-x^2/2}dx$

the greatest integral amongst I_1, I_2, I_3, I_4 is

(A) I_1 (B) I_2 (C) I_3 (D) I_4

Q.15 For any integer n, the integral

$\int_0^{\pi} e^{\cos^2 x}\cos^3(2n+1)x\,dx$ has the value-

(A) π (B) 1
(C) 0 (D) None of these

Q.16 If f(x) and g(x) are continuous functions satisfying f(x) = f(a – x) and g (x) + g(a – x) = 2, then $\int_0^a f(x)\,g(x)\,dx$ is equal to-

(A) $\int_0^a g(x)\,dx$ (B) $\int_0^a f(x)\,dx$
(C) 0 (D) none of these

Q.17 If I = $\int_0^{2\pi}\sin^2 x\,dx$, then

(A) $I = 2\int_0^{\pi}\sin^2 x\,dx$ (B) $I = 4\int_0^{\pi/2}\sin^2 x\,dx$
(C) $I = \int_0^{\pi 2}\cos^2 x\,dx$ (D) (A) and (B)

Q.18 The value of $\int_0^{\sin^2 x}\sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt$ is-

(A) $\frac{\pi}{2}$ (B) 1
(C) $\frac{\pi}{4}$ (D) None of these

Q.19 If for every integer n, $\int_n^{n+1} f(x)dx = n^2$, then the value of $\int_{-2}^{4} f(x)dx$ is-

(A) 16 (B) 14
(C) 19 (D) none of these

Q. 20 If $a_n = \int_0^{\pi/2}\frac{\sin^2 nx}{\sin x}dx$ then $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$ are in-

(A) A.P. (B) G.P.
(C) H.P. (D) A.G.P.

Q.21 The value of the integral

$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}}\frac{x^4}{1-x^4}\cos^{-1}\frac{2x}{1+x^2}\,dx$$

(A) 0

(B) $\frac{\pi}{\sqrt{3}} + \log\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(C) $\frac{1}{\sqrt{3}} + \frac{\pi}{2}\log\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(D) None of these

Q.22 If f is a continuous periodic function with period T, then $\int_a^{a+T} f(x)\,dx$

(A) depends upon a
(B) is independent of T
(C) independent of a
(D) None of these

Q.23 $\int_0^2 \left|\cos\frac{\pi x}{2}\right| dx =$

(A) 1/π (B) 2/π
(C) 3/π (D) 4/π

Q.24 $\int_1^4 \{x\}^{[x]}dx$ where $\{x\} \to$ fractional parts ; $[\,] \to$ greatest integer is given to

(A) $\frac{13}{12}$ (B) $\frac{15}{12}$
(C) $\frac{11}{12}$ (D) None of these

Q.25 Given $\int_0^{\pi/2}\frac{dx}{1+\sin x+\cos x} = \ell n2$ then the value of the definite integral $\int_0^{\pi/2}\frac{\sin x}{1+\sin x+\cos x}\,dx =$

(A) $\frac{1}{2}\ell n2$ (B) $\frac{\pi}{2} - \ell n2$
(C) $\frac{\pi}{4} - \frac{1}{2}\ell n2$ (D) $\frac{\pi}{2} + \ell n2$

Q.26 $\int_{1/e}^{\tan x}\frac{t}{1+t^2}dt + \int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}$ is equal to

(A) 1 (B) –1
(C) 0 (D) None of these

Q.27 The points of extremum of $\int_0^{x^2} \frac{t^2-5t+4}{2+e^t}\, dt$ are

(A) x = 0, ± 1, – 2 (B) x = 0, ± 1, 2
(C) x = 0, ± 2, 1 (D) x = ± 1, ± 2, 0

Q.28 For $x \in R$ and a continuous function f, let

$I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x f\,\{x(2-x)\}\,dx$ and

$I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\,\{x(2-x)\}\,dx$. Then I_1/I_2 is

(A) 0 (B) 1 (C) 2 (D) 3

Statement type Questions

Each of the questions given below consists of Statement-I (Assertion) and Statement-II (Reason). Use the following key to choose the appropriate answer.

(A) If both Statement-I Statement-II are true, and Statement-II is the correct explanation of Statement-I.

(B) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I

(C) If Statement-I is true but Statement-II is false

(D) If Statement-I is false but Statement-II is true.

Q.29 **Statement- I :** $\int_0^{100.5} e^{(x-[x])}dx = 100e - e^{1/2} - 99$

Statement- II : x – [x] is a periodic function of period 1. Therefore.

$$\int_0^{100.5} e^{x-[x]}dx = 100\int_0^1 e^{x-[x]}dx + \int_{100}^{100.5} e^{x-[x]}dx$$

Q.30 **Statement- I :**

$$\int_{-\pi/2}^{\pi/2} [\sin(\log(-x+\sqrt{1+x^2}))]\,dx = 0$$

Statement- II : $\int_{-a}^{a} f(x)\,dx = 0$ when f(x) is even.

Q.31 **Statement- I :** If $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ \sqrt{x}, & 1 \le x \le 2 \end{cases}$ Then

$$\int_0^2 f(x)dx = \frac{4}{3}(\sqrt{2}-1)$$

Statement- II : f(x) is continuous in [0, 2].

Q. 32 **Statement- I :** $\int_0^{\pi/2} (\sin^6 x + \cos^6 x)dx = \frac{5\pi}{16}$

Statement- II : $\sin^6 x + \cos^6 x$ is periodic with period $\pi/2$

Q.33 **Statement- I :** $\int_{-2}^{2} \frac{|x|}{x}dx = 4$

Statement- II: $\frac{|x|}{x} = \begin{cases} -1 & \text{if } x<0 \\ \text{undefined} & \text{if } x=0 \\ 1 & \text{if } x>0 \end{cases}$

Q.34 **Statement- I :** $\int_0^{\pi/2} \frac{\sin x}{x}dx < \frac{\pi}{2}$

Statement- II : $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Q.35 **Statement I** : $\int_2^8 \frac{[x^2]dx}{[x^2-20x+100]+[x^2]} = 3$,

where [.] = G.IF

Statement II : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$

Passage Based Questions

Passage :

