MATHEMATICS

Class-VI

Topic-03

PLAYING WITH NUMBERS

INDEX

CHAPTER 03

PLAYING WITH NUMBERS

TERMINOLOGIES

Factors, multiples, types of numbers, prime factorisation, divisibility test, H.C.F., L.C.M., V-BODMAS

INTRODUCTION

Suppose a teacher has 21 apples. He wants to distribute these apples (without cutting) equally to his students. What are the possibilities ?
The simplest way to give all the apples to 1 student.
Another simple way is to give 1 apple each to 21 students.
Is there any other possibility ? YES
If there are 7 students , he can give 3 apples to each of them. If there are 3 students , he can give 7 apples to each of them.
Thus we can find that he can only divide 21 apples in such a way that a child gets either 1 or 21 or 3 or 7 apples. In this chapter we shall do exact division of numbers.

3.1 FACTORS AND MULTIPLES

(a) Factors

Any number which is an exact divisor of a given number is called a factor of the given number.
For example factor of 6 are 1,2,3 and 6
Important results :
(a) 1 is a factor of every number.
(b) Every number is a factor of itself.
(c) Every factor of a number is always equal to or less than the number.
(d) Every number has a finite number of factors.

(b) Multiples

Just as 1,2,3, and 6 are factors of 6 , we say that 6 is multiple of 1,2,3, and 6
A number is a multiple of each of its factors
Important results :
(a) Every number is a multiple of itself.
(b) Every multiple of a number is equal to or greater than the number.
(c) Every number has as infinite number of multiples.

(c) Types Of Numbers

(i) Even Number
A number which is exactly divisible by 2 is called an even number.
Examples of even numbers are : 0, 12, 34, 56, 78,....

(ii) Odd Number
A number which is not exactly divisible by 2 is called an odd number.
Examples of odd numbers are : 1, 13, 15, 25, 29,....
(iii) Prime Numbers

A natural number greater than 1, which has no factors except 1 and itself is called a prime number.
Examples of prime numbers are : 2, 3, 5, 11, 13, 17, ...

NOTE:

Every even number greater than 4 can be expressed as a sum of two odd prime numbers, e.g., 6 = 3 + 3; 18 = 5 + 13; 44 = 13 + 31.

(iv) Composite Numbers

A number is composite if it has at least one factor other than 1 and itself.
Examples of composite numbers are : 4, 6, 8, 9,10, 12, 14,...

NOTE:

1. 1 is neither prime nor composite.
2. Every natural number except 1 is, either a prime number or a composite number.
3. 2 is the only prime number which is even. All other prime numbers are odd.

(v) Twin primes

Pairs of prime numbers that have a difference of 2 are called twin primes.
Examples of twin primes are : (3, 5),(5, 7), (11, 13), (17, 19),....

(vi) Perfect Numbers

If the sum of all the factors of a number is twice the number, then number is called a perfect number. For example, 6 is a perfect number since the factors of 6 are 1, 2,3, 6 and their sum 1 + 2 + 3 + 6 = 2 × 6.

(vii) Coprime Numbers :

Two numbers are said to be coprime if they do not have a common factor other than 1.
Examples of coprime numbers are : (8, 15); (5, 9); (2, 11)

NOTE:

1. Two prime numbers are always coprime.
2. Two coprime numbers need not be both prime numbers.

(viii) Prime Triplet

A set of three successive prime numbers differing by 2 is called a prime triplet. The only example of a prime triplet is (3, 5, 7).

(d) Divisibility Test

(i) Test of divisibility by 2

A number is divisible by 2 if its one's digit is an even digit. For example 206 is divisible by 2 because 6 is even.

(ii) Test of divisibility by 3

A number is divisible by 3 if the sum of its digits is divisible by 3.
For example 567 is divisible by 3 because sum of digit
of 567(5+6+7= 18) is divisible by 3

(iii) Test of divisibility by 4

A number is divisible by 4 if the number formed by its last two digits (One's and Ten's) is divisible by 4. For example 312 is divisible by 4 because 12 is divisible by 4.

(iv) Test of divisibility by 5

A number is divisible by 5 if its one's digit is 0 or 5.
For example 3075 is divisible by 5 because its one digit is 5.

(v) Test of divisibility by 6

A number is divisible by 6 if it is divisible by both 2 and 3.
For example 456 is divisible by 6 because its one's digit is even so it is divisible by 2, and the sum of digit of 456(4+5+6=15) is divisible by 3. So 456 is divisible by 2 and 3 both , so we can say that it is divisible by 6

(vi) Test of divisibility by 8

A number is divisible by 8 if the number formed by its last three digits (One's, Ten's, Hundred's) is divisible by 8. For example 8864 is divisible by 8 because 864 is divisible by 8.

(vii) Test of divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.
For example 4563 is divisible by 9 because sum of digit of 4563(4+5+6+3= 18) is divisible by 9

(viii) Test of divisibility by 10

A number is divisible by 10 if its one's digit is 0.
For example 3760 is divisible by 10 because its one's digit is 0

(ix) Test of divisibility by 11

A number is divisible by 11 if the difference of the sums of the digits at the alternate places is zero or divisible by 11.

For example 45672 is divisible by 11 because sum of digit 4,6 and 2 at alternate place
$4 + 6 + 2 = 12$
sum of digit 5,7 at alternate place $5+7=12$
So their difference ($12 – 12 = 0$) is zero, so 45672 is divisible by 11.

NOTE:

A given number will be divisible by any other number say, n, if it is divisible by the coprime factors of n.
For example, 9624 is divisible by 12, because it is divisible by 4 and 3
(the coprime factors of 12).

(e) Sieve of Eratosthenes

How can we list all the prime numbers between say, 1 and 100 ? Eratosthenes (274 B.C. – 194 B.C.), a Greek mathematician, gave a simple method to mark out primes. His method is known as the Sieve of Eratosthenes.
We first list the numbers upto 100, except 1

SIEVE OF ERATOSTHENES

	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

1. Begin with 2 which is prime. So keep it but cross out all its multiples.

2. Next, the number 3 is prime. Thus we keep it but cross out all its multiples. Some of these numbers have already been crossed out.

3. The next number not crossed out is 5. It is also prime. So keep it and cross out all its multiples.

4. Continue this process keeping only the primes and striking off their multiples until we cannot strike off any more numbers.

Thus the prime numbers from 1 to 100 are:
2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61, 67,71,73,79, 83,89,97.
Eratosthenes, probably, made holes in the paper instead of crossing out the numbers. Therefore, his paper must have look like a sieve. That is why perhaps this method is known as sieve method.

Observations. Some observations about prime numbers are:

(i) 2 is the smallest prime number.

(ii) All prime numbers (except 2) are odd numbers.

(iii) The number of primes is unlimited.

(iv) Both the numbers 13 and 31 have the same digits and are prime. Other such numbers between 1 and 100 are: 17, 71; 37, 73 and 79, 97.

(v) Every odd prime number can be expressed as a product of even plus 1.

For example
3 = 2 + 1 ; 5 = 2 x 2 + 1
7 = 2 x 3 + 1 ; 43 = 2 x 3 x 7 + 1

(i) To Find Prime Numbers Between 100 and 400

We know that 20 x 20 = 400
So we adopt the following rule:

Rule : The given number will be prime if it is not divisible by any prime number less than 20

Illustration 3.1

Find out whether 397 is a prime number or not.

