MATHEMATICS

Class-VIII

Topic-9

ALGEBRAIC IDENTITIES

INDEX

CH-09

ALGEBRAIC IDENTITIES

TERMINOLOGIES

Identity, factorisation, factors, monomial, binomial, quadratic, trinomial, perfect square.

INTRODUCTION

In the previous chapter, we have discussed about the multiplication of algebraic expressions by various methods. In this unit, we will discuss various identities for multiplication of algebraic expressions.

9.1 ALGEBRAIC IDENTITIES

An identity is an equality, which is true for all values of the variables. The following three identities are very important.

Identity 1 : $(a + b)^2 = a^2 + 2ab + b^2$

Proof : we have :

$$(a + b)^2 = (a + b)(a + b)$$
$$= a(a + b) + b(a + b)$$
$$= a^2 + ab + ba + b^2$$
$$= a^2 + 2ab + b^2 \quad \text{[Since ba = ab]}$$

$\therefore (a + b)^2 = a^2 + 2ab + b^2$.

Identity 2 : $(a - b)^2 = a^2 - 2ab + b^2$.

Proof : we have

$$(a - b)^2 = (a - b)(a - b)$$
$$= a(a - b) - b(a - b)$$
$$= a^2 - ab - ba + b^2$$
$$= a^2 - ab - ab + b^2 \quad \text{[Since ba = ab]}$$
$$= a^2 - 2ab + b^2.$$

$\therefore (a - b)^2 = (a^2 - 2ab + b^2)$

Identity 3 : $(a + b)(a - b) = a^2 - b^2$

Proof We have :

$$(a + b)(a - b) = a(a - b) + b(a - b)$$
$$= a^2 - ab + ba - b^2$$
$$= a^2 - b^2 \quad \text{[Since ba = ab]}$$

$\therefore (a + b)(a-b) = a^2 - b^2$.

(a) Applications of the above identities

Illustration 9.1

Find each of the following products :

(i) $(3x + 2y)(3x + 2y)$ **(ii)** $(4x^2 + 5)(4x^2 + 5)$

(iii) $(2x - 5y)^2$ **(iv)** $(3x^2 + 2y^2)(3x^2 - 2y^2)$

Sol. **(i)** $(3x+2y)(3x+2y) = (3x+2y)^2 = (3x)^2 + (2y)^2 + 2(3x)(2y)$

[Using $(a+b)^2 = a^2 + b^2 + 2ab$]

$= 9x^2 + 4y^2 + 12xy.$

$\therefore (3x+2y)(3x+2y) = 9x^2 + 4y^2 + 12xy.$

(ii) $(4x^2+5)(4x^2+5) = (4x^2+5)^2 = (4x^2)^2 + 5^2 + 2(4x^2)5$ [Using $(a+b)^2 = a^2 + b^2 + 2ab$]

$= 16x^4 + 25 + 40x^2.$

(iii) $(2x-5y)^2 = (2x)^2 + (5y)^2 - 2(2x)(5y)$ [using $(a-b)^2 = a^2 + b^2 - 2ab$]

$= 4x^2 + 25y^2 - 20xy.$

(iv) $(3x^2+2y^2)(3x^2-2y^2) = (3x^2)^2 - (2y^2)^2$ [$\therefore (a+b)(a-b) = (a^2-b^2)$]

$= (9x^4 - 4y^4).$

Illustration 9.2

Evaluate the following, using identities :

(i) $(105)^2$ **(ii)** $(47)^2$ **(iii)** (8.3×7.7)

Sol. We have:

(i) $(105)^2 = (100+5)^2$

$= (100)^2 + (5)^2 + 2 \times 100 \times 5$

$= 10000 + 25 + 1000 = 11025.$

(ii) $(47)^2 = (50-3)^2$

$= (50)^2 + (3)^2 - 2(50)(3)$

$= 2500 + 9 - 300 = 2209.$

(iii) $(8.3 \times 7.7) = (8+0.3)(8-0.3)$

$= (8)^2 - (0.3)^2$

$= 64 - 0.09 = 63.91.$

Illustration 9.3

Find the value of the expression $25x^2 + 9y^2 + 30xy$. when $x = 8$ and $y = 10$.

Sol. $25x^2 + 9y^2 + 30xy = (5x)^2 + (3y)^2 + 2(5x)(3y)$

$= (5x+3y)^2 = (5 \times 8 + 3 \times 10)^2$ [when $x = 8$ and $y = 10$]

$= (40+30)^2 = (70)^2 = 4900.$

Illustration 9.4

Find the value of the expression $(81x^2 + 16y^2 - 72xy)$, when $x = \frac{2}{3}$ and $y = \frac{3}{4}$.