A function $f : R \to R$ satisfies the equation f(x + y) = f(x) + f(y) ∀ x, y ∈ R and is continuous through out the domain. If $I_1 + I_2 + ... + I_5 = 450$

when $I_n = n\int_0^n f(x)\,dx$

Q.36 f(x) =

(A) 4x (B) $\log_e x$
(C) e^{2x} (D) None of these

Q.37 Area bounded by f(x), x-axis and x = 1 is

(A) 2 unit^2 (B) 1 unit^2
(C) 4 unit^2 (D) None of these

Q.38 Interval in which f(x) increases

(A) (0, ∞) (B) (–∞, 0)
(C) (–∞, ∞) (D) None of these

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 If $I_n = \int_0^{\pi/4} \tan^n x \, dx$ then the value of $n(I_{n-1} + I_{n+1})$ is- **[AIEEE-2002]**

(A) 1 (B) $\pi/2$
(C) $\pi/4$ (D) n

Q.2 $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx =$ **[AIEEE-2002]**

(A) π^2 (B) $\pi^2/4$
(C) $\pi/8$ (D) $\pi^2/8$

Q.3 $\int_{\pi}^{10\pi} |\sin x| \, dx =$ **[AIEEE-2002]**

(A) 9 (B) 10
(C) 18 (D) 20

Q.4 $\int_0^{\sqrt{2}} [x^2] \, dx =$ **[AIEEE-2002]**

(A) $\sqrt{2} - 1$ (B) $2(\sqrt{2} - 1)$
(C) $\sqrt{2}$ (D) None of these

Q.5 $\lim_{n\to\infty} \frac{1^P + 2^P + 3^P + ... + n^P}{n^{P+1}}$ equals- **[AIEEE-2002]**

(A) 1 (B) $\frac{1}{P+1}$
(C) $\frac{1}{P+2}$ (D) P^2

Q.6 Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx$ = F(k) – F(1), then one of the possible values of k, is- **[AIEEE-2003]**

(A) 64 (B) 15
(C) 16 (D) 63

Q.7 If f(a + b – x) = f(x), then $\int_a^b x f(x) dx$ is equal to- **[AIEEE-2003]**

(A) $\frac{a+b}{2} \int_a^b f(a+b-x) \, dx$

(B) $\frac{a+b}{2} \int_a^b f(b-x) \, dx$

(C) $\frac{a+b}{2} \int_a^b f(x) \, dx$

(D) $\frac{b-a}{2} \int_a^b f(x) \, dx$

Q.8 The value of the integral $I = \int_0^1 x(1-x)^n dx$ is **[AIEEE-2003]**

(A) $\frac{1}{n+1} + \frac{1}{n+2}$ (B) $\frac{1}{n+1}$
(C) $\frac{1}{n+2}$ (D) $\frac{1}{n+1} - \frac{1}{n+2}$

Q.9 The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t \, dt}{x \sin x}$ is- **[AIEEE-2003]**

(A) 0 (B) 3
(C) 2 (D) 1

Q.10 $\lim_{n\to\infty} \frac{1+2^4+3^4+.....+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+.....+n^3}{n^5}$ is equal to- **[AIEEE-2003]**

(A) 1/5 (B) 1/30
(C) zero (D) 1/4

Q.11 If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y) \, g(y) dy$, then **[AIEEE-2003]**

(A) $F(t) = te^{-t}$ (B) $F(t) = 1 - e^{-1}(1+t)$
(C) $F(t) = e^t - (1+t)$ (D) $F(t) = t e^t$

Q.12 Let f(x) be a function satisfying f'(x) = f(x) with f(0) = 1 and g(x) be a function that satisfies f(x) + g(x) = x^2. Then the value of the integral $\int_0^1 f(x)g(x)\,dx$, is- **[AIEEE-2003]**

(A) $e + \frac{e^2}{2} + \frac{5}{2}$ (B) $e - \frac{e^2}{2} - \frac{5}{2}$

(C) $e + \frac{e^2}{2} - \frac{3}{2}$ (D) $e - \frac{e^2}{2} - \frac{3}{2}$

Q.13 $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is- **[AIEEE-2004]**

(A) e (B) e – 1 (C) 1 – e (D) e + 1

Q.14 The value of $\int_{-2}^{3} \left|1-x^2\right| dx$ is- **[AIEEE-2004]**

(A) 28/3 (B) 14/3 (C) 7/3 (D) 1/3

Q.15 The value of I = $\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ is-

[AIEEE-2004]

(A) 0 (B) 1 (C) 2 (D) 3

Q.16 If $\int_0^{\pi} x f(\sin x)\,dx = A \int_0^{\pi/2} f(\sin x)\,dx$, then A is-

[AIEEE-2004]

(A) 0 (B) π

(C) π/4 (D) 2π

Q.17 If f(x) $= \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is-

[AIEEE-2004]

(A) 2 (B) – 3

(C) – 1 (D) 1

Q.18 $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ equals- **[AIEEE-2005]**

(A) $\frac{1}{2}$ sec 1 (B) $\frac{1}{2}$ cosec 1

(C) tan 1 (D) $\frac{1}{2}$ tan 1

Q.19 If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then - **[AIEEE-2005]**

(A) $I_2 > I_1$ (B) $I_1 > I_2$

(C) $I_3 = I_4$ (D) $I_3 > I_4$

Q.20 Let f : R → R be a differentiable function having f(2) = 6, f ′(2) = $\left(\frac{1}{48}\right)$. Then $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals - **[AIEEE-2005]**

(A) 24 (B) 36 (C) 12 (D) 18

Q.21 The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0, is -

[AIEEE-2005 IIT-97,2000]

(A) aπ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{a}$ (D) 2π

Q.22 The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$ is -

[AIEEE-2006]

(A) $\frac{3}{2}$ (B) 2

(C) 1 (D) $\frac{1}{2}$

Q.23 $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]\,dx$ is equal to -

[AIEEE 2006]

(A) $(\pi^4/32) + (\pi/2)$ (B) π/2

(C) (π/4) – 1 (D) $\pi^4/32$

Q.24 $\int_0^{\pi} x f(\sin x)\,dx$ is equal to- **[AIEEE 2006]**

(A) $\pi\int_0^{\pi} f(\sin x)\,dx$ (B) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)\,dx$

(C) $\pi\int_0^{\pi/2} f(\cos x)\,dx$ (D) $\pi\int_0^{\pi} f(\cos x)\,dx$

Q.25 The value of $\int_1^a [x]f'(x)dx$, a > 1, where [x] denotes the greatest integer not exceeding x is- **[AIEEE-2006]**

(A) [a] f(a) – {f(1) + f(2) + ... + f([a])}
(B) [a] f([a]) – {f(1) + f(2) + ... + f(a)}
(C) a f([a]) – {f(1) + f(2) + ... + f(a)}
(D) a f(a) – {f(1) + f(2) + ... + f([a])}

