MATHEMATICS

Class-VIII

Topic-3

CUBES & CUBE ROOTS

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CH-03

CUBES & CUBE ROOTS

TERMINOLOGIES

Perfect cube, Cube root, Prime factorization, Column method.

INTRODUCTION

In the last class, we have discussed square and square roots of a number. In this class we will study methods to find cubes of a number and cube roots of a number by prime factorisation, Unit digit and successive subtraction.

3.1 CUBES

The cube of a number is that number raised to the power 3. Thus, $2^3 = (2 \times 2 \times 2) = 8$ and we say that the cube of 2 is 8, $3^3 = (3 \times 3 \times 3) = 27$ and we say that the cube of 3 is 27, and so on.

(a) Perfect Cube

A natural number is said to be a perfect cube if it is the cube of some natural number.
We have, $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$, $10^3 = 1000$, etc. So, it follows that each of the numbers 1, 8, 27, 64, 125, 216, 343, 512, 729 and 1000 is a perfect cube.

(b) Testing for Perfect Cubes

In order to test whether a given natural number is a perfect cube or not, we follow the following steps :

(i) Obtain the natural number.
(ii) Express the given natural number as a product of prime factors.
(iii) Group the factors in triplets in such a way that all the three factors in each triplet are equal.
(iv) If no factor is left over in grouping in step III, then the number is a perfect cube, otherwise not. To find the natural number whose cube is the given number, take one factor from each triplet and multiply them. The cube of the number so obtained will be the given number.

Illustration 3.1

Is 64000 Perfect cube ?

Sol. Prime Factorizing 64000 .

2	64000
2	32000
2	16000
2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$64000 = \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{5 \times 5 \times 5}$$

No Factor is left over $\therefore$ 64000 is a perfect cube

Illustration 3.2

What is the smallest number by which 1323 be multiplied to make it a perfect cube.

Sol. Prime Factorization 1323 .

3	1323
3	441
3	147
7	49
7	7
	1

Grouping factors into triplet

$$1323 = \{\overline{3 \times 3 \times 3}\} \times 7 \times 7$$

Here, 7 occurs only twice so $\therefore$ to make 7's triplet, 1323 should be multiplied by 7 .

Illustration 3.3

What is the smallest number by which 2808 be divided so that quotient is a perfect cube ?

Sol.

2	2808
2	1404
2	702
3	351
3	117
3	39
13	13
	1

Grouping factors into triplet

$$2808 = \overline{\{2 \times 2 \times 2\}} \times \overline{\{3 \times 3 \times 3\}} \times 13$$

Clearly if we divide 2808 by 13 the quotient becomes a perfect cube.

(c) Properties Of Cubes Of Numbers

The cubes of numbers have some interesting properties, given below :

(i) Cubes of all even natural numbers are even. i.e. $2^3 = 8$, $6^3 = 216$, $8^3 = 512$ etc.

(ii) Cubes of all odd natural numbers are odd. i.e. $3^3 = 27$, $5^3 = 125$, $9^3 = 729$ etc.

(iii) Cubes of negative integers are negative.
$(-1)^3 = (-1) \times (-1) \times (-1) = -1$, $(-2)^3 = (-2) \times (-2) \times (-2) = -8$
Similarly, $(-3)^3 = -27$, $(-4)^3 = -64$, $(-5)^3 = -125$, and so on.

(iv) For any rational number $\frac{a}{b}$, we have $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

For example : $\left(\frac{3}{5}\right)^3 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{3 \times 3 \times 3}{5 \times 5 \times 5} = \frac{27}{125}$.

And $\left(\frac{-2}{3}\right)^3 = \frac{(-2) \times (-2) \times (-2)}{3 \times 3 \times 3} = \frac{-8}{27}$

(v) The sum of the cubes of first n natural numbers is equal to the square of their sum.
i.e. $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3 = (1 + 2 + 3 + \ldots\ldots\ldots n)^2$

(vi) Cubes of the numbers ending in digits 1, 4, 5, 6 and 9 are the number ending in the same digit. Cubes of numbers ending in digit 2 ends in 8, and cube of numbers ending in digit 8 ends in 2. The cubes of the numbers ending in digits 3 and 7 ends in 7 and 3 respectively.

