SOLVED EXAMPLES

Ex.1 The area bounded by the curve $y = 3/x^2$, x-axis and the lines x = 1 and x = 2 is-

(A) 3/2 (B) 1/2

(C) 2 (D) 1

Sol. Area = $\int_1^2 y\,dx = \int_1^2 \frac{3}{x^2}\,dx$

$= -\left[\frac{3}{x}\right]_1^2 = -3\left(\frac{1}{2}-1\right)$

= 3/2 **Ans.[A]**

Ex.2 The area between the curve $y = \sin^2 x$, x-axis and the ordinates x = 0 and $x = \frac{\pi}{2}$ is-

(A) π (B) $\pi/2$

(C) $\pi/4$ (D) $\pi/8$

Sol. Required area = $\int_0^{\pi/2} \sin^2 x\,dx$

$= \int_0^{\pi/2} \frac{1-\cos 2x}{2}\,dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{\pi}{4}$ **Ans.[C]**

Ex.3 The area between the curve $y = 4 + 3x - x^2$ and x-axis is-

(A) 125/6 (B) 125/3

(C) 125/2 (D) None of these

Sol. Putting y = 0, we get,

$x^2 - 3x - 4 = 0$

$\Rightarrow (x-4)(x+1) = 0$

$\Rightarrow x = -1$ or $x = 4$

$\therefore$ required area = $\int_{-1}^{4} (4+3x-x^2)\,dx$

$= \left(4x + \frac{3x^2}{2} - \frac{x^3}{3}\right)_{-1}^{4} = \frac{125}{6}$ **Ans.[A]**

Ex.4 The area bounded by the curve $y^2 = 4x$, y-axis and y = 3 is-

(A) 2 units (B) 9/4 units

(C) 7/3 units (D) 3 units

Sol. Area = $\int_0^3 x\,dy = \int_0^3 \frac{y^2}{4}\,dy$

$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{12}(27-0)$

= 9/4 units **Ans.[B]**

Ex.5 The area bounded by the curve $x = a\cos^3 t$, $y = a\sin^3 t$, is-

(A) $\frac{\pi a^2}{8}$ (B) $\frac{\pi a^2}{4}$

(C) $\frac{3\pi a^2}{8}$ (D) $\frac{2\pi a^2}{3}$

Sol. Given curve $\left(\frac{x}{a}\right)^{1/3} = \cos t$, $\left(\frac{y}{a}\right)^{1/3} = \sin t$

Squaring and adding $x^{2/3} + y^{2/3} = a^{2/3}$

Clearly it is symmetric with respect to both the axis, so whole area is

$= 4\int_0^a y\,dx$

$= 4\int_{\pi/2}^{0} a\sin^3 t\, 3a\cos^2 t\,(-\sin t)\,dt$

By given equation at x = 0; $t = \frac{\pi}{2}$ at x=a; t = 0

$= 12a^2 \int_0^{\pi/2} \sin^4 t\cos^2 t\,dt$

$= 12a^2 . \frac{3.1.1}{6.4.2} . \frac{\pi}{2} = \frac{3\pi a^2}{8}$ **Ans.[C]**

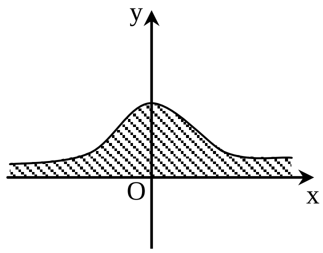
Ex.6 The area between the curve y = sech x and x-axis is-

(A) ∞ (B) π

(C) 2π (D) $\pi/2$

Sol. Given curve is symmetrical about y-axis as shown in the diagram.

