MATHEMATICS

Class-VIII

Topic-2

SQUARES AND SQUARE ROOTS

INDEX

CH-02

SQUARES AND SQUARE ROOTS

TERMINOLOGIES

Perfect square, Pythogorean Triplets, Square roots, Repeated Subtraction, Rationalization, Factors, Column Method, Diagonal Method.

INTRODUCTION

In previous class, we have learnt 4 × 4 can be written as 4^2 and read as square of 4, where 4 is called base, 2 is called exponent. The square of a number is obtained by multiplying it by self. So, a natural number is called a square number or perfect square, if it is the square of some natural number.

In this class we will study methods to find square of numbers and square roots by prime factorization method and long division method.

2.1 SQUARES

We are familiar with exponential notation.

For example : $2 \times 2 = 2^2 = 4$

$3 \times 3 = 3^2 = 9$

$4 \times 4 = 4^2 = 16$

$6 \times 6 = 6^2 = 36$

$9 \times 9 = 9^2 = 81$

Hence, when a number is multiplied with itself, the product is called the **square** of that number.

(a) Perfect Squares

A natural number is called a perfect square if it is the square of some natural number.

For example : 4 = 2 × 2 or square of 2

9 = 3 × 3 or square of 3

36 = 6 × 6 or square of 6

we see that 4, 9 and 36 are squares of 2, 3 and 6 respectively, so they are perfect squares. However, in case of 30, we are not able to write this as a square of some natural number so, 30 is not a perfect square.

(b) To Test a Given Number is a Perfect Square or Not

For testing a given number is a perfect square or not we write the given number as the product of prime factors then we make pairs of same factors. If there are factors all of which have pair, then given number is a perfect square otherwise not.

Illustration 2.1

Is 336 a perfect square ?

Sol. Given number is 336

First we factorise it

2	336
2	168
2	84
2	42
3	21
7	7
	1

$336 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times 7$

Clearly 3 and 7 have no pair therefore it is not a perfect square.

Illustration 2.2

Find the smallest number by which 180 must be multiplied so that the product is a perfect square.

Sol. Given number is 180, first we resolve it into prime factors.

2	180
2	90
3	45
3	15
5	5
	1

$\therefore 180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

Clearly 5 has no pair. Thus if we multiply it by 5 then product will be a perfect square.

$\therefore$ Required smallest number is 5.

Illustration 2.3

Find the smallest number by which 28812 must be divided so that the quotient becomes a perfect square.

Sol. Given number is 28812, first we write it as the product of prime factors.

2	28812
2	14406
3	7203
7	2401
7	343
7	49
7	7
	1

$\therefore 28812 = \underline{2 \times 2} \times 3 \times \underline{7 \times 7} \times \underline{7 \times 7}$

Clearly, 3 has no pair, so if we divide it by 3 then quotient become a perfect square.

Illustration 2.4

Is 1575 a perfect square.

Sol. Given number is 1575, resolve it into prime factors.

3	1575
3	525
5	175
5	35
7	7
	1

∴ 1575 = $\underline{3 \times 3} \times \underline{5 \times 5} \times 7$

7 has no pair, therefore 1575 is not a perfect square.

(c) Table of Squares of Natural Numbers

The following table contains the squares of first thirty natural numbers. Students are advised to keep these to memory.

Number	Square
1	$1^2 = 1 \times 1 = 1$
2	$2^2 = 2 \times 2 = 4$
3	$3^2 = 3 \times 3 = 9$
4	$4^2 = 4 \times 4 = 16$
5	$5^2 = 5 \times 5 = 25$
6	$6^2 = 6 \times 6 = 36$
7	$7^2 = 7 \times 7 = 49$
8	$8^2 = 8 \times 8 = 64$
9	$9^2 = 9 \times 9 = 81$
10	$10^2 = 10 \times 10 = 100$
11	$11^2 = 11 \times 11 = 121$
12	$12^2 = 12 \times 12 = 144$
13	$13^2 = 13 \times 13 = 169$
14	$14^2 = 14 \times 14 = 196$
15	$15^2 = 15 \times 15 = 225$
16	$16^2 = 16 \times 16 = 256$
17	$17^2 = 17 \times 17 = 289$
18	$18^2 = 18 \times 18 = 324$
19	$19^2 = 19 \times 19 = 361$
20	$20^2 = 20 \times 20 = 400$
21	$21^2 = 21 \times 21 = 441$
22	$22^2 = 22 \times 22 = 484$
23	$23^2 = 23 \times 23 = 529$
24	$24^2 = 24 \times 24 = 576$
25	$25^2 = 25 \times 25 = 625$
26	$26^2 = 26 \times 26 = 676$
27	$27^2 = 27 \times 27 = 729$
28	$28^2 = 28 \times 28 = 784$
29	$29^2 = 29 \times 29 = 841$
30	$30^2 = 30 \times 30 = 900$

It is evident from the above table that the numbers 1, 4, 9, 16, ...841, 900 are squares of the numbers 1, 2, 3, ...30. Such numbers are called **perfect squares** or square numbers.

(d) Properties of Perfect Squares

(i) A number ending with 2, 3, 7 or 8 can never be a perfect square.

(ii) The number of zeros at the end of a perfect square is always even.
For example : $10000 = 100^2$, $2500 = 50^2$, $490000 = 700^2$

(iii) Squares of even numbers is always even and square of odd numbers is always odd.
For example : $2^2 = 4$, $8^2 = 64$, $40^2 = 1600$
$5^2 = 25$, $9^2 = 81$, $17^2 = 289$

(iv) For any two consecutive natural numbers n and (n + 1), we have
$(n + 1)^2 - n^2 = (n + 1 + n)(n + 1 - n) = (n + 1) + n$
For example : $11^2 - 10^2 = 11 + 10 = 21$
$15^2 - 14^2 = 15 + 14 = 29$
$19^2 - 18^2 = 19 + 18 = 37$ etc.