Q.26 Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}dt$. Then F(e) equals- **[AIEEE-2007]**

(A) $\frac{1}{2}$ (B) 0
(C) 1 (D) 2

Q.27 The solution for x of the equation

$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ is- **[AIEEE-2007]**

(A) 2 (B) π
(C) $\sqrt{3}/2$ (D) $2\sqrt{2}$

Q.28 Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true? **[AIEEE-2008]**

(A) $I < \frac{2}{3}$ and J < 2 (B) $I < \frac{2}{3}$ and J > 2
(C) $I > \frac{2}{3}$ and J < 2 (D) $I > \frac{2}{3}$ and J > 2

Q.29 $\int_0^{\pi} [\cot x]dx$ where [.] denotes the greatest integer function, is equal to- **[AIEEE-2009]**

(A) $\frac{\pi}{2}$ (B) 1 (C) –1 (D) $-\frac{\pi}{2}$

Q.30 Let p(x) be a function defined on R such that p'(x) = p'(1 – x), for all x ∈ [0, 1], p(0) = 1 and p(1) = 41. Then $\int_0^1 p(x)dx$ equals - **[AIEEE-2010]**

(A) $\sqrt{41}$ (B) 21
(C) 41 (D) 42

Q.31 The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is **[AIEEE-2011]**

(A) $\pi \log 2$ (B) $\frac{\pi}{8}\log 2$
(C) $\frac{\pi}{2}\log 2$ (D) log 2

Q.32 Let [.] denote the greatest integer function then the value of $\int_0^{1.5} x[x^2]dx$ is - **[AIEEE-2011]**

(A) 0 (B) $\frac{3}{2}$
(C) $\frac{3}{4}$ (D) $\frac{5}{4}$

Q.33 If $g(x) = \int_0^x \cos 4t\, dt$, then g(x + π) equals: **[AIEEE-2012]**

(A) g(x) + g(π) (B) g(x) – g(π)
(C) g(x) . g(π) (D) $\frac{g(x)}{g(\pi)}$

Q.34 **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. **[JEE Main - 2013]**

(A) Statement-I is true; Statement-II is false.
(B) Statement-I is false; Statement-II is true.
(C) Statement-I is true; Statement-II is true; Statement-II is a correct explanation for Statement-I.
(D) Statement-I is true; Statement-II is true; Statement-II is not a correct explanation for Statement-I.

SECTION-B

Q.1

$$\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)........\left(1+\frac{(n-1)^2}{n^2}\right)\right]^{1/n}$$

(A) $e^{\frac{4-\pi}{2}}$ (B) $e^{\frac{\pi-4}{2}}$ **[IIT 1993]**
(C) $2e^{\frac{\pi-4}{2}}$ (D) None of these

Q.2 If $I_{m,n} = \int_0^1 t^m (1+t)^n dt$ then expression for $I_{m,n}$ in terms of $I_{(m+1,\, n-1)}$ is **[IIT 1993]**

(A) $\frac{2^n}{m+1} - \frac{n}{m+1} I_{m+1,\, n-1}$

(B) $\frac{n}{m+1} I_{(m+1,\, n-1)}$

(C) $\frac{2^n}{m+1} + \frac{n}{m+1} I_{m+1,\, n-1}$

(D) $\frac{m}{n+1} I_{m+1,\, n-1}$

Q.3 The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is- **[IIT- 1993]**

(A) 0 (B) 1 (C) $\pi/2$ (D) $\pi/4$

Q.4 The value of $\int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi} d\phi$ is ...... **[IIT - 1993]**

(A) $\pi(\sqrt{2}-1)$ (B) $\pi(\sqrt{2}+1)$

(C) $\pi(\sqrt{2}-2)$ (D) None

Q.5 $\int_2^3 \frac{\sqrt{x}}{\sqrt{(5-x)}+\sqrt{x}} dx =$ **[IIT- 1994]**

(A) 1/2 (B) 1/3

(C) 1/5 (D) None

Q.6 If $f(x) = A \sin(\pi x/2) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then the constants A and B are- **[IIT - 1995]**

(A) $\pi/2$ and $\pi/2$ (B) $2/\pi$ and 3π

(C) 0 and $-4/\pi$ (D) $4/\pi$ and 0

Q.7 The value of $\int_{\pi}^{2\pi} [2 \sin x]\, dx$, where [ ] the represents greatest integer function is - **[IIT-1995]**

(A) $-\frac{5\pi}{3}$ (B) $-\pi$ (C) $\frac{5\pi}{3}$ (D) -2π

Q.8 The function $L(x) = \int_1^x \frac{dt}{t}$ satisfies the equation **[IIT-1996]**

(A) $L(x + y) = L(x) + L(y)$

(B) $L\left(\frac{x}{y}\right) = L(x) + L(y)$

(C) $L(xy) = L(x) + L(y)$

(D) None of these

Q.9 If for a non-zero x, a f(x) + b$f\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then $\int_1^2 f(x)\,dx =$ **[IIT-1996]**

(A) $\frac{1}{a^2+b^2}\left(a \log 2 + 5a + \frac{7b}{2}\right)$

(B) $\frac{1}{a^2-b^2}\left(a \log 2 - 5a + \frac{7b}{2}\right)$

(C) $-\frac{1}{a^2+b^2}\left(a \log 2 + 5a - \frac{7b}{2}\right)$

(D) None of these

Q.10 Let $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = F(K) - F(1)$, then one of the possible values of K is- **[IIT-1997]**

(A) 2 (B) 4 (C) 8 (D) 16

Q.11 If $g(x) = \int_0^x \cos^4 t\, dt$, then $g(x + \pi)$ equals - **[IIT-1997]**

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$

(C) $g(x)\, g(\pi)$ (D) $g(x)/g(\pi)$

Q.12 Let f be a positive function, let $I_1 = \int_{1-k}^{k} x.\, f[x(1-x)]dx$ & $I_2 = \int_{1-k}^{k} f[x(1-x)]dx$, where $(2k-1) > 0$, then is- **[IIT-1997]**

(A) 2 (B) k

(C) 1/2 (D) 1

Q.13 If $\int_0^x f(t)\, dt = x + \int_x^1 t f(t)\, dt$, then the value of f (1) is- **[IIT-1998]**

(A) 1/2 (B) 0

(C) 1 (D) $-1/2$

Q.14 $\int_0^1 \tan^{-1}(1-x+x^2)\, dx =$ **[IIT-1998]**

(A) log 2 (B) $\log \frac{1}{2}$

(C) $\pi \log 2$ (D) $\frac{\pi}{2} \log \frac{1}{2}$

Q.15 For $n > 0$ $\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx =$ **[IIT-1998]**