Sol. $(81x^2 + 16y^2 - 72xy) = (9x)^2 + (4y)^2 - 2 \times 9x \times 4y = (9x-4y)^2$

$= \left(9 \times \frac{2}{3} - 4 \times \frac{3}{4}\right)^2$ [when $x = \frac{2}{3}$ and $y = \frac{3}{4}$]

$= (6-3)^2 = (3)^2 = 9.$

Illustration 9.5

If $x + \frac{1}{x} = 5$, find the values of :

(i) $x^2 + \frac{1}{x^2}$ **(ii)** $x^4 + \frac{1}{x^4}$

Sol. **(i)** $x + \frac{1}{x} = 5 \Rightarrow \left(x + \frac{1}{x}\right)^2 = (5)^2$ [on squaring both sides]

$\Rightarrow x^2 + \frac{1}{x^2} + 2(x) \times \frac{1}{x} = 25$ $\Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = (25 - 2)$ $\Rightarrow x^2 + \frac{1}{x^2} = 23$

(ii) $x^2 + \frac{1}{x^2} = 23$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (23)^2$ [on squaring both sides]

$\Rightarrow x^4 + \frac{1}{x^4} + 2(x^2) \times \frac{1}{x^2} = 529$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 529$ $\Rightarrow x^4 + \frac{1}{x^4} = (529 - 2)$

$\Rightarrow x^4 + \frac{1}{x^4} = 527$

Ask yourself

1. Expand the following :

(i) $(10x - 3)^2$ **(ii)** $\left(6x + \frac{y}{3}\right)^2$

2. What least value should be added to the following algebraic expressions to make them perfect squares ?
$36x^2 - 65xy + 25y^2$

3. Find the square of
(i) $12x^2 - 13y^2$ **(ii)** $9x^2 + 15y$

4. Simplify & using identities : $\frac{(3.72)^2 - (1.96)^2}{(3.72 + 1.96)}$

5. If $x - \frac{1}{x} = 5$, find the value of

(i) $x^2 + \frac{1}{x^2}$ **(ii)** $x^4 + \frac{1}{x^4}$

6. Show that
(i) $(4x + 5y)^2 - (4x - 5y)^2 = 80xy$
(ii) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Answers

1. **(i)** $100x^2 - 60x + 9$ **(ii)** $36x^2 + \frac{y^2}{9} + 4xy$ **2.** $5xy$

3. **(i)** $144x^4 + 169y^4 - 312x^2y^2$ **(ii)** $81x^4 + 225y^2 + 270x^2y$

4. 1.76 **5.** **(i)** 27 **(ii)** 727

9.2 FACTORISATION

Factors : When an algebraic expression can be written as the product of two or more expressions, then each of these expressions is called a factor of the given expression.

Factorisation : The process of finding two or more expressions whose product is the given expression is called **factorisation**.

NOTE :

Factorisation is the reverse process of multiplication.

Example :
Look at the examples given below :

Product	Factorisation
(i) $3x(4x - 5y) = 12x^2 - 15xy$	$12x^2 - 15xy = 3x(4x - 5y)$
(ii) $(x+3)(x-2) = x^2 + x - 6$	$x^2 + x - 6 = (x + 3)(x - 2)$
(iii) $(2a + 3b)(2a - 3b) = 4a^2 - 9b^2$	$4a^2 - 9b^2 = (2a + 3b)(2a - 3b)$

(a) Factorisation when a common monomial factor occurs in each term

METHOD :

Step 1. Find the HCF of all the terms of the given expression.

Step 2. Divide each term of the given expression by this HCF.

Step 3. Write the given expression as the product of this HCF and the quotient obtained in step 2.

Illustration 9.6

Factorise each of the following :

(i) $3x^2y - 6xy^2$ **(ii)** $6ab - 9bc$ **(iii)** $25x^2 + 35x^3y - 20xy^3$

Sol. **(i)** $3x^2y - 6xy^2 = 3xy(x - 2y)$

(ii) $6ab - 9bc = 3b(2a - 3c)$

(iii) $25x^3 + 35x^2y - 20xy^3 = 5x(5x^2 + 7xy - 4y^3)$

(b) Factorisation when a binomial is common

METHOD :

Step 1. Find the common binomial.

Step 2. Write the given expression as the product of this binomial and the quotient obtained on dividing the given expression by this binomial.

Illustration 9.7

Factorise : $5a(2x - 3y) + 2b(2x - 3y)$.

Sol. $5a(2x - 3y) + 2b(2x - 3y) = (2x - 3y)(5a + 2b)$.