(d) Methods For Determining The Cubes Of Number

(a) Column Method : We have $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
Method for finding the cube of a two digit number with the ten's digit **a** and the unit's digit **b,** we make four columns. a^3, $(3a^2b)$, $(3ab^2)$ and b^3
The rest of the procedure is the same as followed in squaring a number by the column method.

Illustration 3.4

Find the value of $(31)^3$ by the column method ?

Sol. Here, a = 3, b = 1

$a^2 \times a$	$a^2 \times 3b$	$b^2 \times 3a$	$b^2 \times b$
$3^2 \times 3$	$3^2 \times 3 \times 1$	$1^2 \times 3 \times 3$	$1^2 \times 1$
27	27	9	1
29	7	9	1

$(31)^3 = 29791$

Illustration 3.5

Using column method find the cube of (42).

Sol. Here, a = 4, b = 2

$a^2 \times a$	$a^2 \times 3b$	$b^2 \times 3a$	$b^2 \times b$
$4^2 \times 4$	$4^2 \times 3 \times 2$	$2^2 \times 3 \times 4$	$2^2 \times 2$
64	96	48	8
+ 10 74 74	+ 4 100 0	8	8

$\therefore (42)^3 = 74088$

Illustration 3.6

Show that 2197 is a perfect cube

Sol. Prime Factorization

13	2197
13	169
13	13
	1

$\therefore \quad 2197 = \overline{13 \times 13 \times 13}$

Illustration 3.7

Is $\frac{343}{729}$ a cube of a rational number

Sol. Resolve both in prime factors i.e. 343 and 729

7	343
7	49
7	7
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

$343 = \underline{7 \times 7 \times 7}$ $\quad$ $729 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

Both or perfect cubes

$\therefore$ $\frac{343}{729}$ is a cube of rational number.

A sk yourself

1. **(i)** Find the cube of (– 9).
   (ii) Find the cube root of 1331000
2. Is 1188 a perfect cube ? If not, by which smallest natural number should 1188 be divided so that the quotient is a perfect cube ?
3. Is 243 a pefect cube ? If not find the smallest natural number by which 243 must be multiplied so that the product is a perfect cube.
4. Check whether 106480 is a perfect cube or not. If not, find the smallest number by which 106480 be divided so that quotient is a perfect cube?
5. Is 40500 a perfect cube?

Answers.
1. **(i)** – 729 **(ii)** 110 **2.** No, 44 **3.** No, 3 **4.** No, 10 **5.** No

3.2 CUBE ROOTS

The cube root of a number x is that number whose cube gives x.

The cube root of x is denoted by the symbol $\sqrt[3]{x}$.

Thus, $\sqrt[3]{8} = 2$, $\sqrt[3]{27} = 3$, $\sqrt[3]{64} = 4$, $\sqrt[3]{125} = 5$ and so on.

(a) Cube Roots Through Pattern

Like squares of natural numbers, cubes too have some interesting patterns:

$1^3 = 1$

$2^3 = 8$

$\therefore\ 2^3 - 1^3 = 7 = 1 + 2 \times 1 \times 3$

$3^3 = 27$

$\therefore\quad 3^3 - 2^3 = 19 = 1 + 3 \times 2 \times 3$

$4^3 = 64$

$\therefore\quad 4^3 - 3^3 = 37 = 1 + 4 \times 3 \times 3$

: - :

: - :

$9^3 = 729$

$9^3 - 8^3 = 217 = 1 + 9 \times 8 \times 3$

Also, $1 = 1^3$

$1 + 7 = 2^3$

$1 + 7 + 19 = 3^3$

$1 + 7 + 19 + 37 = 4^3$

-

-

$1 + 7 + 19 + \ldots + 217 = 9^3$

Note, that 2^3 is the sum of first 2 numbers of 1, 7, 19, 37, 61, 91, ...
The number of times the subtraction is carried out, gives the cube root.