(v) A triplet (x, y, z) of three natural numbers x, y, and z is called a **Pythagorean triplet**, if $x^2 + y^2 = z^2$.
For example : (6, 8, 10) is a Pythagorean triplet. Since $6^2 + 8^2 = 36 + 64 = 100$ and $10^2 = 100$.
For any number n greater than 1, the Pythagorean triplet is given by $(2n, n^2 - 1, n^2 + 1)$

(vi) The square of a natural number m is equal to the sum of the first m odd numbers.
Thus,
$1^2 = 1$ = sum of the first 1 odd number.
$2^2 = 4 = 1 + 3$ = sum of the first 2 odd numbers.
$3^2 = 9 = 1 + 3 + 5$ = sum of the first 3 odd numbers. $5^2 = 25 = 1 + 3 + 5 + 7 + 9$ = sum of the first 5 odd numbers and so on.

(vii) Study the following patterns :

$2^2 = 4 = 3 \times 1 + 1$	or	$2^2 = 4 = 4 \times 1$
$3^2 = 9 = 3 \times 3$		$3^2 = 9 = 4 \times 2 + 1$
$4^2 = 16 = 3 \times 5 + 1$		$4^2 = 16 = 4 \times 4$
$5^2 = 25 = 3 \times 8 + 1$		$5^2 = 25 = 4 \times 6 + 1$
$6^2 = 36 = 3 \times 12$		$6^2 = 36 = 4 \times 9$

From the above we can conclude that :

(A) Squares of numbers (greater than 1) can be written as multiples of 3 or multiples of 3 plus 1.

(B) Squares of numbers (greater than 1) can also be written as multiples of 4 or multiples of 4 plus 1.

(viii) Square of any real number is always positive.

(e) Patterns

(i) The squares of numbers like 1, 11 , 111 etc. which are composed of digit 1 alone have a nice pattern as shown below
$1^2 = 1$
$11^2 = 1\ 2\ 1$
$111^2 = 1\ 2\ 3\ 2\ 1$
$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$
$111111111^2 = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1$

(ii) We have another interesting pattern related to squares of numbers.

$1^2 = 1$

$11^2 = 1\ 2\ 1$ and $1 + 2 + 1 = 2^2$

$111^2 = 1\ 2\ 3\ 2\ 1$ and $1 + 2 + 3 + 2 + 1 = 3^2$

(iii) One more pattern given below :

$1\ 2\ 1 \times (1 + 2 + 1) = 4\ 8\ 4 = 22^2$

$1\ 2\ 3\ 2\ 1 \times (1 + 2 + 3 + 2 + 1) = 110889 = 333^2$

i.e., $11^2 \times$ (sum of digits in 11^2) $= 22^2$

$111^2 \times$ (sum of digits in 111^2) $= 333^2$

111111111^2 x (sum of digit in 111111111^2) $= 999999999^2$

Illustration 2.5

Using suitable patterns, complete the following :

(i) $\frac{(333)^2}{12321} = ...$ **(ii)** $\frac{(666666)^2}{12345654321} = ...$

Sol. We have the following pattern :

$(11)^2 = 121$

$(111)^2 = 12321$

$(1111)^2 = 1234321$

$(11111)^2 = 123454321$

$(111111)^2 = 12345654321$

(i) $\frac{(333)^2}{12321} = \frac{(3\times 111)^2}{(111)^2}$ [$\because (111)^2 = 12321$]

$= \frac{3^2 \times (111)^2}{(111)^2} = 3^2 = 9.$

(ii) $\frac{(666666)^2}{12345654321} = \frac{(6\times 111111)^2}{(111111)^2}$ [$(111111)^2 = 12345654321$]

$= \frac{6^2 \times (111111)^2}{(111111)^2} = 6^2 = 36.$

(f) Methods For Finding The Squares Of a Numbers

(a) Column Method

This method uses the identity $a^2 + 2ab + b^2$ for finding the square of a two digit number (where a is the ten's digit and b is the unit's digit). We follow the following steps :

STEP (i) : Make three columns and write the values of a^2, $2 \times a \times b$ and b^2 respectively in these column as follows : Let us take ab = 57

a = 5 and b = 7

Column-I	Column-II	Column-III
a^2	(2 x a x b)	b^2
25	70	49

STEP (ii) : Underline the unit digit of b^2 and add its ten's digit if any, to 2×a×b (in column II)

Column-I	Column-II	Column-III
a^2	(2 x a x b)	b^2
25	70	4<u>9</u>
	+4	
	74	

STEP (iii) Underline the unit digit in column II and add the number formed by tens and other digit, if any, to a^2 in Column I.

Column-I	Column-II	Column-III
a^2	(2 x a x b)	b^2
25	70	49
+7	+4	
32	74	

Step (iv) : Underline the number in column -I

Column-I	Column-II	Column-III
a^2	(2 x a x b)	b^2
25	70	4<u>9</u>
+7	+4	
<u>32</u>	7<u>4</u>	
32	4	9

STEP (v) : Write the underlined digit at the bottom of each column to obtain the square of the given number. In this case, we have $57^2 = 3249$

Illustration 2.6

Find the square of (i) 47 (ii) 86

Sol. **(i)** Given number = 47

a = 4 and b = 7

I	II	III
a^2	(2 x a x b)	b^2
16	56	4<u>9</u>
+6	+4	
<u>22</u>	6<u>0</u>	

$\therefore (47)^2 = 2209$

(ii) Given number = 86.

a = 8 and b = 6

a^2	(2 x a x b)	b^2
64	96	3<u>6</u>
+9	+3	
<u>73</u>	9<u>9</u>	

$(86)^2 = 7396$

(b) Diagonal Method

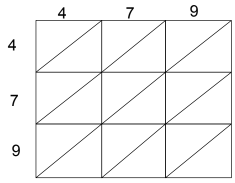
This method is applicable to find the square of any number irrespective of the number of digits in the number. We follow the following steps to find the square of a number by this method.