(c) Factorisation by grouping

Sometimes the terms of the given expression need to be arranged in suitable groups in such a way that all the groups have a common factor. After this arrangement factorisation becomes easy.

METHOD

Step 1. Arrange the terms of the given expression in groups in such a way that all the groups have a common factor.

Step 2. Factorise each group.

Step 3. Take out the factor which is common to each group.

Illustration 9.8

Factorise :

(i) $px + qy + py + qx$ **(ii)** $1 + a + ac + a^2c$

Sol. **(i)** $px + qy + py + qx = px + py + qx + qy$

$= p(x + y) + q(x + y)$

$= (x + y)(p + q)$.

(ii) $1 + a + ac + a^2c = 1(1 + a) + ac(1 + a)$

$= (1 + a)(1 + ac)$.

(d) Factorisation when a binomial is the difference of two square.

In this case, we use the formula :

$(a^2 - b^2) = (a + b)(a - b)$

Illustration 9.9

Factorise :

(i) $49x^2 - 16y^2$ **(ii)** $64 - x^2$ **(iii)** $48a^2 - 243b^2$ **(iv)** $3x^3 - 48x$

Sol. **(i)** $49x^2 - 16y^2 = (7x)^2 - (4y)^2 = (7x + 4y)(7x - 4y)$.

(ii) $64 - x^2 = (8)^2 - x^2 = (8 + x)(8 - x)$.

(iii) $48a^2 - 243b^2 = 3(16a^2 - 81b^2) = 3\{(4a)^2 - (9b)^2\} = 3(4a + 9b)(4a - 9b)$.

(iv) $3x^3 - 48x = 3x(x^2 - 16) = 3x(x + 4)(x - 4)$.

Illustration 9.10

Factorise : $25(x + y)^2 - 36(x - 2y)^2$

Sol. $25(x + y)^2 - 36(x - 2y)^2$

$= \{5(x + y)\}^2 - \{6(x - 2y)\}^2$

$= \{5(x + y) + 6(x - 2y)\}\{5(x + y) - 6(x - 2y)\}$

$= (11x - 7y)(17y - x)$.

Illustration 9.11

Factorise : $4x^2 - y^2 + 6y - 9$.

Sol. $4x^2 - y^2 + 6y - 9$

$= 4x^2 - (y^2 - 6y + 9)$

$= (2x)^2 - (y - 3)^2$

$= (2x + y - 3)\{2x - (y - 3)\}$

$= (2x + y - 3)(2x - y + 3)$.

Illustration 9.12

Evaluate :

(i) $(502)^2 - (498)^2$ **(ii)** $(8.6)^2 - (1.4)^2$

Sol. **(i)** $(502)^2 - (498)^2 = (502 + 498)(502 - 498)$

$= (1000 \times 4) = 4000$

(ii) $(8.6)^2 - (1.4)^2 = (8.6 + 1.4)(8.6 - 1.4) = (10 \times 7.2) = 72.$

(e) Factorisation when the given expression is a perfect square

We know that **(i)** $a^2 + b^2 + 2ab = (a + b)^2$

(ii) $a^2 + b^2 - 2ab = (a - b)^2$.

Illustration 9.13

Factorise :

(i) $x^2 + 10x + 25$ **(ii)** $x^2 - 20x + 100$

(iii) $4x^2 + 9y^2 + 12xy$ **(iv)** $x^4 + 25y^4 - 10x^2y^2$

Sol. **(i)** $x^2 + 10x + 25 = x^2 + (5)^2 + 2(x)(5) = (x + 5)^2$

(ii) $x^2 - 20x + 100 = x^2 + (10)^2 - 2(x)(10) = (x - 10)^2$

(iii) $4x^2 + 9y^2 + 12xy = (2x)^2 + (3y)^2 + 2(2x)(3y) = (2x + 3y)^2$

(iv) $x^4 + 25y^4 - 10x^2y^2 = (x^2)^2 + (5y^2)^2 - 2(x^2)(5y^2) = (x^2 - 5y^2)^2$

(f) Factorisation of quadratic trinomials

First Form : $x^2 + px + q$.

Suppose we are given a quadratic trinomial $x^2 + px + q$

Then, we use the identity :

$x^2 + (a + b)x + ab = (x + a)(x + b)$

In order to factorise $x^2 + px + q$, we find two numbers 'a' and 'b' such that $(a + b) = p$ and $ab = q$.

Then, $x^2 + px + q = x^2 + (a + b)x + ab = (x + a)(x + b)$.