For example :
216 – 1 = 215, 215 – 7 = 208, 208 – 19 = 189, 189 – 37 = 152, 152 – 61 = 91, 91 – 91 = 0.
Since we have subtracted six times to get 0, therefore $\sqrt[3]{216} = 6$.

Illustration 3.8

Find the cube root of 343.

Sol.

$$\begin{array}{r} 343 \\ -1 \\ \hline 342 \\ -7 \\ \hline 335 \\ -19 \\ \hline 316 \\ -37 \\ \hline 279 \\ -61 \\ \hline 218 \\ -91 \\ \hline 127 \\ -127 \\ \hline 0 \end{array}$$

Since the subtraction is performed seven times. Therefore, $\sqrt[3]{343} = 7$.

(b) Cubes roots using unit digit

m	1	2	3	4	5	6	7	8	9	10
m^3	1	8	27	64	125	216	343	512	729	1000

We shall now describe a method that can be used to find cube roots of perfect cubes having at the most six digits. By looking at table, we observe that the cube of a number ending in 0, 1, 4, 5, 6 and 9 ends in 0, 1, 4, 5, 6 and 9 respectively. However, the cube of number ending in 2 ends in 8 and vice versa. Similarly, the cube of a number ending in 3 or 7 ends in 7 or 3 respectively. Thus, by looking the unit digit of a perfect cube number, we can determine the unit digit of its cube root.

Now consider a number which is a perfect cube and has at the most six digits. The cube root of such a number has at the most two digits, because the least seven digit number is 1000000 (= 100^3) and its cube root 100 is a three digit number. We determine the two digits of the cube root as follows

Step (i) : Look at the digit at the unit place of the perfect cube and determine the digit at the unit place in the cube root as discussed above.

Step (ii) : Strike out from the right, last three (i.e., units, tens and hundreds) digits of the number. If nothing is left, we stop. The digit in Step 1 is the cube root.

Step (iii) : Consider the number left from Step 2. Find the largest single digit number whose cube is less than or equal to this left over number. This is the ten's digit of the cube root.

Illustration 3.9

Find the cube roots of the following numbers :

(i) 512 **(ii)** 2197 **(iii)** 117649 **(iv)** 636056

Sol. **(i)** **512** The unit digit of 512 is 2. Therefore, the digit at the unit place in the cube root is 8. Since no number is left after striking out the units, tens and hundreds digits of the number, the required cube root is 8.

(ii) **2197** Here, unit digit is 7. Therefore, unit digit of the cube root is 3. After striking out the last three digits from the right, we are left with the number 2. Now 1 is the largest number whose cube is less than 2. Therefore, the ten's digit is 1.
Thus, the required cube root is 13.

(iii) **117649 :** Here, unit digit is 9. Therefore, the unit digit of the cube root is 9. Striking out the last three digit from the right, the number left is 117. Now $4^3 = 64 < 117$ and $5^3 = 125 > 117$.
Hence, the tens digit of the cube root is 4.
$\therefore \sqrt[3]{117649} = 49$

(iv) **636056 :** Here, unit digit of the cube root is 6.
Also, $8^3 < 636$ and $9^3 > 636$.
Hence, ten's digit of the cube root is 8
$\therefore \sqrt[3]{636056} = 86$

(c) To find the cube roots of a perfect cube by prime factorisation

(i) Find the prime factors of the given perfect cube.

(ii) Make triplets (groups of three) of similar factors.

(iii) Take one factor from each triplet and multiply.

(iv) The product will be the cube root of the given number.

Illustration 3.10

Find the cube root of 5832

Sol. Prime factorize 5832

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$5832 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$\sqrt[3]{5832} = 2 \times 3 \times 3 = 18$

Illustration 3.11

Find the cube root of : 531441.

Sol. Resolving 531441 into prime factors, we get

3	531441
3	177147
3	59049
3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
	3

$531441 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$\therefore \sqrt[3]{531441} = 3 \times 3 \times 3 \times 3 = 81$

(d) Cube roots of rational number and negative number

(i) Cube Root of a Negative Perfect Cube :

If a is positive integer then – a is a negative integer.