STEP (i) : Obtain the number and count the number of digits in it. Let there be *n* digits in the number to be squared.

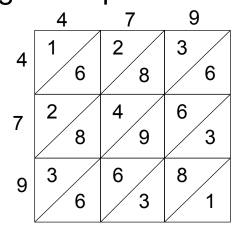
STEP (ii) : Draw square and divide it into n^2 sub-squares of the same size by drawing (n – 1) horizontal and (n – 1) vertical lines.

STEP (iii) : Draw the diagonal of each sub-square. As an illustration, let the number to be squared be 479.

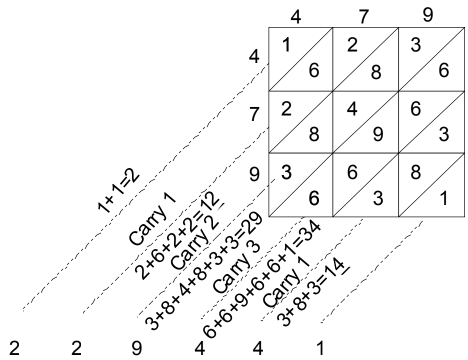
STEP (iv) : Write the digits of the number to be squared along the left vertical side and top horizontal side of the square as shown below.



STEP (v) : Multiply each digit on the left of the square with each digit on top of the column one - by - one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding sub square as shown below.



STEP (vi) : Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the unit's digit of the sum and carry the ten's digit (if any) to the diagonal above as shown below.



STEP (vii) : Obtain the required square by writing the digits from the left-most side.

∴ $479^2 = 229441$.

(c) Some Particular Methods

(i) The square of a number of the form a5 (where a is ten's digit and 5 is unit's digit) is the number which ends in 25 and has the number a (a + 1) before 25.

For example :

85^2 = 8 × (8 + 1) 25 (∴ a = 8)
= 7225

105 = 10 × (10 + 1) 25 (∴ a = 10)
= 11025

(ii) The square of a number of the form 5a where a is unit's digit and 5 is ten's digit is equal to $(25 + a) \times 100 + a^2$.

For example :

$58^2 = (25 + 8) \times 100 + 8^2 = 3300 + 64 = 3364$

(iii) The square of a three digit number 5ab where b is unit's digit, a is ten's digit and 5 is hundred's digit, given by $(250 + ab) \times 1000 + (ab)^2$

For example : $(514)^2 = (250 + 14) \times 1000 + (14)^2$
$= 264000 + 196$
$= 264196$

Ask yourself

1. Find the value of the following

(i) $\left(1\frac{2}{5}\right)^2$ **(ii)** $\left(2\frac{4}{5}\right)^2$ **(iii)** $(3.05)^2$

2. Find the perfect square obtained by multiplying each of the following numbers by the smallest possible number.

(i) 882 **(ii)** 1690

3. Divide the following numbers by the smallest possible number to make each one a perfect square.

(i) 2187 **(ii)** 3267

4. Write a pythogorean triplet whose smallest member is 3

5. Without Actual squaring, find the value of

(i) $42^2 - 41^2$ **(ii)** $108^2 - 107^2$

6. How many numbers lie between

(i) 7^2 and 8^2 **(ii)** 15^2 and 16^2

Answers

1. **(i)** $\frac{49}{25}$ **(ii)** $\frac{196}{25}$ **(iii)** 9.3025 **2.** **(i)** 2 **(ii)** 10

3. **(i)** 3 **(ii)** 3 **4.** 3 , 4 , 5

5. **(i)** 83 **(ii)** 215 **6.** **(i)** 14 **(ii)** 30

2.2 SQUARE ROOTS

As we say square of 2 is 4, then we can also say that square root of 4 is 2

If, $6 \times 6 = 36 = 6^2$ and $8 \times 8 = 64 = 8^2$

Then, square root of 36 is 6 and square root of 64 is 8.

The square root of a number x is that number which when multiplied by itself gives x as the product.

Remarks :

(i) We use the sign $\sqrt{\ }$ to indicate square root of a number.

i.e. $\sqrt{81} = 9$, $\sqrt{225} = 15$, etc.

(ii) We can calculate the square root of positive numbers only.

(a) Properties of Square Roots

(i) If the unit digit of a number is 2, 3, 7 or 8, then it does not have a square root in N. Where N is the set of Natural number.

(ii) If a number ends in an odd number of zeros, then it does not have a square root in N.

(iii) The square root of an even number is even and square root of an odd number is odd.

For example : $\sqrt{64} = 8$, $\sqrt{256} = 16$, $\sqrt{324} = 18$ etc.

$\sqrt{81} = 9$, $\sqrt{169} = 13$, $\sqrt{289} = 17$ etc.

(iv) Negative numbers have no square root in set of real numbers.

(b) Methods for finding square roots

(i) Prime Factorisation Method :

To find the square root of a perfect square, first we find the prime factors of the given perfect square. Since the number is a perfect square, therefore, we get pairs of similar prime factors. Choose one factor from each pair and multiply together. The result will be the square root of the given number.

Ilustration 2.7

Find the square root of 784.

Sol. Given number is 784, first we factorize it.

2	784
2	392
2	196
2	98
7	49
7	7
	1

$784 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

$\sqrt{784} = 2 \times 2 \times 7 = 28$

(ii) Division Method :

Square root of a perfect square by the long division method. When numbers are very large, the method of finding their square roots by factorization becomes lengthy and difficult. So , we use long division method which is explained in the following steps :

(a) Group the digits in pairs, starting with the digit in the unit place. Each pair and the remaining digit (if any) is called a period.

(b) Think of the largest number whose square is equal to or just less than the first period.

(c) Subtract the product of the divisor and the quotient from the first period and bring down the next period to the right of the remainder. This becomes the next dividend.