(A) π^2 (B) π (C) 2π (D) 3π

Q.16 Let f(x) = x – [x], for every real number x, where [x] is the integral part of x. Then $\int_{-1}^{1} f(x)\,dx$ is- **[IIT-1998]**

(A) 1 (B) 2 (C) 0 (D) $\frac{1}{2}$

Q.17 $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to- **[IIT-1999]**

(A) 2 (B) – 2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Q.18 If for a real number y, [y] is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2}$ [2 sin x] dx is **[IIT-1999]**

(A) –π (B) 0 (C) –π /2 (D) π/2

Q.19 The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is- **[IIT-2000]**

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) 5

Q.20 If $f(x) = \begin{cases} e^{\cos x} \sin x; & |x| < 2 \\ 2; & \text{otherwise} \end{cases}$

Then $\int_{-2}^{3} f(x)\,dx =$ **[IIT-2000]**

(A) 0 (B) 1 (C) 2 (D) 3

Q.21 Let f : (0, ∞) → R and $F(x^2) = \int_0^{x^2} f(t)\,dt$.

If $F(x^2) = x^2 (1+ x)$, then f(4) equals- **[IIT-2001]**

(A) 5/4 (B) 7 (C) 4 (D) 2

Q.22 The integral $\int_{-\frac{1}{2}}^{1/2} \left([x] + \ell n\left(\frac{1+x}{1-x}\right)\right) dx$ equals- **[IIT Scr. 2001]**

(A) –1/2 (B) 0

(C) 1 (D) 2 ℓn (1/2)

Q.23 Let T > 0 be a fixed real number. Suppose f is a continuous function such that for all x ∈ R, f(x +T) = f(x). If $I = \int_0^T f(x)\,dx$ then the value of $\int_3^{3+3T} f(2x)\,dx$ is- **[IIT-2002]**

(A) – 3/2 I (B) 2I (C) 3I (D) 6I

Q.24 Let $f(x) = \int_1^x \sqrt{2-t^2}\,dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are- **[IIT-2002]**

(A) ±1 (B) $\pm 1/\sqrt{2}$

(C) $\pm\frac{1}{2}$ (D) 0 and 1

Q.25 If $\int_0^{t^2} x f(x)\,dx = \frac{2}{5} t^5$ for t > 0, then f(4/25) is- **[IIT Scr. 2004]**

(A) $-\frac{2}{5}$ (B) 0 (C) $\frac{2}{5}$ (D) 1

Q.26 $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ equals to- **[IIT Scr. 2004]**

(A) $\frac{\pi}{2} + 1$ (B) $\frac{\pi}{2} - 1$ (C) 1 (D) π

Q.27 $\int_{-2}^{0} [x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x + 1)]\,dx =$ **[IIT Scr. 2005]**

(A) 4 (B) 0 (C) –1 (D) 1

Q.28 $\int_{\sin x}^{1} t^2 f(t)\,dt = 1 - \sin x,\ 0 \le x \le \frac{\pi}{2}$ then $f\left(\frac{1}{\sqrt{3}}\right)$ is- **[IIT Scr. 2005]**

(A) 3 (B) $\frac{1}{3}$ (C) 1 (D) $\sqrt{3}$

Q.29 $\int_0^{\pi/2} (\sin x)^{\cos x} (\cos x.\cot x - \log(\sin x)^{\sin x})\,dx$

(A) 2 (B) 1 (C) 3 (D) 4

Q.30 $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2 - \frac{\pi^2}{16}}$ equals- **[IIT- 2007]**

(A) $\frac{8}{\pi} f(2)$ (B) $\frac{2}{\pi} f(2)$

(C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ (D) 4f(2)

Q.31 Match the integrals in **Column I** with the values in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS **[IIT-2007]**

Column I	Column II
(a) $\int_{-1}^{1} \frac{dx}{1+x^2}$	(p) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(b) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	(q) $2\log\left(\frac{2}{3}\right)$
(c) $\int_{2}^{3} \frac{dx}{1-x^2}$	(r) $\frac{\pi}{3}$
(d) $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	(s) $\frac{\pi}{2}$

(A) **a → p; b → r ; c → p; d→ r**
(B) **a → s; b → s ; c → p; d→ r**
(C) **a → r; b → p ; c → q; d→ r**
(D) **a → s; b → s ; c → r; d→ q**

Q.32 If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, n = 0, 1, 2, ….., then **[IIT-2009]**

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

Q.33 Let f : R → R be a continuous function which satisfies f(x) = $\int_0^x f(t)\,dt$. Then the value of f(ℓn 5) is… **[IIT-2009]**

(A) 0 (B) 3 (C) 2 (D) 5

Q.34 The value of $\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\,\ell n(1+t)}{t^4+4} dt$ is **[IIT-2010]**

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

Q.35 The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is (are) - **[IIT-2010]**

(A) $\frac{22}{7}-\pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15}-\frac{3\pi}{2}$

Q.36 The value of $\int_{\sqrt{\ell n 2}}^{\sqrt{\ell n 3}} \frac{x\sin x^2}{\sin x^2 + \sin(\ell n\, 6 - x^2)} dx$ is **[IIT-2011]**

(A) $\frac{1}{4}\ell n\frac{3}{2}$ (B) $\frac{1}{2}\ell n\frac{3}{2}$

(C) $\ell n\frac{3}{2}$ (D) $\frac{1}{6}\ell n\frac{3}{2}$

Q.37 Let f : [–1, 2] → [0, ∞) be a continuous function such that f(x) = f(1 – x) for all x∈[–1, 2]. Let $R_1 = \int_{-1}^{2} x\,f(x)dx$, and R_2 be the area of the region bounded by y = f(x), x = – 1, x = 2, and the x-axis. Then **[IIT-2011]**

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$
(C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Q.38 If $\int_a^b (f(x)-3x)dx = a^2-b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is **[IIT-2011]**

(A) 4π/3 (B) 3π/2
(C) π/6 (D) π/4

Q.39 The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is **[IIT-2011]**

(A) 4π/3 (B) 3π/2
(C) π (D) π/4

Q.40 The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is **[IIT-2012]**

(A) 0 (B) $\frac{\pi^2}{2}-4$ (C) $\frac{\pi^2}{2}+4$ (D) $\frac{\pi^2}{2}$

Q.41 Let f : $\left[\frac{1}{2}, 1\right]$ → R (the set of all real numbers) be a positive, non-constant and differentiable function such that f '(x) < 2 f(x) and $f\left(\frac{1}{2}\right)$ = 1. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval – **[JEE - Advance 2013]**