We know that $(-a)^3 = -a^3$.

So, $\sqrt[3]{-a^3} = -a$

In general, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$.

(ii) Cube Root of Product of Integers

For any two integers a and b, we have

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

(iii) Cube Roots of Rational Numbers

We know that $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

Similarly, $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$.

Illustration 3.12

Find the cube root of – 2744.

Sol. We have $\sqrt[3]{-2744} = -\sqrt[3]{2744}$

Now, we resolve 2744 into prime factors and find that

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$2744 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$\therefore \sqrt[3]{2744} = (2 \times 7) = 14$.

Hence, $\sqrt[3]{-2744} = -\sqrt[3]{2744} = -14$.

Illustration 3.13

Show that $\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$.

Sol. $125 \times 64 = \underline{5 \times 5 \times 5} \times \underline{4 \times 4 \times 4.}$

$\therefore \sqrt[3]{125 \times 64} = (5 \times 4) = 20$

Now, $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$ and, $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$

$\therefore \sqrt[3]{125} \times \sqrt[3]{64} = (5 \times 4) = 20$

Hence, $\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$

Illustration 3.14

Show that $\sqrt[3]{216 \times (-343)} = \sqrt[3]{216} \times \sqrt[3]{-343}$.

Sol. We have :

$216 \times (-343) = -(216 \times 343) = -(6 \times 6 \times 6 \times 7 \times 7 \times 7)$.

$\therefore \sqrt[3]{216 \times (-343)} = -(6 \times 7) = -42.$

Again, $\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$ and $\sqrt[3]{-343} = -\sqrt[3]{343} = -\sqrt[3]{7 \times 7 \times 7} = -7$

$\therefore \sqrt[3]{216} \times \sqrt[3]{-343} = 6 \times (-7) = -42.$

Hence, $\sqrt[3]{216 \times (-343)} = \sqrt[3]{216} \times \sqrt[3]{-343}$

Illustration 3.15

Show that

(i) $\sqrt[3]{\dfrac{216}{2197}} = \dfrac{\sqrt[3]{216}}{\sqrt[3]{2197}}$ **(ii)** $\sqrt[3]{\dfrac{-125}{512}} = \dfrac{\sqrt[3]{-125}}{\sqrt[3]{512}}$

Sol. **(i)** $\dfrac{216}{2197} = \dfrac{6 \times 6 \times 6}{13 \times 13 \times 13} = \dfrac{6}{13} \times \dfrac{6}{13} \times \dfrac{6}{13} = \left(\dfrac{6}{13}\right)^3$

$\therefore \sqrt[3]{\dfrac{216}{2197}} = \dfrac{6}{13}$,

Again, $\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$ and $\sqrt[3]{2197} = \sqrt[3]{13 \times 13 \times 13} = 13.$

$\therefore \dfrac{\sqrt[3]{216}}{\sqrt[3]{2197}} = \dfrac{6}{13}$.

Hence, $\sqrt[3]{\dfrac{216}{2197}} = \dfrac{\sqrt[3]{216}}{\sqrt[3]{2197}}$.

(ii) $\dfrac{-125}{512} = \dfrac{(-5) \times (-5) \times (-5)}{8 \times 8 \times 8} = \left(\dfrac{-5}{8}\right) \times \left(\dfrac{-5}{8}\right) \times \left(\dfrac{-5}{8}\right) = \left(\dfrac{-5}{8}\right)^3$.

$\therefore \sqrt[3]{\dfrac{-125}{512}} = \dfrac{-5}{8}$

Again, $\sqrt[3]{-125} = \sqrt[3]{(-5) \times (-5) \times (-5)} = (-5)$ and $\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8$

$\therefore \dfrac{\sqrt[3]{-125}}{\sqrt[3]{512}} = \dfrac{-5}{8}$.

Hence, $\sqrt[3]{\dfrac{-125}{512}} = \dfrac{\sqrt[3]{-125}}{\sqrt[3]{512}}$.