(d) Now, the new divisor is obtained by taking two times the quotient and annexing with it a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of the new divisor and the digit is equal to or just less than the new dividend.

Repeat steps (b), (c) and (d) till all the period have been taken up. Now, the quotient so obtained is the required square root of the given number.

Illustration 2.8

Find the square root of 390625.

Sol. By the long-division method, we have

	625
6	$\overline{39}\ \overline{06}\ \overline{25}$
	-36
122	306
	-244
	6225
1245	-6225
	X

$\therefore \quad \sqrt{390625} = 625$

(i) Mark periods by placing bars on every pair, starting with the digit in the unit's place.

(ii) $6 \times 6 = 36 < 39$. So, take 6 as the divisor and 6 as the quotient.

(iii) Subtracting 36 from 39, we get 3 as the remainder. Bring down the next period 06 to the right of 3, to obtain 306 as the new dividend.

(iv) For the next divisor, take 2 times of 6. i.e. 12 as the left two digits of the new divisor.

(v) Now, 2 is the largest digit such that $122 \times 2 = 244$ is just less than 306.
Thus, take 122 as the new divisor and 2 as the second digit of the quotient.

(vi) Now, subtract 244 from 306 to get 62 as the remainder. Bring down the next period 25 to the right of 62 to get 6225 as the new dividend.

(vii) For the next divisor, take 2 times of 62, i.e., 124 as the left three digits of the new divisor.

(viii) Now, 5 is the largest digit such that $1245 \times 5 = 6225$.

Thus, take 1245 as the new divisor and 5 as the new digit of the quotient.
(ix) Thus, = 625.

Illustration 2.9

Find the cost of erecting a fence around a square field whose area is 9 hectares. If fencing costs Rs. 3.50 per metre.

Sol. The area of the square field = $(9 \times 10000)\ m^2$ [$\therefore$ 1 hectare = $10000\ m^2$]
= $90000\ m^2$

Side of the field = $\sqrt{90000}$ = 300 m. Now, the perimeter = (300×4) m = 1200 m
Cost of fencing = Rs (1200×3.50) = Rs 4200.

Illustration 2.10

What least number must be subtracted from 16160 to get a perfect square ? Also find the square root of this perfect square.

Sol. Let us try to find the square root of 16160.

```
          127
     |-----------
   1 | 1  61 60
     | -1
     |------
  22 |    61
     |   -44
     |-------
 247 |   1760
     |  -1729
     |--------
     |     31
```

This shows that $(127)^2$ is less than 16160 by 31. So in order to get a perfect square, 31 must be subtracted from the given number.

Required perfect square number = (16160 – 31) = 16129

Also, $\sqrt{16129}$ = 127.

Illustration 2.11

What least number must be added to 5607 to make the sum a perfect square ? Find this perfect square and its square root.

Sol. We try to find out the square root of 5607.

```
          74
     |----------
   7 | 56 07
     | -49
     |--------
 144 |   707
     |  -576
     |--------
     |   131
```

We observe here that $(74)^2 < 5607 < (75)^2$.

The required number to be added

$= (75)^2 – 5607 = (5625 – 5607) = 18$

The required perfect square = (5607 + 18) = 5625

Clearly, $\sqrt{5625}$ = 75.

Illustration 2.12

Find the greatest number of six digits, which is a perfect square. Find the square root of this number.

Sol. The greatest number of six digits = 999999

Now, we must find the least number which when subtracted from 999999, give a perfect square. Now, we find out the square root of 999999.

```
           999
      |-----------
    9 | 99 99 99
      | -81
      |-----------
  189 |  1899
      | -1701
      |-----------
 1989 |   19899
      |  -17901
      |-----------
      |    1998
```

Thus, $(999)^2$ < 999999 by 1998.

So, the least number to be subtracted is 1998.

The required number = (999999 – 1998) = 998001.

Also, it is clear that = 999.

(iii) Square Root by Successive Subtractions :

We know that 1 + 3 + 5 + 7 + ... + (2n – 1) = n^2

i.e. The sum of first n odd natural numbers is n^2. This property of natural numbers can be used to find the square roots of small natural number. For this we subtract 1, 3, 5, 7, 9, ...successively from the given number till we get zero, then number of times the subtraction is performed to arrive at zero is the square root of given number.

Illustration 2.13

Find the square root of 64 by successive subtraction method.

Sol. Here
64 – 1 = 63
63 – 3 = 60
60 – 5 = 55
55 – 7 = 48
48 – 9 = 39
39 – 11 = 28
28 – 13 = 15
15 – 15 = 0

Clearly, we have performed subtraction eight times $\sqrt{64} = 8$

(iv) Square Roots of Fractions :

For any positive numbers a and b, we have

(a) $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **(b)** $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Illustration 2.14

Find the square root of $\dfrac{529}{841}$.

Sol. We have $\sqrt{\dfrac{529}{841}} = \dfrac{\sqrt{529}}{\sqrt{841}}$

Now, we find the square roots of 529 and 841 separately, as shown below :

```
      23                  29
 2 | 5 29            2 | 8 41
   |-4                 |-4
43 |  129           49 |  441
   |- 129              |- 441
   |   X               |   X
```

Thus, $\sqrt{529} = 23$ and $\sqrt{841} = 29$.

$\therefore \sqrt{\dfrac{529}{841}} = \dfrac{\sqrt{529}}{\sqrt{841}} = \dfrac{23}{29}$.

Illustration 2.15

Find the square root of $42\dfrac{583}{1369}$.

