

MATHEMATICS

Class-VIII

Topic-5

QUADRILATERALS



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CH-05 QUADRILATERALS

TERMINOLOGIES

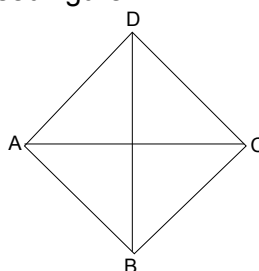
Polygon, Parallel Lines, Transversal, Collinear, Vertices, Line Segments, Sides, Angles, Convex and concave quadrilateral, diagonals, parallelogram, Rhombus, Rectangle, Square, Trapezium, Kite, Isosceles trapezium, congruent.

INTRODUCTION

'Poly' means many and 'gon' means sides. So a polygon is a closed figure of many sides. A polygon of 'n' sides is also called n-gon. Polygon can be classified according to the number of sides like triangle (3 sides), Quadrilateral (4 sides), Pentagon (5 sides).

5.1 QUADRILATERAL

A quadrilateral is four sided closed figure.



Let A, B, C and D be four points in a plane such that :

- (i) No three of them are collinear.
- (ii) The line segments AB, BC, CD and DA do not intersect except at their end points, then **figure** obtained by joining A, B, C & D is called a **quadrilateral**.

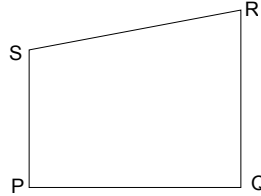
(a) Definitions

- (i) **Vertices** : The point A, B, C and D are called vertices.
- (ii) **Opposite vertices** : The vertices A and C; B and D are called the opposite vertices.
- (iii) **Sides** : The line segment AB, BC, CD and AD are called sides.
- (iv) **Opposite sides** : AB and DC; AD and BC are called opposite sides.
- (v) **Adjacent sides** : AD and AB; AB and BC, BC and CD, CD and AD are called the adjacent sides.
- (vi) **Angles** : $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are the angles of the quadrilateral ABCD.
- (vii) **Opposite angles** : $\angle A$ and $\angle C$; $\angle B$ and $\angle D$ are opposite angles.
- (viii) **Adjacent angles** : $\angle A$ and $\angle B$; $\angle B$ and $\angle C$; $\angle C$ and $\angle D$; $\angle D$ and $\angle A$ are the adjacent angles.

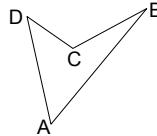
(ix) **Diagonals** : Line segment joining the opposite vertices of a quadrilateral ABCD are called its diagonal. In the above figure AC and BD are two diagonals of the quadrilateral ABCD.

(b) Convex and Concave Quadrilaterals

(i) A quadrilateral in which the measure of each interior angle is less than 180° is called a convex quadrilateral. In **figure**, PQRS is convex quadrilateral.

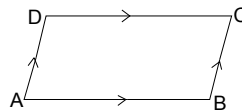


(ii) A quadrilateral in which the measure of one of the interior angle is more than 180° is called a concave quadrilateral. In **figure**, ABCD is concave quadrilateral.

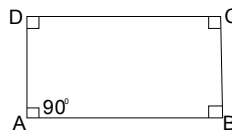


(c) Special Quadrilaterals

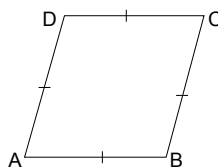
(i) **Parallelogram** : A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. In **figure**, $AB \parallel DC$, $AD \parallel BC$ therefore, ABCD is a parallelogram.



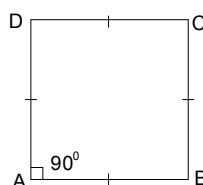
(ii) **Rectangle** : A rectangle is parallelogram, but each of its angle is right angle. If ABCD is a rectangle then $\angle A = \angle B = \angle C = \angle D = 90^\circ$.



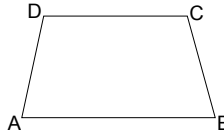
(iii) **Rhombus** : A rhombus is a parallelogram but all its sides are equal in length. If ABCD is a rhombus then $AB = BC = CD = DA$.



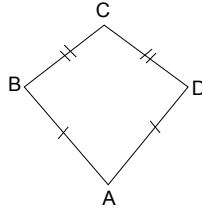
(iv) **Square** : A square is a parallelogram having all sides equal and each angle equal to right angle. If ABCD is a square then $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90^\circ$.



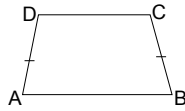
(v) **Trapezium** : A trapezium is a quadrilateral with only one pair of opposite sides parallel. In **figure**, ABCD is a trapezium with $AB \parallel DC$.



(vi) **Kite** : A kite is a quadrilateral in which two pairs of adjacent sides are equal. If ABCD is a kite then $AB = AD$ and $BC = CD$.



(vii) **Isosceles trapezium** : A trapezium is said to be an isosceles trapezium, if its non-parallel sides are equal. Thus a quadrilateral ABCD is an isosceles trapezium, if $AB \parallel DC$ and $AD = BC$ and $\angle A = \angle B$ and $\angle D = \angle C$.



Ask yourself

- It is possible to have a quadrilateral whose angles are of measures 105° , 165° , 55° and 45° ? Give reason.
- The angles of a quadrilateral are respectively 20° , 100° , 80° . Find the fourth angle.
- What will be the sum of all angles of a convex polygon which has
 - 6 sides
 - 8 sides
- How many sides has a regular polygon, each angle of which is of measure 108° ?
- What is the sum of all the angles of
 - A hexagon
 - An octagon
 - A regular decagon
- It is possible to have a regular polygon whose interior angle measures 124° ? Justify

Answers

- No
- 160°
- (i) 720° (ii) 1080°
- 5
- (i) 720° (ii) 1080° (iii) 1440°
- No

5.2 PROPERTIES OF VARIOUS SPECIAL TYPES OF QUADRILATERALS

(a) Parallelogram

Properties in the form of theorems have been given.

Theorem-1 : The sum of the four angles of a quadrilateral is 360° .

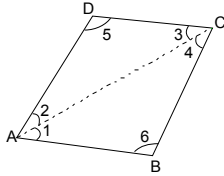
Given : Quadrilateral ABCD.

To Prove : $\angle A + \angle B + \angle C + \angle D = 360^\circ$.

Construction : Join AC.

Proof : In $\triangle ABC$, we have

$$\angle 1 + \angle 4 + \angle 6 = 180^\circ \quad \dots(i)$$



In $\triangle ACD$, we have

$$\angle 2 + \angle 3 + \angle 5 = 180^\circ \quad \dots(ii)$$

Adding (i) and (ii) we get

$$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + \angle 5 + \angle 6 = 180^\circ + 180^\circ$$

$$\angle A + \angle C + \angle D + \angle B = 360^\circ.$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ.$$

Theorem-2 : A diagonal of a parallelogram divides it into two congruent triangles.

Given : A parallelogram ABCD.

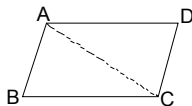
To Prove : A diagonal, say, AC of parallelogram ABCD divides it into congruent triangles ABC and CDA i.e. $\triangle ABC \cong \triangle CDA$.

Construction : Join AC.

Proof : Since ABCD is a parallelogram. Therefore, $AB \parallel DC$ and $AD \parallel BC$

Now, $AD \parallel BC$ and transversal AC intersects them at A and C respectively.

$$\angle DAC = \angle BCA \quad \dots(i) \quad [\text{Alternate interior angles}]$$



Again, $AB \parallel DC$ and transversal AC intersects them at A and C respectively.