Illustration 3.16

Find the cube root of 4.096.

Sol. $\sqrt[3]{4.096} = \sqrt[3]{\dfrac{4096}{1000}} = \dfrac{\sqrt[3]{4096}}{\sqrt[3]{1000}}$

Resolving 4096 into prime factors,
we get
4096 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$.

$\therefore \sqrt[3]{4096} = (2 \times 2 \times 2 \times 2) = 16.$

Also, $\sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = 10.$

So, $\dfrac{\sqrt[3]{4096}}{\sqrt[3]{1000}} = \dfrac{16}{10} = 1.6$

Hence, $\sqrt[3]{4.096} = 1.6$

Ask yourself

1. Find the cube root of the following :

(i) 0.000343 **(ii)** $4^{12} \times 6^{15} \times 7^{21}$

2. What is an edge of a cube whose volume is 91125 m^3 ?

3. Find the cube root of – 8 / 3375.

4. Evaluate $\dfrac{\sqrt[3]{729}+\sqrt[3]{343}}{\sqrt[3]{512}}$.

5. Evaluate $\sqrt[3]{512 \times 2197}$

6. Find $\dfrac{\sqrt[3]{512}+\sqrt[3]{216}}{\sqrt[3]{343}}$

7. Find $\sqrt[3]{0.027 \times 2.744}$

Answres.

1. **(i)** 0.07 **(ii)** $2^{13} \times 3^{5} \times 7^{7}$ **2.** 45 **3.** $\dfrac{-2}{15}$ **4.** 2

5. 104 **6.** 2 **7.** 0.42

Add to Your Knowledge

1. Sum and difference of the cubes of 2 numbers is divisible by the sum and difference of the numbers itself.

2. Product of 3 consecutive natural numbers is always divisible by '6'.

Concept Map

Cube & Cube roots

The cube of no. is obtained when no. is multiplied by itself 3 times. Cube of x is x × x × x

Perfect cube

A natural no. is said to be a perfect cube if it is the cube of same natural no.

Properties of perfect cube

(i) Cube of even no is even.
(ii) Cube of odd no. is odd
(iii) Cube of negative no is negative.
(iv) The sum of the cube of first n natural no. is equal to the squre of their sum.
$1^3 + 2^3 + 3^3 + ... + n^3 = (1 + 2 + 3 + ... n)^2$

(v) Cubes of the numbers ending in digits 1, 4, 5, 6 and 9 are the number ending in the same digit. Cubes of numbers ending in digit 2 ends in 8, and cube of numbers ending in digit 8 ends in 2. The cubes of the numbers ending in digits 3 and 7 ends in 7 and 3 respectively.

Cube by column method

to find 25^3, take a = 2, b = 5

a^3	$3a^2b$	$3ab^2$	b^3
2^3	$3\times 2^2\times 5$	$3\times 2\times 5^2$	5^3
8 +7 15	60 +16 76	150 +12 162	125

25^3 = 15625

Cube roots

The cube root of a no. is x that no whose cube gives x.
Ex. Cube root of 8 is 2 because
$2^3 = 8$
$\sqrt[3]{8} = 2$

Cube root by prime factorization

$\sqrt[3]{216}$

2	216
2	108
2	54
3	27
3	3
3	3

$\sqrt[3]{216} = \sqrt[3]{2\times 2\times 2\times 3\times 3\times 3} = 2\times 3 = 6$

Ex.
$\sqrt[3]{\frac{-512}{125}} = \frac{\sqrt[3]{-512}}{\sqrt[3]{125}}$
$\frac{\sqrt[3]{-8\times -8\times -8}}{\sqrt[3]{5\times 5\times 5}}$
$= -\frac{8}{5}$

Cube root by pattern

We have to successively subtract 1,7,19,37,61,91 .... from number till we get zero. The no of time we subtract give the cube root.