(A) (2e – 1, 2e) (B) (e – 1, 2e – 1)
(C) $\left(\frac{e-1}{2}, e-1\right)$ (D) $\left(0, \frac{e-1}{2}\right)$

Q.42 For a ∈ R (the set of all real numbers), a ≠ – 1,

$$\lim_{n\to\infty} \frac{(1^a+2^a+.....+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+......+(na+n)]} = \frac{1}{60}.$$

Then a = **[JEE - Advance 2013]**

(A) 5 (B) 7 (C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	A	C	D	C	C	C	D	C	B	D	D	D	A	B	B	A	C	B
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	D	A	C	D	C	C	A	C	A	B	A	A	B	C	B	D	C	C	C	B
Q.No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	A	C	C	A	A	B	B	D	C	D	B	C	B	A	C	B	C	A	D	B
Q.No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	C	C	B	B	C	B	B	D	C	C	B	A	C	D	C	D	A	C	D	D
Q.No.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	A	B	D	B	D	C	A	B	C	C	A	C	A	B	C	A	B	D	B	A
Q.No.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115					
Ans.	D	B	B	A	A	B	C	D	D	B	B	B	B	B	A					

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	C	C	C	A	A	B	C	B	A	B	C	A	B	C	A	A	A	D
Q.No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	C	D	D	C	B	C	D	A	B	A	B	B	D	C	C	A	D	A	C	B
Q.No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	A	B	A	A	A	D	C	D	A	A	D	D	D	B	A	C	C	B	**B**

LEVEL- 3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	A	D	B	B	B	B	B	C	C	B	C	D	C	B	D	C	C	C
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38		
Ans.	D	C	D	A	C	A	D	B	D	C	D	B	D	B	A	A	A	C		

LEVEL- 4

SECTION-A

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	C	A	B	A	C	D	D	A	C	D	B	A	C	B	A	D	B	D
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34						
Ans.	B	A	B	C	A	A	A	A	D	B	A	C	A	B						

SECTION-B

1.[C] $y = \lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)........\left(1+\frac{(n-1)^2}{n^2}\right)\right]^{1/n}$

$$\log y = \lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$= \int_0^1 \log(1+x^2)\,dx$$

$$= [\,x\log(1+x^2)]_0^1 - \int_0^1 \frac{2x^2}{1+x^2}\,dx$$

$$= \log 2 - 2\,(x - \tan^{-1}x)_0^1 = \log 2 - 2\left(1-\frac{\pi}{4}\right)$$

$$y = e^{\log 2 + 2(\pi/4-1)}$$

$$y = 2\,e^{\frac{\pi-4}{2}}$$

2.[A] $I_{m,n} = \int_0^1 t^m (1+t)^n \, dt$

$= \left[\frac{t^{m+1}}{m+1}(1+t)^n\right]_0^1 - \int_0^1 \frac{t^{m+1}}{m+1} . n(1+t)^{n-1} dt$

$= \frac{2^n}{m+1} - \frac{n}{m+1} I_{m+1,\, n-1}$

3.[D] $I = \int_0^{\pi/2} \frac{1}{1+\tan^3 x} dx$

$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx$ … (1)

$I = \int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx$ … (2)

From eq^n (1) + (2)

$2I = \int_0^{\pi/2} dx$

$2I = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

4.[A] $I = \int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi} d\phi$ …(1)

$I = \int_{\pi/4}^{3\pi/4} \frac{\pi-\phi}{1+\sin\phi} d\phi$ …(2)

$2I = \pi \int_{\pi/4}^{3\pi/4} \frac{1}{1+\sin\phi} d\phi$

$I = \frac{\pi}{2} \int_{\pi/4}^{3\pi/4} \frac{1-\sin\phi}{\cos^2\phi} d\phi$

$= \frac{\pi}{2} \int_{\pi/4}^{3\pi/4} (\sec^2\phi - \sec\phi\tan\phi)\, d\phi$

$= \frac{\pi}{2} (\tan\phi - \sec\phi)_{\pi/4}^{3\pi/4} = \pi(\sqrt{2} - 1)$

5.[A] $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{(5-x)} + \sqrt{x}} dx$

$I = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx$

$2I = \int_2^3 dx$

$2I = [x]_2^3$

$I = \frac{1}{2}$

6.[D] $f(x) = A \sin(\pi x/2) + B,\ f'\left(\frac{1}{2}\right) = \sqrt{2}$

$I = \int_0^1 f(x)\, dx = \frac{2A}{\pi}$

$I = \int_0^1 \left(A \sin\frac{\pi x}{2} + B\right) dx$

$\Rightarrow \frac{2A}{\pi}\left[-\cos\frac{\pi x}{2}\right]_0^1 + B[x]_0^1$

$\Rightarrow \frac{2A}{\pi}[1] + B = \frac{2A}{\pi} \Rightarrow B = 0$

and

$f'(x) = \frac{\pi A}{2} \cos\frac{\pi x}{2}$

$f'\left(\frac{1}{2}\right) = \frac{\pi}{2} A . \frac{1}{\sqrt{2}} = \frac{\pi A}{2\sqrt{2}}$

But $f'\left(\frac{1}{2}\right) = \sqrt{2}$

$\Rightarrow \frac{\pi A}{2\sqrt{2}} = \sqrt{2} \Rightarrow A = \frac{4}{\pi}$

7.[A] $\int_\pi^{2\pi} [2 \sin x]\, dx$

$\because \pi < x < \frac{7\pi}{6}\ \&\ \frac{11\pi}{6} < x < 2\pi;\ -\frac{1}{2} < \sin x < 0$

and $\frac{7\pi}{6} < x < \frac{11\pi}{6},\ -1 < \sin x < -\frac{1}{2}$

$I = \int_\pi^{7\pi/6} -1 dx + \int_{7\pi/6}^{11\pi/6} -2 dx + \int_{11\pi/6}^{2\pi} -1 dx$

$= \left(\pi - \frac{7\pi}{6}\right) + 2\left(\frac{7\pi}{6} - \frac{11\pi}{6}\right) + \left(\frac{11\pi}{6} - 2\pi\right)$

$= -\frac{5\pi}{3}$

8.[C] $L(x) = \int_1^x \frac{dt}{t}$

$L(x) = \log x$

$\Rightarrow L(xy) = \log(xy) = \log(x) + \log(y)$

$L(xy) = L(x) + L(y)$

9.[B] $a\,f(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$ … (1)