Therefore,

$$\angle BAC = \angle DCA \quad \dots(ii) \quad [\text{Alternate interior angles}]$$

Now, in $\triangle ABC$ and $\triangle CDA$, we have

$$\angle BCA = \angle DAC \quad [\text{From (i)}]$$

$$AC = AC \quad [\text{Common side}]$$

$$\angle BAC = \angle DCA \quad [\text{From (ii)}]$$

So, by ASA congruence criterion, we have

$$\triangle ABC \cong \triangle CDA$$

Theorem-3 : In a parallelogram, opposite sides are equal.

Given : A parallelogram ABCD.

To Prove : $AB = CD$ and $DA = BC$.

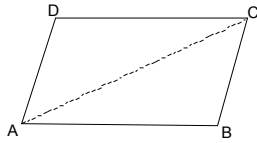
Construction : Join AC.

Proof : Since ABCD is a parallelogram. Therefore

$$AB \parallel DC \text{ and } AD \parallel BC$$

Now, $AD \parallel BC$ and transversal AC intersects them at A and C respectively.

$$\angle DAC = \angle BCA \quad \dots(i) \quad [\text{Alternate interior angles}]$$



Again, $AB \parallel DC$ and transversal AC intersects them at A and C respectively.

$$\angle BAC = \angle DCA \quad \dots(ii) \quad [\text{Alternate interior angles}]$$

Now, in $\triangle ADC$ and $\triangle CBA$, we have

$$\angle DCA = \angle BAC \quad [\text{From (ii)}]$$

$$AC = AC \quad [\text{Common side}]$$

$$\text{And } \angle DAC = \angle BCA \quad [\text{From (i)}]$$

So, by ASA criterion congruence

$$\triangle ADC \cong \triangle CBA$$

$$AD = CB \text{ and } DC = BA \quad [\text{Corresponding parts of congruent triangles are equal}]$$

Theorem - 4 : The opposite angles of a parallelogram are equal.

Given : A parallelogram $ABCD$

To Prove : $\angle A = \angle C$ and $\angle B = \angle D$

Proof : Since $ABCD$ is a parallelogram. Therefore,

$$AB \parallel DC \text{ and } AD \parallel BC$$

Now, $AB \parallel DC$ and transversal AD intersects them at A and D respectively.



$$\angle A + \angle D = 180^\circ \quad \dots(i) \quad \left[\begin{array}{l} \because \text{Sum of consecutive} \\ \text{interior angles is } 180^\circ \end{array} \right]$$

Again, $AD \parallel BC$ and DC intersects them at D and C respectively.

$$\angle D + \angle C = 180^\circ \quad \dots(ii) \quad \left[\begin{array}{l} \because \text{Sum of consecutive} \\ \text{interior angles is } 180^\circ \end{array} \right]$$

From (i) and (ii) we get

$$\angle A + \angle D = \angle D + \angle C \quad \Rightarrow \quad \angle A = \angle C.$$

Similarly, $\angle B = \angle D$

Hence, $\angle A = \angle C$ and $\angle B = \angle D$.

Theorem - 5 : The diagonals of a parallelogram bisect each other.

Given : A parallelogram $ABCD$ such that its diagonals AC and BD intersect at O .

To Prove : $OA = OC$ and $OB = OD$.

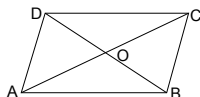
Proof : Since $ABCD$ is a parallelogram. Therefore,

$$AB \parallel DC \text{ and } AD \parallel BC$$

Now, $AB \parallel DC$ and transversal AC intersects them at A and C respectively.

$$\angle BAC = \angle DCA \quad [\text{Alternate interior angles are equal}]$$

$$\angle BAO = \angle DCO \quad \dots(i)$$



Again, $AB \parallel DC$ and BD intersects them at B and D respectively.

$$\angle ABD = \angle CDB \quad [\text{Alternate interior angles are equal}]$$

$$\angle ABO = \angle CDO \quad \dots(\text{ii})$$

Now, in $\triangle AOB$ and $\triangle COD$, we have

$$\angle BAO = \angle DCO \quad [\text{From (i)}]$$

$$AB = CD \quad [\text{Opposite sides of a parallelogram are equal}]$$

$$\text{And, } \angle ABO = \angle CDO \quad [\text{From (ii)}]$$

So, by ASA congruence criterion

$$\triangle AOB \cong \triangle COD$$

$$OA = OC \text{ and } OB = OD \quad [\text{By CPCT}]$$

Hence, $OA = OC$ and $OB = OD$

Illustration 5.1

In a quadrilateral $ABCD$, the angles A , B , C and D are in the ratio $1 : 2 : 3 : 4$. Find the measure of each angles of the quadrilateral.

Sol. We have $\angle A : \angle B : \angle C : \angle D = 1 : 2 : 3 : 4$.

So, let $\angle A = x^\circ$, $\angle B = 2x^\circ$, $\angle C = 3x^\circ$, $\angle D = 4x^\circ$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$x + 2x + 3x + 4x = 360$$

$$10x = 360$$

$$x = 36$$

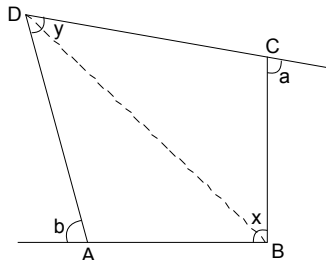
Thus, the angles are :

$$\angle A = 36^\circ, \angle B = (2 \times 36)^\circ = 72^\circ, \angle C = (3 \times 36)^\circ = 108^\circ$$

$$\text{And, } \angle D = (4x)^\circ = (4 \times 36)^\circ = 144^\circ.$$

Illustration 5.2

The sides BA and DC of a quadrilateral $ABCD$ are produced as shown in **figure**, prove that $a + b = x + y$.



Sol. Join BD . In $\triangle ABD$, we have

$$\angle ABD + \angle ADB = b^\circ \quad \dots(\text{i})$$

[Exterior angle theorem]

In $\triangle CBD$, we have

$$\angle CBD + \angle CDB = a^\circ \quad \dots(\text{ii})$$

[Exterior angle theorem]

Adding (i) and (ii) we, get

$$(\angle ABD + \angle CBD) + (\angle ADB + \angle CDB) = a^\circ + b^\circ$$

$$\Rightarrow x^\circ + y^\circ = a^\circ + b^\circ$$

Hence, $x + y = a + b$.

Illustration 5.3

In a parallelogram $ABCD$ diagonals AC and BD intersect at O and $AC = 6.8$ cm and $BD = 5.6$ cm. Find the measures of OC and OD .

Sol. Since, the diagonals of a parallelogram bisect each other. Therefore, O is the mid-point of AC and BD .

$$\therefore OC = \frac{1}{2} AC = \frac{1}{2} \times 6.8 \text{ cm} = 3.4 \text{ cm}$$

$$\text{And, } OD = \frac{1}{2} BD = \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm}$$

Illustration 5.4

Given $\triangle ABC$, lines are drawn through A, B and C parallel respectively to the sides BC, CA and AB, forming $\triangle PQR$ show that $BC = \frac{1}{2} QR$.

Sol. We have, $AQ \parallel CB$ and $AC \parallel QB$

\Rightarrow AQBC is parallelogram

$$BC = AQ \quad \dots(i)$$

[\because Opposite sides of a \parallel^m are equal]

Again $AR \parallel BC$ and $AB \parallel RC$

\Rightarrow ARCB is a parallelogram.