Sol. Because 397 < 400, we check whether 397 is divisible by any prime number less than 20.

The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. Let us test the divisibility of 397 by each of them. 397 is not divisible by 2 because the digit in the ones place is odd.

397 is not divisible by 3 because 3 + 9 + 7 = 19, but 19 is not divisible by 3. 397 is not divisible by 5 because the digit in the ones place is neither 5 nor 0. 397 is not divisible by 7 because 397 ÷ 7 gives quotient 56 and remainder 5. 397 is not divisible by 11 because the difference of the sums of the digits at the alternate places is 1 which is not divisible by 11. Now 397 is not divisible by 13 because 397 ÷ 13 gives quotient 30 and remainder 7.

397 is not divisible by 17 because 397 ÷ 17 gives quotient 23 and remainder 6.

397 is not divisible by 19 because 397 ÷ 19 gives quotient 20 and remainder 7.

Since 397 is not divisible by any prime number less than 20, so 397 is a prime number.

Ask yourself

1. Which of the following have 15 as their factor ?
(i) 15625 **(ii)** 151230

2. Without actual division show that 11 is a factor of 1100011.

3. Test the divisibility of following number by 11.
(a) 3178965 **(b)** 9020814

4. In each of the following numbers, replace * by smallest number to make it divisible by 3.
(a) 75*5 **(b)** 66784* **(c)** 18*71

5. In each of the following numbers, replace * by smallest number to make it divisible by 9 .
(a) 67*19 **(b)** 35*64 **(c)** 538*8

6. Express each of the following numbers as the sum of two odd primes :
(a) 180 **(b)** 12

Answers 1. **(i)** No **(ii)** Yes **3.** **(a)** No **(b)** Yes
4. **(a)** 1 **(b)** 2 **(c)** 1
5. **(a)** 4 **(b)** 0 **(c)** 3
6. **(a)** 173+7 (b) 5 +7

3.2 PRIME FACTORISATION OR COMPLETE FACTORISATION

A factorisation is prime if all the factors are prime.

For example : prime factorisation of 120 is
$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$
The prime factorisation is unique
$60 = 60 \times 1 = 30 \times 2 = 15 \times 2 \times 2 = 5 \times 3 \times 2 \times 2$
In writing 60 as 5 x 3 x 2 x 2, we see that each of the factors of 60 is a prime number. When we factorise a number into prime factors, we say that we have written the prime factorisation or complete factorisation of the number.

Fundamental Theorem of Arithmetic

Every composite number can be factorised into primes in only one way, except, for the order of primes.

Common Factors

Numbers which exactly divide two or more numbers are called their common factors.
All factors of 18 are : 1, 2, 3, 6, 9, 18
All factors of 24 are : 1, 2, 3, 4, 6, 8, 12, 24
Common factors of 18, 24 are : 1, 2, 3, 6 as they divide both 18 and 24.

(a) Highest Common Factor (H.C.F.)

To find highest common factor (H.C.F.) or greatest common divisor (G.C.D.) of two or more numbers, we adopt the following method. Let us find H.C.F. of two numbers say 16 and 40.
All possible factors of 16 are : 1, 2, 4, 8, 16.
All possible factors of 40 are : 1, 2, 4, 5, 8, 10, 20, 40
Now the common factors of 16 and 40 are : 1, 2,4, 8.
The highest of all these common factors is 8.

(i) Finding HCF by Prime Factorisation

STEP 1. Find the prime factorisation of the given numbers.

STEP 2. Find the common factors and circle them.

STEP 3. Multiply the common factors to get HCF.

Illustration 3.2

Let us now find the HCF of 72, 64, and 48.

STEP 1. Find the prime factorisation of each the numbers.

We can factorise the numbers as follows :
72 = 2 x 2 x 2 x 3 x 3
64 = 2 x 2 x 2 x 2 x 2 x 2
48 = 2 x 2 x 2 x 2 x 3

STEP 2. Take the common factors in all the given numbers.
72 = **2 x 2 x 2** x 3 x 3
64 = **2 x 2 x 2** x 2 x 2 x 2
48 = **2 x 2 x 2** x 2 x 3

STEP 3. Multiply the common factor to get the HCF.
The HCF of 72, 64 and 48 = $2 \times 2 \times 2 = 8$

(ii) Finding HCF using Division by Common Factors

Divide all the three numbers by any factor common to all of them. If there are still any common factors, again divide the quotients by them and keep dividing until there is no common factor for all three of them. The product of these common factors will give the highest common factor (HCF) of these numbers.

(iii) H.C.F. By Division Method

In this method , we divide the greater number by the smaller number. Then the remainder is treated as divisor and the first divisor as dividend . We continue this operation till we get the remainder zero. The last divisor is the H.C.F. of the two given numbers. We illustrate this method by the following examples.

Illustration 3.3

Find the H.C.F. of 345 and 506.

Sol.

```
345 ) 506 ( 1
     – 345
     -----------
      161 ) 345 ( 2
          – 322
          ----------
           23 ) 161 ( 7
               –161
               -------
                  0
```

The last divisor is 23.

∴ H.C.F. of 345 and 506 is 23.

To find the H.C.F. of three numbers , first we find the H.C.F. of any two numbers. Then treating this H.C.F.as one number and third number as another number, we find their H.C.F. by the method stated above. The H.C.F. so found will be the H.C.F. of the three numbers.

Illustration 3.4

Find the H.C.F. of 219, 2628, 2190 and 8833.

Sol. First we find the H.C.F. of 219 and 2628.

```
219 ) 2628 ( 12
     –219
     -----
      438
     – 438
     -----
        0
```

Now we find the H.C.F. of 219 and 2190.

```
219 ) 2190 ( 10
     –219
     -----
        0
```

∴ H.C.F. of 219, 2628 and 2190 = 219.

Now we find the H.C.F. of 219 and 8833.

```
219 ) 8833 ( 40
     – 876
     -----------
        73 ) 219 ( 3
            –219
            -----
               0
```

Hence the H.C.F. of 219, 2628, 2190 and 8833 is 73.

Illustration 3.5

Find the greatest number that will divide 18 and 48 without leaving a remainder.

Sol. Required number is the HCF of 18 and 48 .

HCF of 18 and 48 is 6

18 = 2 x 3 x 3
48 = 2 x 2 x 2 x 2 x 3
Required HCF = 2 x 3
the greatest number that will divide 18 and 48 without leaving a remainder is 6.

Illustration 3.6

Find the greatest number which divides 43 and 91 leaving remainder 7 in each case.

Sol. It is given that the required number when divides 43 and 91, the remainder is 7 in each case . This means that 43 – 7 = 36 and 91– 7 = 84
are completely divisible by required number .
Also, the required number is the greatest number satisfying the above property .
It is the HCF of 36 and 84.
36 = 2 x 2 x 3 x 3
84 = 2 x 2 x 3 x 7
Required HCF = 2 x 2 x 3 = 12
Hence, the required number = 12

Illustration 3.7

Find the largest number that will divide 20, 57 and 85 leaving remainders 2, 3 and 4 respectively.