$a\,f\left(\frac{1}{x}\right) + bf(x) = x - 5$ …(2)

from (1) and (2)

$(a^2 - b^2)\,f(x) = a\left(\frac{1}{x} - 5\right) - b(x - 5)$

$(a^2 - b^2)\,f(x) = \frac{a}{x} - bx + 5(b - a)$

$\int_1^2 f(x) = \frac{1}{(a^2 - b^2)}\int_1^2\left[\frac{a}{x} - bx + 5(b-a)\right]dx$

$= \frac{1}{a^2 - b^2}\left[a\log x - \frac{bx^2}{2} + (b-a)5x\right]_1^2$

$= \frac{1}{a^2 - b^2}\left[a\log 2 - 5a + \frac{7b}{2}\right]$

10.[D] $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$

$I = \int_1^4 \frac{2e^{\sin x^2}}{x}dx = F(k) - F(1)$

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$I = \int_1^{16}\frac{e^{\sin t}}{t}dt = [F(t)]_1^{16} = F(16) - F(1)$

$k = 16$

11.[A] $g(x) = \int_0^x \cos^4 t\,dt$

$g(x + \pi) = \int_0^{x+\pi} \cos^4 t\,dt$

$= \int_0^x \cos^4 t\,dt + \int_x^{x+\pi}\cos^4 t\,dt$

$= \int_0^x \cos^4 t\,dt + \int_0^{\pi}\cos^4 t\,dt = g(x) + g(\pi)$

$\because \cos^4(\pi - t) = \cos^4 t$

12.[C] $I_1 = \int_{1-k}^{k} xf\,(x(1-x))\,dx$

$I_1 = \int_{1-k}^{k}(1-x)\,f\,[x(1-x)]\,dx$

$2I_1 = \int_{1-k}^{k} f\,[x(1-x)]\,dx$

$2I_1 = I_2$

$\frac{I_1}{I_2} = \frac{1}{2}$

13.[A] $\int_0^x f(t)\,dt = x + \int_x^1 tf(t)\,dt$

Differentiate both side

$f(x) = 1 - x\,f(x)$

at $x = 1$

$f(1) = 1 - f(1)$

$\Rightarrow f(1) = \frac{1}{2}$

14.[A] $I = \int_0^1 \tan^{-1}(1 - x + x^2)\,dx$

$\Rightarrow I = \int_0^1 \cot^{-1}\left(\frac{1}{1 - x(1-x)}\right)dx$

$= \int_0^1 \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{1 - x(1-x)}\right)dx$

$\Rightarrow I = \int_0^1 \frac{\pi}{2}.dx - \left[\int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}x\,dx\right]$

$= \frac{\pi}{2} - 2\int_0^1 \tan^{-1}x.dx$ …(1)

$\because \int_0^1 \tan^{-1}x.dx = (\tan^{-1}x.x)_0^1 - \int_0^1 \frac{1.x}{(1+x^2)}.dx$

$= \frac{\pi}{4} - \frac{1}{2}.(\log(1+x^2))_0^1$

$\Rightarrow \frac{\pi}{4} - \frac{1}{2}\log 2$ …(2)

Now, from eq^n (1) & (2)

$\Rightarrow I = \frac{\pi}{2} - 2.\int_0^1 \tan^{-1}x\,dx$

$I = \frac{\pi}{2} - 2.\frac{\pi}{4} + 2.\frac{1}{2}.\log 2$

$\Rightarrow I = \log 2$

15.[A] $I = \int_0^{2\pi} \frac{x\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}\,dx$ …(1)

$I = \int_0^{2\pi} \frac{(2\pi-x)\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}\,dx$ …(2)

(1) + (2)

$2I = 2\pi\int_0^{2\pi} \frac{\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}\,dx$

$I = 4\pi\int_0^{\pi/2} \frac{\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}\,dx$ ..(3)

$I = 4\pi\int_0^{\pi/2} \frac{\cos^{2n}x}{\sin^{2n}x+\cos^{2n}x}$ …(4)

(3) + (4)

$2I = 4\pi\int_0^{\pi/2} dx$

$I = 2\pi\left[\frac{\pi}{2}\right] = \pi^2$

16.[A] $f(x) = x - [x]$

$I = \int_{-1}^{1} f(x)\,dx$

$= \int_{-1}^{1}(x-[x])\,dx = \int_{-1}^{1} x\,dx - \int_{-1}^{1}[x]\,dx$

$= 0 + \int_{-1}^{0} 1\,dx - \int_0^1 0\,dx = \int_{-1}^{0} dx$

$= [x]_{-1}^{0} = 1$

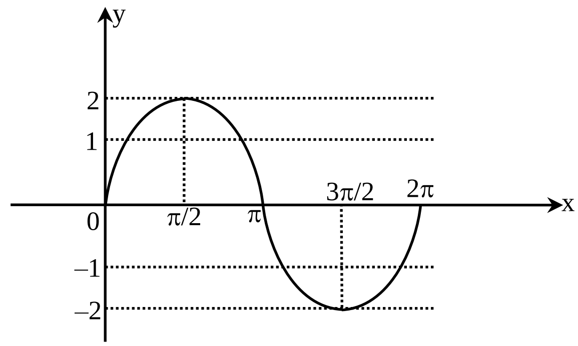
17.[A] $I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}\,dx$

$= \int_{\pi/4}^{3\pi/4} \frac{1-\cos x}{\sin^2 x}\,dx$

$= \int_{\pi/4}^{3\pi/4} (\mathrm{cosec}^2 x - \mathrm{cosec}\,x\cot x)\,dx$

$= [-\cot x + \mathrm{cosec}\,x]_{\pi/4}^{3\pi/4} = 2$

18.[C]



so $\int_{\pi/2}^{3\pi/2}[2\sin x]\,dx = \int_{\pi/2}^{5\pi/6} 1.dx + \int_{5\pi/6}^{\pi} o\,dx$

$+ \int_{\pi}^{7\pi/6}(-1)\,dx + \int_{7\pi/6}^{3\pi/2}(-2)\,dx$

$= \frac{\pi}{3} + 0 - \frac{\pi}{6} - 2.\frac{\pi}{3}$

$= -\frac{\pi}{2}$

19.[B] $I = \int_{1/e}^{e^2} \left|\frac{\log_e x}{x}\right| dx$

$= -\int_{1/e}^{1} \frac{\log x}{x}\,dx + \int_1^{e^2} \frac{\log x}{x}\,dx$

$= -\int_{-1}^{0} t\,dt + \int_0^2 t\,dt$

$= -\left[\frac{t^2}{2}\right]_{-1}^{0} + \left[\frac{t^2}{2}\right]_0^2 = \frac{1}{2} + 2 = \frac{5}{2}$