\Rightarrow $BC = AR \quad \dots(ii)$ [Opposite sides of \parallel gm are equal] From (i) and (ii), we get
 $AQ = AR$

$$\Rightarrow AQ = AR = \frac{1}{2} QR$$

$$\Rightarrow BC = \frac{1}{2} QR.$$

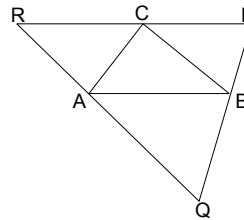


Illustration 5.5

If ABCD is a quadrilateral in which $AB \parallel CD$ and $AD = BC$, prove that $\angle A = \angle B$.

Sol. Extend AB and draw a line CE parallel to AD as shown in **figure**, since $AD \parallel CE$ and transversal AE cuts them at A and E respectively.

$$\therefore \angle A + \angle E = 180^\circ \quad \dots\dots (i)$$

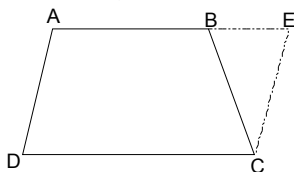
Since $AE \parallel CD$ and $AD \parallel CE$. Therefore, AECD is parallelogram.

$$\Rightarrow AD = CE$$

$$\Rightarrow BC = CE$$

[$\because AD = BC$ (given)]

Thus, in $\triangle BCE$, we have



$$BC = CE$$

$$\Rightarrow \angle CBE = \angle CEB \text{ (} \triangle BCE \text{ is isosceles triangle)}$$

$$\Rightarrow 180 - \angle B = \angle E \quad \dots\dots(ii)$$

$$\Rightarrow 180 - \angle E = \angle B$$

From (i) and (ii), we get

$$\angle A = \angle B$$

Illustration 5.6

In a parallelogram ABCD, $\angle D = 115^\circ$, determine the measure of $\angle A$ and $\angle B$.

Sol. Since the sum of any two consecutive angles of a parallelogram is 180° .

Therefore,

$$\angle A + \angle D = 180^\circ \text{ and } \angle A + \angle B = 180^\circ$$

Now, $\angle A + \angle D = 180^\circ$

$$\angle A + 115^\circ = 180^\circ \quad [\angle D = 115^\circ \text{ (Given)}]$$

$$\angle A = 65^\circ$$

And, $\angle A + \angle B = 180^\circ$

$$65^\circ + \angle B = 180^\circ \quad \angle B = 115^\circ$$

Thus, $\angle A = 65^\circ$ and $\angle B = 115^\circ$

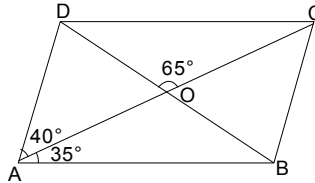
Illustration 5.7

In **fig.**, ABCD is a parallelogram in which $\angle DAO = 40^\circ$, $\angle BAO = 35^\circ$ and $\angle COD = 65^\circ$.

Find:

(i) $\angle ABO$ (ii) $\angle ODC$ (iii) $\angle ACB$ (iv) $\angle CBD$

Sol. Since $\angle AOB$ and $\angle COD$ are vertically opposite angles.



$$\therefore \angle AOB = \angle COD$$

$$\angle AOB = 65^\circ$$

(i) In $\triangle AOB$, we have

$$\angle OAB + \angle AOB + \angle ABO = 180^\circ$$

$$35^\circ + 65^\circ + \angle ABO = 180^\circ$$

$$100^\circ + \angle ABO = 180^\circ$$

$$\angle ABO = 180^\circ - 100^\circ = 80^\circ$$

(ii) Since $\angle ABO$ and $\angle ODC$ are alternate interior angles and alternate interior angles are always equal.

$$\therefore \angle ODC = \angle ABO$$

$$\angle ODC = 80^\circ$$

(iii) Since $\angle ACB$ and $\angle DAC$ are alternate interior angles.

$$\therefore \angle ACB = \angle DAC$$

$$\angle ACB = 40^\circ$$

(iv) Since $\angle A$ and $\angle B$ are adjacent interior angles of parallelogram ABCD and adjacent interior angles are supplementary.

$$\therefore \angle A + \angle B = 180^\circ$$

$$\angle B = 180^\circ - \angle A$$

$$\Rightarrow \angle B = 180^\circ - (40^\circ + 35^\circ) = 105^\circ$$

$$\Rightarrow \angle ABD + \angle CBD = 105^\circ$$

$$\Rightarrow \angle ABO + \angle CBD = 105^\circ$$

$$\Rightarrow 80^\circ + \angle CBD = 105^\circ$$

$$\Rightarrow \angle CBD = 105^\circ - 80^\circ = 25^\circ$$

Illustration 5.8

The ratio of two sides of a parallelogram is as 3 : 5, and its perimeter is 48 m. Find sides of a parallelogram.

Sol. Let the two sides of the parallelogram be $3x$ metres and $5x$ metres in length.

Then,

$$\text{Perimeter} = 2 (\text{length} + \text{breadth})$$

$$\text{Perimeter} = 2 (3x + 5x) \text{ metres}$$

$$= 2 \times 8x \text{ metres}$$

$$= 16x \text{ metres.}$$

But, the perimeter is given as 48 metres.

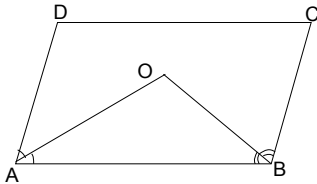
$$\therefore 16x = 48 \quad \Rightarrow \quad \frac{16x}{16} = \frac{48}{16} \quad \Rightarrow \quad x = 3$$

Hence, the sides of the parallelogram are $3 \times 3 \text{ m} = 9 \text{ m}$ and $5 \times 3 \text{ m} = 15 \text{ m}$.

Illustration 5.9

In a parallelogram ABCD, the bisectors of $\angle A$ and $\angle B$ meet at O. Find $\angle AOB$.

Sol. Since OA and OB are the bisectors of $\angle A$ and $\angle B$ respectively.



$$\therefore \angle OAB + \angle AOB + \angle OBA = 180^\circ$$

$$\Rightarrow \frac{1}{2} \angle A + \angle AOB + \frac{1}{2} \angle B = 180^\circ$$

$$\Rightarrow \angle AOB = 180^\circ - \frac{1}{2} (\angle A + \angle B)$$

$$\Rightarrow \angle AOB = 180^\circ - \frac{1}{2} (180^\circ) \quad \left[\because \angle A \text{ and } \angle B \text{ are adjacent angles of } \right. \\ \left. \text{parallelogram ABCD } \therefore \angle A + \angle B = 180^\circ \right]$$

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ = 90^\circ.$$

$$\Rightarrow \angle ABO + \angle CBD = 105^\circ$$

$$\Rightarrow 80^\circ + \angle CBD = 105^\circ$$

$$\Rightarrow \angle CBD = 105^\circ - 80^\circ = 25^\circ$$

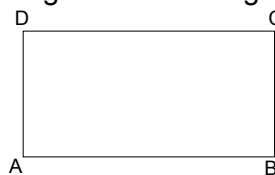
$$\Rightarrow \angle AOB = 180^\circ - \frac{1}{2} (180^\circ) \quad \left[\because \angle A \text{ and } \angle B \text{ are adjacent angles of } \right. \\ \left. \text{parallelogram ABCD } \therefore \angle A + \angle B = 180^\circ \right]$$

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ = 90^\circ.$$

(b) Rectangle

Some properties of rectangles, rhombus and squares have been given in the form of theorems:

Theorem - 6 : Each of the four angles of a rectangle is a right angle.



Given : A rectangle ABCD such that $\angle A = 90^\circ$.

To Prove : $\angle A = \angle B = \angle C = \angle D = 90^\circ$.