Sol. Clearly, the required number is the HCF of the number 20 – 2 = 18, 57 – 3 = 54 and 85 – 4 = 81.
18 = 2 x 3 x 3
54 = 2 x 3 x 3 x 3
81 = 3 x 3 x 3 x 3
Required HCF = 3 x 3 = 9
Hence, the required number = 9

Illustration 3.8

The length, breadth and height of a room are 8 m 25cm, 6m 75cm and 4m 50 cm respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

Sol. We have, 8 m 25cm = 825cm, 6m 75cm = 675cm
4 m 50 cm = 450 cm
Length of the longest rod in cm is the HCF of 825,675 and 450.
825 = 3 x 5 x 5 x 11
675 = 3 x 3 x 3 x 5 x 5
450 = 2 x 3 x 3 x 5 x 5
HCF 825, 675 and 450 = 3 x 5 x 5 = 75
Hence, the required length of rod = 75 cm

Illustration 3.9

A rectangular courtyard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose ?

Sol. Largest size of the tile = H.C.F of 378 cm and 525 cm = 21 cm.

(b) Least Common Multiple (L.C.M.)

To find LCM of two or more number we adopt the following steps:

(1) Find the multiples of the given numbers.
(2) Select their common multiples

(3) Take the smallest of the above common multiples

Let us find the LCM of 6 and 8
The multiples of 6 are 6,12,18,24,30,36,42,48,....
The multiples of 8 are 8,16,24,32,40,48,.....
So common multiples of 6,8 are 24, 48,......
The lowest common multiple of 6 and 8 is 24
Hence 24 is the LCM of 6 and 8

(i) Finding LCM by Prime Factorisation

STEP 1. Find the prime factorisation of the given numbers.
STEP 2. LCM is the product of all the prime factors with greatest powers.

Illustration 3.10

Find the LCM of 84 and 96.

Sol. $84 = 2 \times 2 \times 3 \times 7$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

LCM of 84,96 is = 672

(ii) Finding LCM by common Division

STEP 1. Write the given numbers in a row separated by commas

STEP 2. Divide these numbers by the least prime numbers which divides at least one of the given numbers.

STEP 3. Write the quotients and the numbers that are not divisible by the prime numbers in the second row. Then repeat Steps 2 and 3 with the rows and continue till the numbers in a row are all 1.
STEP 4. The LCM is found out by multiplying all the prime divisors and quotients other than 1.

Illustration 3.11

Find the L.C.M. of 28, 36, 45 and 60.

Sol.

2	28, 36, 45, 60
2	14, 1 8, 45, 30
3	7, 9, 45, 15
3	7, 3 , 15 , 5
5	7, 1 , 5 , 5
	7 , 1 , 1 , 1

L.C.M. $= 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$

Illustration 3.12

Find the least number which when divided by 6, 7, 8, 9 and 10 leaves remainder 1.

Sol. As the remainder is same
Required number = LCM of divisors + Remainder
= LCM (6, 7, 8, 9, 10) +1 = 2520 + 1 = 2521

Illustration 3.13

Find the least number which when decreased by 7 is exactly divisible by 12, 16, 18, 21 and 28.

Sol.

2	12,	16,	18,	21,	28
7	6,	8,	9,	21,	14
3	6,	8,	9,	3,	2
2	2,	8,	3,	1,	2
	1,	4,	3,	1,	1

LCM = 2 × 7 × 3 × 2 × 4 × 3 = 1008
Required number = 1008 + 7 = 1015.

Illustration 3.14

When 21 is added to a number it is divided exactly by 3,8,9,12,16 and 18. How many such numbers exist ? Find the least of them .

Sol. We know that the least number divisible by 3,8,9,12,16 and 18 is their LCM .
Therefore, the required number must be 21 less then their LCM

2	3,	8,	9,	12,	16,	18
2	3,	4,	9,	6,	8,	9,
2	3,	2,	9,	3,	4,	9
3	3,	1,	9,	3,	2,	9
3	1,	1,	3,	1,	2,	3
	1,	1,	1,	1,	2,	1

LCM = 2 × 2 × 2 × 3 × 3 × 2 =144
Hence, the required number = (144 – 21) = 123
There exists many such numbers (i.e., all the multiples of 123) and least of them is 123.

Illustration 3.15

In a morning walk four boys steps off together. Their steps measure 70 cm , 65 cm , 75 cm and 80 cm respectively . At what distance from the starting point will they step off together again ?

Sol. The distance covered by each one of them is required to be same and minimum both .The required minimum distance each should walk must be the LCM of their steps in cm.

2	70,	65,	75,	80
5	35,	65,	75,	40
	7,	13,	15,	8

LCM = 2 × 5 × 7 × 13 × 15 × 8 =109200
They will step off together again after a distance of 109200 cm = 1092 m.

(c) Relationship between HCF and LCM

Let us take two numbers, say 16 and 24.
The HCF of 16 and 24 is 8.
The LCM of 16 and 24 is 48.
Since 8 is factor of 48, so we can say that HCF of the numbers is a factor of their LCM.
Product of HCF and LCM = 8 x 48 = 384
Product of Numbers = 16 x 24 = 384
So we can say that the product of two numbers is equal to the product of their HCF and LCM.
Let a and b are two numbers then $a \times b$ = HCF x LCM

Illustration 3.16

The HCF of two number is 29 and their LCM is 1160. If one of the number is 145 , find the other.

Sol. We know that
Product of the number = Product of their HCF and LCM
Required No. x 145 = 29 x 1160

Required No.= $\frac{29 \times 1160}{145} = 232$

(i) Properties of HCF and LCM

1. The HCF of 6 and 10 is 2. So, 2 is a factor of both 6 and 10. Also, 2 is the smallest amongst 2, 6, and 10.

2. The LCM of 6 and 10 is 30. 30 is a multiple of both 6 and 10. Also, 30 > 10 and 6, i.e., it is the greatest amongst 6, 10, and 30.

3. Consider two numbers 35 and 39.
Now, 35 = 1 x 5 x 7
39 = 1 x 3 x 13
Common factor = 1
$\therefore$ 35 and 39 are co-prime.
HCF of 35 and 39 = 1
Thus HCF of two or more co-prime numbers is 1.

4. Again consider 35 and 39 which are coprime.
LCM of 35 and 39 = 3 x 5 x 7 x 13 = 35 x 39
Thus the LCM of co-prime numbers = the product of the co-primes.

5. HCF of 6, 10 = 2
LCM of 6, 10 = 30
Also, 30 = 2 x 15 = 2 x 3 x 5
i.e., 2 is a factor of LCM.
Thus, HCF is a factor of LCM. In other words, LCM is a multiple of HCF.

6. 2 and 3 are prime numbers.
HCF of 2 and 3 is 1.
HCF of two or more prime numbers is 1.

Some More Divisibility Rules

Let us observe a few more rules about the divisibility of numbers.

- One of the factor of 18 is 9. A factor of 9 is 3. Is 3 a factor of 18? Yes it is. Take any other factor of 18, say 6. Now, 2 is a factor of 6 and it also divides 18. Check this for the other factors of 18. Consider 24. It is divisible by 8 and the factors of 8 i.e. 1,2,4 and 8 also divide 24.
  So, we may say that if a number is divisible by another number then it is divisible by each of the factors of that number.

- The number 80 is divisible by 4 and 5. It is also divisible by 4 × 5 = 20, and 4 and 5 are co-primes.
  Similarly, 60 is divisible by 3 and 5 which are co-primes.60 is also divisible by 3 × 5 = 15,

If a number is divisible by two co-prime numbers then it is divisible by their product also.