Sol. $\sqrt{42\dfrac{583}{1369}} = \sqrt{\dfrac{58081}{1369}} = \dfrac{\sqrt{58081}}{\sqrt{1369}}$

$$\begin{array}{r|l} & \;\; 241 \\ \hline 2 & \overline{5}\;\overline{80}\;\overline{81} \\ & -4 \\ \hline 44 & \;\;180 \\ & -176 \\ \hline 481 & \;\;\;\;481 \\ & \;\;-481 \\ \hline & \;\;\;\;\;\text{X} \end{array} \qquad \begin{array}{r|l} & \;\;37 \\ \hline 3 & \overline{13}\;\overline{69} \\ & -9 \\ \hline 67 & \;\;469 \\ & -469 \\ \hline & \;\;\;\text{X} \end{array}$$

$\therefore \sqrt{58081} = 241$, and $\sqrt{1369} = 37$

So, $\sqrt{42\dfrac{583}{1369}} = \dfrac{\sqrt{58081}}{\sqrt{1369}} = \dfrac{241}{37} = 6\dfrac{19}{37}$.

Illustration 2.16

Find the value of $\sqrt{\dfrac{3}{7}}$ up to four decimal places.

Sol. $\sqrt{\dfrac{3}{7}} = \sqrt{\dfrac{3 \;\; \times 7}{7 \times 7}} = \dfrac{\sqrt{21}}{\sqrt{7 \times 7}} = \dfrac{\sqrt{21}}{7}$.

Now, we evaluate $\sqrt{21}$ up to four places of decimal as given below.

$$\begin{array}{r|l} & \;\;4.5825 \\ \hline 4 & \overline{21}.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00} \\ & -16 \\ \hline 85 & \;\;\;500 \\ & -\;425 \\ \hline 908 & \;\;\;\;\;7500 \\ & \;\;-7264 \\ \hline 9162 & \;\;\;\;\;\;23600 \\ & \;\;\;-18324 \\ \hline 91645 & \;\;\;\;\;\;\;527600 \\ & \;\;\;\;-458225 \\ \hline & \;\;\;\;\;\;\;\;69375 \end{array}$$

Thus, $\sqrt{\dfrac{3}{7}} = \dfrac{\sqrt{21}}{7} = \dfrac{4.5825}{7} = 0.6546$

Illustration 2.17

Find the value of $\sqrt{15625}$ and from this value, evaluate $\sqrt{156.25} + \sqrt{1.5625}$.

Sol. We find the value of $\sqrt{15625}$ as shown below :

$$\begin{array}{r|l} & \;125 \\ \hline 1 & \overline{1}\;\overline{56}\;\overline{25} \\ & -1 \\ \hline 22 & \;\;\;56 \\ & -44 \\ \hline 245 & \;\;\;1225 \\ & -1225 \\ \hline & \;\;\;\;\text{X} \end{array}$$

$\therefore \sqrt{15625} = 125$.

So, $\sqrt{156.25} + \sqrt{1.5625} = \sqrt{\dfrac{15625}{100}} + \sqrt{\dfrac{15625}{10000}} = \dfrac{\sqrt{15625}}{\sqrt{100}} + \dfrac{\sqrt{15625}}{\sqrt{10000}}$

$= \dfrac{125}{10} + \dfrac{125}{100} = 12.5 + 1.25 = 13.75$

Illustration 2.18

Find the value of $\sqrt{99} \times \sqrt{396}$.

Sol. $\sqrt{99} \times \sqrt{396} = \sqrt{99 \times 396} = \sqrt{3 \times 3 \times 11 \times 3 \times 3 \times 2 \times 2 \times 11} = 3 \times 3 \times 2 \times 11 = 198.$

(v) Square Root of Numbers in Decimal Form :

Make the number of decimal places even by affixing a zero, if necessary. Now, mark period and find out the square root by the long-division method. Put the decimal point in the square root as soon as the integral part is exhausted.

Illusration 2.19

Find the square root of 176.252176

Sol. Here, the number of decimal places is already even. So, mark the periods and proceed as follows :

```
              13.276
        ----------------
     1  | 1 76.25 21 76
        |-1
        |------
    23  |  76
        | -69
        |-------
   262  |    725
        |   -524
        |---------
  2647  |     20121
        |    -18529
        |-----------
 26546  |      159276
        |     -159276
        |     --------
        |          X
```

$\therefore \sqrt{176.252176} = 13.276.$

Illustration 2.20

Find the square root of 0.00059049.

Sol. Here, the number of decimal places is even. So, we mark the periods and find the square roots as shown below.

```
          .0243
      -------------
   2  | 0.00 05 90 49
      |      -4
      |     ----
  44  |      190
      |     -176
      |     -----
 483  |      1449
      |     -1449
      |     -----
      |        X
```

$\therefore \sqrt{0.00059049} = 0.0243.$

Illustration 2.21

Find the value of $\sqrt{2}$ correct up to three places of decimal.

Sol. Since we have to find the value of $\sqrt{2}$ correct up to three decimal places. We shall find its value up to four decimal places. We may write 2 = 2.00000000.

Now, mark the period and proceed as follows.

```
           1.4142
1     | 2. 00 00 00 00
      | -1
24    |  100
      | -96
281   |   400
      | -281
2824  |   11900
      |  -11296
28282 |    60400
      |   -56564
      |     3836
```

$\sqrt{2}$ = 1.4142 up to four places of decimal.

= 1.414 correct up to three places of decimal. Hence, $\sqrt{2}$ = 1.414

Ask yourself

1. Find the square roots of the following natural numbers by the prime factorisation method.
   (i) 2401 **(ii)** 7225
2. Find the square roots of the following fractions by the prime factorisation method
   (i) $\frac{49}{64}$ **(ii)** $\frac{484}{625}$
3. Find the square roots of the following decimal fractions by the division method.
   (i) 44.89 **(ii)** 4.5796
4. Find the square roots of the following numbers, correct up to 3 decimal places
   (i) 13 **(ii)** 1458
5. Find the smallest and greatest 5-digit numbers that are perfect squares.
6. The area of square is 729 sqm. find the dimension of the square?
7. Find the smallest square number that is divisible by each of the numbers 2,3,6,10 ?