Illustration 9.14

Factorise :

(i) $x^2 + 8x + 15$ **(ii)** $x^2 + 15x + 56$ **(iii)** $x^2 + x - 56$

Sol. **(i)** The given expression is $x^2 + 8x + 15$

Find two number whose sum = 8 and product = 15

Clearly, the number are 5 and 3.

$\therefore x^2 + 8x + 15 = x^2 + 5x + 3x + 15$

$= x(x + 5) + 3(x + 5)$

$= (x + 5)(x + 3)$.

(ii) The given expression is $x^2 + 15x + 56$.

Find two numbers whose sum = 15 and product = 56

Clearly, such numbers are 8 and 7.

$\therefore x^2 + 15x + 56 = x^2 + 8x + 7x + 56$

$= x(x + 8) + 7(x + 8)$

$= (x + 8)(x + 7)$.

(iii) The given expression is $x^2 + x - 56$.
Find two numbers whose sum = 1 and product = – 56
Clearly, such numbers are 8 and – 7
$\therefore x^2 + x - 56 = x^2 + 8x - 7x - 56 = x(x + 8) - 7(x + 8) = (x + 8)(x - 7)$.

Second Form : $ax^2 + bx + c$.
In this case, split it into two parts whose sum = b and product = ac. Now proceed as in the first case.

Illustration 9.15

Factorise:

(i) $2x^2 + 9x + 10$ **(ii)** $6x^2 + 7x - 3$ **(iii)** $15x^2 - 26x + 8$

(iv) $3x^2 - 4x - 4$

Sol. **(i)** The given expression is $2x^2 + 9x + 10$.
Find two numbers whose sum = 9 and product = $(2 \times 10) = 20$.
Clearly, such numbers are 5 and 4.
$\therefore 2x^2 + 9x + 10 = 2x^2 + 5x + 4x + 10 = x(2x + 5) + 2(2x + 5) = (2x + 5)(x + 2)$.

(ii) The given expression is $6x^2 + 7x - 3$.
Find two numbers whose sum = 7 and product = $6 \times (-3) = -18$.
Clearly, such numbers are 9 and – 2.
$\therefore 6x^2 + 7x - 3 = 6x^2 + 9x - 2x - 3 = 3x(2x + 3) - (2x + 3) = (2x + 3)(3x - 1)$.

(iii) The given expression is $15x^2 - 26x + 8$.
Find two numbers whose sum = –26 and product = $(15 \times 8) = 120$.
Clearly. such numbers are – 20 and – 6.
$\therefore 15x^2 - 26x + 8 = 15x^2 - 20x - 6x + 8 = 5x(3x - 4) - 2(3x - 4) = (3x - 4)(5x - 2)$.

(iv) The given expression is $3x^2 - 4x - 4$.
Find two numbers whose sum = –4 and product = $3 \times (-4) = -12$.
Clearly, such numbers are – 6 and 2.
$\therefore 3x^2 - 4x - 4 = 3x^2 - 6x + 2x - 4 = 3x(x - 2) + 2(x - 2) = (x - 2)(3x + 2)$.

Ask yourself

Factorise the following :