- The numbers 16 and 20 are both divisible by 4. The number 16 + 20 = 36 is also divisible by 4. **If two given numbers are divisible by a number, then their sum is also divisible by that number.**
- The numbers 35 and 20 are both divisible by 5. Their difference 35 – 20 = 15 also divisible by 5 .

If two given numbers are divisible by a number, then their difference is also divisible by that number.

Ask yourself

1. Write prime factorisation of the following
(a) 180 **(b)** 65

2. Find by the method of prime factors , the LCM of :
(a) 10,12,18 **(b)** 9,15,18,20

3. Find the LCM of the following numbers by division method:
(a) 18,36,60,72 **(b)** 3,12,24,36,56

4. Find the smallest number which when divided by 25,40 and 60 leaves remainder 7 in each case.

5. For each of the following pairs of numbers , show that the product of their HCF and LCM equals their
product :
(a) 27, 90 **(b)** 117 , 221 **(c)** 14,21

6. The HCF and LCM of two numbers are 13 and 1989 respectively. If one of the numbers is 117, determine the other.

Answers

1.	**(a)**	$2\times2\times3\times3\times5$	**(b)**	5×13	**2.**	**(a)** 180	**(b)** 180
3.	**(a)**	360	**(b)**	504	**4.**	607	
5.	**(a)**	2430	**(b)**	25857	**(c)**	294	**6.** 221

3.3 OPERATOR PRECEDENCE

Generally, the order in which we perform operations sequentially from left to right is : division, multiplication, additions & subtraction. This order is expressed in short as 'DMAS' where 'D' stands for division, 'M' for multiplication, 'A' for addition and , 'S' for subtraction.

Illustration 3.17

Simplify : (– 20) + (– 8) ÷ (– 2) × 3.

Sol. We have,
$(-20) + (-8) \div (-2) \times 3 = (-20) + 4 \times 3 = (-20) + 12 = -8.$

(a) Use Of Brackets

In order to simplify expression involving more than one brackets, we use the following steps.

STEP 1. See whether the given expression contains a vinculum or not. If a vinculum is present, then perform operations under it. Otherwise go to next step.

STEP 2. See the innermost bracket and perform operations within it.

STEP 3. Remove the innermost bracket by using following rules:

Rule 1 : If a bracket is preceded by a plus sign, remove it by writing its terms as they are.

Rule 2 : If a bracket is preceded by minus sign, change positive signs within it to negative and vice-versa.

Rule 3 : If there is no sign between a number and a grouping symbol, then it means multiplication.

Rule 4 : If there is a number before some brackets then we multiply the number inside the brackets with the number outside the brackets.

STEP-4. See the next innermost bracket and perform operations within it. Remove the second innermost bracket by using the rules given in step III. Continue this process till all the brackets are removed.

Illustration 3.18

Simplify

(i) $39 - [23 - \{29 - (17 - \overline{9-3})\}]$

(ii) $15 - (-3)\{4 - \overline{7-3}\} \div [3\{5 + (-3) \times (-6)\}]$

Sol. **(i)** $39 - [23 - \{29 - (17 - 6)\}]$

$= 39 - [23 - \{29 - 11\}]$

$= 39 - [23 - 18]$

$= 39 - 5$

$= 34$

(ii) $15 - (-3)\{4 - 4\} \div [3\{5 + 18\}]$

$15 - (-3) \times 0 \div 69$

$15 - (-3) \times 0 = 15$

Ask yourself

Simplify :

1. $2-\left[3-\{6-(5-\overline{4-3})\}\right]$

2. 73 of [45 – {6 × 7 + (23 – 4 of 5)}]

3. $\left\{5(18\div\overline{8-5})-30\right\}+2\times 10\div 5$

4. $3\frac{1}{12}-\left[1\frac{3}{4}+\left\{2\frac{1}{2}-\left(1\frac{1}{2}-\frac{1}{3}\right)\right\}\right]$

5. [7.2 ÷ 0.8 – 1.2 × 0.9 + 0.08]

Answers

1. 1 **2.** 0 **3.** 4 **4.** 0 **5.** 8

Add your knowledge

(a) There are just four numbers (after 1) which are the sums of the cubes of their digits.

$153 = (1)^3 + (5)^3 + (3)^3$
$370 = (3)^3 + (7)^3 + (0)^3$
$371 = (3)^3 + (7)^3 + (1)^3$
$407 = (4)^3 + (0)^3 + (7)^3$

(b) Every even number greater or equal to 4 can be expressed as a sum of exactly two (not necessarily distinct) prime numbers. Eg: 4 = 2+2 , 6=3+3, 8=3+5 , 10=5+5 , 12=5+7 etc

(c) **Divisibility Rule for 7 :** Double the last digit of given number and subtract from remaining number the result should be zero or divisible by 7.

(d) **Divisibility Rule for 13 :** Four times the last digit and add to remaining number the result should be divisible by 13.

(e) **Divisibility Rule for 17 :** Five times the last digit of the number and subtract from previous number the result obtained should be either 0 or divisible by 17.

(f) **HCF and LCM of fractions :**

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

Make sure the fractions are in the most reducible form.

Concept Map

Playing with Numbers

Types of Numbers
- Even Number
- Odd Number
- Prime Number
- Composite Number
- Twin Prime
- Prime Triplet

Factors
Factors of 2
1, 2
Factors of 12 →
1, 2, 3, 4, 6, 12

Multiples
Multiples of 2→
2, 4, 6, 8,.........

Multiples of 8→
8, 10, 24, ...

Prime Factorisation
Prime factorisation of
60 = 2 × 2 × 3 × 5

Divisibility Test

Relation between HCF and LCM

Product of two numbers = HCF × LCM

H.C.F. and L.C.M.

H.C.F.

By Prime Factorisation Method

H.C.F. of 72 and 64
72 = 2 × 2 × 2 × 3 × 3
64 = 2 × 2 × 2 × 2 × 2 × 2
HCF = 2 × 2 × 2 = 8

By Division Method

HCF 825 and 675

```
675)825(1
    675
    150)675(4
        600
         75)675(9
            675
              0
```

HCF = 75

L.C.M.

By Prime Factorisation

LCM of 12 and 16
12 = 2 × 2 × 3 = 2^2 × 3
16 = 2 × 2 × 2 × 2 = 2^4
LCM = 2^4 × 3
16 × 3 = 48

By Common Division

LCM of 24, 36, 40

2	24, 36, 40
2	12, 18, 20
2	6, 9, 10
3	3, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

LCM = 2 × 2 × 2 × 3 × 3 × 5 = 360

Summary

1. A factor of a number is an exact divisor of that number.
2. 1 is a factor of every number.
3. A multiple of any natural number is obtained by multiplying this number by natural numbers 1,2,3...
4. A prime number is a whole number greater than 1 which is divisible by 1 and by itself . A natural number greater than 1 which is not prime is called a composite number. The number 1 is neither prime nor composite
5. Two natural numbers which do not have a common prime factor are called co-primes, e.g. (2,3) , (3,4) , (16,25), etc.
6. Prime numbers that differs by 2 are called twin primes.
7. A set of three successive prime numbers differing by 2 is called a prime triplet.
8. A number which is equal to twice the sum of all its factors is called a perfect number.
9. A number is divisible by
   (a) 2 if its one's digit is even.
   (b) 3 if the sum of its digits is divisible by 3.
   (c) 4 if the number formed by its last two digits (One's and Ten's) is divisible by 4.
   (d) 5 if its one's digit is 0 or 5.
   (e) 6 if it is divisible by both 2 and 3.
   (f) 9 if the sum of its digits is divisible by 9.
   (g) 8 if the number formed by its last three digits is divisible by 8.
   (h) 11 if the difference of the sums of the digits at the alternate places is divisible by 11.
10. The process of writing a composite number as a product of prime factors is called **prime factorisation** of the given numbers**.**
11. The H.C.F. of two or more numbers is the greatest number that divides each one of them exactly.
12. The L.C.M. of two or more numbers is the least number that is divisible by all these numbers.
13. The product of two numbers is equal to the product of their H.C.F. and L.C.M.

