MATHEMATICS

Class-VIII

Topic-8

ALGEBRAIC EXPRESSION

CH-08

ALGEBRAIC EXPRESSION

TERMINOLOGIES

Literals, constant, variables, terms, binomial, trinomial, algebraic expression, factors, coeffiicients, multinomial, polynomial, like terms, unlike terms, degree, exponent, factor method, monomial.

INTRODUCTION

In this class, we will study various definitions of literals, constant and variable, factors, degree of an algebraic expressions various operations like addition, Subtraction, multiplication and division of algebraic expression.

8.1 VARIOUS DEFINITIONS & CONCEPTS

(a) Literals

The letters which are used to represent numbers are called literal numbers or **literals**. In 2xy, x & y are the literals.
Literal numbers obey all the rules (and signs) of addition, subtraction, multiplication and division of numbers along with the properties of these operations. $a \times b = ab$, $2 \times a = 2a$, $1 \times a = a$, $x \times 3 = 3x$ and $a \times a \times a \times ......\times a$ (15 times) $= a^{15}$.
In a^5, 5 is called the index or exponent and 'a' is called the base.

(b) Constant

A term of the expression having no literal factor is called a **constant** term.

(i) In the binomial expression 5x + 7, the constant term is 7. In short, a symbol having a fixed numerical value is called a constant.

(ii) In the trinomial expression $x^2 - y^2 - \frac{3}{2}$, the constant term is $-\frac{3}{2}$.

(c) Variable

A symbol which takes various numerical values is called a **variable**.

(d) Algebraic expression

A combination of constants and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an **algebraic expression**.

(e) Terms, factors and coefficients

Take the expression 3x – 5y + 8. This expression is made up of three terms, 3x, – 5y and 8. Terms are added to form expressions. Terms themselves can be formed as the product of factors. The 3x is the product of its factors 3 and x. The numerical factor of term is called its numerical coefficients or simply coefficients. The coefficients in the term 3x is 3 and the coefficient in the term – 5y is – 5.

(f) Types of algebraic expressions

(i) **Monomial :** An expression is said to be a monomial if it has only one term.
For example : x, $9x^2$, $-5x^2$ are all monomials.

(ii) **Binomial :** An expression is said to be a binomial if it contains two terms.
For example : $2x^2 + 3x$, $\sqrt{3}\,x + 5x^4$, $-8x^3 + 3$ etc. are all binomials.

(iii) **Trinomial :** An expression is said to be a **trinomial** if it contains three terms.
For example : $3x^3 - 8x + \frac{5}{2}$, $\sqrt{7}\,x^{10} + 8x^4 - 3x^2$ etc. are all trinomials.

(iv) **Multinomial :** An expression is said to be a **multinomial** if it contains more than three terms. All these monomial, binomial, trinomial, multinomial are called **polynomial.**

(g) Like and unlike terms

Terms having same combination of literal numbers are called like terms otherwise unlike terms. for example,

(i) In the algebraic expression : $12a^2 - 15b^2 + b^2 - 17a^2 + 8ab + 9$, we have, $12a^2$ and $-17a^2$ as like terms. Also, $-15b^2$ and b^2 are like terms.

(ii) In the algebraic expression : $3p^2q + 5pq^2 - 7pq - 9qp^2$, $5pq^2$ and $-7pq$ are unlike terms.

(h) Degree of an algebraic expression

If an algebraic expression contain only one variable say x, then it is called an **algebraic expression** in one variable and its degree is the highest exponent (degree) of that variable in the expression.

For example : $2x^3 + 3x^2 - 6x + 4$. The highest power of x in all terms of polynomial is 3. Hence, the degree of the polynomial is 3.
If an algebraic expression contains two variables say x and y, then it is called an algebraic expression in two variables and its degree is the highest exponent (power) of the term obtained by adding the exponents of the variables.

For example : $8xy - 7y^3 + 9y^2x^2$ is an algebraic expression in x and y. Its degree is 4 which is the highest of all the terms.
An algebraic expression in x is said to be in **standard form** when the terms are written either in increasing order or decreasing order of the powers of x in various terms.