(i) $2x^2 + 6xy$

(ii) $24x^2y^2 - 36x^2y$

(iii) $5y^3(x^2 - 5) - 2y^2(x^2 - 5) + 3y(x^2 - 5)$

(iv) $x^2y(x - z) + y^2x(z - x)$

(v) $10xy + 5x - 14y - 7$

(vi) $15x^2 - 10xy + 12x - 8y$

(vii) $63xy - 72y - 48 + 42x$

(viii) $x^2 - 16$

(ix) $49x^4 - 121y^2$

(x) $16x^4 - 1$

(xi) $x^2 + 8xy + 16y^2$

(xii) $4x^2 - 24xy + 36y^2$

(xiii) $121 - 66y^2x + 9y^4x^2$

(xiv) $x^2 - 5x + 6$

(xv) $x^2 - 8x + 15$

Answers

1. **(i)** $2x(x + 3y)$ **(ii)** $12x^2y(2y – 3)$

(iii) $y(x^2 – 5)(5y^2 – 2y + 3)$ **(iv)** $xy(x – z)(x – y)$

(v) $(5x – 7)(2y + 1)$ **(vi)** $(5x + 4)(3x – 24)$

(vii) $(9y + 6)(7x – 8)$ **(viii)** $(x – 4)(x + 4)$

(ix) $(7x^2 – 11y)(7x + 11y)$ **(x)** $(2x – 1)(2x + 1)(4x^2 + 1)$

(xi) $(x + 4y)^2$ **(xii)** $4[x – 3y]^2$

(xiii) $(11 – 3y^2x)^2$ **(xiv)** $(x – 3)(x – 2)$

(xv) $(x – 5)(x – 3)$

Add your knowledge

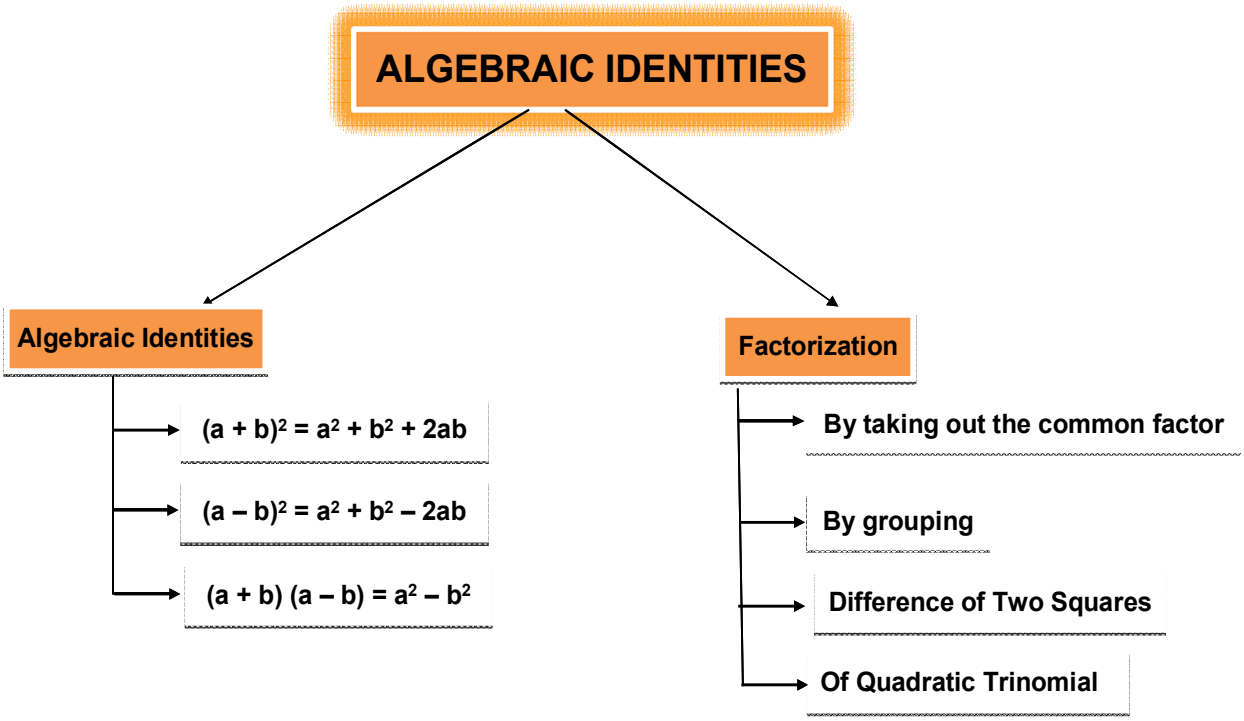
1. Some more identities :-

(i) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

(ii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ or $(a – b)^3 = a^3 – b^3 – 3ab(a – b)$

(iii) If $a^2 + b^2 = 0$, then $a = 0$ and $b = 0$ simultaneously.

Concept Map



Summary

Some standard identities :

1. $(a + b)^2 = a^2 + 2ab + b^2$

2. $(a – b)^2 = a^2 – 2ab + b^2$

3. $(a + b)(a – b) = a^2 – b^2$

4. $(x + a)(x + b) = x^2 + ax + bx + ab$

OR

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

1. $(5x+7)(5x+7)$ is :
(A) $25x^2+70x+49$ (B) $25x^2-70x+49$ (C) $5x^2+70x+7$ (D) $5x^2-70x+7$

2. Square of $3x^2-6y^2$ is :
(A) $9x^4-36y^4$ (B) $9x^4+36y^4-36x^2y^2$
(C) $9x^4-36y^4+36x^2y^2$ (D) $9x^4-36y^4-36x^2y^2$

3. $(4x^2+3y^2)(4x^2-3y^2)$ is :
(A) $16x^4+9y^4-24x^2y^2$ (B) $16x^4+9y^4+24x^2y^2$
(C) $16x^4+9y^4$ (D) $16x^4-9y^4$