EXERCISE 01

SECTION -A (FIXED RESPONSE TYPE)

MULTIPLE CHOICE QUESTIONS

1. Which one of the following is a composite number ?
(A) 1 (B) 2 (C) 5 (D) 8

2. The only even prime number is :
(A) 2 (B) 3 (C) 4 (D) 0

3. Which of the following is a pair of twin primes ?
(A) (7, 9) (B) (17, 19) (C) (51, 53) (D) (31, 33)

4. The smallest digit to make the number 5703_2 divisible by 4 is :
(A) 2 (B) 4 (C) 8 (D) 1

5. Which of the following is a pair of co-prime ?
(A) (55, 57) (B) (46, 50) (C) (72, 78) (D) none of these

6. Number of even between 58 and 80 is
(A) 10 (B) 11 (C) 12 (D) 13

7. The number of distinct prime factors of the largest 4-digit number is
(A) 2 (B) 3 (C) 5 (D) 11

8. The number of distinct prime factors of the smallest 5-digit number is
(A) 2 (B) 4 (C) 6 (D) 8

9. The largest 3-digit number which is exactly divisible by 3 is :
(A) 998 (B) 992 (C) 999 (D) None of these

10. Sum of the number of primes between 16 to 80 and 90 to 100 is
(A) 20 (B)18 (C) 17 (D)16

11. The prime factorisation of 54 is :
(A) $2 \times 2 \times 3 \times 3$ (B) $2 \times 2 \times 2 \times 3$ (C) $2 \times 3 \times 3 \times 3$ (D) 2×27

12. Which of the following statements is not true ?
(A) The HCF of two distinct prime numbers is 1
(B) The HCF of two co prime number is 1
(C) The HCF of two consecutive even numbers is 2
(D) The HCF of an even and an odd number is even

13. LCM of 15, 20 and 30 is :
(A) 50 (B) 20 (C) 60 (D) 15

14. The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, then the other is
(A) 279 (B) 283 (C) 308 (D) 318

15. The length and breadth of a room are 18 m and 28 m respectively. What is the greatest length of the side of a square tile required for pairing the floor of the room (No space is to be left in the room without a tile)
(A) 2.4 m (B) 4 m (C) 2 m (D) 3.2 m

16. Three city tour buses leave the bus stop at 9.00 AM. Bus A returns every 30 minutes, but B returns every 20 minutes and Bus C returns every 45 minutes. What is the next time, the buses will all return at the same time to the bus stop.
(A) 1.:00 PM (B) 12 noon (C) 7 PM (D) 11 : 30 PM

17. The least number of 5-digits which is exactly divisible by 16,24,36 and 54 is
(A) 10638 (B) 10368 (C) 13068 (D) 1084

18. Find the greatest number of 5-digits which when divided by 3,5,8 and 12 will have 2 as remainder
(A) 99999 (B) 99958 (C) 99960 (D) 99962

19. The LCM and HCF of two numbers are 4125 and 25 respectively. One number is 375. Find by how much is the second number less than the first ?
(A) 100 (B) 50 (C) 75 (D) 25

20. Product of two co-prime numbers is 117. Their LCM should be
(A) 1 (B) 117
(C) equal to their HCF (D) can’t be calculated.

21. Simplify : $\frac{3}{8}-\frac{-2}{9}+\frac{-1}{36}$
(A) $\frac{1}{8}$ (B) $\frac{41}{72}$ (C) $\frac{43}{72}$ (D) $\frac{11}{72}$

22. Evaluate : $\frac{8-[5-(-3+2)]\div 2}{|5-3|-|5-8|\div 3}$
(A) 2 (B) 3 (C) 4 (D) 5

23. Simplify : $18-[5-\{6+2(7-\overline{8-5})\}]$.
(A) 13 (B) 15 (C) 27 (D) 32

FILL IN THE BLANKS

1. 1 is neither _____ nor _______

2. The smallest prime number is ______

3. The smallest composite number is ______

4. Two perfect numbers are ______ and _____

5. The HCF of two consecutive odd numbers is ____

6. The LCM of 24 and 8 is ____

7. In BODMAS rule B stands for ____

8. In BODMAS rule D stands for ____

TRUE / FALSE

1. Every multiple of a number is greater than or equal to the number.
2. Every number is a multiple of itself.
3. Every prime number is odd.
4. Every even number is composite.
5. The sum of two odd numbers is always odd.
6. The sum of two even numbers is always even.
7. Sum of two consecutive odd numbers is always divisible by 4.
8. If a number divides three numbers exactly, it must divide their sum exactly.
9. If a number exactly divides the sum of three numbers. It must exactly divide the numbers separately.
10. The HCF of two given numbers is always a factor of their LCM
11. The HCF of two distinct prime numbers is 1.
12. The HCF of two co-prime numbers is 1.
13. The HCF of an even and an odd number is even.
14. The HCF of two consecutive even number is 2.
15. The HCF of two consecutive odd numbers is 2.
16. In BODMAS first we divide the numbers
17. In BODMAS first we open small bracket

MATCH THE COLUMN

1.

	Column-I		Column-II
(A)	45	**(p)**	8^{th} multiple of 3
(B)	15	**(q)**	factor of 40
(C)	24	**(r)**	multiple of 7
(D)	20	**(s)**	factor of 30
(E)	35	**(t)**	5^{th} multiple of 9

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. List all the factors of :

(i) 23 **(ii)** 48 **(iii)** 168

2. Write the following :
(i) The first 3-digit even multiple of 7
(ii) Odd multiples of 17, less than 100
(iii) Multiples of 5 between 52 and 76.

3. Write all the prime numbers between :

(a) 5 and 35 **(b)** 70 and 100 **(c)** 40 and 80 **(d)** 77 and 158

4. Can a composite number be odd ? If yes, write the smallest odd composite number.

5. Express each of the following numbers as sum of two odd primes :
(a) 36 **(b)** 42 **(c)** 84 **(d)** 98

6. Express each of the following odd numbers as the sum of three odd prime numbers :
(a) 31 **(b)** 35 **(c)** 49 **(d)** 63

7. What least number should be subtracted from 26492518 so that the resulting number is divisible by 3, but not by 9 ?

8. Find the LCM of 80, 96, 125, 160.

9. Find the prime factors of :
(a) 2520 **(b)** 2145

10. Write the smallest 5-digit number and express it as a product of primes.

11. What is the value of $64 \div 8 \div 4 \div 2$?

12. $8 - (4 \times 2) \div 8$

SHORT ANSWER TYPE

13. **(a)** Is there any natural number having no factor at all?
(b) Find all the numbers having exactly one factor.
(c) Find numbers between 1 and 100 having exactly three factors.