Ask yourself

1. Identify the like terms of the expression $3x^2y - 10xy^2 - \frac{5}{6}x^2y + 2x^2y - 7y$

2. What is the coefficient of xyz in $13\,xy^2z^2$?

3. Give an example of a binomial?

4. The expression $5y + 3y^2 + 12y^2 - 7y$ is a
(a) Monomial **(b)** Binomial **(c)** Trinomial **(d)** Polynomial

5. What is an expression containing 3 terms called ?

Answers

1. $3x^2y$, $-\frac{5}{6}x^2y$, $2x^2y$ **2.** $13\,yz$ **3.** $2x + 3$

4 Binomial **5.** Trinomial

8.2 OPERATIONS

(a) Addition and Subtraction of algebraic expressions

For addition of two algebraic expression, first we arrange both the algebraic expression in standard form and then we add the coefficients of like powers of variable. For example suppose we have to find the sum of two algebraic expression $f(x) = x + x^2 + 1$ and $g(x) = 2x^2 - 4x + 3$, for addition first we arrange both the algebraic expression in standard form as follows $f(x) = x^2 + x + 1$, $g(x) = 2x^2 - 4x + 3$.

Now $f(x) + g(x) = (x^2 + x + 1) + (2x^2 - 4x + 3)$ after arranging both the algebraic expression as above we find the sum of coefficients of like power of x as follows :

$f(x) + g(x) = (1 + 2)x^2 + (1 - 4)x + (1 + 3) = 3x^2 - 3x + 4$.

Thus we get the required sum.

NOTE:

The process of subtraction is same as addition. In subtraction after arrangement, we find the difference of coefficients of like powers of variable.

For example :

$f(x) - g(x) = (x^2 + x + 1) - (2x^2 - 4x + 3) = (1 - 2)x^2 + (1 + 4)x + (1 - 3) = -x^2 + 5x - 2$.

Illustration 8.1

Find the sum of f(x) & g(x) where,

$f(x) = 4x^5 + 3x^3 + 4x^2 + x + 1$ & $g(x) = 5x^4 + x^5 + x^3 + 3$.

Sol. Arrange in standard form

$f(x) = 4x^5 + 3x^3 + 4x^2 + x + 1$ or $f(x) = 4x^5 + 0.x^4 + 3x^3 + 4x^2 + x + 1$

and $g(x) = x^5 + 5x^4 + x^3 + 3$ or $g(x) = x^5 + 5x^4 + x^3 + 0.x^2 + 0.x + 3$

$f(x) + g(x) = (4x^5 + 0.x^4 + 3x^3 + 4x^2 + x + 1) + (x^5 + 5x^4 + x^3 + 0.x^2 + 0.x + 3)$

$f(x) + g(x) = (4 + 1)x^5 + (0 + 5)x^4 + (3 + 1)x^3 + (4 + 0)x^2 + (1 + 0)x + (1 + 3)$

$= (5x^5 + 5x^4 + 4x^3 + 4x^2 + x + 4)$.

Illustration 8.2

Subtract g(x) from f(x) where $f(x) = 2 + x^2 + 4x^3$, $g(x) = x^4 + x^2 + 3x + 5$.

Sol. $f(x) = 4x^3 + x^2 + 0.x + 2 = 0.x^4 + 4x^3 + x^2 + 0.x + 2$

$g(x) = x^4 + 0.x^3 + x^2 + 3x + 5$

$f(x) - g(x) = (0.x^4 + 4x^3 + x^2 + 0.x + 2) - (x^4 + 0x^3 + x^2 + 3x + 5)$

$f(x) - g(x) = (0 - 1)x^4 + (4 - 0)x^3 + (1 - 1)x^2 + (0 - 3)x + (2 - 5)$

$= -x^4 + 4x^3 + 0.x^2 - 3x - 3 = -x^4 + 4x^3 - 3x - 3$.

Illustration 8.3

Subtract h(x) from the sum of f(x) & g(x)

where $f(x) = x^3 + x^2 + x + 1$, $g(x) = 2x^3 - 3x^2 + 1$, $h(x) = 3x^2 - 4x^3 + 5x + 7$.

Sol. $f(x) = x^3 + x^2 + x + 1$; $g(x) = 2x^3 - 3x^2 + 0.x + 1$

$h(x) = -4x^3 + 3x^2 + 5x + 7$

$f(x) + g(x) - h(x) = (x^3 + x^2 + x + 1) + (2x^3 - 3x^2 + 0.x + 1) - (-4x^3 + 3x^2 + 5x + 7)$

$f(x) + g(x) - h(x) = (1 + 2 + 4)x^3 + (1 - 3 - 3)x^2 + (1 + 0 - 5)x + (1 + 1 - 7)$

$= 7x^3 - 5x^2 - 4x - 5$

(b) Multiplication of Algebraic Expression

To get the product of two algebraic expression, carry out the following steps :

Step (i) Multiply each term of the first algebraic expression with each term of the second algebraic expression.

Step (ii) Add all the products obtained is step I.