Reqd. area $= 2\int_0^{\infty} \text{sech}\, x\,dx$



$$= 2\int_0^{\infty} \frac{2}{e^x + e^{-x}} dx = 4 \int_0^{\infty} \frac{e^x}{e^{2x}+1} dx$$

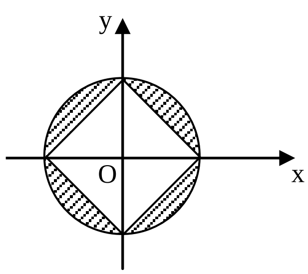
$$= 4 \left[\tan^{-1}(e^x)\right]_0^{\infty} = 4\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \pi$$ **Ans.[B]**

Ex.7 The area bounded by the circle $x^2 + y^2 = 1$ and the curve $|x| + |y| = 1$ is-

(A) $\pi - 2$ (B) $\pi - 2\sqrt{2}$

(C) $2(\pi - 2\sqrt{2})$ (D) None of these

Sol. By changing x as – x and y as – y, both the given equation remains unchanged so required area will be symmetric w.r.t both the axis, which is shown in the fig., so required area is



$$= 4\int_0^1 \left[\sqrt{1-x^2} - (1-x)\right] dx$$

$$= 4\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$$

$$= 4\left[0 + \frac{1}{2}.\frac{\pi}{2} - 1 + \frac{1}{2}\right] = \pi - 2$$ **Ans.[A]**

Ex.8 The area bounded by the curve $y = \sin x$, $x = 0$ and $x = 2\pi$ is-

(A) 4 units (B) 0 units

(C) 4π units (D) 2 units

Sol. $f(x) = y = \sin x$

when $x \in [0, \pi]$, $\sin x \geq 0$

and when $x \in [\pi, 2\pi]$, $\sin x \leq 0$

$$\therefore \text{ required area} = \int_0^{\pi} y\, dx + \int_{\pi}^{2\pi} (-y)\, dx$$

$$= \int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} (-\sin x)\, dx$$

$$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$$

$$= (-\cos\pi + \cos 0) + (\cos 2\pi - \cos \pi)$$

$$= (1 + 1) + (1 + 1)$$

= 4 units **Ans.[A]**

Ex.9 The area between the curves $y = \sqrt{x}$ and $y = x$ is-

(A) 1/3 (B) 1/6

(C) 2/3 (D) 1

Sol. The points of intersection of curves are $x = 0$ and $x = 1$.

$$\therefore \text{ required area} = \int_0^1 (\sqrt{x} - x)\, dx$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$ **Ans.[B]**

Ex.10 The area between the parabola $x^2 = 4y$ and line $x = 4y - 2$ is-

(A) 9/4 (B) 9/8

(C) 9/2 (D) 9

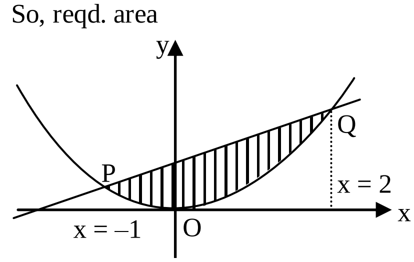
Sol. Solving the equation of the given curves for x, we get

$x^2 = x + 2$

$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = -1, 2$

So, reqd. area



$$= \int_{-1}^{2} \left[\frac{x+2}{4} - \frac{x^2}{4}\right] dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}[(2 + 4 - 8/3) - (1/2 - 2 + 1/3)] = 9/8$$

Ans.[B]

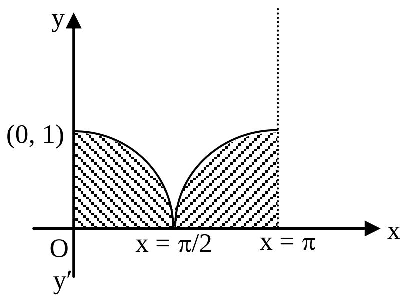
Ex.11 The area between the curve $y = \cos^2 x$, x-axis and ordinates $x = 0$ and $x = \pi$ in the interval $(0, \pi)$ is-

(A) π (B) $\pi/4$

(C) $\pi/2$ (D) 2π

Sol. Required area = $\int_0^{\pi} \cos^2 x \, dx$

$= \int_0^{\pi/2} \cos^2 x \, dx + \int_{\pi/2}^{\pi} \cos^2 x \, dx$



$= \frac{1}{2} \times \frac{\pi}{2} + \frac{1}{2} \int_{\pi/2}^{\pi} (1 + \cos 2x) \, dx$

$= \frac{\pi}{4} + \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right]_{\pi/2}^{\pi}$

$= \frac{\pi}{4} + \frac{1}{2} \left[\left(\pi - \frac{\pi}{2} \right) \right]$

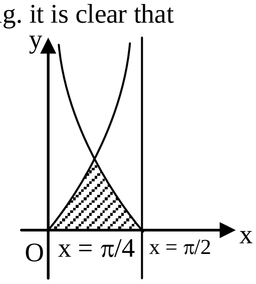
$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ **Ans.[C]**