14. Write the seven consecutive composite numbers less than 100.

15. What least value should be given to * so that the number 653 * 47 is divisible by 11 ?

16. What least value should be given to * so that the number 153 * 48 is divisible by 9 ?

17. What least value should be given to * so that the number 8456 * 4107 is divisible by 3 ?

18. Without actual division, check the divisibility of 376948 with 11.

19. Replace the star() by the smallest number, so that
(i) 78 * 964 may be divisible by 9. **(ii)** 75 * may be divisible by 4.
(iii) 2 * 345 may be divisible by 3.

20. Use prime factorisation to find HCF of the following :

(a) 66, 330 **(b)** 45, 75 **(c)** 54, 81 **(d)** 64, 80
(e) 64, 96 and 216 **(f)** 70, 105, 175 **(g)** 91, 175, 49 **(h)** 18,54, 81

21. Use division method to find HCF of the following :

(a) 72 ,126 **(b)** 36, 84 **(c)** 34, 102 **(d)** 27, 63
(e) 924, 1463, 1925 **(f)** 1134, 1344, 1638

22. Find the LCM of the following by division method :

(i) 20, 25, 30, 50 **(ii)** 9, 12, 18, 24, 27
(iii) 22, 54, 108, 135, 198

23. Find the LCM of the following by prime factorization method :

(a) 24, 36, 40 **(b)** 24, 63, 70 **(c)** 91, 65, 75, 39
(d) 42, 78, 104, 112 **(e)** 36, 40, 126

24. Find the LCM by the method of division :

(a) 8, 24, 48, 80, 120 **(b)** 11, 22, 44, 66, 88 **(c)** 105, 315, 693, 1287
(d) 21, 28, 36, 45 **(e)** 180, 384, 144

25. Find the least number of square tiles that will be needed to pave a plot 225 m by 30m

26. Three different tankers contain 496 litres, 403 litres and 713 litres of milk. Find the maximum capacity of a container that can measure the milk of any tanker an exact number of times.

27. Simplify : $\{3 + (4 \times 5) \div 2 - 6\} \div 7$

28. Simplify : $2 - \left[3 - \{6 - (5 - \overline{4 - 3})\}\right]$

LONG ANSWER TYPE

29. Find a 4-digit odd number using each of the digits 1,2,4 and 5 only once such that when the first and the last digits are interchanged, it is divisible by 4.

30. Using each of the digits 1,2,3 and 4 only once, determine the smallest 4-digit number divisible by 4.

31. Find the largest number that divides 220, 313 and 716 leaving remainder 3 in each case.

32. Find the largest number that will divide 623, 729 and 841 leaving remainders 3,9 and 1 respectively.

33. Find the least number which when divided by 12,16,24 and 36 leaves a remainder 7 ?

34. Find the greatest number of 4-digit exactly divisible by 12, 16, 24, 28 and 36.

35. Find the greatest length of a rod which can measure exactly 42 m , 49m and 84 m . Find also the number of times the rod is contained in each length.

36. Find the greatest number such that if 245 and 1029 be divided by it, the remainder in each case is 5.

37. Find the greatest number that will divide 1750 and 2000 leaving 48 and 2 as remainders respectively.

38. In a walking competition, three person step off together. Their steps measure 85 cm, 90 cm and 80 cm respectively. At what distance from the starting point will they again step off together?

39. Find the largest size of a square tile that can be used for paving a rectangular plot 84m by 162 m. Find also the number of tiles that will be needed.

40. There are 527 apples , 646 pears and 748 oranges. These are to be put in heaps of equal quantities. Find the maximum number of fruits in each heap. How many such heaps would be formed ?

41. Five bells ring simultaneously and afterwards at intervals of 4, 6, 8, 10 and 12 minutes respectively. At what interval will they all ring together ?

42. Find the least number which when divided by 12, 15, 18 and 30 leaves the remainder 6, 9, 12 and 24 respectively.

43. Determine the least number which when divided by 3, 4 and 5 leaves remainder 2 in each case.

44. Find the least number which when diminished by 9 is exactly divisible by 12, 16, 24 and 48.

45. Find the least number which when increased by 3 is exactly divisible by 9, 12, 15 and 21.

46. Find the least number of 4-digits which is exactly divisible by 2, 3, 4,5, 6 and 7.

47. 73 of [45 – {6 × 7 + (23 – 4 of 5)}]

48. $\left\{ 5(18 \div \overline{8-5}) - 30 \right\} + 2 \times 10 \div 5$

EXERCISE 02

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

1. The sum of all prime numbers between 10 to 25 is
(A) 83. (B) 84. (C) 85 (D) 86

2. By Goldbach's conjecture, every even number greater than 4 can be expressed as a sum of
(A) Two even numbers (B) two odd prime numbers
(C) two composite numbers (D) Two co-prime numbers

3. The number 10 has four factors : 1, 2, 5 and 10. The table below lists the number of factors for some numbers

Number	Number of factors
21	4
23	2
25	3
27	4
29	2

From this we can say that the number of prime numbers between 20 and 30 is
(A) 0 (B) 2 (C) 3 (D) 4

4. A number is divisible by 5 and 6. It may not be divisible by
(A) 10 (B) 15 (C) 30 (D) 60

5. The number of distinct prime factors of the largest 4-digit number is :
(A) 2 (B) 3 (C) 5 (D)11

6. The number of distinct prime factors of the smallest 5-digit number is
(A) 2 (B) 4 (C) 6 (D) 8

7. Which pair of numbers has a HCF that is not a prime number ?
(A) 60,231 (B) 15,80 (C) 24,52 (D) 30,42

8. Which of the following statements is not true ?
(A) The HCF of two distinct prime numbers is 1
(B) The HCF of two co prime number is 1
(C) The HCF of two consecutive even numbers is 2
(D) The HCF of an even and an odd number is even

9. Which team of same size are formed from three groups of 512,430 and 489 students separately, 8, 10 and 9 students respectively are left out. What could be the largest size of the team ?
(A) 6 (B) 12 (C) 18 (D) 20

10. The maximum number of students among whom 1001 pens and 910 pencils can be distributed, such that each student gets the same number of pens and same number of pencils is :
(A) 910 (B) 1001 (C) 91 (D) 191

11. The length and breadth of a room are 16.58 m and 8.32 m respectively. What is the greatest length of the side of a square tile required for pairing the floor of the room.
(No space is to be left in the room without a tile)
(A) 2.4m (B) 4m (C) 0.02m (D) 3.2m

12. The greatest 4-digit number which is divisible by 4, 6, 12 is
(A) 9998. (B) 9997. (C) 9996. (D) 9995.

13. The LCM of two numbers is x and their HCF is y. The product of two numbers is
(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) x + y (D) xy

14. The least number which when decreased by 7 is exactly divisible by 12, 16, 18, 21 and 28 is
(A) 1012 (B) 1008 (C) 1015 (D) 1022

15. The least number of 4 digits which in exactly divisible by 13 is
(A) 1052 (B) 1039 (C) 1032 (D) 1001

16. The correct option is
(A) if two numbers are co-prime, then one of them must be prime.
(B) a number is divisible by 185, then it is divisible by 3 and 6.
(C) a number is divisible by both 3 and 7, so it divisible by 21 also.
(D) numbers which are divisible by 6, must be divisible by 12 also.