Illustration 8.4

Find the product of $(x + y)$ & $(x^2 + xy + y^2)$

Sol. **Step (i)** $(x + y)(x^2 + xy + y^2) = x(x^2 + xy + y^2) + y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 + yx^2 + xy^2 + y^3$

Step (ii) Product $= x^3 + 2x^2y + 2xy^2 + y^3$ (Grouping like terms)

Thus, the product $= x^3 + 2x^2y + 2xy^2 + y^3$.

Illustration 8.5

Find the product of f(x) & g(x) where $f(x) = (x^2 + 2x + 2)$, $g(x) = 3x^2 - x - 1$.

Sol. **Step (i)**

$f(x) . g(x) = (x^2 + 2x + 2)(3x^2 - x - 1)$

$= x^2(3x^2 - x - 1) + 2x(3x^2 - x - 1) + 2(3x^2 - x - 1)$

$= 3x^4 - x^3 - x^2 + 6x^3 - 2x^2 - 2x + 6x^2 - 2x - 2$

Step (ii)

$f(x).g(x) = 3x^4 + (-1 + 6)x^3 + (-1 - 2 + 6)x^2 + (-2 - 2)x - 2 = 3x^4 + 5x^3 + 3x^2 - 4x - 2$

(c) Division of Algebraic Expression

Division algorithm :

Dividend = quotient × divisor + remainder.

The process of division of an algebraic expression by another algebraic expression is also same as we use in number system. The process of division may be divided in three cases in polynomials.

(i) Division of a monomial by another monomial.

(ii) Division of a polynomial by monomial.

(iii) Division of a polynomial by another polynomial.

We have two methods for division of a polynomial by another polynomial.

(a) Factor method : Factor method is generally used when remainder is zero.

(b) Long division : This method generally used when remainder is not zero. This method can also be used when remainder is zero. We shall discuss these three cases one by one.

Case1. Division of a monomial by another monomial

Illustration 8.6

Divide : **(i)** $15a^5$ by $5a^3$ **(ii)** $36a^3b^5$ by $-12a^2b$ **(iii)** $2x^3$ by $\sqrt{2}\,x$

Sol. **(i)** Quotient $= \frac{15a^5}{5a^3} = \left(\frac{15}{5}\right)\left(\frac{a^5}{a^3}\right) = 3a^2$

(ii) Quotient $= \dfrac{36a^3b^5}{-12a^2b} = \left(\dfrac{36}{-12}\right)\left(\dfrac{a^3}{a^2}\right)\left(\dfrac{b^5}{b}\right) = -3ab^4$

(iii) Quotient $= \dfrac{2x^3}{\sqrt{2}x} = \left(\dfrac{2}{\sqrt{2}}\right)\left(\dfrac{x^3}{x}\right) = \sqrt{2}\ x^2$

Case 2. Division of a polynomial by a monomial.

Divide each term of the polynomial by the monomial and then write the resulting quotients with their proper signs.

Illustration 8.7

Divide : $-4x^3 - 6x^2 + 8x$ by $2x$.

Sol. Dividing each term of the dividend by the divisor, we get

Quotient $= \dfrac{-4x^3 - 6x^2 + 8x}{2x} = \dfrac{-4x^3}{2x} - \dfrac{6x^2}{2x} + \dfrac{8x}{2x} = -2x^2 - 3x + 4$

Case 3. Division of a polynomial by another polynomial.

It is advisable in this case to rearrange the dividend and the divisor in descending order of powers of variable .

Illustration 8.8

Divide $x^2 + 5x + 6$ by $x + 3$.

Sol.

$$\begin{array}{r|l} x+3 & x^2 + 5x + 6 \ (x+2 \\ & x^2 + 3x \\ & -\quad - \\ \hline & \qquad 2x + 6 \\ & \qquad 2x + 6 \\ & \qquad -\quad - \\ \hline & \qquad\quad 0 \end{array}$$

Quotient = $x + 2$
Remainder = 0

❖ **EXPLANATION :**

(i) Divide the first term (x^2) of the dividend by the first term (x) of the divisor. The result $x^2 \div x = x$ is the first term of the quotient.

(ii) Multiply the divisor $x + 3$ by x, the first term of the quotient.

(iii) Subtract the product $(x + 3)\ x = x^2 + 3x$ from the dividend $x^2 + 5x + 6$. i.e. $(x^2 + 5x + 6) - (x^2 + 3x) = 2x + 6$

(iv) Proceed with this remainder $2x + 6$ as with the original dividend i.e., divide $2x$ by x, The result $2x \div x = 2$ is the second term of the quotient.

(v) Multiply the divisor $(x + 3)$ by 2, the second term of the quotient.
Now subtract $2(x + 3)$ from $2x + 6$ i.e., $2x + 6 - 2(x + 3) = 2x + 6 - 2x - 6 = 0$. The remainder is 0.
Hence the required quotient = $x + 2$.