Proof : ABCD is a rectangle

ABCD is a parallelogram

$AD \parallel BC$

Now, $AD \parallel BC$ and line AB intersects them at A and B.

$\therefore \angle A + \angle B = 180^\circ$ [\because Sum of the interior angles on the same side of a transversal is 180°]

$\Rightarrow 90^\circ + \angle B = 180^\circ$ [$\because \angle A = 90^\circ$ (Given)]

$\Rightarrow \angle B = 90^\circ$

Similarly, we can show that $\angle C = 90^\circ$ and $\angle D = 90^\circ$

Hence, $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Theorem - 7 : The diagonals of a rectangle are of equal length.

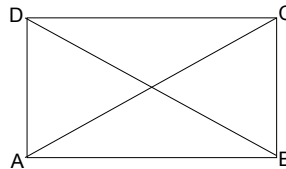
Given : A rectangle ABCD with AC and BD as its diagonals.

To Prove : $AC = BD$.

Proof : ABCD is a rectangle.

\Rightarrow ABCD is a parallelogram such that one of its angles, say, $\angle A$ is a right angle.

$\Rightarrow AD = BC$ and $\angle A = 90^\circ$...(i)



Now, $AD \parallel BC$ and AB intersects them at A and B respectively.

$\therefore \angle A + \angle B = 180^\circ$

$\Rightarrow 90^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = 90^\circ$ [$\because \angle A = 90^\circ$]

Now, in Δ s ABD and BAC, we have

$AB = BA$ [Common side]

$\angle A = \angle B$ [Each equal to 90°]

$AD = BC$ [From (i)]

So, by SAS criterion of congruence

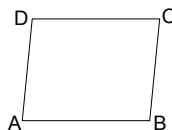
$\Delta ABD \cong \Delta BAC$

$BD = AC$ [\because Corresponding parts of congruent triangles are equal]

Hence, $AC = BD$.

(c) Rhombus

Theorem - 8 : Each of the four sides of a rhombus is of the same length.



Given : A rhombus ABCD such that $AB = BC$

To Prove : $AB = BC = CD = DA$

Proof : ABCD is a rhombus

⇒ ABCD is a parallelogram

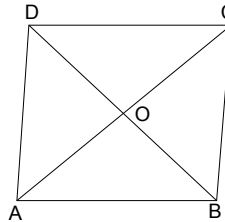
⇒ AB = CD and BC = AD

But AB = BC (Given)

∴ AB = BC = CD = AD

Hence, all the four sides of a rhombus are equal.

Theorem -9 : The diagonals of a rhombus are perpendicular to each other.



Given : A rhombus ABCD whose diagonals AC and BD intersect at O.

To Prove : $\angle BOC = \angle DOC = \angle AOD = \angle AOB = 90^\circ$

Proof : We know that a parallelogram is rhombus, if all of its sides are equal. So, ABCD is a rhombus

⇒ ABCD is a \parallel^m such that AB = BC = CD = DA ... (i)

Since the diagonals of a parallelogram bisect each other.

∴ OB = OD and OA = OC ... (ii)

Now, in Δ s BOC and DOC, we have

BO = OD [From (ii)]

BC = DC [From (i)]

OC = OC [Common]

So, by SSS criterion of congruence

$\Delta BOC \cong \Delta DOC$ [∴ Corresponding parts of congruent triangles are equal]

⇒ $\angle BOC = \angle DOC$

But, $\angle BOC + \angle DOC = 180^\circ$ [Linear pair axiom]

∴ $\angle BOC = \angle DOC = 90^\circ$ [∴ $\angle BOC = \angle DOC$]

Similarly, $\angle AOB = \angle AOD = 90^\circ$

Hence, $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$.

(d) Square

Theorem - 10 : Each of the angles of a square is a right angle and each of the four sides is of the same length.

Given : A square ABCD such that AB = BC

To Prove : AB = BC = CD = DA and $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Proof : ABCD is a square

⇒ ABCD is a rectangle

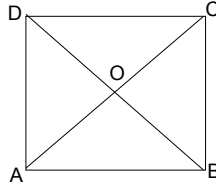
⇒ $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Again, ABCD is a square

⇒ ABCD is a parallelogram such that AB = BC

⇒ AB = BC = CD = AD.

Theorem - 11 : The diagonals of a square are equal and perpendicular to each other.



Given : A square ABCD

To Prove : $AC = BD$ and $AC \perp BD$.

Proof : In $\triangle ADB$ and $\triangle BCA$, we have

$$AD = BC \quad [\because \text{Sides of a square are equal}]$$

$$\angle BAD = \angle ABC \quad [\text{Each equal to } 90^\circ]$$

$$\text{And, } AB = BA \quad [\text{Common}]$$

So, by SAS criterion of congruence

$$\triangle ADB \cong \triangle BCA \quad [\because \text{Corresponding parts of congruent triangles are equal}]$$

$$\Rightarrow AC = BD$$

Now, in $\triangle AOB$ and $\triangle AOD$, we have

$$OB = OD \quad [\because \text{Diagonals of } \square \text{ bisect each other}]$$

$$AB = AD \quad [\because \text{Sides of a square are equal}]$$

$$\text{And, } AO = AO \quad [\text{Common}]$$

So, by SSS criterion of congruence

$$\triangle AOB \cong \triangle AOD \quad [\because \text{Corresponding parts of congruent triangles are equal}]$$

$$\Rightarrow \angle AOB = \angle AOD$$

$$\text{But, } \angle AOB + \angle AOD = 180^\circ$$

$$\therefore \angle AOB = \angle AOD = 90^\circ$$

$$AO \perp BD \Rightarrow AC \perp BD$$

Hence, $AC = BD$ and $AC \perp BD$.

Illustration 5.10

AB, CD are two parallel lines and a transversal l intersects AB at X and CD at Y. Prove that the bisectors of the interior angles form a parallelogram, with all its angles right angles.

Sol. Given : AB, CD are two parallel lines which are cut by a transversal l in points X and Y respectively. The bisectors of interior angles intersect in P and Q.

To Prove : XPYQ is a rectangle.

Proof : Since $AB \parallel CD$ and transversal l intersects them.

$$\therefore \angle AXY = \angle DYX \quad [\text{Alternate angles are equal}]$$

$$\frac{1}{2} \angle AXY = \frac{1}{2} \angle DYX \quad \Rightarrow \angle 1 = \angle 2 \quad [\text{XP and YQ are the bisectors of } \angle AXY \text{ and } \angle DYX \text{ respectively}]$$

Thus, XY intersects PX and QY at X and Y respectively such that $\angle 1 = \angle 2$ i.e. alternate interior angles are equal.

$$PX \parallel QY$$

$$\text{Similarly, } YP \parallel QX$$

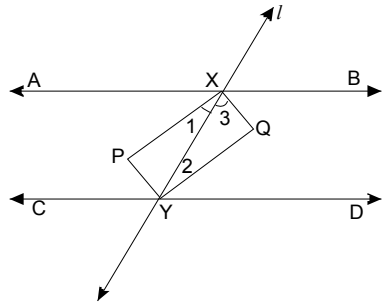
Hence, PYQX is a parallelogram.

Now, we shall show that each angle of the \parallel^{gm} PYQX is right angle.

Since, the sum of the interior angle on the same side of a transversal is 180° .