Answres.

1.	**(i)** 49	**(ii)** 85	**2. (i)** $\frac{7}{8}$	**(ii)** $\frac{22}{25}$	**3. (i)** 6.7	**(ii)** 2.14
4.	**(i)** 3.606	**(ii)** 38.184	**5.** 10000, 99856		**6.** 27	**7.** 900

Add to Your Knowledge

(1) **Rationalisation of denominator :-** When the denominator of an expression contains a term with a square root the process of converting to an equivalent expression whose denominator is a rational number

For example :-

Rationalize the denominator of $\frac{1}{7+5\sqrt{3}}$

$$= \frac{1}{7+5\sqrt{3}} \times \frac{7-5\sqrt{3}}{7-5\sqrt{3}} = \frac{7-5\sqrt{3}}{(7)^2-(5\sqrt{3})^2} = \frac{7-5\sqrt{3}}{-26}.$$

Concept Map

Square and Square Roots

When a number is multiplied with it self
Square of 7 is 7× 7 = 49.

Perfect Square

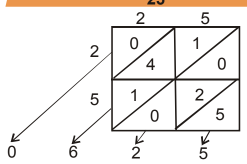
A natural no. is perfect sq.
if it is sq. of some natural no.
36 is perfect sq because
it is the me sq. of 6

Properties of perfect square

No. ending with 2,3,7,8 never be a perfect square
No. ending with odd no of zero never be a perfect sq.
diff of square of two consecutive natural no. is equal to their sum. $(n+1)^2 - n^2 = (n+1) + (n)$
Pythogorean triplet (x,y,z) if $z^2 = x^2 + y^2$

Square by diagonal method.

25^2



$25^2 = 625$

Square by column method

to find 25^2, take a = 2, b = 5

a^2	2ab	b^2
2^2	2 5	5^2
4 +2 6	20 +2 22	25

$25^2 = 625$

Squre Root

Squre root of no x, in that no whose square is x
Ex. Sq. root of 64 is 8 because $8^2 = 64$. $\sqrt{64} = 8$

Prime factorization

Q. Find sq. root of 36
Sol. $\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3}$
$= 2 \times 3 = 6$

2	36
2	18
3	9
3	3
	1

Long Division method.

Q. find sq. root 58081

	241
2	58081
2	4
44	180
+4	176
481	481
	481
	×

$\sqrt{58081} = 241$

Successive subtraction

We successive subtract odd no from the given no. till we get zero. The number of time we subtract is the square root of the no.

Ex. $\sqrt{16}$
16 – 1 = 15
15 – 3 = 2
12 – 5 = 7
7 – 7 = 0 so $\sqrt{16} = 4$

Squre root of fraction

$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Ex. $\sqrt{\frac{529}{841}} = \frac{\sqrt{529}}{\sqrt{841}} = \frac{23}{29}$

Ex. $\sqrt{36 \times 49} = \sqrt{36} \times \sqrt{49} = 6 \times 7 = 42$

Summary

1. A perfect square number is never negative.

2. A square number never ends in 2, 3, 7 or 8.

3. A perfect square number leaves a remainder 0 or 1 on division by 3.

4. A number m is a square root of n, if n = m × m = m^2. Positive square root of n is written as $\sqrt{n}$.

5. If p and q are perfect squares ($q \neq 0$), then

(i) $\sqrt{p \times q} = \sqrt{p} \times \sqrt{q}$ **(ii)** $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$.

6. The pairing of numbers in the division method starts from the decimal point. For the integral part, it goes from right to left and for the decmial part, it goes from left to right.

7. If a positive number is not a perfect square, then an approximate value of its square root may be obtained by the division method.

8. If p and q are not perfect squares, then to find $\sqrt{\frac{p}{q}}$, we may express $\frac{p}{q}$ as a decimal number and then use the division method.

9. If n is not a perfect square, then $\sqrt{n}$ is not a rational number.

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

1. Which one is perfect square :
(A) 356 (B) 225 (C) 146 (D) 1064

2. The square of 64 is :
(A) 4026 (B) 4096 (C) 4076 (D) 4086

3. The value of $(501)^2 - (500)^2$ is :
(A) 1 (B) 101 (C) 1001 (D) None of these

4. The value of 1 + 3 + 5 + 7 + 9 + ...+ 25 is :
(A) 196 (B) 625 (C) 225 (D) 169

5. If a square number ends in 6, the preceding number is :
(A) An even number (B) An odd number (C) A prime number (D) A composite number

6. The least number which must be subtracted from 2509 to make it a perfect square is :
(A) 6 (B) 9 (C) 12 (D) 14

7. The least perfect square which is divisible by 2, 4, and 6 is :
(A) 36 (B) 64 (C) 16 (D) 144

8. The value of $\sqrt{0.25}$ is :
(A) 0.5 (B) 0.6 (C) 0.9 (D) 0.4

9. Evaluate : $\sqrt{41-\sqrt{21+\sqrt{19-\sqrt{9}}}}$.
(A) 3 (B) 6 (C) 5 (D) 6.4

10. If $\sqrt{\frac{16}{49}} = \frac{n}{49}$, then n =
(A) 4 (B) 7 (C) 16 (D) 28

11. If $\sqrt{1+\frac{27}{169}} = 1 + \frac{x}{13}$, then x =
(A) 1 (B) 14
(C) Cannot be determined (D) None

12. A four digit square number whose first two digits and last two digits taken respectively are also perfect square numbers is :
(A) 1681 (B) 1636 (C) 3664 (D) 6481

13. Simplify : $\left(\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{144}}\right) \div \sqrt{\frac{16}{81}}$.
(A) $\frac{1}{48}$ (B) $\frac{5}{48}$ (C) $\frac{5}{16}$ (D) None of these

14. If m is a perfect square number, then the next immediate square number is :

(A) m + 5 (B) $m + 2\sqrt{m} + 1$ (C) $m^2 + 2m$ (D) None of these

15. There are 540 students in a school. For a P.T drill, they have to stand in such a manner that the number of rows is equal to number of Columns. How many minimum number of students would be left out in this arrangements.