Ex. $\sqrt[3]{64}$
64 – 1 = 63
63 – 7 = 56
56 – 19 = 37
37 – 37 = 0
So $\sqrt[3]{64} = 4$

Summary

1. A number n is a perfect cube, if there is an integer m such that $n = m^3$.

2. If n is a perfect cube and $n = m^3$, then m is a cube root of n. A cube root of n is written as $\sqrt[3]{n}$.

3. The units digit of the cube root of a perfect cube can be determined with the help of the units digit of the perfect cube.

4. The cube root of a perfect cube can be obtained by prime factorisation of the number.

5. The cube root of a product of two perfect cubes is the product of the cube roots of the perfect cubes, i.e., $\sqrt[3]{ab} = \sqrt[3]{a} \cdot \sqrt[3]{b}$.

6. The cube root of a quotient of two perfect cubes is the quotient of their cube roots, i.e., $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$, $b \neq 0$, where a and b are perfect cubes.

7. The cube root of a negative perfect cube is negative.

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

1. Total number of perfect cubes from 1 to 100 are :
(A) 2 (B) 5 (C) 6 (D) 4

2. The length of each side of the cubical box is 2.4 m. Its volume is :
(A) 13.824 m^3 (B) 14.824 m^3 (C) 16.934 m^3 (D) 18.824 m^3

3. How many such numbers are there which are equal to their cube but not equal to their square ?
(A) 2 (B) 1 (C) 0 (D) 3

4. If 72 × k is a perfect cube, then the value of k is :
(A) 1 (B) 2 (C) 3 (D) 4

5. The smallest number by which 32 must be multiplied to get a perfect cube is :
(A) 16 (B) 4 (C) 2 (D) 8

6. The smallest number by which 32 must be divided to get a perfect cube is :
(A) 16 (B) 4 (C) 2 (D) 8

7. Choose the correct statement ?
(A) Cubes of odd natural numbers are odd
(B) Cubes of even natural numbers are even
(C) Cubes of negative integers are negative
(D) All of the above

8. The largest four digit number which is perfect cube, is:
(A) 8000 (B) 9261 (C) 9999 (D) None of these

9. The cube root of 1.331 is :
(A) 0.11 (B) 0.011 (C) 11 (D) 1.1

10. Find the cube root of – (8000).
(A) – 20 (B) – 40 (C) – 60 (D) – 15

11. The value of $(3^3 + 4^3 + 5^3)^{1/3}$ is equal to :
(A) 6 (B) 12 (C) 36 (D) 216

12. The value of $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$ is :
(A) 120 a^2 (B) 100 a^2 (C) 100 a (D) – 125 a

13. Evaluate : $\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} - 1.$

(A) 0 (B) 1 (C) 9 (D) 4

14. $\sqrt[3]{16\frac{16}{27}} = \frac{4}{3} \times \sqrt[3]{x}$, then x is :

(A) 7 (B) 16 (C) 10 (D) None

15. The cube of a number is 8 times the cube of another number. If the sum of the cubes of numbers is 243, the difference of the numbers is :

(A) 3 (B) 4 (C) 6 (D) None of these

16. Three numbers are in ratio 2 : 3 : 4. The sum of their cubes is 33957, then greatest number is :

(A) 14 (B) 21 (C) 28 (D) 35

17. If $\sqrt[3]{18} = 2.621$, then $\sqrt[3]{0.018}$ is :

(A) 0.2621 (B) 0.02621 (C) 262.1 (D) 26.21

FILL IN THE BLANKS

1. ________ is the least number which is a perfect square as well as a perfect cube .

2. Number of digits in the cube of a two-digit number may be _________.

3. If a number ends in two 9's, then its cube ends in _________ number of 9's.

4. cube root of a odd number is always a ________ number .

5. the cube of a number is 27 times the cube of other number and sum of both the number is 16, then the difference of both the number is ________.

6. cube root of 0.729 is ________.

7. Cube root of a perfect even cube is _________ and the perfect odd cube is_________.

8. $\sqrt[3]{a^6 . b^9}$ = _________.