Therefore,

$$\begin{aligned} \therefore \quad \angle BXY + \angle DYX &= 180^\circ \\ \Rightarrow \quad 2\angle 3 + 2\angle 2 &= 180^\circ \\ \Rightarrow \quad \angle 3 + \angle 2 &= 90^\circ. \end{aligned}$$



Now, in $\triangle XQY$, we have

$$\begin{aligned} \angle 2 + \angle 3 + \angle XQY &= 180^\circ \\ \Rightarrow \quad 90^\circ + \angle XQY &= 180^\circ \\ \Rightarrow \quad \angle XQY &= 90^\circ \end{aligned} \quad \text{[Using (i)]}$$

Since XPYQ is a parallelogram.

$$\begin{aligned} \therefore \quad \angle XQY &= \angle XPY \\ \Rightarrow \quad \angle XPY &= 90^\circ \end{aligned} \quad \text{[}\because \angle XQY = 90^\circ\text{]}$$

$$\text{Now, } \angle PXQ + \angle XQY = 180^\circ \quad \left[\because \text{Adjacent angles in a } \parallel^{\text{gm}} \text{ are supplementary} \right]$$

$$\begin{aligned} \Rightarrow \quad \angle PXQ + 90^\circ &= 180^\circ \\ \Rightarrow \quad \angle PXQ &= 90^\circ \\ \Rightarrow \quad \angle PYQ &= 90^\circ \end{aligned} \quad \text{[}\because \angle PXQ = \angle PYQ\text{]}$$

Hence, all the interior angles are right angles.

Illustration 5.11

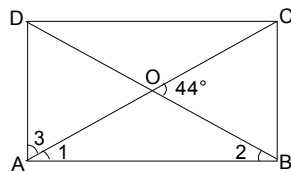
The diagonals of a rectangle ABCD meet at O. If $\angle BOC = 44^\circ$, find $\angle OAD$.

Sol. We have,

$$\begin{aligned} \angle BOC + \angle BOA &= 180^\circ \quad \text{[Linear pairs]} \\ \Rightarrow \quad 44^\circ + \angle BOA &= 180^\circ \\ \Rightarrow \quad \angle BOA &= 136^\circ \end{aligned}$$

Since diagonals of a rectangles are equal and they bisect each other. Therefore, in , we have

$$\begin{aligned} OA &= OB \quad \text{[}\because \text{Angles opp. to equal sides are equal]} \\ \Rightarrow \quad \angle 1 &= \angle 2 \end{aligned}$$



Now, in $\triangle OAB$, we have

$$\begin{aligned} \angle 1 + \angle 2 + \angle BOA &= 180^\circ \\ \Rightarrow \quad 2\angle 1 + 136 &= 180^\circ \end{aligned}$$

$$\Rightarrow 2\angle 1 = 44^\circ$$

$$\Rightarrow \angle 1 = 22^\circ$$

Since each angle of a rectangle is a right angle. Therefore,

$$\angle BAD = 90^\circ$$

$$\Rightarrow \angle 1 + \angle 3 = 90^\circ$$

$$\Rightarrow 22^\circ + \angle 3 = 90^\circ$$

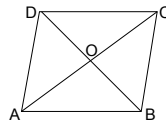
$$\Rightarrow \angle 3 = 68^\circ$$

$$\Rightarrow \angle OAD = 68^\circ$$

Illustration 5.12

The diagonals of a rhombus are 6 cm and 8 cm. Find the length of a side of the rhombus.

Sol. Let ABCD be the rhombus whose diagonals AC and BD are of lengths 8 cm and 6 cm respectively. Let AC and BD intersect at O. Since the diagonals of a rhombus bisect each other at right angles.



$$\therefore AO = \frac{1}{2}AC = \frac{1}{2} \times 8 \text{ cm} = 4\text{cm} \quad \text{and} \quad BO = \frac{1}{2}BD = \frac{1}{2} \times 6\text{cm} = 3 \text{ cm.}$$

Since $\triangle AOB$ is a right triangle, right angled at O. Therefore, by pythagoras theorem

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = 4^2 + 3^2$$

$$\Rightarrow AB^2 = 16 + 9$$

$$\Rightarrow AB = 5$$

Hence, the length of each side of the rhombus is 5 cm.

Illustration 5.13

In figure ABCD is a rectangle. BM and DN are perpendiculars from B and D respectively on AC. Prove that

(i) $\triangle BMC \cong \triangle DNA$

(ii) $BM = DN$

Sol. (i) Since BM and DN are perpendiculars from B and D respectively on AC.

$$\therefore BM \parallel DN$$

Also, $AD \parallel BC$.

$$\angle ADN = \angle CBM.$$

Now in $\triangle s$ ADN and BCM, we have

$$\angle ADN = \angle CBM$$

$$AD = BC$$

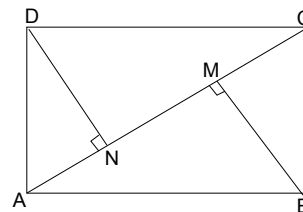
$$\angle DAN = \angle BCM \text{ [Alternate angles as } AD \parallel BC \text{]}$$

So, by ASA congruence condition, we have

$$\triangle BMC \cong \triangle DNA \quad \dots(ii)$$

$$\therefore \triangle BMC \cong \triangle DNA$$

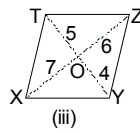
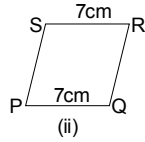
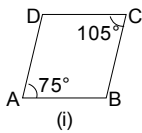
$$\Rightarrow BM = DN$$



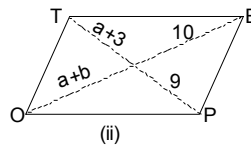
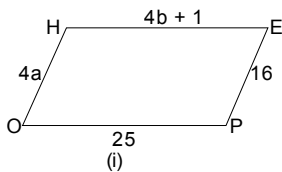
[\therefore Corresponding parts of congruent triangles are equal]

Ask yourself

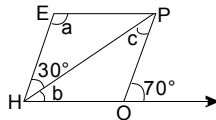
- Adjacent angles of a parallelogram are in 7 : 2. Find all the angles.
- Can the following figures be parallelogram ? Justify your answer.



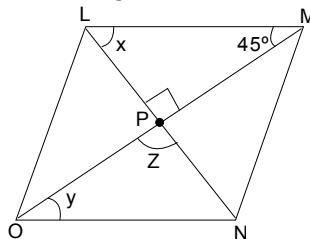
- The following figures HOPE and TOPE are parallelogram. Find 'a' and 'b'.



- In the figure below, HOPE is a parallelogram. Find the measures of angles a, b and c.



- In the given parallelogram, find missing values?



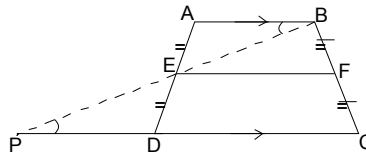
- The perimeter of a square is 40 cm. Find the length of its diagonal?
- ABCD is a rectangle with diagonals AC and BD meeting at point O. Find x if $OA = 5x - 7$ and $OD = 4x - 5$.

Answers

- | | | | | | |
|----|--|------|-----------------|----------|---|
| 1. | $140^\circ, 40^\circ, 140^\circ, 40^\circ$ | 2. | (i) No | (ii) Yes | (iii) No |
| 3. | (i) $a = 4, b = 6$ | (ii) | $a = 6, b = 4$ | 4. | $a = 110^\circ, b = 40^\circ, c = 30^\circ$ |
| 5. | $x = 45^\circ, y = 45^\circ, z = 90^\circ$ | 6. | $10\sqrt{2}$ cm | 7. | 2 |

Add to Your Knowledge

- In the given figure, E and F are respectively, the mid-points of non-parallel sides of a trapezium ABCD.



Then :

- $EF \parallel AB$
- $EF = \frac{1}{2} (AB + DC)$.

- The figure formed by joining the midpoints of the pairs of consecutive sides of a quadrilateral is a parallelogram.
- ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Then the quadrilateral PQRS is a rectangle.