(A) 22 Students (B) 11 Students (C) 40 Students (D) 29 Students

16. The suare root of a perfect square containing n digits has .... digits.

(A) $\frac{n+1}{2}$ (B) n/2 (C) A or B (D) None

FILL IN THE BLANKS

1. _____ is the least number which should be subtracted from 1029 to make it a perfect square.

2. 6 , 8 and ______ are pythagorean triplet .

3. ______ is the least perfect square number .

4. ______is the sum of first 10 perfect squares .

5. The number of zeroes at end of a perfect square is always __________.

6. Square root of 14641 is ______.

7. Squar root of 390625 is ______.

8. If $\frac{\sqrt{25}}{\sqrt{81}}$ is $\frac{x}{10}$, then x is ______.

9. The square root of an even number is _______ and square root of an odd number is _____

TRUE / FALSE

1. 3,4,5 are pythagorean triplet

2. 9 is a perfect square .

3. 80 is a perfect square .

5. A perfect square is always a even number .

6. square root of 21 is 4.5825 .

7. A perfect square number can also be a prime number.

8. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

9. Square root of 1.5625 is 1.25.

10. Square root of negative number is real number.

11. Square root of any positive number is always integer.

MATCH THE COLUMN

1. Match the squares in column –I with their square roots in column–II.

Column – I		Column – II	
(A)	841	(p)	19
(B)	361	(q)	17
(C)	64	(r)	18
(D)	289	(s)	29
(E)	324	(t)	8

2. Match the least value , that should be subtracted from the values of column–I , to make them a perfect square, with the values in column–II.

Column – I		Column – II	
(A)	229	(p)	5
(B)	103	(q)	6
(C)	448	(r)	4
(D)	329	(s)	3
(E)	87	(t)	7
(F)	129	(u)	8

3.

Column – I		Column – II	
(A)	Units digit of 36^2	(p)	1
(B)	Number of first consecutive odd natural numbers whose sum is 64	(q)	3
(C)	Unit digit of the square 89	(r)	6
(D)	Non-perfect square numbers between 5^2 and 6^2	(s)	8
(E)	Number of digits in the square root of a 5-digit square number	(t)	10

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. Is 53361 a perfect square ?

2. Find the square of 121 by diagonal method.

3. Find the square of 37 using column method.

4. Without actual adding find the sum of

(a) 1 + 3 + 5 + 7 + 9 + 11 + 13 **(b)** 15 + 17 + 19 + 21 + 23 + 25

5. Find the smallest number by which 252 must be multiplied to get a perfect square. Also find the square root of the perfect square so obtained.

6. Find the smallest number by which 108 must be divided to get a perfect square. Also find the square root of the perfect square so obtained.

SHORT ANSWER TYPE

7. Find Pythagorean triplet one of whose numbers is :

(a) 10 **(b)** 14

8. Find x. If $\frac{\sqrt{x}}{\sqrt{5}} = \sqrt{2}+1$.

9. If $x + \frac{1}{x} = 2$, then find $x^2 + \frac{1}{x^2}$.

10. Find the square root of 0.00088804.

11. A general, wishing to arrange his 120419 men in the form of a square, found that he had 10 extra men, find the number of men in the front row.

12. The products of two numbers is 1296. If one number is 16 times the other, find the numbers.

13. There are 1521 students in a school. P.T teacher wants them to stand in rows and columns such that the number of rows is equal to the number of columns. Find the number of rows.

14. Find the least number that must be subtracted from 5607 so as to get a perfect square. Also find the square root of the perfect square.

LONG ANSWER TYPE

15. Write each of the following as the difference of the squares of consecutive natural numbers:

(a) 23 **(b)** 15

16. Find the square root of $\frac{2}{3}$ up to 3 decimal places.

17. If $\sqrt{0.04 \times 0.4 \times a} = 0.004 \times 4 \times \sqrt{b}$, then find the value of $\frac{a}{b}$.

18. Find the least four digit number which is a perfect square.

19. The area of a square field is 60025 m^2. A man cycles along its boundary at 18 km/h. In how much time will he return to the starting point.

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

OBJECTIVE QUESTIONS

1. Given $\sqrt{5}$ = 2.236 the value of $\sqrt{45}+\sqrt{605}-\sqrt{245}$ correct to 3 decimal places is :
(A) 15.652 (B) 11.180 (C) 18.652 (D) 16.652

2. If x ∗ y = $\sqrt{x^2+y^2}$, the value of $(1*2\sqrt{2})\,(1*-2\sqrt{2})$ is :
(A) –7 (B) 0 (C) 2 (D) 9

3. If A and B are real numbers and $A^2 + B^2 = 0$, then :
(A) A > 0, B < 0 (B) A < 0, B > 0 (C) A = 0 = B (D) A = – B

4. $\dfrac{1+2+3+.......+12+11+10+....+2+1}{1+2+....+6+5+....+2+1}$. =
(A) 2 (B) 3 (C) 4 (D) 6

5. The value of $\sqrt{214+\sqrt{130-\sqrt{88-\sqrt{44+\sqrt{25}}}}}$:
(A) 14 (B) 15 (C) 16 (D) 17

6. The value of $\sqrt{1\frac{1}{2}-[1\frac{1}{2}-1\frac{1}{2}+(1\frac{1}{2}-1\overline{\frac{1}{2}-1\frac{1}{4}})]}$ is :
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{16}$ (D) $1\frac{1}{5}$

7. The sum of the squares of two number is 3341 and the difference of their squares is 891. The two numbers are :
(A) 46, 35 (B) 45, 36 (C) 36, 15 (D) 46, 36