9. $\sqrt[3]{2^6 \times 5^3 \times 3^9}$ is _________

10. $\dfrac{\left(\dfrac{\sqrt[3]{729} + \sqrt[3]{216} + \sqrt[3]{27}}{3}\right)}{3}$ is ________.

TRUE / FALSE

1. A perfect cube can end with even number of zeroes.

2. A perfect cube does not end with two zeros.

3. The cube of a 2 digit number may be a 3 digit number .

4. The cube of a 2 digit number may have seven or more digits .

5. If n is a multiple of 2, then n^3 is also a multiple of 2.

6. The cubes of the digits 1,4,5,6 and 9 are the numbers ending in the same digits 1,4,5,6 and 9 respectively.

7. If n ends in 3, then n^3 ends in 7.

8. There is no perfect cube which ends in 8 .

9. The cube root of a perfect cube can be obtained by prime factorisation of a number.

10. The cube root of 8 is 3.

MATCH THE COLUMN

1. Match the cubes in column–I with their cube roots in column–II

Column – I		**Column – II**	
(A)	1331	(p)	4
(B)	64	(q)	8
(C)	343	(r)	10
(D)	216	(s)	11
(E)	1000	(t)	6
(F)	512	(u)	7

2. What should be subtracted from column–I to make them a perfect cube

Column – I		**Column – II**	
(A)	65	(p)	5
(B)	31	(q)	0
(C)	343	(r)	2
(D)	130	(s)	1
(E)	11	(t)	4
(F)	3	(u)	3

SECTION -B (FREE RESPONSE TYPE)

SUBJECTIVE QUESTIONS

VERY SHORT ANSWER TYPE

1. Find the smallest number which is a perfect cube and also multiple of 9 .

2. What number should 1512 be divided to make it a perfect cube.

3. Evaluate : $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$.

4. Evaluate : $\sqrt[3]{125 \times 64}$

5. Evaluate : $\sqrt[3]{4^3 \times 6^3}$.

SHORT ANSWER TYPE

6. Find the smallest number by which 2560 must be multiplied, so that the product is perfect cube.

7. Find the smallest number by which 10368 must be divided, so that the quotent is perfect cube.

8. Show that : $\sqrt[3]{-125 \times 216} = \sqrt[3]{-125} \times \sqrt[3]{216}$.

9. Find the cube root of 0.001728.

10. Find the cube root of 233,744.896

LONG ANSWER TYPE

11. By what smallest natural number should – 6125 be multiplied so that the product becomes a perfect cube ?

12. What is the smallest number by which 1600 must be divided, so that the quotient is perfect cube.

13. If 3p is a perfect cube then prove that p is a multiple of 9.

14. Prove that if a number is tripled then its cubes will be 27 times the cube of the given number.

15. Find the cube roots of number 941192.

16. Evaluate : $\dfrac{3\sqrt[3]{729} + 5\sqrt[3]{-0.008}}{\sqrt[3]{\sqrt[3]{512}} - \sqrt[3]{64}}$

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

OBJECTIVE QUESTIONS

1. Cube root of $\frac{0.125}{64}$ is

(A) $\frac{5}{4}$ (B) $\frac{0.5}{4}$ (C) $\frac{0.05}{4}$ (D) $\frac{0.005}{4}$

2. The value of $\sqrt[3]{\frac{343}{125}} \times \sqrt{64}$ is

(A) 11.2 (B) 1.12 (C) 112 (D) None of these

3. The perfect cube nearest to 2750 is

(A) 2249 (B) 2747 (C) 2744 (D) 2754

4. If $\sqrt[3]{50}$ = 3.684 the cube of 0.3684 is

(A) 5 (B) 50 (C) 0.5 (D) 0.05

5. Total surface area of a cube is 294 cm^2 then its volume is

(A) 7 (B) 49 (C) 343 (D) None of these

6. The volume of a cube is numerically equal to its surface area then the side of the cube is

(A) 1 cm (B) 6 cm (C) 9 cm (D) None of these

7. We know that $\sqrt{1^3 + 2^3 + 3^3 + ... + n^3} = \frac{n(n+1)}{2}$ for all natural numbers. Using the above result the value of $\sqrt{1^3 + 2^3 + 3^3} \div \sqrt{1^3 + 2^3}$ is