Concept Map

Quadrilaterals
A quadrilateral is four sided closed figure.

Property of parallelogram

- Opposite angle are equal
- Opposite sides are equal
- Diagonal bisect each other
- Diagonal divide parallelogram in to two congruent triangle.

Angle sum property

sum of all interior angle of quadrilateral is 360°

Q. Angle of Quadrilateral are in the ratio $1 : 2 : 3 : 4$ find angle

Sol. Let angle are $x, 2x, 3x, 4x$

$$x + 2x + 3x + 4x = 360$$

$$10x = 360$$

$$x = 36$$

So, angle are $36, 2 \times 36$
 $3 \times 36, 4 \times 36$
i.e. $36^\circ, 72^\circ, 108^\circ, 144^\circ$

Polygon

Sum of interior angle = $(n - 2)180$

Sum of exterior angle = 360°

For Regular polygon

Each Interior angle = $\frac{(n-2)180}{n}$

Each Interior angle = $\frac{360}{n}$

Types of Quadrilateral

Trapezium

Quadrilateral with one pair of opposite side is parallel.

If $AD = BC$, it is known as isosceles trapezium

Parallelogram

Quadrilateral in which both pair of opposite side is parallel.

Kite

Quadrilateral in which adjacent sides equal but unequal opp. side

Rectangle

\parallel^m with all angle 90°

*Length of diagonal are equal

Rhombus

\parallel^m with all side equal

*diagonal bisect each other at 90°

Square

\parallel^m with all side equal and all angle 90°

*Length of diagonal are equal

*Diagonal bisect at 90°

Q. ABCD is \parallel^m find all angle of \parallel^m

Sol. $\angle C = \angle A = 50^\circ$

$$\therefore \angle C = 50^\circ$$

as $AB \parallel CD$

$$\therefore \angle A + \angle D = 180^\circ$$

$$50^\circ + \angle D = 180^\circ$$

$$\angle D = 130^\circ$$

$$\angle B = \angle D = 130^\circ$$

$$\therefore \angle B = 130^\circ$$

Q. If ABCD is \parallel^m find x,y

Sol. In \parallel^m diagonal bisect each other

$$\therefore OA = OC$$

$$x - 5 = 7$$

$$x = 12$$

and $OB = OD$

$$15 = x + y$$

$$15 = 12 + y$$

$$y = 3$$

Summary

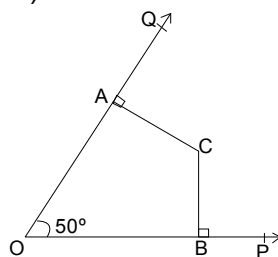
1. A quadrilateral two of whose opposite sides are parallel is called a trapezium.
2. A quadrilateral in which opposite sides are parallel is called a parallelogram.
3. A parallelogram with a pair of adjacent sides equal is called a rhombus. In fact, all the sides of a rhombus are equal.
4. A parallelogram with one angle a right angle is called a rectangle. In fact, all the angles of a rectangle are right angles.
5. A parallelogram with a pair of adjacent sides equal and one angle a right angle is called a square. In fact, all the sides of a square are equal and all its angles are right angles.
6. In a parallelogram,
 - (i) opposite sides are equal,
 - (ii) opposite angles are equal, and
 - (iii) diagonals bisect each other.
7. Diagonals of a rhombus bisect each other at right angles.
8. Diagonals of a rectangle are equal and bisect each other.
9. Diagonals of a square are equal and bisect each other at right angles

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

- If all the angles of a quadrilateral are less than 180° , then the quadrilateral is a :
 (A) Convex quadrilateral (B) Parallelogram
 (C) Concave quadrilateral (D) Trapezium
- If one angle of a quadrilateral is greater than 180° , then the quadrilateral is a :
 (A) Concave quadrilateral (B) Trapezium
 (C) Rectangle (D) Convex quadrilateral
- If the opposite sides and the opposite angles of a quadrilateral are equal, then the quadrilateral is a :
 (A) Trapezium (B) Concave quadrilateral
 (C) Convex quadrilateral (D) parallelogram
- A quadrilateral whose opposite sides and all the angles are equal is a :
 (A) Square (B) Rectangle (C) Rhombus (D) Parallelogram
- A quadrilateral whose all the sides, diagonals and angles are equal is a :
 (A) Square (B) Rhombus (C) Trapezium (D) Rectangle
- If the adjacent angles of a parallelogram are equal, then the parallelogram is a :
 (A) Trapezium (B) Rectangle (C) Rhombus (D) All of these
- If the diagonals of a quadrilateral are equal and bisect each other (not at right angles), then the quadrilateral is a :
 (A) Square (B) Rhombus (C) Parallelogram (D) Rectangle
- If the diagonals of a quadrilateral bisect each other at right angles, then it is a :
 (A) Trapezium (B) Parallelogram (C) Rectangle (D) Rhombus
- A quadrilateral whose all the sides and opposite angles are equal and the diagonals bisect each other at right angles is a :
 (A) Square (B) Rhombus (C) Rectangle (D) Parallelogram
- The quadrilateral having only one pair of opposite sides parallel is called a :
 (A) Kite (B) Rhombus (C) Trapezium (D) Parallelogram
- The measure of $\angle BCA$ (in **figure**) :



- (A) 180° (B) 130° (C) 110° (D) 108°

12. The sum of adjacent angles of a parallelogram is :
 (A) 180° (B) 120° (C) 360° (D) 90°
13. In a quadrilateral ABCD, $\angle A = 35^\circ$, $\angle B = 65^\circ$, $\angle C = 65^\circ$ the $\angle D$ is :
 (A) 100° (B) 120° (C) 195° (D) 180°
14. Adjacent angles of a parallelogram are in the ratio of 2 : 7, their values will be :
 (A) 20, 160° (B) 30, 150° (C) 40, 140° (D) 60, 120°
15. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4, the angles are :
 (A) 36° , 72° , 108° , 144° (B) 15° , 130° , 45° , 150°
 (C) 45° , 110° , 55° , 150° (D) None of these
16. ABCD is a parallelogram and E is the mid point of BC. DE and AB when produced meet at F. Then,
 (A) $AF = \frac{3}{2} AB$ (B) $AF = 2AB$ (C) $AF = 3AB$ (D) $AF^2 = 2AB^2$

FILL IN THE BLANKS

- A quadrilateral in which the measure of each interior angles is less than 180° is called a _____ quadrilateral.
- A _____ is a quadrilateral with only one pair of opposite sides parallel.
- A _____ is a quadrilateral in which two pairs of adjacent sides are equal and pairs of adjacent unequal sides.
- A diagonal of a parallelogram divides it two _____ triangles.
- The diagonal of a rhombus are _____ to each other.
- A _____ is a parallelogram having all sides equal and each angle equal to right angle.
- A quadrilateral in which the measure at one of interior angle is greater than 180° is called a _____ quadrilateral.
- A line segment joining the opposite vertices of a quadrilateral are called its _____.
- A trapezium is said to be an _____ trapezium, if its non-parallel sides are equal.
- A _____ is a parallelogram but all its sides are equal in length.

TRUE / FALSE

- If all the angles of a quadrilateral are equal, it is a rectangle.
- The adjacent angles of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.
- In a parallelogram, the diagonals are equal.
- The diagonals of a rectangle are of equal length.
- The diagonals of a square are equal and perpendicular to each other.
- The diagonals of a rhombus are perpendicular bisectors.