4. If $x+\frac{1}{x}=7$, the the value of $x^2+\frac{1}{x^2}$ is :
(A) 49 (B) 51 (C) 47 (D) 7

5. The value of $\frac{97x97-87x87}{10}$ is :
(A) 10 (B) 97 (C) 87 (D) 184

6. The value of is $(47)^2+(43)^2-2(47)(43)$:
(A) 4 (B) 8 (C) 16 (D) 32

7. The factors of $2x^2-32$ are :
(A) $2(x+4)(x-4)$ (B) $2(x+4)(x+4)$ (C) $2(x-4)(x-4)$ (D) none of these

8. The factors of $x^2+18x+81$ are :
(A) $(x-9)^2$ (B) $(x+9)^2$ (C) $(x-9)(x+9)$ (D) $(x+81)^2$

9. The factors of $x^2-10x+25$ are :
(A) $(x-5)^2$ (B) $(x+5)^2$ (C) $(x-5)(x+5)$ (D) $(x+25)^2$

10. The factors of $16x^2-24xy+9y^2$ are :
(A) $(4x+3y)^2$ (B) $(4x-3y)^2$ (C) $(4x+3y)(4x-3y)$ (D) none of these

FILL IN THE BLANKS

1. _____ is the reverse process of multiplication.

2. $(ax+by)(cx+dy) = acx^2+(\ \_\_\_\_\_\ )xy+bdy^2$

3. _____ must be added to $(49x^2-42x)$ to make it a perfect square.

4. A rectangular parking lot has area $(5x^2+17x+6)$, then sides of the lot are _____ and _____ .

5. If $x^2+\frac{1}{x^2}=27$, then the positive value of $x-\frac{1}{x}$ is _____.

TRUE / FALSE

1. An identity is an equality, which is true for all values of the variables.

2. $(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2}$

3. If $a + b = p$ and $ab = q$, then $x^2 + px + q = (x + a)(x + b)$

4. $(a + b)^2 - (a - b)^2 = 4ab$

5. $(m + n)(m + n)(m - n)(m - n) = (m^4 - n^4)$

MATCH THE COLUMN

1.

Column–I		**Column–II**	
(A)	$(a + b)^2$	(p)	$a^2 - b^2$
(B)	$(a - b)^2$	(q)	$a^2 + b^2 + 2ab$
(C)	$(a + b)(a - b)$	(r)	$a^2 - 2ab + b^2 - c^2$
(D)	$(x + a)(x + b)$	(s)	$a^2 + b^2 - 2ab$
(E)	$(a - b - c)(a - b + c)$	(t)	$x^2 + (a + b)x + ab$

2.

Column–I		**Column–II**	
(A)	$(a + 3)^2 - (a-3)^2$	(p)	2
(B)	$(a+2b-5c)^2 - (a - 2b + 5c)^2$	(q)	12a
(C)	$\left(2x + \frac{1}{2x}\right)^2 - \left(4x^2 + \frac{1}{4x^2}\right)$	(r)	8ab – 20ac
(D)	$(1 + x)(1 + x^2)(1 - x)$	(s)	$a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
(E)	$(a - b - c)^2$	(t)	$1 - x^4$

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. Find the following products:

(i) $(2x + 5y)(2x + 5y)$

(ii) $\left(\frac{5}{6}a^2 + 2\right)\left(\frac{5}{6}a^2 + 2\right)$

(iii) $(3x - 7y)(3x - 7y)$

(iv) $\left(\frac{1}{3}x^2 - 9\right)\left(\frac{1}{3}x^2 - 9\right)$

(v) $(x + 3)(x - 3)$

(vi) $\left(\frac{4x}{5} - \frac{5y}{3}\right)\left(\frac{4x}{5} + \frac{5y}{3}\right)$

2. Without multiplication find the values of :

(i) 107^2

(ii) 96^2

(iii) $57 \times 57 - 47 \times 47$

(iv) $\frac{5.6 \times 5.6 - 3.6 \times 3.6}{9.2}$

3. Simplify :

(i) $(2p - 3q)^2 - (2p + 3q)^2$ **(ii)** $\left(\frac{1}{2}a^2 + \frac{3}{5}b^3\right)^2 - \left(\frac{1}{2}a^2 - \frac{3}{5}b^3\right)^2$

4. Find the value of the expression $(9x^2 + 24x + 16)$, when $x = 12$.

5. Simplify : $0.645 \times 0.645 + 2 \times 0.645 \times 0.355 + (0.355)^2$

SHORT ANSWER TYPE

6. If $\left(x + \frac{1}{x}\right) = 4$, find the value of :

(i) $\left(x^2 + \frac{1}{x^2}\right)$ **(ii)** $\left(x^4 + \frac{1}{x^4}\right)$