Ex.12 The area between the curves $y = \tan x$, $y = \cot x$ and x-axis in the interval $[0, \pi/2]$ is-

(A) log 2 (B) log 3

(C) $\log \sqrt{2}$ (D) None of these

Sol. From the fig. it is clear that



$= \int_0^{\pi/4} \tan x \, dx - \int_{\pi/4}^{\pi/2} \cot x \, dx$

$= [\log \sec x]_0^{\pi/4} + [\log \sin x]_{\pi/4}^{\pi/2}$

$= \log \sqrt{2} - \log \frac{1}{\sqrt{2}}$

$= \log 2$ **Ans.[A]**

Ex.13 The area between the curves $y = \cos x$ and the line $y = x + 1$ in the second quadrant is-

(A) 1 (B) 2

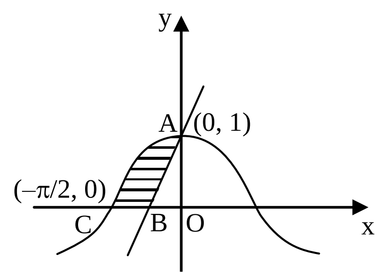
(C) 3/2 (D) 1/2

Sol. Let the line $y = x + 1$, meets x-axis at the point A (0, 1). Also suppose that the curve $y = \cos x$ meets x-axis and y-axis respectively at the points C and A. From the adjoint figure it is obvious that

Required area = area of ABC

= area of OAC – area of OAB

$= \int_{-\pi/2}^{0} \cos x \, dx - \frac{1}{2} \times OB \times OA$



$= [\sin x]_{-\pi/2}^{0} - \frac{1}{2} \times 1 \times 1$

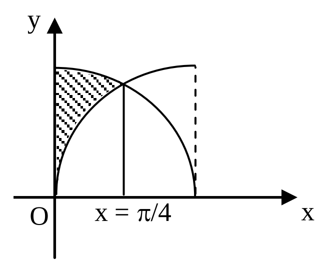
$= 1 - (1/2) = (1/2)$. **Ans. [D]**

Ex.14 The area bounded by the curves $y = \sin x$, $y = \cos x$ and y-axis in first quadrant is-

(A) $\sqrt{2} - 1$ (B) $\sqrt{2}$

(C) $\sqrt{2} + 1$ (D) None of these

Sol. In first quadrant sin x and cos x meet at $x = \pi/4$. The required area is as shown in the diagram. So



Required area = $\int_0^{\pi/4} (\cos x - \sin x) \, dx$

$= [\sin x + \cos x]_0^{\pi/4}$

$= (1/\sqrt{2} + 1/\sqrt{2}) - (0 + 1)$

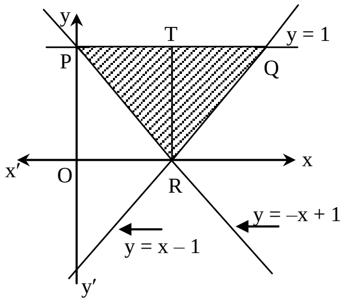
$= \sqrt{2} - 1$ **Ans.[A]**

Ex.15 The area bounded by curve y = |x – 1| and y = 1 is-

(A) 1 (B) 2

(C) 1/2 (D) None of these

Sol. $y = |x-1| = \begin{cases} x-1 & \text{when } x \ge 1 \\ 1-x & \text{when } x < 1 \end{cases}$



Point of intersection of y = x – 1, y = 1 is (2, 1)

Point of intersection of y = 1 – x, y = 1 is (0, 1)

Required area = Area of Δ PQR

$= \frac{1}{2}$ (PQ) . (RT)

$= \frac{1}{2}$. 2.1 = 1 **Ans.[A]**

Ex.16 If area bounded by the curve $y = 8x^2 - x^5$ and ordinate x = 1, x = k is $\frac{16}{3}$ then k =