Illustration 8.9

Divide $2x^3 + x^2 – 3x – 3$ by $2x – 1$ and verify your answer.

Sol.

$$2x - 1 \overline{\big) 2x^3 + x^2 - 3x - 3 \big(} \; x^2 + x - 1$$

$$\begin{array}{l} 2x^3 - x^2 \\ - \quad + \\ \hline 2x^2 - 3x \\ 2x^2 - x \\ - \quad + \\ \hline -2x - 3 \\ -2x + 1 \\ + \quad - \\ \hline -4 \\ \hline \end{array}$$

Quotient = $x^2 + x – 1$, Remainder = $– 4$

❖ **VERIFICATION :**

Dividend = Divisor × Quotient + Remainder

$= (2x – 1)\ (x^2 + x –1) + (–4)$

$= 2x^3 + x^2 – 3x + 1 + (–4)$

$= 2x^3 + x^2 – 3x + 1 + (–4)$

$= 2x^3 + x^2 – 3x – 3.$

Hence, the answer is correct.

❖ **REMARK :**

When the dividend and the divisor are polynomials of one variable, the degree of the polynomial in the remainder is always less than the degree of the polynomial of the divisor.

Illustration 8.10

Divide $3x^4 + 5x^3 – x^2 + 13x + 9$ by $3x + 2$ and verify that :

Dividend = Divisor Quotient + Remainder

Sol. First we divide $3x^4 + 5x^3 – x^2 + 13x + 9$ by $3x + 2$.

$$3x + 2 \overline{\big) 3x^4 + 5x^3 - x^2 + 13x + 9 \big(} \; x^3 + x^2 - x + 5$$

$$\begin{array}{l} 3x^4 + 2x^3 \\ - \quad - \\ \hline 3x^3 - x^2 \\ 3x^3 + 2x^2 \\ - \quad - \\ \hline -3x^2 + 13x \\ -3x^2 - 2x \\ + \quad + \\ \hline 15x + 9 \\ 15x + 10 \\ - \quad - \\ \hline -1 \\ \hline \end{array}$$

Quotient = $x^3 + x^2 – x + 5$, Remainder = $–1$

Now, divisor Quotient + Remainder

$= (3x + 2)\ (x^2 + x^2 – x + 5) – 1$

$= 3x\ (x^3 + x^2 – x + 5) + 2\ (x^3 + x^2 – x + 5) – 1$

$= 3x^4 + 3x^3 – 3x^2 + 15x + 2x^3 + 2x^2 – 2x + 10 – 1$

$= 3x^4 + 5x^3 – x^2 + 13x + 9$

= Dividend

NOTE :

When the remainder is zero, the divisor is called a factor of the dividend.

Illustration 8.11

Find the value of a if $2x - 3$ is a factor of $2x^4 - x^3 - 3x^2 - 2x + a$.

Sol. First we divide $2x^4 - x^3 - 3x^2 - 2x + a$ by $2x - 3$.

$2x - 3)\ 2x^4 - x^3 - 3x^2 - 2x + a\ (\ x^3 + x^2 - 1$

$2x^4 - 3x^3$

$-\quad +$

$2x^3 - 3x^2$

$2x^3 - 3x^2$

$-\quad +$

$-2x + a$

$-2x + 3$

$+\quad -$

$a - 3$

$2x - 3$ is a factor of $2x^4 - x^3 - 3x^2 - 2x + a$ if, $a - 3 = 0$. Hence, $a = 3$.

Ask yourself

1. Add the following :

(i) $3x^2y^2 - 4xy + 4,\ 6 + 8xy - 4x^2y^2$

(ii) $ab - bc,\ bc - ca,\ ca - ab$

(iii) $6x^2 - 5y^2 + 7y - 4\ ,\ 8x^2 - 5xy + 9y^2 + 6x - 4y$

(iv) $2xy + 5yz - 8zx\ ,\ 4xy - yz - xz + 9xyz$

2. Find the product of the following :

(i) $-3x,\ 8yz$ **(ii)** $\frac{5}{3}p^2qr \times (-3pqr^2) \times \left(\frac{1}{5}pq^2r\right)$

3. Carry out of the following divisions

(i) $16x^6y^6 \div (-8x^4y^2)$ **(ii)** $(-x^5y^9) \div (-xy^4)$

4. Divide the given polynomials by the given monomials $a^2 + 2ab + ac$ by a

5. Divide as directed : $(m^2 - 14m - 32) \div (m + 2)$

6. What should be added to $15x + 13xy + 5y^2$ to obtain $-7x + 12xy - 6y^2$?

7. Multiply $(9x^2 - 2xy)$ and $(5xy^2 + 3y)$?

Answers

1. **(i)** $-x^2y^2 + 4xy + 10$ **(ii)** 0 **(iii)** $14x^2 + 4y^2 + 6x + 3y - 5xy - 4$