7. If $x + y = 10$ and $xy = 9$, find the value of $(x^2 + y^2)$.

8. If $x - y = 5$ and $xy = 14$, find the value of $(x^2 + y^2)$.

9. If $a + b = 5$ and $a^2 + b^2 = 13$, find the value of $(a - b)$.

10. Without actual multiplication find the square of :

(i) 1003 **(ii)** 99.99

LONG ANSWER TYPE

11. Find the continued product :

(i) $(x + 2)(x - 2)(x^2 + 4)$ **(ii)** $(2p + 3)(2p - 3)(4p^2 + 9)$

12. Prove that: $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$.

13. If $(ax^3 + 2)^2 = a^2x^6 + x^3 + 4$, then find a.

14. Factorize:

(i) $15ab^2 - 20a^2b$ **(ii)** $14x^3 + 21x^4y - 28x^2y^2$ **(iii)** $5x(x - 4) - 7(x - 4)$

(iv) $2a + 6b - 3(a + 3b)^2$ **(v)** $6ab - b^2 + 12ac - 2bc$ **(vi)** $16x^2 - 25y^2$

(vii) $16x^2 - \frac{1}{144}$ **(viii)** $(2a + 3b)^2 - 16c^2$

15. Factorize:

(i) $4x^2 - 20xy + 25y^2$ **(ii)** $49x^2 - 14x + 1$ **(iii)** $a^2 - 2ab + b^2 - c^2$

(iv) $x^2 + 8x + 16$ **(v)** $16x^2 - 24x + 9$ **(vi)** $a^2b^2 - 6abc + 9c^2$

(vii) $x^2 - 10x + 21$ **(viii)** $5x^2 - 13x + 6$

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

1. If $9x^2 + 25y^2 = 30xy$, then the value of x : y is :

(A) 5 : 3 (B) 3 : 5 (C) 9 : 25 (D) 25 : 9

2. If $a^2 + b^2 = 7$ and $(a - b)^2 = 3$, what is the value of ab?

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) 3

3. If $a^2 + b^2 + c^2 = 20$ and $a + b + c = 0$, then the value of $ab + bc + ca$ is :

(A) 0 (B) – 5 (C) – 10 (D) None of these

4. The value of $\left[\frac{a^2 - 5ab}{a^2 - 6ab + 5b^2} \times \frac{a^2 - b^2}{a^2 + ab}\right]$

(A) – 1 (B) $\frac{a}{b}$ (C) $\frac{1}{a}$ (D) 1

5. The value of $(x + 2y + 2z)^2 + (x - 2y - 2z)^2$ is :

(A) $2x^2 + 8y^2 + 8z^2$ (B) $2x^2 + 8y^2 + 8z^2 + 8xyz$

(C) $2x^2 + 8y^2 + 8z^2 - 8yz$ (D) $2x^2 + 8y^2 + 8z^2 + 16yz$

6. If $x + \frac{1}{x} = 5$, the value of $\frac{x^4 + 1}{x^2}$ is

(A) 21 (B) 23 (C) 25 (D) 30

7. What must be added to $9x^2 - 24x + 10$ to make a perfect square ?

(A) 16 (B) 26 (C) 6 (D) –26

8. Factors of $(x^2 - 13x + 42)$ are :

(A) $(x - 7)(x - 6)$ (B) $(x + 7)(x - 6)$ (C) $(x + 7)(6 - x)$ (D) $(x + 7)(x + 6)$

9. Factors of $\left(x^2 + \frac{x}{6} - \frac{1}{6}\right)$ are :

(A) $\frac{1}{6}(2x + 1)(3x + 1)$ (B) $\frac{1}{6}(2x + 1)(3x - 1)$

(C) $\frac{1}{6}(2x - 1)(3x - 1)$ (D) $\frac{1}{6}(2x - 1)(3x + 1)$

10. If $(a - 5)^2 + (b - 6)^2 = 0$, then find the value of $(a + b - 3)$?

(A) 8 (B) – 8 (C) 10 (D) –10

SECTION -B (TECHIE STUFF)

1. Factorise : $8x^3 + 27y^3 + 36x^2y + 54xy^2$.

(A) $(2x + 3y)^3$ (B) $(2x - 3y)^3$ (C) $(3x + 2y)^3$ (D) $(3x - 2y)^3$

2. Evaluate $(1001)^3$ by using suitable identity.
(A) 1003003001 (B) 1004004001 (C) 1002003001 (D) 1002002001

4. Simplify $(x + 4y)^3 - (x - 4y)^3$.
(A) 16 xy (B) –16xy (C) $128y^3 - 24yx^2$ (D) $128y^3 + 24yx^2$

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

1. If $x + \frac{1}{x} = 3$, then $\frac{x}{x^2+1}$ is **[Aryabhatta - 2002]**
(A) 9 (B) $\frac{1}{3}$ (C) 13 (D) $\frac{1}{13}$