(A) 2 (B) $[8-\sqrt{17}]^{1/3}$

(C) $[\sqrt{17}-8]^{1/3}$ (D) –1

Sol. $\int_1^k (8x^2 - x^5)\,dx = \frac{16}{3}$

$\Rightarrow \left[\frac{8x^3}{3} - \frac{x^6}{6}\right]_1^k = \frac{16}{3}$

$\Rightarrow \frac{8}{3}(k^3-1) - \left(\frac{k^6-1}{6}\right) = \frac{16}{3}$

$\Rightarrow 16k^3 - k^6 - 15 = 32$

$\Rightarrow k^6 - 16k^3 + 47 = 0$

$\Rightarrow k^3 = 8 \pm \sqrt{17}$

$\Rightarrow k = (8 \pm \sqrt{17})^{1/3}$ **Ans.[B]**

Ex.17 The area bounded by curve y = ex log x and $y = \frac{\log x}{ex}$ is-

(A) $\frac{e^2-5}{4}$ (B) $\frac{e^2+5}{4e}$

(C) $\frac{e}{4} - \frac{5}{4e}$ (D) None of these

Sol. Solving the equation of curves

$ex \log x = \frac{\log x}{ex}$

$\Rightarrow \log x \left(ex - \frac{1}{ex}\right) = 0$

$\Rightarrow x = 1, 1/e$

$\therefore$ required area $= \int_{1/e}^{1} \left(\frac{\log x}{ex} - ex\log x\right) dx$

$= \left[\frac{1}{e}\frac{(\log x)^2}{2} - e\left((\log x).\frac{x^2}{2} - \frac{x^2}{4}\right)\right]_{1/e}^{1}$

$= \frac{1}{2e}\,[0-(-1)^2] - e\left[0 - \frac{1}{4} - \left(-\frac{1}{2e^2} - \frac{1}{4e^2}\right)\right]$

$= -\frac{1}{2e} - \frac{1}{2e} + \frac{e}{4} - \frac{1}{4e} = \frac{e}{4} - \frac{5}{4e}$

Ans.[C]

Ex.18 If $0 \le x \le \pi$; then the area bounded by the curve y = x and y = x + sin x is-

(A) 2 (B) 4

(C) 2π (D) 4π

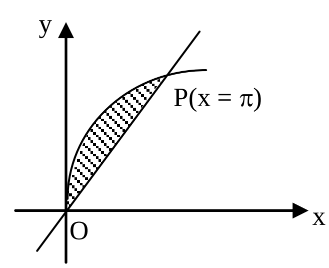
Sol. For the points of intersection of the given curves

x = x + sin x

$\Rightarrow \sin x = 0$

$\Rightarrow x = 0, \pi$

$\therefore$ required area



$= \int_0^{\pi} [(x + \sin x) - x]\,dx$

$= \int_0^{\pi} \sin x\,dx = -[\cos x]_0^{\pi} = 2$ **Ans.[A]**

Ex.19 The area bounded by curves $3x^2 + 5y = 32$ and y = |x – 2| is-

(A) 25 (B) 17/2

(C) 33/2 (D) 33

Sol. Here the first curve can be written in the following form

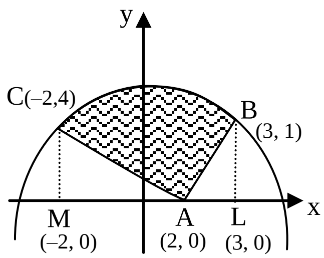
$$x^2 = -\frac{5}{3}\left(y - \frac{32}{5}\right)$$

which is a parabola whose vertex lies on the y-axis.

Again second curve is given by

$$y = \begin{cases} x - 2, x \geq 2 \\ -(x-2), x < 2 \end{cases}$$

which consists of two perpendicular lines AB and AC as shown in the fig.



These lines meet the parabola at B(3,1) and C(–2,4) .

Hence the reqd. area A is given by

$$A = \int_{-2}^{3} y\, dx - \Delta\, ABL - \Delta ACM$$

$$\int_{-2}^{3} \frac{1}{5}(32 - 3x^2)\, dx - \frac{1}{2} . 1.1 - \frac{1}{2}(4 . 4)$$

$$= \frac{1}{5}\left[32x - x^3\right]_{-2}^{3} - \frac{17}{2}$$

$$= \frac{1}{5}[69 + 56] - \frac{17}{2} = \frac{33}{2}$$ **Ans.[C]**