20.[C] $I = \int_{-2}^{3} f(x)\,dx$

$= \int_{-2}^{2} e^{\cos x}\sin x\,dx + \int_2^3 2\,dx$

$= 0 + 2\int_2^3 dx$

$= 2[x]_2^3 = 2$

21.[C] $F(x^2) = \int_0^{x^2} f(t)\,dt$

$F(x^2) = x^2(1+x) \Rightarrow x^2(1+x) = \int_0^{x^2} f(t)\,dt$

Diff.w.r. to x

$x^2 + 2x(1+x) = f(x^2).\,2x$

$f(x^2) = \frac{x}{2} + (1+x)$

Put x = 2

f(a) = 1 + 3 = 4

22.[A] $I = \int_{-1/2}^{1/2} \left([x] + \ell n\left(\frac{1+x}{1-x}\right)\right) dx$

$= \int_{-1/2}^{1/2} [x]\,dx + 0 = \int_{-1/2}^{0} -1\,dx + 0 = [-x]_{-1/2}^{0} = -\frac{1}{2}$

23.[C] $f(x+T) = f(x),\ I = \int_0^T f(x)\,dx$

$= \int_3^{3+3T} f(x)\,dx = \int_3^{3T} f(x)\,dx = 3\int_0^T f(x)\,dx = 3I$

24.[A] $f(x) = \int_1^x \sqrt{2-t^2}\,dt$

$f'(x) = \sqrt{2-x^2}$

Equation is

$x^2 - \sqrt{2-x^2} = 0$

$x^4 = (2-x^2)$

$x^4 + x^2 - 2 = 0$

$(x^2-1)(x^2+2) = 0$

$x^2 = 1,\ x^2 = -2$

$x = \pm 1$

25.[C] $\int_0^{t^2} x f(x)\,dx = \frac{2}{5}t^5 \Rightarrow$ diff. w.r. to t

$t^2 f(t^2).2t = 2t^4 \Rightarrow f(t^2) = t$

put $t = \frac{2}{5} \Rightarrow f\left(\frac{4}{25}\right) = \frac{2}{5}$

26.[B] $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$

$= \int_0^1 \frac{1}{\sqrt{1-x^2}} - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$

$= \left[\sin^{-1} x\right]_0^1 + \int_1^0 dt$ where $1 - x^2 = t^2$

$= \frac{\pi}{2} - 0 + [x]_1^0$

$= \frac{\pi}{2} - 1$

27.[A] $\int_{-2}^{0} [x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)]\,dx$

$= \int_{-2}^{0} [(x+1)^3 + 2 + (x+1)\cos(x+1)]\,dx$

$= \int_{-2}^{0} 2dx + \int_{-2}^{0} [(x+1)^3 + (x+1)\cos(x+1)]\,dx$

$= 2[x]_{-2}^{0} + \int_{-1}^{1} (t^3 + t\cos t)\,dt$ where x + 1 = t

I = 4 + 0 = 4 ($\because$ $t^3 + t\cos t$ is an odd function)

28.[A] $\int_{\sin x}^{1} t^2 f(t)\,dt = 1 - \sin x$

diff. w.r. to x

$-\sin^2 x\, f(\sin x).\cos x = -\cos x$

$f(\sin x) = \frac{1}{\sin^2 x}$

Put $\sin x = \frac{1}{\sqrt{3}}$

$f\left(\frac{1}{\sqrt{3}}\right) = 3$

29.[B] $I = \int_0^{\pi/2} (\sin x)^{\cos x}(\cos x.\cot x - \log(\sin x)^{\sin x})\,dx$

Let $(\sin x)^{\cos x} = t$

$\Rightarrow (\sin x)^{\cos x}(\cos x \cot x - \log(\sin x)^{\sin x})\,dx = dt$

$I = \int_0^1 dt = 1$

30.[A] $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2 - \frac{\pi^2}{16}}$ $\qquad \frac{0}{0}$ form

$= \lim_{x \to \frac{\pi}{4}} \frac{f(\sec^2 x).2\sec^2 x.\tan x}{2x}$

$= \frac{f(2).2.2.1}{2.\frac{\pi}{4}} = \frac{8}{\pi} f(2)$

31.[B] (a) $a \to s$

$$\int_{-1}^{1}\frac{1}{1+x^2}dx = 2\int_{0}^{1}\frac{1}{1+x^2}dx$$

$= 2[\tan^{-1} x]_0^2 = \frac{\pi}{2}$

(b) $b \to s$

$$\int_{0}^{1}\frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]_0^1 = \frac{\pi}{2}$$

(c) $c \to p$

$$\int_{2}^{3}\frac{1}{1-x^2}dx = \frac{1}{2}\left[\log\left(\frac{1+x}{1-x}\right)\right]_2^3$$

$= \frac{1}{2}[\log 2 - \log 3] = \frac{1}{2}\log\frac{2}{3}$

(d) $d \to r$

$$\int_{1}^{2}\frac{dx}{x\sqrt{x^2-1}} = [\sec^{-1} x]_1^2 = [\sec^{-1} 2 - \sec^{-1} 1]$$

$= \frac{\pi}{3} - 0 = \frac{\pi}{3}$

32.[A, B, C]

$$I_n = \int_{-\pi}^{\pi}\frac{\sin nx\, dx}{\sin x(1+\pi^x)} \Rightarrow I_n = \int_{-\pi}^{\pi}\frac{\pi^x \sin nx}{(1+\pi^x)\sin x}dx$$

$$\Rightarrow 2I_n = \int_{-\pi}^{\pi}\frac{\sin nx}{\sin x}dx$$

$$I_n = \int_{0}^{\pi}\frac{\sin nx}{\sin x}dx \Rightarrow I_{n+2} = \int_{0}^{\pi}\frac{\sin(n+2)x}{\sin x}dx$$

$$\Rightarrow I_{n+2} - I_n = \int_{0}^{\pi}\frac{\sin(n+2)x - \sin nx}{\sin x}dx$$

$$= \int_{0}^{\pi}\frac{2\cos(n+1)x\sin x}{\sin x}dx = 0 \Rightarrow I_{n+2} = I_n$$

Again $\sum_{m=1}^{10} I_{2m+1} = I_3 + I_4 + \ldots I_{21} \Rightarrow 10I_3 = 10I_1 = 10\pi$