8. If $(1^2 +2^2 +3^2 +4^2 +5^2)^2 = \frac{1}{P}$, then P is :
(A) $\frac{1}{3125}$ (B) 3125 (C) $\frac{1}{3025}$ (D) 3025

9. Physical Instructor wants to arrange boys in rows to form a perfect square. He finds that in doing so, 25 boys are left out. If the total number of boys is 1250 then find the number of boys in each row is :
(A) 25 (B) 125 (C) 45 (D) 35

10. The number of value of n for which 10n +8 is a perfect square where n is any natural number
(A) 0 (B) 1 (C) 4 (D) infinite values

SECTION -B (TECHIE STUFF)

11. The rationalising factor of $5+2\sqrt{6}$ is

(A) $-2\sqrt{6}-5$ (B) $\sqrt{6}-10$ (C) $6-2\sqrt{5}$ (D) $5-2\sqrt{6}$

12. If $\sqrt{5}$ = 2.236 and $\sqrt{2}$ = 1.414, then find $\frac{3}{\sqrt{5}+\sqrt{2}} + \frac{2}{\sqrt{5}-\sqrt{2}}$ = ?

(A) 2.2653 (B) 2.2553 (C) 3.2653 (D) 3.2553

13. The number $\frac{3-\sqrt{3}}{3+\sqrt{3}}$ is

(A) rational (B) irrational (C) both (D) can't say

Exercise-3

SECTION -A (PREVIOUS YEAR EXAMINATION QUESTIONS)

1. If $x+\frac{1}{x}$ = 3 then $\frac{x}{x^2+1}$ is **[Aryabhatta – 2002**

(A) 9 (B) $\frac{1}{3}$ (C) 13 (D) $\frac{1}{13}$

2. $\sqrt{5+\sqrt{10+\sqrt{x}}}$ = 3, then the value of x is **[Aryabhatta - 2006]**

(A) 49 (B) 36 (C) 16 (D) 25

3. If $\frac{x^2-1}{x} = \frac{1}{2}$, then $4x^2 + \frac{4}{x^2}$ is **[Aryabhatta - 2007]**

(A) – 7 (B) 7 (C) 9 (D) – 9

4. $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}}}$ is equal to **[Aryabhatta - 2007]**

(A) 0 (B) 7 (C) $7^{63/64}$ (D) $7^{31/32}$

5. Between which two consecutive whole number $\sqrt{1,000}$ lies? **(IMO 2010)**

(A) 29 and 30 (B) 30 and 31 (C) 31 and 32 (D) 32 and 33

6. The Wright brothers had their first successful flight near Kitty Hawk, North Carolina Mr. Satish finds it easy to remember the year in which the flight occurred because the number $\sqrt{3,621,502}$, the square root of his telephone number is nearest to a whole number. In which year did the flight occur? **(IMO 2010)**

(A) 1902 (B) 1903 (C) 1904 (D) 1905

7. Which of the following is a pythagorean triplet ? **[NSTSE 2011]**

(A) 16, 18, 20 (B) 1, 2, 3 (C) 30, 40, 50 (D) 50, 51, 52

8. Mehul plants 15376 apple trees in his garden and arranges them so that there are as many rows as there are apple trees in each row. The number of rows is. **(IMO 2011)**

(A) 124 (B) 126 (C) 134 (D) 144

9. The square of which number below will contain the first nine natural numbers **[NSTSE 2012]**

(A) 1111111 (B) 11111111 (C) 111111111 (D) 1111111111

10. Which of the following describes a square root of 41? **(IMO 2012)**

(A) Between 5 and 6 (B) Between 6 and 7
(C) Between 20 and 21 (D) Between 40 and 42

11. Garima wants to plant 50625 roses and arranges them in such a way that there are as many rows as there are roses in a row. The number of roses in a row are ________. **(IMO 2012)**

(A) 365 (B) 165 (C) 625 (D) 225

12. Identify the value of the given expression $\sqrt{\left(\left(\left(\left(\frac{3}{4}\right)^2 \div \frac{2}{5}\right) \times \frac{8}{9}\right) \div \frac{5}{16}\right)}$ **[NSTSE - 2013]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\sqrt{10}$ (C) $\sqrt{4}$ (D) $\frac{1}{\sqrt{5}}$

Answer Key

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	B	B	C	D	B	B	A	A	B	D	A	A	C	B	B	C

FILL IN THE BLANKS

1. 5 **2.** 10 **3.** 1 **4.** 385 **5.** Even

6. 121 **7.** 625 **8.** $\frac{50}{9}$ **9.** Even, Odd

TRUE / FALSE

1. True **2.** True **3.** False **4.** False **5.** True

6. False **7.** True **8.** True **9.** False **10.** False

MATCH THE COLUMN

1. (A) – s, (B) – p, (C) – t, (D) – q, (E) – r

2. (A) – r, (B) – s, (C) – t, (D) – p, (E) – q, (F)–u

3. (A) – r, (B) – s, (C) – p, (D) – t, (E) – q

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. Yes **2.** 14641 **3.** 1369

4. **(a)** 49 **(b)** 120 **5.** 7, 42 **6.** 3 , 6

SHORT ANSWER TYPE

7. **(a)** (10,24,26) **(b)** (14,48,50) **8.** $15 + 10\sqrt{3}$ **9.** 2

10. 0.0298 **11.** 347 **12.** 144, 9 **13.** 39

14. 131,74

LONG ANSWER TYPE

15. **(a)** $12^2 – 11^2$ **(b)** $8^2 – 7^2$ **16.** 0.816 **17.** 16×10^{-3}

18. 1024 **19.** 3 min. 16 sec.

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	A	D	C	C	B	A	B	C	D	A	D	D	B

Exercise-3

SECTION -A (PREVIOUS YEAR EXAMINATION QUESTIONS)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	B	B	C	C	C	B	C	A	C	B	D	C