17. If the number 7254 * 98 is divisible by 22, the digit at * is
(A) 1 (B) 2 (C) 6 (D) 0

18. The largest number which always divides the sum of any pair of consecutive odd number is
(A) 2 (B) 4 (C) 6 (D) 8

SECTION -B (TECHIE STUFF)

19. Which of the following number is divisible by 7
(A) 365 (B) 2356 (C) 6545 (D) 963

20. Which of the following number is divisible by 13
(A) 234 (B) 360 (C) 298 (D) 654

21. Which of the following number is divisible by 17
(A) 300 (B) 698 (C) 982 (D) 357

22. H.C.F. of $\frac{8}{9}, \frac{16}{81}, \frac{2}{3}$ and $\frac{10}{27}$ is
(A) $\frac{2}{3}$ (B) $\frac{10}{27}$ (C) $\frac{80}{3}$ (D) $\frac{2}{81}$

23. L.C.M of $\frac{3}{7}, \frac{4}{21}$ and $\frac{5}{14}$ is
(A) $\frac{60}{7}$ (B) $\frac{1}{21}$ (C) $\frac{1}{42}$ (D) None of these

EXERCISE 03

(PREVIOUS YEAR EXAMINATION QUESTIONS)

1. Match List- I with List- II and select the correct answer using the codes given below **(NSTSE 2010)**

List -I (Number)		List - II (Divisible by)	
p.	4926549	**1.**	11
q.	54192039	**2.**	5
r.	394192045	**3.**	4
s.	19706196	**4.**	3

(A) p-4 , q-1 , r-2, s-3 (B) p-4 , q-2, r-3, s-4
(C) p-1 , q-3, r-2, s-4 (D) p-2, q-1, r-4, s-3

2. What is the smallest number of oranges that can be shared equally among 2, 5 or 9 children without leaving any remainder? **(NSTSE 2010)**
(A) 10 (B) 18 (C) 45 (D)90

3. If "a" and "b" are two natural numbers such that thier GCD is same as their LCM, then which of the following is necessarily true? **(NSTSE 2010)**
(A) a = b = 1 (B) a < b
(C) a divides b , but ab (D) a > b

4. A number is always divisible by 40 if: **(NSTSE 2010)**
(A) it is divisible by both 2 and 20 (B) it is divisible by both 4 and 10
(C) it is divisible by both 5 and 8 (D) All the above

5. The given table shows the starting and end time of a movie at a theatre. According to the information in the table, which of the following statements is true? **(IMO 2010)**

Movie times

Start	End
12:30 P.M.	2:45 P.M.
3:00 P.M.	5:15 P.M.
6:45 P.M.	9:00 P.M.
9:20 P.M.	11:35 P.M.

(A) The end time is exactly 2 hours 45 minutes after the start time.
(B) The end time is exactly 2 hours 15 minutes after the start time.
(C)The end time is exactly 3 hours 30 minutes after the start time.
(D) The end time is exactly 3 hours 45 minutes after the start time.

6. Which digit makes the given sentence true?
58,314,70? is divisible by 5. **(IMO 2010)**
(A) 4 (B) 5 (C) 9 (D) 1

7. Look for the pattern of the three sequence of numbers given below.
2, 4, 6, 8, 10
7, 14, 21, 28, 35
11, 22, 33, 44, 55
Each sequence is an example of what kind of numbers? **(IMO 2010)**
(A) Even numbers (B) Multiples (C) Primes (D) Odd numbers

8. The sum of the prime factors of 63 is__________. **(IMO 2011)**
(A) 4 (B) 8 (C) 10 (D) 14

9. If (k – 8) is the highest common factor of 56 and 77, then the value of k is **(IMO 2011)**
(A) 7 (B) 11 (C) 15 (D) 16

10. Which of the following numbers is divisible by 9? **(IMO 2011)**
(A) 9076185 (B) 92106345 (C) 10349576 (D) 95103476

11. Four different electronic devices make a beep after every 30 minutes, 1 hour, hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at_______. **(IMO 2011)**
(A) 12 midnight (B) 3 a.m. (C) 6 a.m. (D) 9 a.m.

12. 5*3523 is exactly divisible by 13 and 77. Find the digit represented by *. **(NSTSE 2012)**
(A) 1 (B) 2 (C) 3 (D) 4

13. Vrunda packed 6 baskets with identical chocolates. It was the greatest number she could pack using all the chocolates. Which of these is her chocolate list ? **(NSTSE 2012)**
(A) 36 Perks, 40 Fivestars , 42 Gems (B) 50 Perks, 54 Fivestars , 60 Gems
(C) 30 Perks, 72 Fivestars , 84 Gems (D) 49 Perks, 40 Fivestars , 90 Gems

14. A prime number can best be described as **(IMO-2012)**
(A) A number with exactly 2 factors
(B) Always an odd number
(C) A number with more than 2 different factors
(D) Always an even number

15. Fill in the blank: **(IMO-2012)**
A number for which sum of all its factors is equal to ____ the number is called perfect number.
(A) Twice (B) Thrice (C) Four times (D) Square

16. Find the lowest natural number which when divided by 112, 140 and 168 leaves a remainder 8 in each case. **(IMO-2012)**
(A) 1440 (B) 1688 (C) 720 (D) 1672

17. Rohan, Amit and Kartik walk around a circular path. Their steps measure 24 cm, 30 cm and 36 cm respectively. How much distance will they cover from the starting point so that they meet again? **(IMO-2012)**
(A) 380 cm (B) 400 cm (C) 360 cm (D) 350 cm

18. Replace " by the smallest digit so that 2315016* is divisible by 8. **(IMO-2012)**
(A) 1 (B) 0 (C) 3 (D) 2

19. Two ropes 16 m and 20 m long are to be cut into small pieces of equal lengths. What will be the maximum length of each piece? **(IMO-2012)**
(A) 80 m (B) 5 m (C) 24 m (D) 4 m

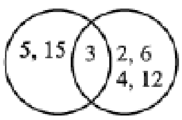
20. In a morning walk, Garima, Latika and Prerna step off together. Their steps measure 50 cm, 65 cm and 75 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps? **(IMO 2012)**
(A) 1595 cm (B) 1955 cm (C) 1185 cm (D) 1950 cm

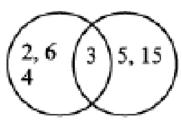
21. What is the L.C.M. of X and Y if X is the first odd prime number and Y is the only even prime number ? **(NSTSE 2013)**
(A) 6 (B) 2 (C)8 (D)12

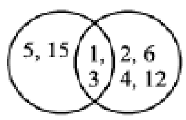
22. The five digit number '24x8y' is divisible by 4,5 and 9. What is the sum of the digits x and y? **(NSTSE 2013)**
(A) 10 (B) 5 (C)9 (D) 4

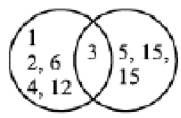
23. Which of the following numbers is completely divisible by 3? **(IMO 2013)**
(A) 297149 (B) 1790184 (C) 6392105 (D) 901352

24. Which of the following diagrams correctly describes the factors of 15 and 12? **(IMO 2013)**

(A) 

(B) 

(C) 

(D) 

25. Which of the following statements is true? **(IMO 2013)**
(A) 1 is a prime number.
(B) 1 is neither a prime nor a composite number.
(C) A prime number will not have the number itself as one of its factors.
(D) A composite number will have only two factors.