$\because I_1 = \pi$ and $\sum I_{2m} = 0$

$\because I_{2m} = \int_{0}^{\pi}\frac{\sin 2mx}{\sin x} = 0$

$\because f(a-x) = -f(x)$

Clearly $I_{n+1} \neq I_n$

33.[A] $f(x) = \int_0^x f(t)dt \Rightarrow f'(x) = f(x)$

$\Rightarrow \ell n f(x) = A + x$

$\Rightarrow f(x) = e^{A+x} = e^A.\, e^X$

$\Rightarrow f(x) = c.e^x.$

But from parent equation $f(0) = 0$ so $f(x)$ can not be $f(x) = c.\, e^x$. The $f(x)$ is constant function and $f(\ell n5) = 0$

34.[B] $\lim_{x\to 0}\frac{1}{x^3}\int_0^x \frac{t\,\ell n(1+t)}{t^4+4}dt$, $\frac{0}{0}$ form

$= \lim_{x\to 0}\frac{x\ell n(1+x)}{(x^4+4)(3x^2)}$

$= \lim_{x\to 0}\frac{1}{3}.\frac{\ell n(1+x)}{x}.\frac{1}{x^4+4} = \frac{1}{12}$

35.[A] $I = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$

$= \int_0^1 \frac{x^8 - 4x^7 + 6x^6 - 4x^5 + x^4}{1+x^2}dx$

$= \int_0^1\left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}\right)dx$

$= \frac{22}{7} - \pi$

36.[A] Given $I = \int_{\sqrt{\ell n2}}^{\sqrt{\ell n3}}\frac{x.\sin x^2}{\sin x^2 + \sin(\ell n6 - x^2)}dx$

$x^2 = t \Rightarrow 2x\,dx = dt$

so $I = \frac{1}{2}\int_{\ell n2}^{\ell n3}\frac{\sin t}{\sin t + \sin(\ell n6 - t)}dt$ …(i)

$I = \frac{1}{2}\int_{\ell n2}^{\ell n3}\frac{\sin(\ell n6 - t)}{\sin(\ell n6 - t) + \sin t}dt$ …(ii)

(i) + (ii) $\because 2I = \frac{1}{2}.\int_{\ell n2}^{\ell n3}dt = \frac{1}{2}.\ \ell n(3/2)$

$\Rightarrow I = \frac{1}{4}.(\ell n(3/2))$

37.[C] $R_1 = \int_{-1}^{2} x.f(x)dx$ …(i)

$R_1 = \int_{-1}^{2}(1-x).f(1-x)dx$ …(ii)

As $f(x) = f(1-x)$

Add. $2R_1 = \int_{-1}^{2} f(x)dx \Rightarrow R_1 = \frac{\int_{-1}^{2} f(x)dx}{2}$

Also $R_2 = \int_{-1}^{2} f(x)dx$

So $R_1 = R_2/2$

38.[C] Given $\int_a^b (f(x) - 3x)dx = a^2 - b^2$

$\Rightarrow \int_a^b f(x)dx - \frac{3}{2}.(b^2 - a^2) = a^2 - b^2$

$\Rightarrow \int_a^b f(x)dx = -\frac{1}{2}$. $(a^2 - b^2)$ = $\frac{b^2 - a^2}{2}$

diff. w.r.t. b

$\Rightarrow$ f(x) = x so $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$

39.[C] Given $I = \frac{\pi^2}{\ell n3}.\int_{7/6}^{5/6} \sec(\pi x)dx$

$\Rightarrow I = \frac{\pi^2}{\ell n3}\left(\frac{\log(\sec \pi x + \tan \pi x)}{\pi}\right)_{7/6}^{5/6}$

$\Rightarrow I = \frac{\pi}{\ell n3} \ell n3 = \pi$

40.[B] $\int_{-\pi/2}^{\pi/2} x^2 \cos x \, dx + \int_{-\pi/2}^{\pi/2} \ell n\left(\frac{\pi - x}{\pi + x}\right) \cos x \, dx$

↓ Even function ↓ Odd function

$= 2\int_0^{\pi/2} x^2 \cos x \, dx + 0$

$= 2[x^2 \sin x + 2x \cos x - 2 \sin x]_0^{\pi/2}$

$= 2\left[\frac{\pi^2}{4} - 2\right]$

$= \frac{\pi^2}{2} - 4$

41.[D] f '(x) – 2f(x) < 0…(i)

Multiply equation (i) by e^{-2x}

f '(x) e^{-2x} – $2e^{-2x}$ f(x) < 0

$\frac{d}{dx}$ (f(x) e^{-2x}) < 0

So f(x) e^{-2x} decreases

So f(x) e^{-2x} < f(1/2) e^{-1} ; for x ∈ [1/2, 1]

f(x) $e^{-2x} < \frac{1}{e}$; for x ∈ [1/2, 1]

f(x) < e^{+2x-1} ; for x ∈ [1/2, 1]

since f(x) > 0 (given)

so $0 < \int_{1/2}^{1} f(x) \, dx < \int_{1/2}^{1} e^{2x-1} \, dx$

so $0 < \int_{1/2}^{1} f(x) \, dx < \left(\frac{e^{2x-1}}{2}\right)_{1/2}^{1}$ dx

so, $0 < \int_{1/2}^{1} f(x) \, dx < \frac{e-1}{2}$

42.[B,D] $\lim_{n\to\infty} \frac{1^a + 2^a + \ldots\ldots\ldots + n^a}{(n+1)^{a-1}\left[n^2 a + \frac{n(n+1)}{2}\right]}$

$= \lim_{n\to\infty} \frac{n^a\left[\left(\frac{1}{n}\right)^a + \left(\frac{2}{n}\right)^a + \ldots\ldots\ldots\ldots + 1\right]}{n^{a-1}\left(1+\frac{1}{n}\right)^{a-1} n^2\left[a + \frac{(1+1/n)}{2}\right]}$

$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{a-1}\left[a + \frac{1+1/n}{2}\right]} . \int_0^1 x^a \, dx$

$= \frac{1}{a+\frac{1}{2}} \left(\frac{x^{a+1}}{a+1}\right)_0^1$

$= \frac{2}{(2a+1)} \times \frac{1}{a+1} = \frac{1}{60}$

$\Rightarrow$ (2a + 1) (a + 1) = 120

$\Rightarrow$ $2a^2 + 3a + 1 = 120$

$\Rightarrow$ $2a^2 + 3a - 119 = 0$

$\Rightarrow$ (a – 7) (2a + 17) = 0

a = 7, $\frac{-17}{2}$