2. If $x - y = 2$, $xy = 24$, then $\left|\frac{1}{x} + \frac{1}{y}\right|$ is equal to **[Aryabhatta - 2008]**
(A) $\frac{5}{2}$ (B) 12 (C) $\frac{5}{12}$ (D) 3

3. $\frac{3.764 \times 3.764 - (1.236)^2}{3.764 - 1.236}$ = ? **[Aryabhatta - 2009]**
(A) 2.5 (B) 5 (C) 1.4 (D) 3

4. If $x + y = 17$ and $x^2 + y^2 = 167$, then what is the value of xy = ? **[NSTSE - 2009]**
(A) $17 + 4\sqrt{114}$ (B) 61 (C) $48 + \sqrt{167}$ (D) 122

5. If $2x - \frac{1}{2x} = 3$, then the value $16x^4 + \frac{1}{16x^4}$ is **[NSTSE - 2009]**
(A) 11 (B) 117 (C) 119 (D) 121

6. If $xy = b$ and $\frac{1}{x^2} + \frac{1}{y^2} = a$, then $(x + y)^2$ equals **[NSTSE - 2010]**
(A) $(a + 2b)^2$ (B) $a^2 + b^2$ (C) $b(ab + 2)$ (D) $\frac{1}{a} + 2b$

7. If $ab = 6$ and $a + b = 5$, then find the value of $a^2 + b^2$? **[NSTSE - 2012]**
(A) 11 (B) 12 (C) 13 (D) 16

8. What is the value of $\frac{(67.542)^2 - (32.458)^2}{75.458 - 40.374}$ **[NSTSE - 2013]**
(A) 1 (B) 10 (C) 100 (D) 1000

Answer Key

Exercise-1

SECTION -A (FIXED RESPONSE TYPE)

OBJECTIVE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	D	C	D	C	A	B	A	B

FILL IN THE BLANKS

1. Factorisation **2.** bc + ad **3.** 9 **4.** (5x + 2), (x + 3)

5. 5

TRUE / FALSE

1. True **2.** False **3.** True **4.** True

5. False

MATCH THE COLUMN

1. (A) – (q), (B) – (s), (C) – (p), (D) – (t), (E) – (r)

2. (A) – (q), (B) – (r), (C) – (p), (D) – (t), (E) – (s)

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. **(i)** $4x^2 + 25y^2 + 20xy$. **(ii)** $\frac{25}{36}a^4 + 4 + \frac{10}{3}a^2$ **(iii)** $9x^2 + 49y^2 - 42xy$

(iv) $\frac{1}{9}x^4 + 81 - 6x^2$. **(v)** $x^2 - 9$. **(vi)** $\frac{16x^2}{25} - \frac{25y^2}{9}$

2. **(i)** 11449 **(ii)** 9216 **(iii)** 1040

(iv) 2.

3. **(i)** (4p) (– 6q) = – 24pq. **(ii)** $(a^2)\left(\frac{6b^3}{5}\right) = \frac{6}{5}a^2b^3$.

4. 1600 **5.** 1

SHORT ANSWER TYPE

6. **(i)** 14 **(ii)** 194 **7.** 82 **8.** 53.

9. + 1 or –1 **10.** **(i)** 1006009 **(ii)** 9998.0001

LONG ANSWER TYPE

11. **(i)** $(x^4 - 16)$ **(ii)** $(16p^4 - 81)$ **13.** $\frac{1}{4}$

14. **(i)** $5ab\,(3b-4a)$ **(ii)** $7x^2(2x+3x^2y-4y^2)$ **(iii)** $(x-4)(5x-7)$

(iv) $(a+3b)(2-3a-9b)$ **(v)** $(6a-b)(b+2c)$ **(vi)** $(4x+5y)(4x-5y)$

(vii) $\left(4x+\frac{1}{12}\right)\left(4x-\frac{1}{12}\right)$ **(viii)** $(2a+3b+4c)(2a+3b-4c)$

15. **(i)** $(2x-5y)^2$ **(ii)** $(7x-1)^2$ **(iii)** $(a-b+c)(a-b-c)$

(iv) $(x+4)^2$ **(v)** $(4x-3)^2$ **(vi)** $(ab-3c)^2$

(vii) $(x-3)(x-7)$ **(viii)** $(5x-3)(x-2)$

Exercise-2

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	A	C	C	D	D	B	C	A	B	A	A	A	D

Exercise-3

PREVIOUS YEAR EXAMINATION QUESTIONS

Ques.	1	2	3	4	5	6	7	8
Ans.	B	C	B	B	C	C	C	C