(iv) $6xy + 4yz - 9zx + 9xyz$

2. **(i)** $-24xyz$ **(ii)** $-p^4q^4r^4$ **3. (i)** $-2x^2y^4$ **(ii)** x^4y^5

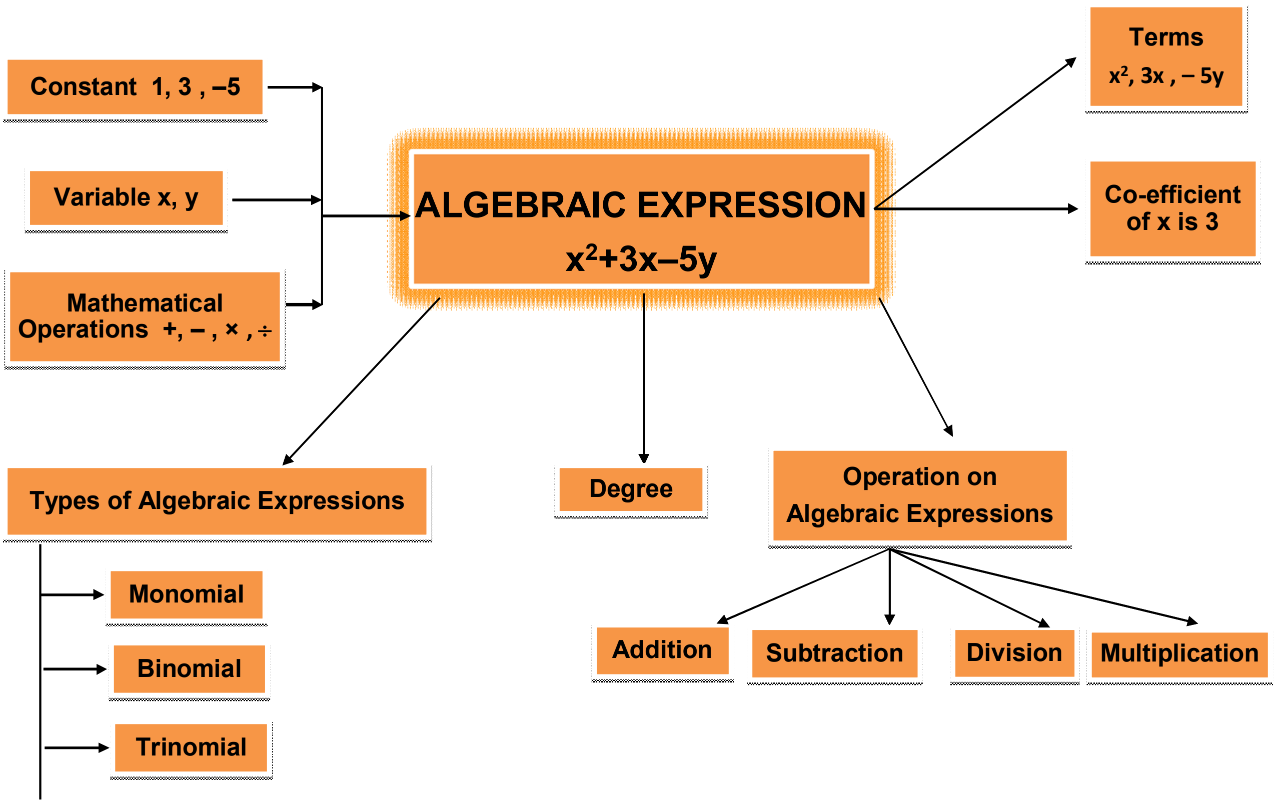
4. $a + 2b + c$ **5.** $(m - 16)$ **6.** $-22x - xy - 11y^2$

7. $45x^3y^2 + 27x^2y - 10x^2y^3 - 6xy^2$

Add your knowledge

1. Let 'p(x)' be any polynomial of degree greater than or equal to one and **a** be any real number and if p(x) is divided by (x – a), then the remainder is equal to p(a).

Concept Map



Summary

1. A term is a constant, a variable or a combination of constants and variables.

2. An algebraic expression is a single term or a combination of two or more terms, connected by symbols (+) or (–).

3. Algebraic expressions in which the variables involved have only non-negative integral exponents are called polynomials.

4. The highest exponent of the variable in various terms of a polynomial in one variable is called its degree.

5. The standard form of a polynomial in one variable is that in which the terms of the polynomials are written in the decreasing order of the exponents of the variable.