8. In a convex quadrilateral all the angle is greater than 180° .
9. A kite is a quadrilateral in which two pair of adjacent sides are equal.
10. In a quadrilateral sum of all interior angle is 360.
11. In a concave quadrilateral all the angle is greater than 180° .
12. In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.
13. If three sides of a quadrilateral are equal, it is a parallelogram.

MATCH THE COLUMN

- | 1. Column – I | Column – II |
|--|-----------------------|
| (A) A quadrilateral in which both pairs of opposite side are parallel | (p) kite |
| (B) A quadrilateral whose all the sides are equal and each angle is 90° | (q) two right |
| (C) Sum of all angles of quadrilateral | (r) parallelogram |
| (D) Sum of the angles on a straight line | (s) square |
| (E) A quadrilateral in which two pairs of adjacent side are equal. | (t) four right angles |

SECTION -B (FREE RESPONSE TYPE)

SUBJECTIVE QUESTIONS

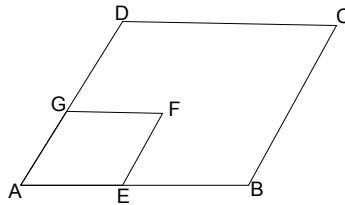
VERY SHORT ANSWER TYPE

1. If an angle of a parallelogram is two third of its adjacent angle, find the angles of the parallelogram.
2. Three angles of a quadrilateral are equal and the fourth angle is equal to 144° . Find each of the equal angle of the quadrilateral.
3. Two opposite angles of a parallelogram are $(3x - 2)^\circ$ and $(50 - x)^\circ$. Find the measure of each angle.
4. The sides of a rectangle are in the ratio 4 : 5. Find its sides if the perimeter is 90 cm.
5. Find the length of the diagonal of a rectangle whose sides are 12 cm and 5 cm.

SHORT ANSWER TYPE

6. In quadrilateral PQRS if $\angle P = 60^\circ$ and $\angle Q : \angle R : \angle S = 2 : 3 : 7$, then find the measure of $\angle S$.
7. PQRS is a trapezium in which $PQ \parallel RS$. If $\angle P = \angle Q = 50^\circ$, what are the measures of the other two angles?

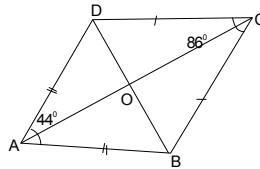
8. The perimeter of a parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram.
9. In **figure**, ABCD and AEGF are each a parallelogram. If $\angle C = 55^\circ$, what is the measure of $\angle F$?



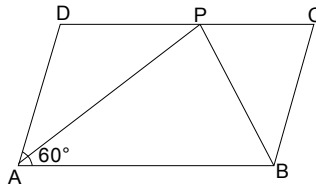
10. EFGH is a square. $\angle E = x + 60$ and $EF = x + 1$ cm. Find the perimeter of EFGH.
11. Diagonal AC of a rhombus ABCD is equal to one of its side BC. Find all the angles of the rhombus.

LONG ANSWER TYPE

12. In figure, ABCD is a kite whose diagonals intersect at O. If $\angle DAB = 44^\circ$ and $\angle BCD = 86^\circ$
Find : (i) $\angle ODA$ (ii) $\angle OBC$



13. The diagonals of a quadrilateral are of lengths 6 cm and 8 cm. If the diagonals bisect each other at right angles, what is the length of each side of the quadrilateral ?
14. In **figure**, ABCD is a parallelogram in which $\angle A = 60^\circ$. If the bisectors of $\angle A$ and $\angle B$ meet at P, prove that $AD = DP$, $PC = BC$ and $DC = 2AD$.



15. The diagonals of a ||gm PQRS intersect at O. A line through O intersects PQ at M and RS at N. Prove that $OM = ON$.

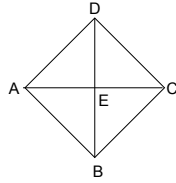
Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

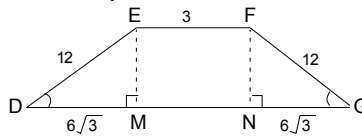
OBJECTIVE QUESTIONS

1. The ratio of two sides of a parallelogram is as 3 : 5, and its perimeter is 48 m, then the sides of the parallelogram is :
 (A) 9 m, 15 m (B) 3 m, 5 m (C) 33 m, 25 m (D) None of these

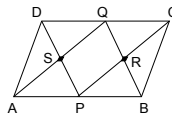
2. In the figure, parallelogram ABCD is composed of four congruent triangles. If $BE = 3$ cm and $CE = 4$ cm then the perimeter of the entire figure is :



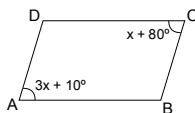
- (A) 20 cm (B) 25 cm (C) 22 cm (D) None of these
3. The diagonals of a square with area 9 m^2 divide the square into four non-overlapping triangles. What is the sum of the perimeter of the four triangles ?
- (A) 12 m (B) $12\sqrt{2}$ m (C) $12 + 12\sqrt{2}$ m (D) none of these
4. In given figure, area of isosceles trapezium DEFG is :



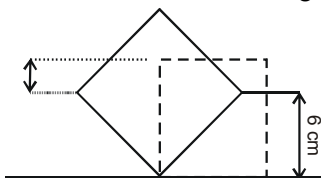
- (A) $18(1 + \sqrt{3})$ (B) $18\sqrt{3}$ (C) $\sqrt{3} + 1$ (D) $18(1 + 2\sqrt{3})$
5. In fig. ABCD is a parallelogram. P and Q are mid points of the sides AB and CD, respectively. Then PRQS is :



- (A) Parallelogram (B) Trapezium (C) Rectangle (D) None of these
6. In the given figure ABCD is parallelogram. Then, find the value of x if $\angle A = 3x + 10$ and $\angle C = x + 80^\circ$.

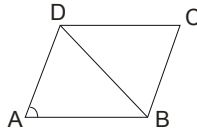


- (A) 40° (B) 35° (C) 60° (D) 115°
7. The diagonals of a parallelogram ABCD intersect each other at the point O. If $\angle DAC = 32^\circ$ and $\angle AOB = 70^\circ$, then $\angle DBC$ is :
- (A) 24° (B) 32° (C) 38° (D) 86°
8. A square board side 10 centimeters, standing vertically, is tilted to the left so that the bottom-right corner is raised 6 centimeters from the ground.



- By what distance is the top-left corner lowered from its original position ?
- (A) 1 cm (B) 2 cm (C) 3 cm (D) 0.5 cm
9. A quadrilateral ABCD has four angles x° , $2x^\circ$, $\frac{5x^\circ}{2}$ and $\frac{7x^\circ}{2}$ respectively. What is the difference between the value of biggest and the smallest angles.
- (A) 40° (B) 100° (C) 80° (D) 20°

10. Diagonal DB of a rhombus ABCD is equal to one of its sides.



The values of $\angle A$ is :

- (A) 30° (B) 60° (C) 120° (D) 90°

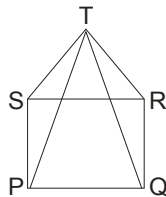
SECTION -B (TECHIE STUFF)

11. LMNO is a trapezium with $LM \parallel NO$. If P and Q are the mid-points of LO and MN respectively and $LM = 5$ cm and $ON = 10$ cm, then $PQ =$
 (A) 2.5 cm (B) 5 cm (C) 7.5 cm (D) 15 cm
12. The figure formed by joining the midpoints of the pairs of consecutive sides of a rectangle is a
 (A) kite (B) rectangle (C) rhombus (D) trapezium

Exercise-3

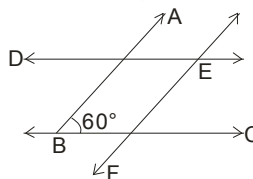
(PREVIOUS YEAR EXAMINATION QUESTIONS)