26. Study the two statements carefully.
Statement 1 : A natural number is divisible by 11 if the difference of the sum of digits at the alternative place (starting from unit's place) is divisible by 11.
Statement 2 : 19487171 is divisible by 11.
Which of the following options hold? **(IMO 2013)**
(A) Statement-1 is true and Statement-2 is false.
(B) Statement-I is false and Statement-2 is true.
(C) Both the statements are true.
(D) Both the statements are false.

27. Fill in the blank.
The highest common factor (HCF) of______ given numbers is the highest (or greatest) number of their common factors. **(IMO 2013)**
(A) Two (B) One (C) Two or more (D) Zero

28. Which of the following statements is INCORRECT? **(IMO 2013)**
(A) 5 hours is of a day. G
(B) Addition and multiplication are commutative for whole numbers.
(C) A triangle having all three unequal sides is called scalene triangle.
(D) A number with 4 or more digits is divisible by 8, if the number formed by last three digits is divisible by 8.

29. What is the sum of all the prime numbers between 90 and 100 ? **(NSTSE 2014)**
(A) 188 (B) 281 (C) 376 (D) 97

30. Which of the following numbers is perfect ? **(NSTSE 2014)**
(A) 6 (B) 28 (C) 34 (D) Both (A) and (B)

31. Find the least number which on adding 9 to it becomes exactly divisible by 15, 25, 30 and 45. **(IMO 2014)**
(A) 410 (B) 450 (C) 380 (D) 441

32. Prateek, Divyanshu and Manish jog daily in the morning around a rectangular park. They take 3 minutes, 10 minutes and 5 minutes respectively to take one complete round. In one morning, all of them start at the same time from the same point and jog in the same direction for an hour.
(a) After how many minutes will all three of them meet again ?
(b) How many times will they meet together during the 1 hour period ? **(IMO 2014)**

	(a)	**(b)**
(A)	20 minutes	3 times
(B)	25 minutes	2 times
(C)	30 minutes	2 times
(D)	60 minutes	1 time

33. Which of the following numbers is divisible by 11? **(IMO 2014)**
(A) 1011011 (B) 1111111 (C) 22222222 (D) 3333333

34. If P and Q represents the prime digits, then find the value of P and Q respectively **(IMO 2014)**

$$\begin{array}{rrrrr} & & P & P & 5 \\ & & \times & Q & Q \\ \hline & 2 & Q & 2 & 5 \\ + & 2 & Q & 2 & 5 & 0 \\ \hline & 2 & 5 & 5 & 7 & 5 \\ \hline \end{array}$$

(A) 3, 3 (B) 7, 5 (C)1,5 (D) 7,3

35. Sanchi distributed the equal number of sweets and the equal number of biscuits to each of her classmates at her birthday party. She gave out 220 sweets and 300 biscuits in total. Find the largest possible number of classmates at the party. **(IMO 2014)**

(A) 10 (B) 20 (C) 30 (D) 25

36. The largest number which divides 38, 46 and 62 leaving remainder 2, 4 and 6 respectively is______ **(IMO 2014)**

(A) 4 (B) 2 (C) 3 (D) 1

37. Which of the following number are divisible by 3? **(IMO 2014)**

(A) 700458 (B) 2345162 (C) 6594832 (D) 7145221

ANSWER KEY

EXERCISE 01

SECTION -A (FIXED RESPONSE TYPE)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	A	B	D	A	A	B	A	C	C	C	D	C	C	C	B	B	D	A	B
Ques.	**21**	**22**	**23**																	
Ans.	B	D	C																	

FILL IN THE BLANKS

1. prime , composite **2.** 2 **3.** 4 **4.** 6 and 28

5. 1 **6.** 24 **7.** Brackets **8.** Division

TRUE / FALSE

1. True **2.** True **3.** False **4.** False **5.** False

6. True **7.** True **8.** True **9.** False **10.** True

11. True **12.** True **13.** False **14.** True **15.** False

16. False **17.** True

MATCH THE COLUMN

1. (A) - (t) , (B) - (s) , (C) - (p) , (D) - (q) , (E) - (r)

SECTION -B (FREE RESPONSE TYPE)

VERY SHORT ANSWER TYPE

1. **(i)** 1,23 **(ii)** 1,2,3,4,6,8,12,16,24,48
(iii) 1,2,3,4,6,7,8,12,14,21,24,28,42,56,84,168

2. **(i)** 112 **(ii)** 17,51,85 **(iii)** 55 ,60,65, 70,75

3. **(a)** 7,11,13,17,19,23,29,31 **(b)** 71,73,79,83,89,97 **(c)**41,43,47,53,59,61

(d) 79,83,89,97,101,103,109,113,127,131,139,149,151,157

4. Yes , 9

5. **(a)** 36 = 7 +29 **(b)** 42 = 5 +37 **(c)** 84 = 17+67 **(d)** 98 = 79 +19

6. **(a)** 31 = 5+7+19 **(b)** 35 = 5+7+23 **(c)** 49 = 3+5+41 **(d)** 63 = 7+13+43

7. 4 **8.** 12000

9. **(a)** $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 5$ **(b)** $2145 = 3 \times 5 \times 11 \times 13$

11. 1 **12.** 7

SHORT ANSWER TYPE

13. **(a)** No **(b)** 1 **(c)** 4, 9, 25, 49

14. 90,91,92,93,94,95,96 **15.** 1 **16.** 6 **17.** 1 **18.** yes

19. **(i)** 2 **(ii)** 2 **(iii)** 1

20.	**(a)**	66	**(b)**	15	**(c)**	27	**(d)**	16
	(e)	8	**(f)**	35	**(g)**	7	**(h)**	9
21.	**(a)**	18	**(b)**	12	**(c)**	34	**(d)**	9
	(e)	77	**(f)**	42				
22.	**(i)**	300	**(ii)**	216	**(iii)**	5940		
23.	**(a)**	360	**(b)**	2520	**(c)**	6825	**(d)**	4368
	(e)	2520						
24.	**(a)**	240	**(b)**	264	**(c)**	45045	**(d)**	1260
	(e)	5760						
25.	30 tiles		**26.**	31L	**27.**	1	**28.**	1

LONG ANSWER TYPE

29.	4521, 2415	**30.**	1324.	**31.**	31	**32.**	20
33.	151	**34.**	9072	**35.**	7m; 6,7,12 respectively		
36.	16	**37.**	74	**38.**	122m 40cm	**39.**	6m by 6m ;378 tiles
40.	17 ,113	**41.**	2 hr	**42.**	174	**43.**_	62.
44.	57	**45.**	1257	**46.**	1260	**47.**	0
48.	4						

EXERCISE 02

SECTION -A (COMPETITIVE EXAMINATION QUESTION)

MULTIPLE CHOICE QUESTIONS

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	B	D	B	A	D	D	B	C	C	C	D	C	D	C	C	B	C	A
Ques.	21	22	23																	
Ans.	D	D	A																	

EXERCISE 03

(PREVIOUS YEAR EXAMINATION QUESTIONS)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	D	A	C	B	B	D	C	C	A	D	B	C	A	A	B	C	B	D	D
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37			
Ans.	A	D	B	C	B	C	C	A	D	D	D	C	C	D	B	B	A			