(A) 2 (B) $3^3 + 4^3$ (C) $3^{\frac{3}{2}} + 4^{\frac{3}{2}}$ (D) None of these

8. Two times the square of a number is three times of its cube. The number is

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{9}{4}$ (D) $\frac{4}{9}$

9. $1^3 + 2^3 + 3^3 + ... + 10^3 =$

(A) 3025 (B) 5050 (C) 1225 (D) 1625

10. $21^3 + 22^3 + 23^3 + ... + 30^3 =$

(A) 44100 (B) 216225 (C) 172125 (D) none of these

SECTION -B

11. $8^3 – 5^3$ is divisible by

(A) 3 (B) 7 (C) 8 (D) 5

12. 1.2.3+2.3.4+3.4.5+4.5.6+........+2015.2016.2017 is divided by 6 then remainder is

(A) 0 (B) 1 (C) 2 (D) 5

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

1. $\dfrac{10+\sqrt[3]{8}}{98.4+\sqrt{2.56}}$ is equal to **[Aryabhatta - 2006]**

(A) 0.12 (B) 0.012 (C) 1.2 (D) 11.2

2. Cube root of $\dfrac{0.216}{27}$ is **[Aryabhatta - 2006]**

(A) 0.2 (B) 0.02 (C) 0.002 (D) 0.06

3. $6^3 - 5^3$ is always divisible by **[Aryabhatta - 2008]**

(A) 9 (B) 8 (C) 1 (D) 5

4. $10 + 10^3$ equals **[NSTSE - 2009]**

(A) 2.0×10^3 (B) 8.0×10^3 (C) 4.0×10^3 (D) 1.01×10^3

5. By what least number by which 3600 be divided to make it a perfect cube ? **[NSTSE - 2010]**

(A) 9 (B) 50 (C) 300 (D) 450

6. Find the value of $\sqrt[3]{\sqrt{0.000064}}$ **(IMO 2011)**

(A) 002 (B) 0.2 (C) 2 (D) None of these

7. The largest four-digit number which is a perfect cube, is__________ . **(IMO 2012)**

(A) 8000 (B) 9261 (C) 9999 (D) None of these

8. The cube of a number is 8 times the cube of another number. If the sum of the cubes of numbers is 243, then what is the difference of the numbers ? **[NSTSE - 2013]**

(A) 3 (B) 4 (C) 6 (D) – 6

9. The product 864 × n is a perfect cube. What is the smallest value of 'n' ? **[NSTSE - 2014]**

(A) 2 (B) 1 (C) 4 (D) 3

Answer Key

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	A	B	C	C	B	D	B	D	A	A	A	A	A	A
Ques.	16	17													
Ans.	C	A													

FILL IN THE BLANKS

1. 1 **2.** 4 or 5 or 6 **3.** two **4.** odd

5. 8 **6.** 0.9 **7.** even, odd **8.** $a^2 b^3$

9. $2^2 \times 5 \times 3^3$ **10.** 2

TRUE / FALSE

1. True **2.** True **3.** True **4.** False **5.** True **6.** True
7. True **8.** False **9.** True **10.** False

MATCH THE COLUMN

1. (A) – s, (B) – p, (C) – u, (D) – t ,(E) – r, (F) – q

2. (A) – s, (B) – t, (C) – q, (D) – p, (E) – u, (F) – r

SECTION -B (FREE RESPONSE TYPE)

SUBJECTIVE QUESTIONS

VERY SHORT ANSWER TYPE

1. 27 **2.** 7 **3.** 3.6 **4.** 20 **5.** 24

SHORT ANSWER TYPE

6. 25 **7.** 6 **9.** 0.12 **10.** 61.6

LONG ANSWER TYPE

11. 7 **12.** 25 **15.** 98 **16.** – 13

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	B	A	C	D	C	B	A	B	A	C	A	A

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9
Ans.	A	A	C	D	D	B	B	A	A