6. If on dividing a polynomial (dividend) by a polynomial (divisor), a zero remainder is obtained, then the divisor is a factor of the dividend. In such cases, quotient is also a factor of the dividend. Further.

Dividend = Divisor × Quotient.

7. In general , Dividend = Divisor × Quotient + Remainder

8. The degree of the remainder is always less than the degree of the divisor.

9. Before performing long division, the divisor and the dividend must be written in the standard form.

10. While performing long division, like terms are written one below the other, leaving gaps whenever necessary.

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

1. Find the coefficient of z^3 in $-7xy^2z^3$.
(A) xy^2 (B) $-7xy^2$ (C) $7xy^2$ (D) -7

2. In the following, which pair contains like terms ?
(A) $2xy, -3xy$ (B) $2x^2y, 2xy^2$ (C) $6, 6x$ (D) $2xy, 3xyz$

3. Which of the following is a trinomial.
(A) $x^2 + 2x + x$ (B) $3x^3 + 2x^2 - x^2$ (C) xyz (D) $x^2 + x + 1$

4. The degree of the polynomial $\frac{x^2 + 8x^3 + 2x + 6x^4}{2x}$ is :
(A) 3 (B) 2 (C) 4 (D) Does not exists

5. Which of the following polynomial is in the standard form.
(A) $x^3 - 2x^4 + x - 1$ (B) $x^8 - x^5 + 1$ (C) $-ax^3 + bx + cx^2$ (D) $7x^{-2} + x^{-1} + 3$

6. Simplify the following : $2x^2 + 3y^2 - 5xy + 5x^2 - y^2 + 6xy - 3x^2$.
(A) $7x^2 + 4y^2 - 11xy$ (B) $7x^2 + 2y^2 - 30xy$ (C) $4x^2 + 2y^2 - 11xy$ (D) $4x^2 + xy + 2y^2$

7. The product of $3xy$ and $-7x^2yz^3$ is :
(A) $21xyz$ (B) $-21xyz$ (C) $21x^3y^2z^3$ (D) $-21x^3y^2z^3$

8. By how much is $a^4 + 4a^2b^2 + b^4$ more than $a^4 - 8a^2b^2 + b^4$.
(A) $12a^2b^2$ (B) $-12a^2b^2$ (C) $2a^4 + 2b^4$ (D) None of these

9. If we divide $x^5 + x^3 + 2x + 1$ by $x^3 + x + 1$, then degree of remainder will be :
(A) 3 (B) Greater than 3 (C) Less than 3 (D) None of these

10. The remainder obtained when $t^6 + 3t^2 + 10$ is divided by $t^3 + 1$ is :
(A) $t^2 - 11$ (B) $3t^2 + 11$ (C) $t^3 - 1$ (D) $1 - t^3$

FILL IN THE BLANKS

1. ____________ = quotient × divisor + remainder.

2. When the dividend and the divisor are polynomials of one variable, the degree of the polynomial in the remainder is always _________ than the degree of the polynomial of the divisor.

3. When the remainder is zero, the divisor is called a _________ of the dividend.

4. The degree of the polynomial $\frac{8x^4 - 3x^2 + 5x^3 - x}{2x}$ is.

5. The coefficient of xy^2 in $-5x^2y^3$ is _________ .

TRUE / FALSE

1. $5x^2 - x^3 + x^2$ is a trinomial

2. $x + x^{-1}$ is a polynomial

3. Coefficient of ab in $-12a^2b$ is $-12a$

4. Degree of $\frac{8x^2 - 3x}{2x}$ is one

5. Degree of a polynomial cannot be negative

MATCH THE COLUMN

1.

	Column–I		Column–II
(A)	$(-4x^5y^3z^2) \div (8x^2yz)$	(p)	$2x^3y^2z$
(B)	$(4x^3y)\left(\frac{-1}{2}zy\right)$	(q)	$\frac{1}{2}x^3y^2z$
(C)	$8x^3y^2z - 6x^3y^2z$	(r)	$-2x^3y^2z$
(D)	HCF of $5x^3y^3z^3$ and $8x^4y^2z$	(s)	$\frac{-1}{2}x^3y^2z$
(E)	$\frac{1}{3}x^3y^2z + \frac{1}{6}x^3y^2z$	(t)	x^3y^2z

2.