1. In the figure PQRS is a square and SRT is an equilateral triangle. Then find $\angle TQR$
 [Aryabhata 2002]



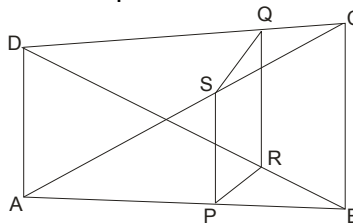
- (A) 60° (B) 90° (C) 30° (D) 15°

2. In figure $AB \parallel EF$, $DE \parallel BC$ & $\angle ABC = 60^\circ$, find $\angle DEF$
 [Aryabhata 2002]



- (A) 60° (B) 120° (C) 30° (D) 80°

3. In the given figure, points P, Q, R and S are respectively the mid points of side AB, side CD, diagonal BD and diagonal AC of quadrilateral ABCD. The quadrilateral PRQS is a

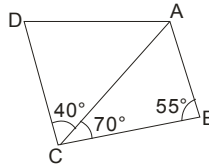


[Aryabhata-2004]

- (A) rectangle (B) rhombus (C) square (D) parallelogram

4. In the diagram $DA = CB$ what is the measure of $\angle DAC$?

[NSTSE - 2009]



- (A) 70° (B) 100° (C) 95° (D) 125°

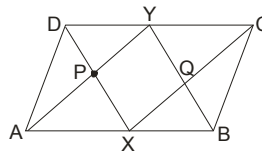
5. Each angle of a rectangle is bisected. Let P, Q, R and S be the points of intersection of the pairs of bisectors adjacent to the same side of the rectangle. Then PQRS is a

[NSTSE - 2009]

- (A) rectangle
 (B) rhombus
 (C) parallelogram with unequal adjacent sides
 (D) quadrilateral with no special property

6. X, Y are the mid points of opposite sides AB and DC of a parallelogram ABCD. AY and DX are joined intersecting in P, CX and BY are joined intersecting in Q. The PXQY is

[NSTSE - 2010]



- (A) rectangle (B) rhombus (C) parallelogram (D) square

7. Of all quadrilaterals of a given perimeter, which has the largest area ? [Aryabhata 2010]

- (A) square (B) rectangle (C) parallelogram (D) rhombus

8. ABCD is a parallelogram. The angle bisectors of $\angle A$ and $\angle D$ meet at O. The measure of $\angle AOD$ is ____.

(IMO 2010)

- (A) 45° (B) 90°
 (C) Depends on the angles A and D (D) Not able to determine from given data

9. The diagonal of a rectangle is thrice its smaller side. The ratio of its sides is

- (A) $\sqrt{2} : 1$ (B) $2\sqrt{2} : 1$ (C) $3 : 2$ (D) $\sqrt{3} : 1$

10. In a quadrilateral ABCD, $AB \parallel CD$ and $AD = BC = 7$ cm. If $\angle A = 70^\circ$ then the measure of $\angle C$ is

[Aryabhata-2011]

- (A) 70° (B) 100° (C) 80° (D) 110°

11. Smallest angle of a triangle is equal to two-third the smallest angle of a quadrilateral. The ratio of the angles of the quadrilateral is $3 : 4 : 5 : 6$. Largest angle of the triangle is twice its smallest angle. What is the sum of second largest angle of the triangle and largest angle of the quadrilateral ?

(IMO 2011)

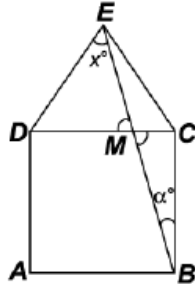
- (A) 160° (B) 180° (C) 190° (D) 170°

12. Which of the following statements is INCORRECT?

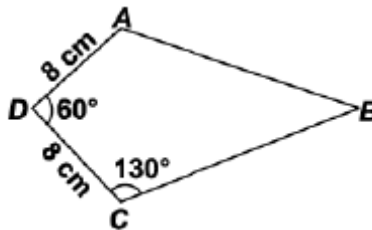
(IMO 2011)

- (A) All rhombuses are parallelograms. (B) All squares are parallelograms.
 (C) All rectangles are not squares. (D) All squares are trapeziums.

13. A quadrilateral that is not a parallelogram but has exactly two equal opposite angles is **[NSTSE - 2012]**
 (A) a rhombus (B) a trapezium (C) a square (D) a kite
14. Find the measure of largest angle of a quadrilateral if the measures of its interior angles are in the ratio of 3 : 4 : 5 : 6. **(IMO 2012)**
 (A) 60° (B) 120° (C) 90° (D) Can't be determined
15. In the given diagram, equilateral triangle EDC surmounts square ABCD. Find $\angle BED$ represented by x , where $\angle EBC = \alpha^\circ$. **(IMO 2012)**



- (A) 45° (B) 60° (C) 30° (D) None of these
16. In the kite ABCD, $AD = CD = 8$ cm, $\angle ADC = 60^\circ$, $\angle DCB = 130^\circ$ and $AB = CB$. Find $\angle ABC$.



- (A) 50° (B) 40° (C) 60° (D) 25° **(IMO 2012)**
17. In a parallelogram, **(IMO 2012)**
 Statement 1 : Diagonals bisect each other.
 Statement 2 : Diagonals divide the parallelogram into two triangles.
 (A) Only statement 1 is true. (B) Only statement 2 is true.
 (C) Both statement 1 and 2 are true. (D) Both statement 1 and 2 are false.

Answer Key

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| Ans. | A | A | D | B | A | B | D | D | B | C | B | A | C | C | A | B |

FILL IN THE BLANKS

- | | | | |
|------------------|--------------|------------|--------------|
| 1. convex | 2. trapezium | 3. kite | 4. Congruent |
| 5. perpendicular | 6. square | 7. Concave | 8. Diagonal |
| 9. isosceles | 10. rhombus | | |

TRUE / FALSE

- | | | | |
|-----------|----------|-----------|-----------|
| 1. True | 2. False | 3. True | 4. False |
| 5. True | 6. True | 7. True | 8. False |
| 9. True | 10. True | 11. False | 12. False |
| 13. False | | | |

MATCH THE COLUMN

1. (A) – r, (B) – s, (C) – t, (D) – q, (E) – p

SECTION -B (FREE RESPONSE TYPE)

SUBJECTIVE QUESTIONS

VERY SHORT ANSWER TYPE

- | | | |
|---|---------------|---|
| 1. $108^\circ, 72^\circ, 108^\circ, 72^\circ$ | 2. 72° | 3. $37^\circ, 143^\circ, 37^\circ, 143^\circ$ |
| 4. 20 cm, 25 cm | 5. 13 cm | |

SHORT ANSWER TYPE

- | | | |
|-----------------------------|---------------------------|--|
| 6. $\angle S = 175^\circ$. | 7. $130^\circ, 130^\circ$ | 8. 50 cm, 25 cm |
| 9. 55° | 10. 124 cm | 11. $120^\circ, 60^\circ, 120^\circ, 60^\circ$ |

LONG ANSWER TYPE

12. (i) 68° (ii) 47° 13. 5 cm.

Exercise-2**SECTION -A (COMPETITIVE EXAMINATION QUESTION)****OBJECTIVE QUESTIONS**

| | | | | | | | | | | | | |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|-----------|-----------|
| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Ans. | A | A | D | D | A | B | C | B | B | B | C | C |

Exercise-3**PREVIOUS YEAR EXAMINATION QUESTIONS**

| | | | | | | | | | | | | | | | | | |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Ques. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| Ans. | D | A | D | A | A | C | A | B | B | D | B | D | D | B | A | B | A |