	Column–I		Column–II
(A)	A binomial of degree 3 is	(p)	$\frac{8}{2x}$
(B)	A monomial of degree zero is	(q)	$x^4 - x^2 + 3$
(C)	A trinomial of degree 4 is	(r)	2^5
(D)	An algebraic expression which is not a polynomial is	(s)	$2x^3 + 9$

SECTION -B (FREE RESPONSE TYPE)

SUBJECTIVE QUESTIONS

VERY SHORT ANSWER TYPE

1. Write the polynomial $x^6 - 3x^4 + \sqrt{2}\,x + 5x^2 + 7x^5 + 4$ in standard form.

2. Write the polynomial $m^7 + 8m^5 + 4m^6 + 6m - 3m^2 - 11$ in standard form.

3. Add 8ab, – 5ab, 3ab, – ab

4. Add $5x - 8y + 2z$, $3x - 2y - 7z$, $x + 5y + 3z$

5. Subtract $7p^2 - 3q + 8r$ from $12p^2 - 5q + 7r$.

6. Subtract $8x^3 + 3x - 7$ from $3x - 9$.

SHORT ANSWER TYPE

7. The perimeter of triangle is $12a + 13b - 7c$ and two of its side are $3a + 7c$ and $4a - 3b + 5c$. Find the third side.

8. Multiply :

(i) $3x^3(5x^5 - 7x + 8)$ **(ii)** $(5x - 7)(3x + 2)$

(iii) $(x - 1)(x^2 + x + 1)$ **(iv)** $(x + \frac{1}{x})(x - \frac{1}{x})$

9. Divide $9x - 6x^2 + x^3 - 2$ by $(x - 2)$ and hence verify the division algorithm.

10. Show that $(x - 1)$ is the factor of $(x^3 - 1)$.

11. If $(x + 5)$ is a a factor of $x^3 + 2x^2 - 14x + k$, then find the value of k.

LONG ANSWER TYPE

12. Divide the polynomial $3y^4 - y^3 + 12y^2 + 2$ by $3y^2 - 1$ and hence find the quotient & remainder.

13. Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.

14. Find the value of K so that $(x - 3)$ is a factor of $3x^2 - 11x + K$.

15. Show that $(x - 2)$ is a factor of $2x^3 + x^2 - 7x - 6$

16. If $x = -2$, $y = 1$ then find the value of $(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$.

17. Divide $12x^3 - 8x^2 - 6x + 10$ by $(3x - 2)$. Also, write the quotient and the remainder.

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

OBJECTIVE QUESTIONS

1. The value of $25x^2 + 16y^2 + 40xy$ at $x = 1$ and $y \;\; -1$ is

(A) 81 (B) -49 (C) 1 (D) None of these

2. The coefficient of x^2 in $(3x^2 - 5)(4 + 4x^2)$ is

(A) 12 (B) 5 (C) -8 (D) 8

3. Find the value of k if $(x+2)$ is the factor of $2x^3+3x^2-5x+2+k$

(A) 12 (B) 5 (C) -8 (D) 8

4. Find the value of a if $2x+3$ is a factor of $6x^3+19x^2+13x+a$

(A) 12 (B) 5 (C) -3 (D) 8

5. If we divide $3y^4 - y^3 + 12y^2 + 2$ by $3y^2 - 1$ then remainder is :

(A) $\frac{y}{3} + \frac{19}{3}$ (B) $-\frac{1}{3}y + \frac{19}{3}$ (C) $-\frac{1}{3}y - \frac{19}{3}$ (D) $\frac{1}{3}y - \frac{19}{3}$

6. The area of a rectangle is given by A = $8x^2 - 2x - 15$. If the length be larger than the breadth, it is:-
(A) $3x + 5$ (B) $5x + 3$ (C) $4x + 5$ (D) $2x - 3$

7. Evaluate : $x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6xz$. If $x = 8, y = -7, z = 6$.
(A) 1200 (B) 500 (C) 1600 (D) 900

8. The value of $\dfrac{\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^b}$ is equal to :
(A) $\left(\frac{x}{y}\right)^{a+b}$ (B) $\left(\frac{y}{x}\right)^{a+b}$ (C) $(xy)^{a+b}$ (D) $\frac{x^a}{y^b}$

9. If $x + 2 = 0$, then the value of $\dfrac{3(x+3)\ \ (x+1)}{x-1}$ is :
(A) 2 (B) 3 (C) 4 (D) 1

10. If $P = a^3 - 4b^3 + 3a^2b$, $Q = -4a^3 + 13a^2b + 7b^3$, $R = -4a^2b + 8b^3 + 3a^3$ and $S = 12a^2b - 5b^3 + 9a^3$, then $P - Q + R - S$ is equal to :
(A) $-a^3 + 2b^3 - 26a^2b$ (B) $a^3 - 2b^3 + 26a^2b$
(C) $a^3 + 2b^3 - 26a^2b$ (D) $-a^3 - 2b^3 - 26a^2b$

SECTION -B (TECHIE STUFF)

11. Find α and β if $x + 1$ and $x + 2$ are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.
(A) $\alpha = -1, \beta = 0$ (B) $\alpha = 0, \beta = 0$
(C) $\alpha = 1, \beta = 1$ (D) $\alpha = 1, \beta = 0$

12. The polynomial $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by $x + 2$ if the remainder in each case is same, find the value of a ?
(A) 0 (B) 5/9 (C) 4/9 (D) 1

13. If $x^{51} + 51$ is divided by $(x + 1)$ the remainder is
(A) 0 (B) 1 (C) 49 (D) 50

14. The remainder obtained when the polynomial p(x) is divided by $(b - ax)$ is
(A) $p\left(\frac{-b}{a}\right)$ (B) $p\left(\frac{a}{b}\right)$ (C) $p\left(\frac{b}{a}\right)$ (D) $p\left(\frac{-a}{b}\right)$

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

1. Each of the numbers 1, 2, 3 and 4 is subsituted in some order for p, q, r and s. The greatest possible value of $p^q + r^s$ is **[NSTSE - 2009]**
(A) 14 (B) 19 (C) 66 (D) 83

2. When simplified and expressed with negative exponents, the expression $(x + y)^{-1} (x^{-1} + y^{-1})$ is equal to **[NSTSE - 2010]**

(A) $x^{-2} + 2x^{-1} y^{-1} + y^{-2}$
(B) $x^{-2} + 2^{-1} x^{-1}y^{-1} + y^{-2}$
(C) $x^{-1} y^{-1}$
(D) $x^{-2} + y^{-2}$

3. The product of x^2y and $\frac{x}{y}$ is equal to the quotient obtained when x^2 is divided by **[NSTSE - 2010]**

(A) 0
(B) 1
(C) x
(D) $\frac{1}{x}$

4. What is the value of $3(x^2 - 4x)$ when $x = 5$? **[IMO - 2010]**

(A) 5
(B) 15
(C) 30
(D) 55

5. Find the product of $(x^2 + 3x + 5)$ and $(x^2 – 1)$ **[NSTSE - 2012]**

(A) $x^4 + 3x^3 – 4x^2 – 3x – 5$
(B) $x^4 + 3x^2 + 4x^2 – 3x – 8$
(C) $x^4 + 3x^3 + 4x^2 – 3x – 5$
(D) $x + 3x^3 + 4x^2 – 3x + 5$

6. The cost of a notebook is Rs. $3a^2 – 4ab – 6b^2$. How much does $5a^2b^2$ notebooks cost ? **[NSTSE - 2014]**

(A) Rs. $15a^4b^2 – 20a^3b^3 + 30 a^2b^4$
(B) Rs. $15a^4b^2 – 20a^3b^3 + 30 a^2b^4$
(C) Rs. $15 a^4b^2 – 20 a^3b^3 – 30 a^2b^4$
(D) Rs. $15 a^4b^2 + 20 a^3b^3 – 30 a^2b^4$

Answer Key

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	D	A	B	D	D	A	C	B

FILL IN THE BLANKS

1. Dividend **2.** less **3.** factor **4.** three

5. $-5xy$

TRUE / FALSE

1. False **2.** False **3.** True **4.** True

5. True

MATCH THE COLUMN

1. (A) – (s); (B) – (r); (C) – (p); (D) – (t); (E) – (q)

2. (A) – (s); (B) – (r); (C) – (q); (D) – (p)

SECTION -B (FREE RESPONSE TYPE)

SUBJECTIVE QUESTIONS

VERY SHORT ANSWER TYPE

1. $x^6 + 7x^5 - 3x^4 + 5x^2 + \sqrt{2}\ x + 4$ **2.** $m^7 + 4m^6 + 8m^5 - 3m^2 + 6m - 11$

3. $5ab$ **4.** $9x-5y-2z$ **5.** $5p^2-2q-r$ **6.** $-8x^3-2$

SHORT ANSWER TYPE

7. $5a+16b-19c$

8. **(i)** $15x^8-21x^4+24x^3$ **(ii)** $15x^2-11x-14$

(iii) x^3-1 **(iv)** $x^2 - \frac{1}{x^2}$

9. $q = x^2-4x+1,\ r = 0$ **11.** $k = 5$

LONG ANSWER TYPE

12. remainder = $-y/3 + 19/3.$, quotient = $y^2 - \frac{y}{3} + \frac{13}{3}$ **13.** $-\frac{35}{8}$

14. $k = 6$ **16.** -46592 **17.** quotient = $4x^2 - 2$, remainder = 6

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	C	C	C	C	B	C	C	A	D	A	A	B	D	C

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

Ques.	1	2	3	4	5	6
Ans.	D	C	D	B	C	C